Lecture 06
Power calculation and Introduction to Smith chart

Hello and welcome, to NPTEL MOOC on electromagnetic guided waves, electromagnetic waves in guided and wireless media, this is module six, where we continue our discussion on transmission lines, I will just show you this

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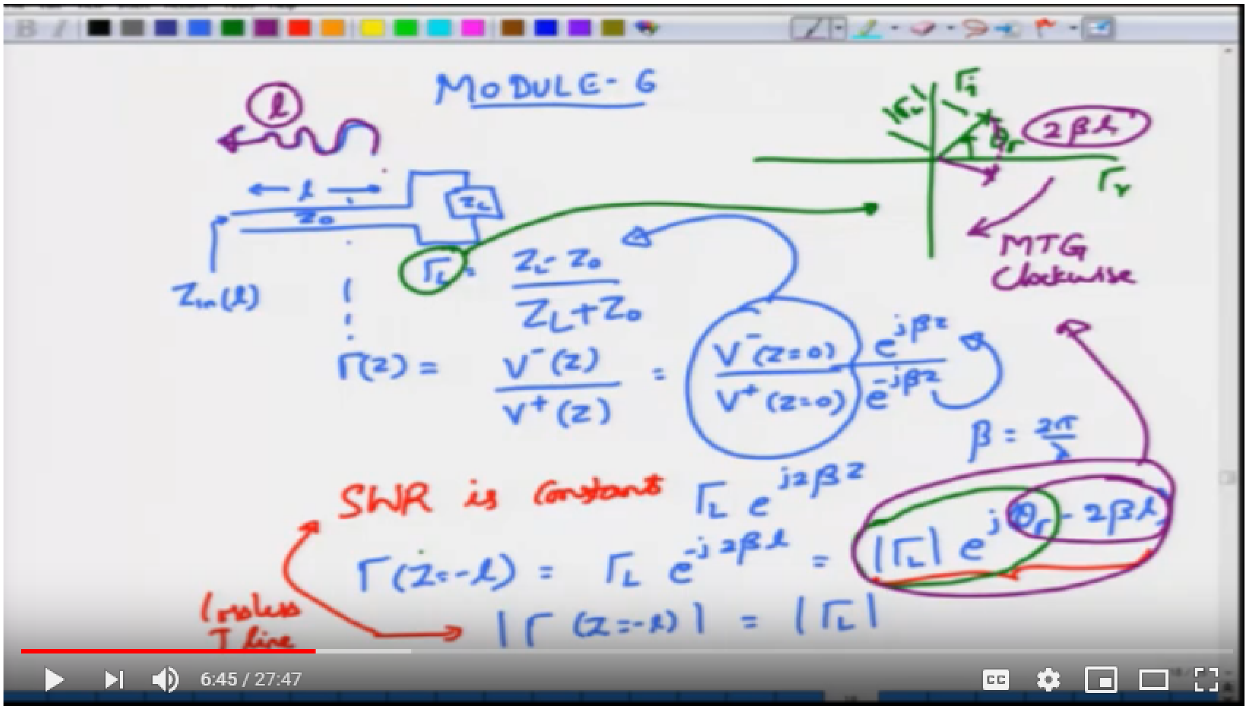

Basic diagram of the transmission line that we have already seen, given transmission line of characteristic impedance Z naught, with a length L terminated in the load impedance Z L, you can determine what is the load reflection coefficient, which is basically the ratio of the, reflected voltage amplitude to the reflected or an incident voltage amplitude and at the load this will be usually a complex number because ZL is usually a complex number, even when you consider a lossless transmission line where Z naught is real. Okay? Then you can also, for a given length of the transmission line, actually find out what would be the equivalent input impedance seen looking into the transmission line, of length L terminated in the impedance ZL. Now for later use, we will make an important observation here, at any point on the transmission line, which is lossless transmission line, that we are considering what would be the reflection coefficient, well the reflection coefficient will simply be the ratio, of the you know reflected voltage which we have which we will denote as B minus Z meaning that this is the phaser at Z on the transmission line, divided by V plus Z. Now we can relate this one to V minus at Z equal to zero divided by V Plus at Z equal to zero. And in addition write down, what is the appropriate transmission formulas as well that is to say, we can we know that the voltage that is reflected at the load, which is V - set equal to zero will propagate, you know with a propagation constant of E power J beta set along, you know on back from the node towards a generator site and the incident wave would also be given essentially as we plus e power minus J beta Z. Right? Now this ratio V minus at Z equal to 0 by VZ, Z equal to zero it's precisely what we have defined as the load reflection coefficient gamma L. And then you can take the C power minus J beta Z in the denominator onto the numerator, to make this one as a power J to beta Z. Okay? And if you now look at, what would be the reflection coefficient, at the input side or after a travelling after traveling a distances at equal to minus L this would actually be equal, to gamma L T power J 2 beta L sorry there's an e power minus J 2 P 2 beta L where beta of course is given by 2 pi by lambda as the propagation constant of this particular transmission line, which is operating at a wavelength of lambda. Right? Now gamma L, can itself be written in terms of its magnitude and an angle so, I have gamma L I power J theta gamma minus 2 beta L. Okay? I will put that one into the exponential and if you now look at, what is the magnitude of the reflection coefficient at Z equal to minus L that is as you move towards the generator. Right? So, this is the movement towards the generator what would be the magnitude of the reflection coefficient along the line, the magnitude of the reflection coefficient along the line, will be exactly equal to the magnitude of the reflection coefficient at the load, this property will apply only for lossless transmission line. If the line is lossy then the magnitude will actually change, in fact it will usually be smaller compared to the reflection coefficient magnitude at the load. Okay? So, because this you know, the magnitude of the reflection coefficient remains the same, it also follows immediately that yes W R also remains the same. So, as W R is constant along the line on a transmission line which is lossless. So, on a lossless transmission line both the magnitude of the reflection coefficient, is constant as well as the SWR is constant. Okay?

Now coming to this other part. Right? The complete reflection coefficient you will actually see that, if I were to come up with you know an interesting graphical picture for this one, I know that this gamma at Z equal to minus L, is a complex number, which has been written in the polar form by giving its magnitude as well as the angle. So, let's first consider the value, at L equal to zero meaning that we are considering gamma L at the load plane that is Z equal to 0 plane, then that will simply be magnitude gamma L, times E power J theta gamma which in the so-called,' Complex Plane' or the argon plane can be expressed as a point. Okay? Whose radius is magnitude gamma L and whose angle with respect to the real axis is given by theta gamma, do you agree with this take a minute to appreciate this graphical picture, all that we are doing with this graphical picture, is to represent this gamma L which we calculate at the load, on to a nice vector or a point whose distance from the origin gives you the magnitude of gamma L and whose angle as measured from the real axis is given by theta gamma. Now what happens as you move towards the generator, well as you move towards the generator and you reach say Z equal to minus L the total angle has actually reduced. Right? So, the angle has actually, reduced meaning the new point is located somewhere at this point on the same circle, of the same radius but then since the angle is reduced you have essentially moved, by moving a length of L you moved a total angle change, of 2 beta L. Okay? So, this is what means when we say, this is moving towards generator. So, I have actually moved towards generator by going clockwise. Okay? So, the overall angle, is now reduced so, this is what I am actually plotting now and this would correspond to the reflection coefficient, after moving a distance L towards the generator. Now please note, you have moved a distance of L, but in terms of an angle you have moved 2 beta L. Right?

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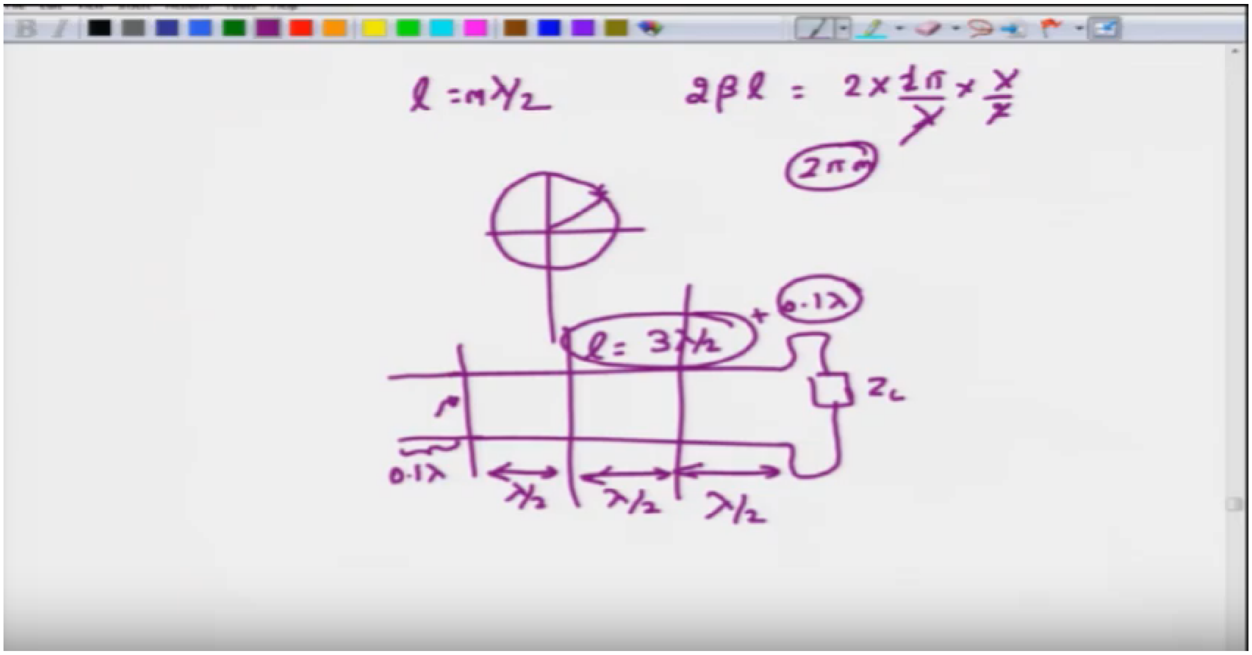

So, clearly if you move a total length of L equal to say lambda by 2 that is when you measure Lenz in terms of the wavelength of the signal, then if you move a length of lambda by two you would actually move on the organ plane, you will your angle will change by an amount of to beta times L, which is 2 into 2 pi by lambda because beta is 2 pi by lambda and L you have moved a distance of lambda by 2. So, you cancel lambda on both sides, will cancel 2 and then what you see is just 2 pi. Right? Of course you don't have to just move lambda by 2 any multiple of lambda by 2 you move an integral multiple of lambda by 2 you move you're going to move a distance change the angle by an amount of 2 M pi which essentially means that if you started off at this point. Right? This is the origin you actually, have come back to the same point and you come back, to the same point again and again so, physically on a lossless transmission line, if the line, of you know line is actually of length say 3 lambda by 2 the properties that it would see, at the load would essentially be the same properties, that it would see at this point as well because you have taken an integral number, of lambda by 2 distance on the other hand if the length is, 3 lambda by 2 plus say some point 1 lambda then the properties, that you would see would only be determined by this excess length 0.1 lambda, of course this is true because, we have assumed a lossless transmission line and therefore there is no power changes, there is no voltage amplitude changes that is happening out there and therefore gamma L is actually a constant. So, on the transmission line all properties on a lossless transmission line all properties are repeating or they will repeat every lambda, by 2 distance. Okay? So, if the physical length of the transmission line is when you represent in terms of lambda is given by an integral multiple of lambda by 2, plus some additional length then the properties impedance transformation and everything is dependent only on this excess length. Okay? So, keep this in mind because, very shortly we are going to use this application. Okay?

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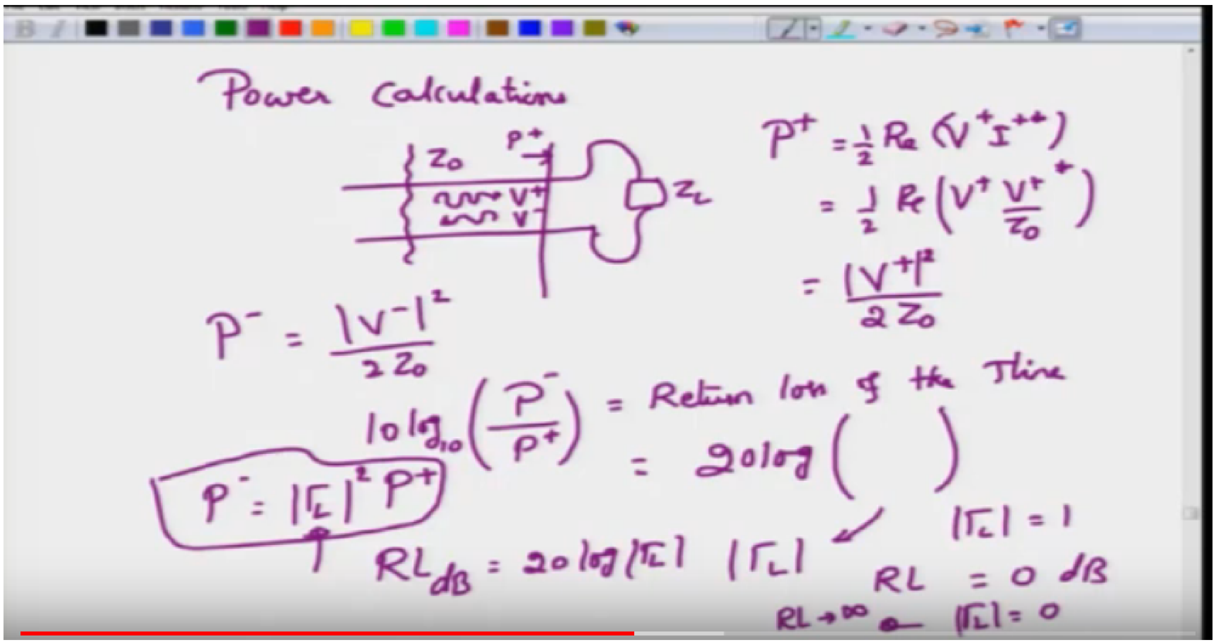

So, that was the point that I wanted to make the transmission line, actually has this thing. Right? Now before we further continue, I want to very quickly give you, simple you know I want to very quickly give you some points about the power calculations so, on a transmission line. Right? We know that this is the transmission line, then you know you have a positively traveling wave or a forward going wave the plus or incident wave and then you have a backward or reflected voltage V- minus you may be interested, to ask how much is the power that is being incident and how much is the power being reflected. Right? Reflection will always arise, when the characteristic impedance is not matched, with respect to the load, that is load and characteristic impedance, of the transmission line are different then they will lead to reflection. So, you are interested in knowing if I transmit or if I incident one watts of power how much power is actually getting reflected. Right? And you can relate that, power reflection or power reflected to with respect to the reflection coefficient, how the incident power which we will denote as P plus at any plane. So, if this is some plane that we are considering, this is the incident power P plus will simply be given by real part half of real part V plus I plus conjugate. Okay? I am assuming that V and I are P quantities not RMS quantities and that is reason why I have a half here and I have a I plus conjugate here because I am dealing with voltage and current phasers, please remember all the quantities that we are writing here, in this module all our phasor relationships. Okay? This is very important to remember otherwise you won't get the conjugate and your power calculations will not be correct. So, I plus is basically v plus divided z not obvious. Right? And then when you conjugate this one because, they're not is real, you conjugate of Z naught will still be equal to Z naught itself and what you get here is real part of

V Plus V Plus conjugate by Z naught into half which is basically V plus magnitude square divided by 2 Z naught but following, the same lines you can easily show that P - will be V - magnitude square by 2 Z naught, now you look for the ratio of P minus 2 P plus. Right? And then express this ratio in terms of log, the base-10 in our course. So, you have 10 log of this quantity, which we will call as return loss of the device or return loss of the transmission line, of course there is nothing specific about transmission line in this definition you can always take any you know device and then call its return loss, for example even if you go to a low frequency scenario, you put a load you have a source, you know equivalent source and a 17 register, then you can always calculate what would be the power that is incident or power that is that is available from the transmitter and the power that is actually dissipated, by the resistor and then the difference between the two is what we can think of as a reflected power. Right? So, this concept of return loss is quite general, but it is used widely in high frequency or transmission line and microwave applications. So, this is your return loss of the transmission line, well so, far so, good I can use the expressions for P minus and P plus 2Z naught is a common thing that gets cancelled. So, what I get is 20 log. Right? Because the square in V plus magnitude square and V minus magnitude square will be taken out so, you get V minus magnitude by, V plus magnitude but this is nothing but the quantity in this bracket is nothing but magnitude of gamma and for a transmission line, the magnitude of gamma for a lossless transmission line that is simply equal to magnitude of gamma L. Right? So, the return loss, in DB is given by 20 log of magnitude of gamma L. So, when you have the return loss expressed in DB in this manner. So, for the completely mismatched case where gamma magnitude of gamma L will be equal to 1 the return loss is equal to 0 DB and when gamma L magnitude is equal to 0 for the case of lossless transmission line, the return loss will be equal to infinity meaning.

So, it's, it's really I mean we should have thought about it a little carefully, nevertheless what it actually shows is that when you I know you have your incident power, then the reflected power P plus can always be expressed as the magnitude of gamma L square, times the incident this is the reflected power. Right? So, the reflected power can be expressed as, of gamma n square times P plus and therefore this quantity magnitude of gamma L Square, will tell you, how much is the power that is actually getting reflected so, when gamma L is equal to 1, which will be completely the case where you have complete mismatch then all of the power is reflected back. Okay? And when gamma L equal to zero, then none of the power incident has will be reflected back everything will be transmitted. So, it could be related to the transmission coefficient, which we can relate as to the power that is delivered to the load, to the power that is actually being incident. So, if you think of the transmission, then having very high written loss is actually good because, that means there will be nothing coming back from the load. Okay? By the way this can be used, to denote the incident and reflected powers or you can use no like talk about, the returned losses at any plane on the transmission line

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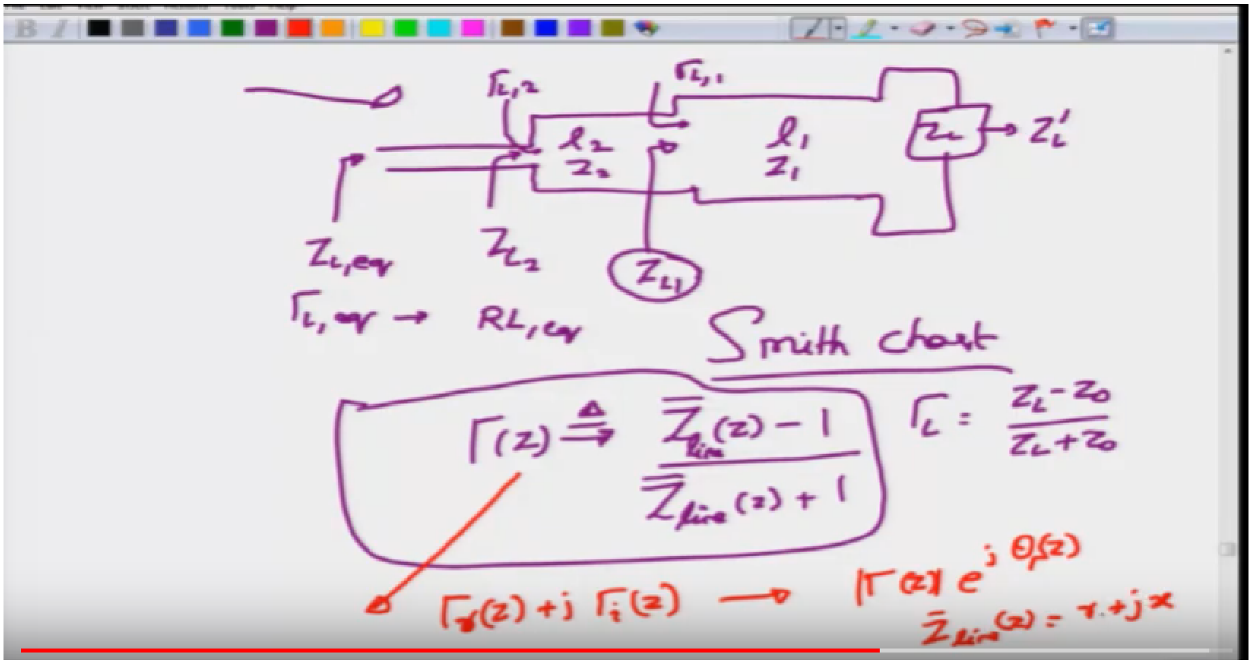

Sometimes you will also encounter scenario where one transmission line, will be connected to another transmission line, which itself may be connected to another transmission line and finally terminated with some load. Okay? So, I just short-circuited here, but you can take any load here. Okay? And then at each plane, you can I mean you will have some incident power and some reflected power and you're more or less quite, often interested in knowing what is the overall return loss Okay? The way to solve this problem would be to first transform the impedance, seen looking at this edge and because, in this case it is just the last transmission line. So, you move it to the transmission line before here you find out what would be the equivalent load impedance, say after transforming the load here. So, we will attach a load now so, we have a load here, which we'll call and said L and after attaching a load ZL of the transmission line whose length is L 1 with the characteristic impedance said 1, you find out what is the equivalent load impedance once you find the equivalent load impedance that L 1 here you find out what would be the equivalent load impedance ZL 2. Okay?

Which would be the impedance seen, at this plane. Okay? For a transmission line of length L 2, with a characteristic impedance Z 2 connected to this transmission line. So, you can successively transform the impedances, to obtain final ZL equivalent calculate what is gamma L equivalent and from this calculate its magnitude and calculate the overall equivalent return loss. Okay? so, you can do that one and if you're interested in knowing how much power is being transmitted from the first transmission line, to the second transmission line no problem here you calculate, what is gamma L 2 similarly, you calculate what is gamma L1 and then work towards this thing. Right? You may already have seen that you know transforming the impedances calculating magnitude of gamma L, may seem very tiresome infer it is true that they are actually going to be tiresome if you have multiple such lines to be connected and if for example if I simply change that L to some other values at n Prime, then you have to repeat the calculations those things are not quite easy and intuitive there is a nice graphical method which actually, can be applied to solve these type of transmission line problems and that graphical method is called as,' Smith Chart', We are going to build up to Smith chart so, all these that we discussed in the last 10-15 minutes was actually to introduce you to Smith chart, where we will start with two basic assumptions that at any point on the transmission line. Right? You can describe what is gamma of Z and at that gamma of Z; you can actually obtain an equivalent Z of z Okay? I know the notations are getting a little confusing so, I will what I am going to write here is Z line of Z .Okay? Just to indicate that this is the line impedance, that I am saying at the Z plane. Okay? And there is a one-to-one relationship between, these two Y remember gamma L is given by ZL minus Z naught by Z L plus Z naught and gamma any Z can similarly be expressed, by writing this as the line impedance at that load point or the load plane Z divided minus Z not divided by Z line of Z plus Z naught. Okay? So, there is a one-to-one relationship between gamma and the line impedances and because, we don't know what transmission line characteristic impedance will be used, in many problems we in fact can work with what is called as,' Normalized Impedances', when you work with normalized impedances you simply divide every impedance by the transmission line, characteristic impedance. Okay? Or some reference impedance if you think of this way so, you have this one-to-one relationship which forms the basis of Smith chart which allows you to solve problems that I described earlier like this multiple connected transmission line problems. And this we will not develop the equations, that is something that we can do it in a different course, but I will give you the basic idea of Smith chart. Okay?

How it would come about for that again we go back to the graphical method, of course this is a graphical method and we realize how we can represent Gamma at any Z. Right? How do you represent this you can represent this in terms of its gamma real part and the imaginary part so, for example I can represent this one as, gamma R of Z plus J gamma I of Z equivalently, I can represent this one by giving its magnitude and the corresponding angle J theta gamma of Z. Okay? Meaning that at every point on the transmission line, I can give the reflection coefficient by giving its magnitude, as well as its angle. Now on the same you know using this relationship, for every reflection coefficient at any point Z, I can calculate the equivalent normalized line impedance, which also can be written in terms of its real and imaginary parts, which we will denote as R plus J X. Okay? R is for small R you know capital R is usually used for resistor, small capital X is used for reactances, we have used small or and small X to denote that these are normalized components. So, if I have this

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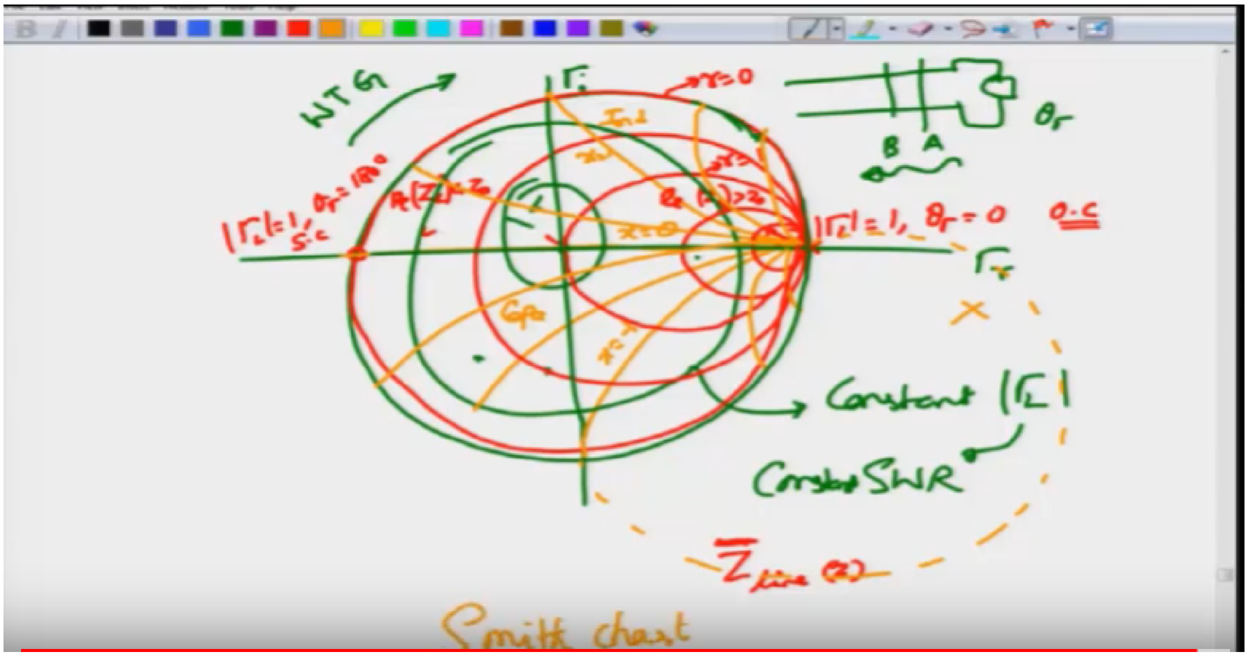

Argand plane or the complex plane that I have, then I can represent so, any point here would then represent, in the gamma R gamma I plane any point here would represent magnitude of gamma L or because you know on the transmission line magnitude of gamma L will be the same as magnitude of gamma any Z. Okay? If you are not happy you can simply write this as magnitude of gamma itself and the angle, which initially will begin with theta gamma and then successively change, as you move towards the generator. So, as you move towards the generator you are essentially moving along the circle. Okay? Whose radius is now magnitude of gamma L and whose angle would be changing as you move towards the no generator so, if you go clockwise you are moving towards generator which we will denote as WT G what is w TG W? Stands for wavelength and you are moving so-and-so number of wavelengths towards generator, that's what essentially means they are moving from the load all the way to the generator. Now what is the maximum radius of this circle? On a piece of paper you can draw any radius but you have to also normalize all these radiuses. Right? So, if you have a radius you you take a pen and then draw a circle of radius 10 centimeter, you and that is the maximum radius and on your paper that you think then that maximum radius will correspond to gamma L equal to 1, because that would be the maximum mismatch that you are going to get. Right? What would be the minimum, minimum value is simply on the origin itself gamma L equal to 1. Right? And then you have different circles. So, you this will be gamma L between 0 & 1 this will be reduced gamma, this will be a gamma which is between this and this value. So, these are all basically circles although they don't really look, nice circles, but each circle here corresponds to a constant, gamma L value. Okay? And because gamma L uniquely determines

SW R these are also called as,' Constant SW R Circles', S WR is VSWR circles. Okay? So, any point as you keep moving would correspond to different points on the transmission line starting from some you know initial load point. So, these are all different loads, I mean the different, different planes on the transmission line. So, if I have this transmission line here, terminated with the load, say this plane corresponds to point A this plane corresponds to point B. And so, on I am moving towards the generator and that corresponds to clockwise movement, on the appropriate constant SWR circle or a constant gamma L circle. However interestingly, every point that I consider say for example I consider at this point. Okay? This point would have an equivalent line impedance. Right? So, this point would actually have an equivalent line impedance, because on this point you have a certain gamma Z. Okay? On some point on the transmission line that I don't know, but because I have this gamma of Z by knowing the angle, as well as the radius I will be able to obtain an equivalent line impedance. Okay? so, any point or all the points on this, I know on the on this circle within this circle, would correspond to equivalent line impedances, from the one-to-one relationship that we have this corresponds to normalized line impedances, of course what would be the line impedance for a point, let's say gamma L equal to magnitude gamma L equal to 1 and an angle theta gamma equal to 0, this of course would correspond to the open circuited point and that will be then so, normalized or unnormalized the value is always you know infinity here. So, this point that we have written with the cross mark, that corresponds to open circuit interestingly, if you go to this point which is gamma L magnitude still equal to 1 but an angle equal to 180 degrees. Right? Theta gamma equal to 180 degrees, this corresponds to short-circuit termination. Right? So, the point here, with coordinates 1 and 180 degrees or 1 and D Bar minus JP Bar J PI would actually correspond to a normalized impedance of 0 interesting. Right? In fact you can now extend this superimpose all the points, on this transmission line. Okay? All the points on this plane, with constant values of R and constant values of X meaning, I can actually have different circles, along those circles the value of gamma can change, but the value of R will always be constant. So, this corresponds to what is called as R equal to 1 or the unit circle or unity circle, the circle on the outer line that you have you know would correspond to a circle which we will call as R equal to 0 because that would b have a impedance of 0 there. Okay? And then you have a well this one I have drawing, this circle would have an R value which is between zero to one please remember R is normalized impedance, meaning that this is the case where the real part of Z L is actually less than Z naught. Okay?

So, the real part of Z L is less than Z naught similarly this case or this region corresponds to real part of Z L greater than Z naught and that would be these curves or these circles you can notice that the radius of the circle keeps on dropping as the real part of Z L keeps on increasing that is as you move from short circuit to open circuit the radius of the circle shrinks Center actually keeps moving away. So, the center for this one will be somewhere here the center for this one is here, the center for this one is here so, the center keeps shifting towards the. Right? It's not towards .Right? And Right? And so, on and then the circle radius also starts to shrink we are not done yet please note that we already on this graph, we have two kinds of circles one circles where we have constant gamma value, where the value of smaller and smaller can change, however the magnitude of gamma L will remain the same and then you have these red circles where the value of R will be constant R is real part of the impedance will be constant, whereas other values can actually change, I mean you can have different values of X, you can have different values of gamma those things will change. Now I am about to draw, another set of circles, they are really not circles they're actually R is because, to make them circles you have to complete them outside this chart. Okay? We don't normally deal with region outside this chart, unless we are dealing with active circuits. Okay? Which is not in this particular course so, therefore this region is forbidden for us. So, what you have are these R is this straight line in fact is the R where you know you have X equal to 0 no susceptance and then this arc is X equal to minus 1, this corresponds to capacitive region this corresponds to inductive region, because the susceptance is here are all positive this is X equal to 1, this is X less than 1 this is X greater than 1 and so, on so, forth. Okay? So, you have these different R is so, this is your full Smith chart it may look very the way I have written but please go to net and download this chart called Smith chart keep it ready for the next module, because this is what we are going to use, I mean this will be we will use to show you how to solve many transmission line problems. Okay? So, until then. Thank you very much.