Lecture - 05

Properties of Transmission Line

(Reflection Coefficient, Input Impedance, Standing Wave Ratio)

Hello and welcome again to, NPTEL MOOC on electromagnetic waves in guided and wireless media. In this module we will collect a number of interesting and important facts about transmission Lines, these important facts are derived by the fact that you have a finite length transmission Line in most practical Cases that is what you will have? And you want to be able to describe, what will be the effect of load on the voltages and Currents? So, let us begin here,

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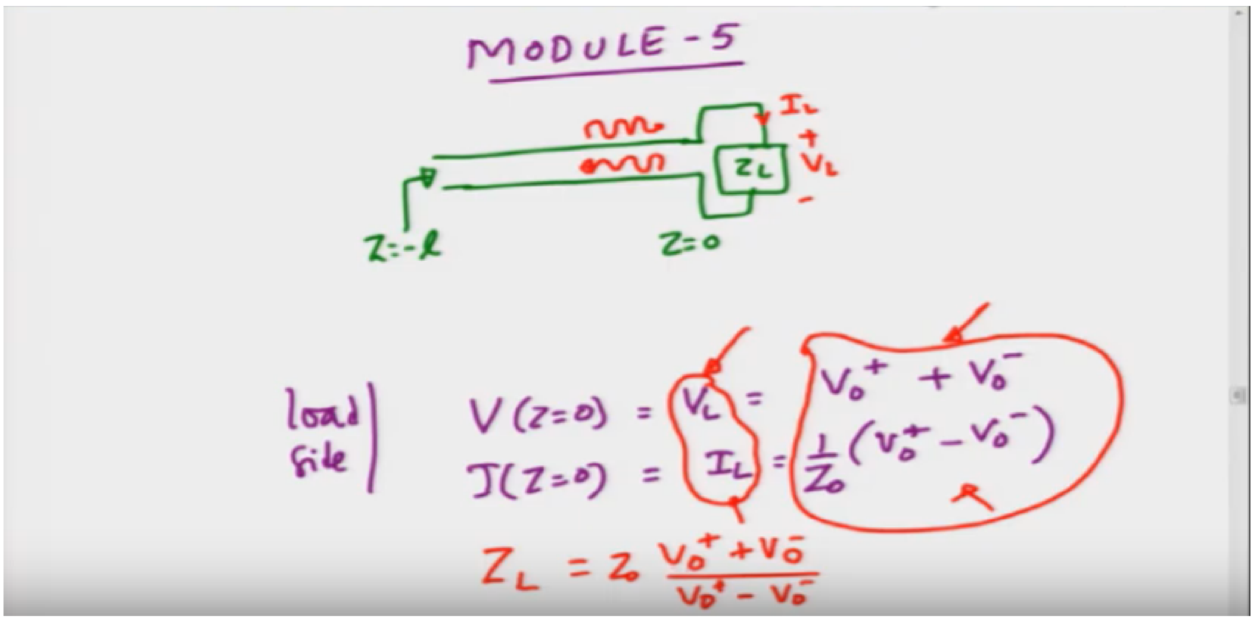

We have already seen, that we have introduced a new coordinate system Z equal to 0 representing the load side and the voltages at those you know, at that plane we have also written the expressions Here, I want to emphasize one point here, the ratio of this I mean, the voltage Value VL and IL, are the voltages across the load and the Volt current through the load. Okay? These have nothing to do with the transmission Lines; this ratio is determined by whatever the Zl Value, that you would actually put in, for instance if instead of having this transmission Line you directly connect the you know, an ideal voltage source here, and then measure the voltage across the load and the current through this particular load, that ratio will be equal to Z L. Okay? So, that is what we have VL and IL? However, from the transmission line point of view, you have a forward and a backward wave the sum of these two or the superposition of these two, this voltage on the transmission Line at Z equal to 0 is given by V 0 plus, plus B 0 minus, similarly the current is this quantity. Okay? Now, the ratio of this Line voltage to the Line current must be equal to the node voltage to load current. Okay? The load voltage to load current is of course Z L and this Right hand side Quantity, which is the ratio of the Line voltage the Line current? is given by V 0 Plus, plus V 0 minus, Z naught is there in the denominator so, it goes up in the numerator, divided by V zero plus minus V zero. Okay? I can now, take out V zero plus and then you know, cancer it from the numerator and the denominator. When I do that, what I get? Is the ratio V zero minus two V zero plus. Okay?

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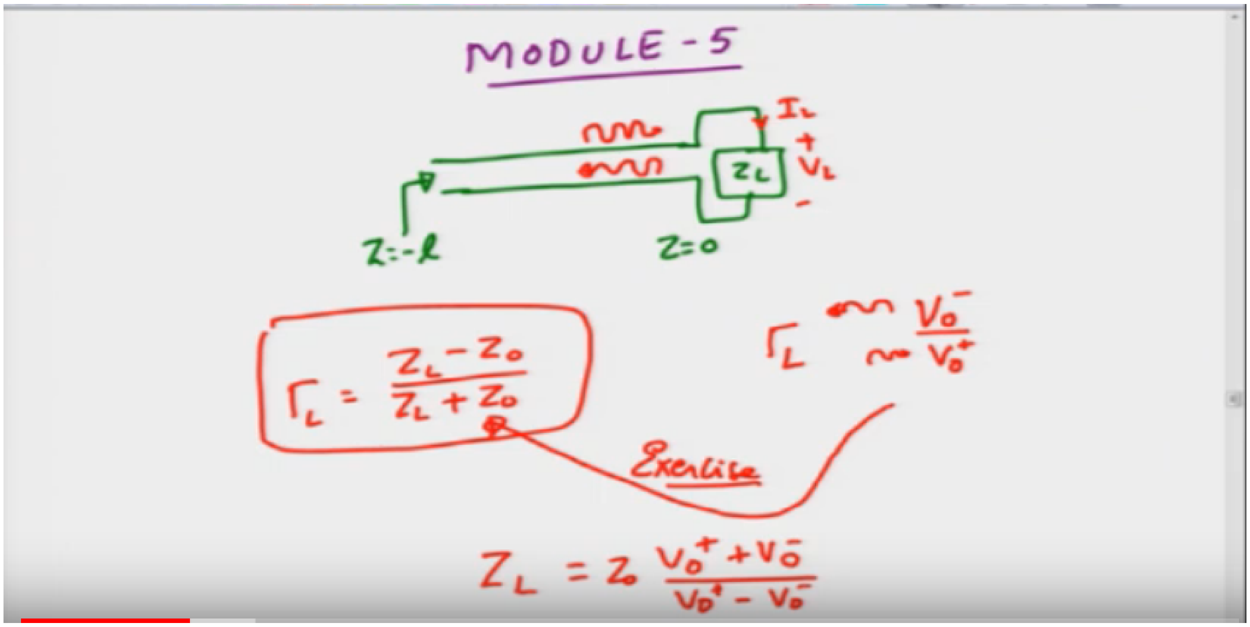

What is the ratio of V zero minus 2 V zero plus? That is in fact, the ratio of the voltage Amplitude of the backward traveling wave, to the voltage amplitude or the phasor, which is forward traveling wave. Okay? So, this ratio of backward wave phasor to, the forward wave phasor ,is denoted by the reflection, coefficient gamma and in fact we denote the reflection, coefficient because, this has been done at the load side as gamma times, and I mean gamma is a subscript L, that L will simply tell us ,what is the ratio? I mean, what is the location at which we are evaluating this ratio? And that ratio is being evaluated at the load. Okay? Now, of course gamma L is what we have denoted this ratio of amplitudes but, we need to obtain an expression for gamma L, in terms of ZL and Z naught, and you can take this as a simple exercise, by substituting this gamma L here, and then know which you can do it by taking V zero plus out and cancelling it out, you can show that the ratio of the reflected voltage amplitudes to the incident voltage amplitudes, V zero minus two V zero plus, can be given by Z L minus Z naught by, Z L plus Z naught. Okay? This is an expression; I will leave this as a Very, very simple exercise for you to show that this is the case. Now, this formula is very important. Okay? In the context of how much reflection can be, can be possible? When ZL and Z naught are different from each other.

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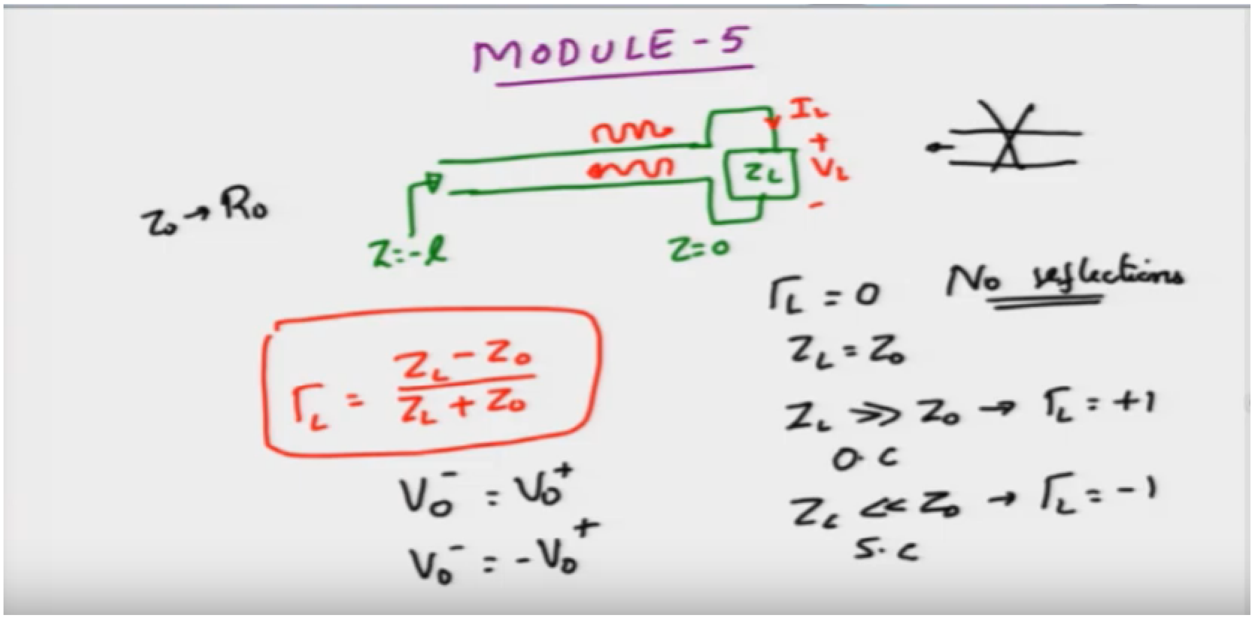

First, what would be the ideal scenario? Ideally I d not want any reflected voltages, why? Because any reflected voltage will be accompanied by a reflected current and together the reflected voltage and the reflected current will be carrying some amount of power. Where is that power coming from? That power is coming from the part of the incident wave itself. So, you had this incident wave, some portion of the power has been taken off from the reflected waves and then that comes back on to the source side. So, which is wastage of power? We want all the power to be delivered into the load itself. So, for that to happen v-0 - should be equal to zero. The only way I can make me zero minus equal to zero is when I set gamma L equal to zero.

So, in this case I will have no reflections. Okay? All the incident power will be transferred to the load as much as possible; however there will be no power reflected back from the load side. Okay? So, gamma L equal to zero obviously means, ZL equal to Z naught. Okay? So, if you know the transmission Line characteristic impedance and that is why I told you that it is an important quantity to note, then you can terminate the transmission Line with Z L in order to have no reflections. Okay? Alternatively you could let the transmission Line continues all the way to infinity. So, that you know there is no chance that there will be any reflected Wave, but that of course is not an ideal scenario and this situation is not being considered here, because we have assumed that the Line has a finite length, and it actually has a load ZL here. Okay? So, gamma L being equal to zero is an important thing to note, it doesn't matter whether Z naught is complex, if Z naught is complex you put Z L also Complex. But, it is equal to Z naught. You don't do that for different reasons but, for no reflection scenario, you can always make gamma L equal to zero, by making Z L equal to Z naught. Okay? Now, let us consider two additional cases what is this Z L is very, very large compared to Z naught? for the lossless case Z naught is real. So, you know sometimes Z naught is actually replaced by R naught to indicate that we are dealing with a lossless transmission Line. But, since I have told you that we are going to do deal with only lossless transmission Lines, I am not writing R naught in place of Z naught. Okay? So, when Z L the load impedance is actually Very, very large compared to Z naught. This, leads to the value of gamma L being equal to plus 1, ok. Approximately equal to plus 1, and this scenario where ZL is much Larger, than Z naught comes when you open circuit the transmission Line .so, the open circuited transmission Line, will have gamma L that is at the load side the reflection, coefficient will be equal to plus1. So, when gamma L is equal to plus 1, what is the meaning of that? V-0 minus is actually equal to v-0 plus, that is entire voltage is reflected, no voltage is being transmitted or no voltage is you know is there on this one. Right? So, gamma V zero minus, would actually be equal to completely V zero plus. But, what would happen to the current in this open circuit case? Well current will be equal to zero. So, there is no power being transferred at Z equal to zero load plane but, the voltage is completely reflected and the current is clear has gone down to zero they're. Exactly opposite situation occurs; when ZL is very, very small compared to Z naught, as in the case of a short circuit of the Line. Okay? When you short-circuit the Line at the load side, you clearly see that gamma L will be equal to minus 1. Right? So, when that happens v-0 minus will be equal to minus v-0 plus. Okay? And here, it is the voltage which goes off to Zero, and the current will be whatever the current that can happen that current will be present. Okay? So, the open circuited loads have gamma L of plus 1, short circuited loads or gamma of minus 1, and in both cases power is not being delivered to the Z equal to 0 Plane, because either the voltage I being 0, or the current is being 0. So, this is for the reflection, coefficient. Okay?

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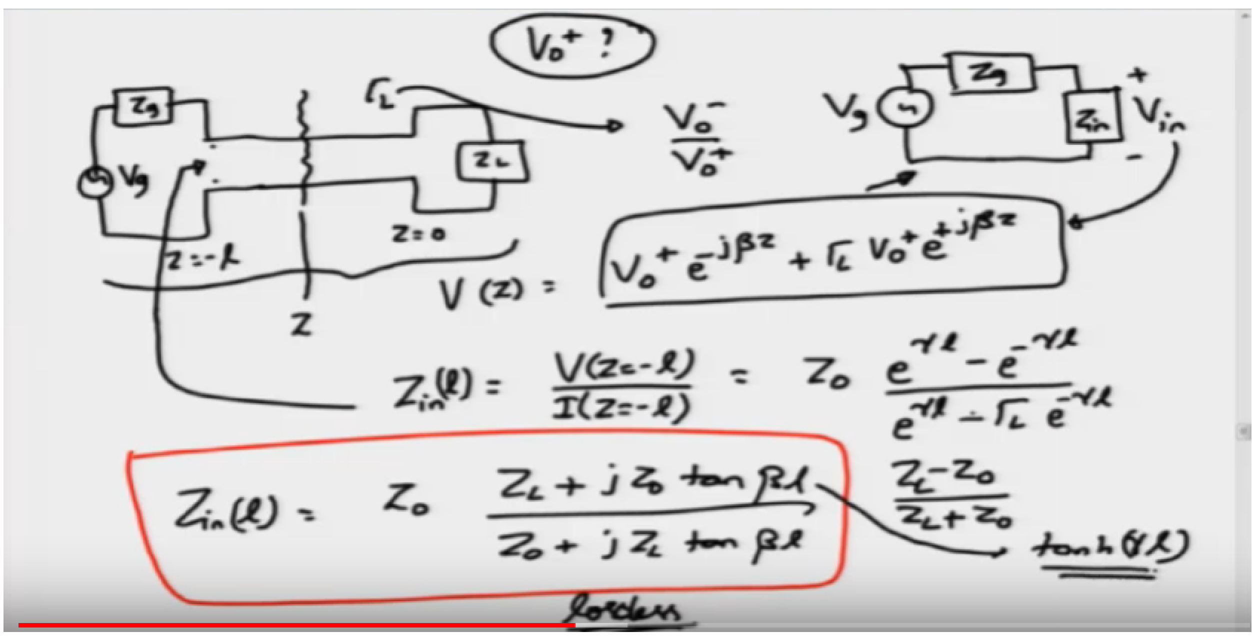

Now, you can ask well we have written all this, we know what is V 0 minus in terms of the ratios? But, what about V 0 plus? I mean we have not talked about how to obtain the incident wave Amplitudes, yes. We have not done that one because, we have not solved this following problem, and the problem is I have the transmission Line connected to the Load. Ok? So, I have this Z equal to 0 load, and then I have this transmission Line connected to the source, why are the generator impedance Z G and the load impedance VG. Okay? So, clearly, oh sorry, should I write it, written it parallel this one. Okay? So, I have this load, which is sorry, I have this transmission Line of length L and I want to solve this problem, well I have not so far solved the problem, all I have done is I know what is gamma L here, I have calculated gamma L.

so, once I know what is gamma L I know the ratio of V 0 minus 2 V 0 plus But, I don't know what is V 0 plus? Okay? But, at any Z plane, I know what the total voltage is? Well what is the total voltage or the Line voltage on this Line? That would be V 0 plus E power minus J beta Z plus gamma L V 0 plus E power plus J beta Z. Right? So, I have this particular you know this expressions out there and similarly the expression for the current as well. Okay?

So, I have a current expression as well .now, the way to solve this problem is to first transform this impedance Z L, such that what would be the equivalent impedance that would appear at this terminals, at Z equal to minus L, what would be the equivalent impedance that would appear? and you can find from that equivalent impedance you can find out what would be the incident wave amplitudes, we will call that equivalent impedance has z in and then you have this expression and solving this circuit, will tell us what will be the value of v-0 plus? Okay? Because, this voltage will be the Line voltage and then you can relate everything to v-0 plus because that is the only quantity that is now, unknown on to this set of equations. Okay? So, I have V here, I have the current at any Z here, and clearly the impedance seen at this terminals Z equal to minus L, will be the impedance z in, will be equal to the Line voltage, that I have at Z equal to minus L to the voltage I at Z equal to minus L, this is the voltage defend currents on the Line at this Z equal to minus L will give me the input impedance of a transmission Line, whose length is L. Okay? now substituting for V and I from these expressions that we have already written and no setting Z equal to minus L ,you can show that this will be equal to Z naught e / you know ,in the general case it will be e / gamma L, minus e power minus gamma L divided by e / gamma L minus, gamma L e power minus gamma L, for the case of transmission I mean ,for this one you can simply replace gamma is equal to J beta, I am writing this gamma because in some problems you may actually have nonzero value of alpha, but if you don't then it's all right.

So, you don't have to worry about that one, and then you already know, what is this gamma L? Gamma L is basically the ZL minus Z naught by Z L plus Z naught .Okay? and you can put that value here, and rearrange this equation, such that you can find out what would be the input impedance Z in of a transmission Line of length L as Z naught, Z L plus, J Zed not tan beta L divided by Z naught plus J said L tan beta L. Okay? this expression is very important, I know it is Little complicated to remember ,but this expression you can derive it in two three minutes and there is actually an important expression because ,it tells you or it actually simplifies the circuit which is on this left-hand side, into an equivalent circuit on this Right hand side and now, you can easily apply you know, the voltage divider formula to find out what would be the voltage that would appear across this input impedance, and that voltage will be equal to this expression when you substitute Z equal to minus L and you once you know, what is the voltage that would appear ?we will call this as V in you can equate V into this one and then find out what would be the amplitudes V zero plus. So, this is an important expression in that sense. But, for the general case where gamma is considered, that is when you have alpha non zero, then you don't have this J set not tan beta L, you replace this J tan beta L by tan hyperbolic of gamma L. Okay? So, since gamma is complex, it will also you know it can be now looked at with expect to tan hyperbolic one, rather than just a tan beta L, when gamma is Imaginary Like gamma equals J beta, then hyperbolic tan is essentially same as J times tan function. Okay? So, this is for the lossless transmission Line, and this is the easiest case, the loss Line will require you to use a nice calculator which will also support diaper bullock function evaluation. Okay?

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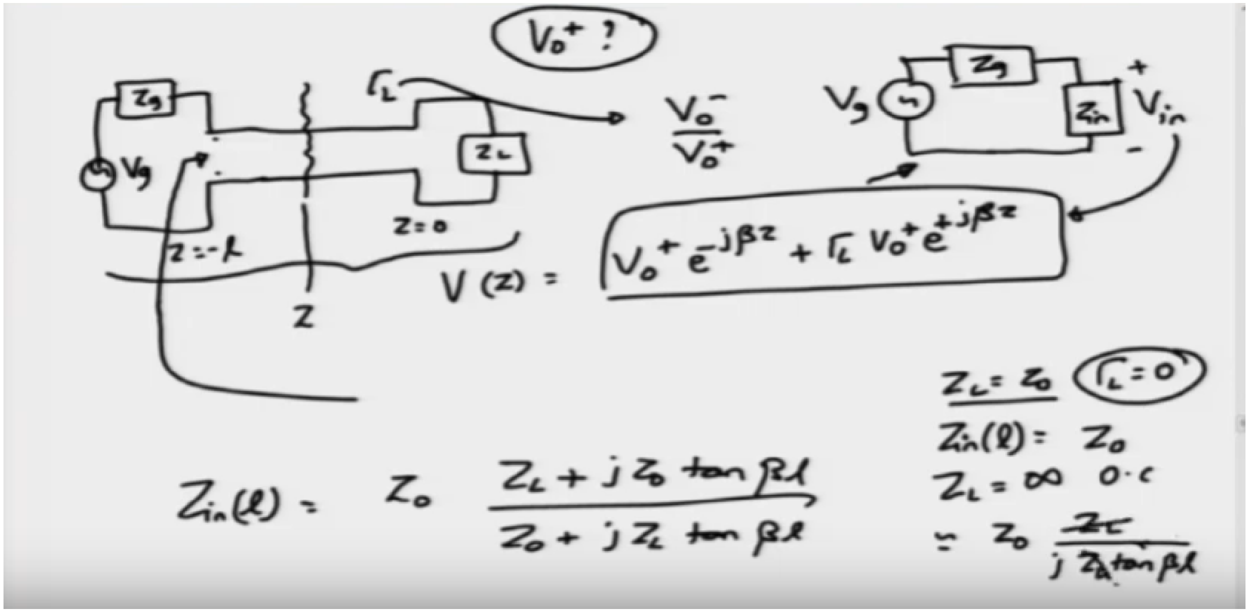

Let’s look at this input impedance expression again, this is important, and we will look at different conditions. Okay? In fact that is one of the ways in which you know that you have actually obtained correct Expressions, let's look at this one, for the simple case where ZL is equal to Z Naught, remember for ZL equal to Z naught gamma L was directly equal to zero, that is there was no deflection whatsoever. Okay? What will then happen to the input impedance? the input impedance of a Line of length L, will be then equal to so, you can substitute ZL equal to Z naught so, from this equation in the numerator, that would be Z naught plus J said not tan beta L, divided by Z naught plus J Z naught tan beta L, the numerator and denominator terms cancel and the input impedance is equal to Z naught. Okay? So, because we are considering lossless transmission Line, the input impedance also will be lossless, I mean it will also be real and it will be equal to Z Naught. So, a transmission Line that has been terminated with Z naught, no wonder it still sees the input impedance as the same as Z naught. Okay? This is a very important expression .now, consider what happens when ZL is infinity that is open circuited or at least very, very large compared to this Z naught value. Okay? So, in that case the input impedance will be approximately Z naught because, ZL is very large in the numerator only ZL will survive and in the denominator J Z L tan beta L will survive for the length L, and you can see that ZL and ZL can Cancel.

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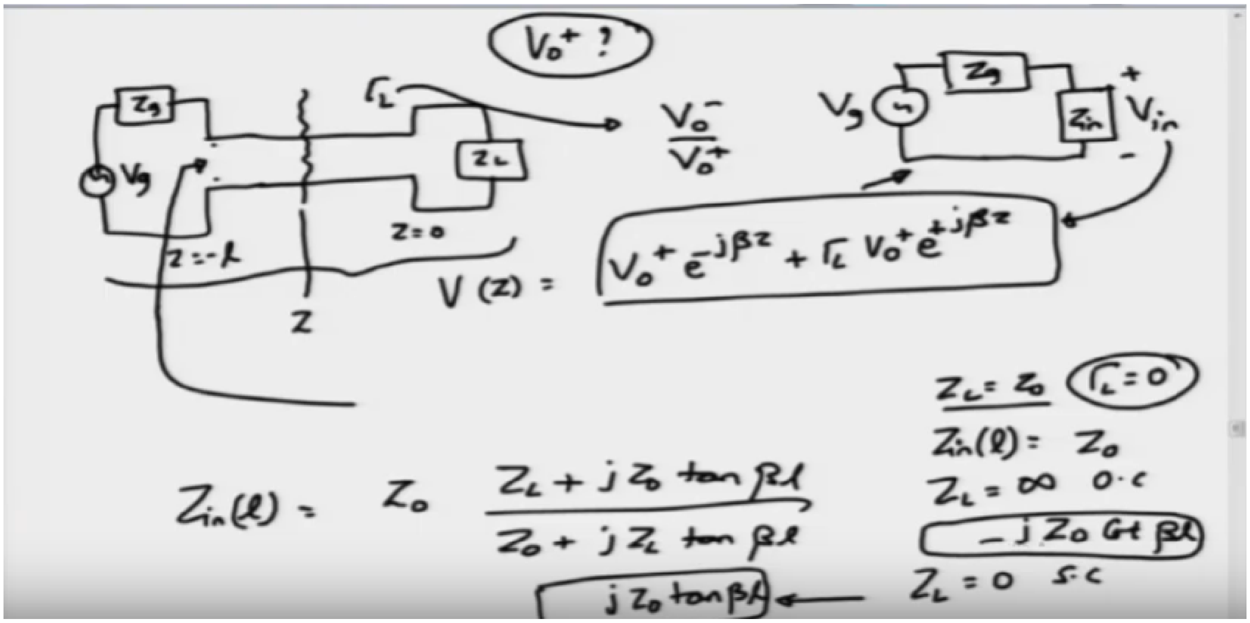

So, for the open circuit case the equivalent input impedance seen ,will actually be equal to minus J Z Naught divided by tan beta L, but tan beta L, 1 by tan beta L is basically cotangent of beta L and therefore this is the Expression. Ok? and for the short circuit case where ZL will be equal to 0 ,you can see that by sub settings at L equal to 0, in the numerator and in the denominator you can see that Z naught, Z naught will Cancel, and for the short-circuit case the equivalent impedance will simply be plus J Z naught tan beta L . Ok? so this is the input impedance for short-circuit Case, in fact the short circuit and open circuit cases are quite common, in many matching can you know circuits, if you have a printed circuit board where you have a micro strip Line and there is a you know, source and the load you normally you know, put these matching circuits and the matching circuit will have a stub as we would call it, which would be? You know, open circuited or it could be short circuited. So, for short circuit you have to drill of wire into from the top layer, to the ground layer but if you don't drill then it becomes a open circuit and now ,the impedance seen or offered by the stub, stub meaning a single trace of micro strip Line so to Speak, will be dependent on the length L. Ok? It will be given by minus J said not caught beta L when it is open circuited. Okay?

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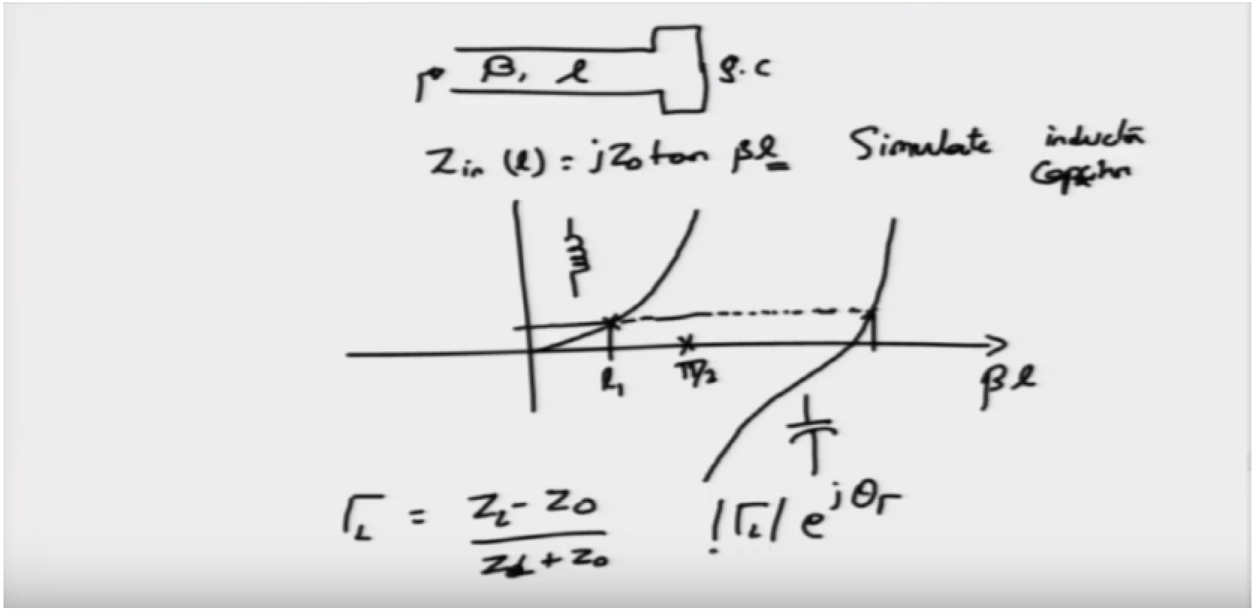

Inside in many other cases transmission Lines are also used of appropriate length, they're also used to make resonators, so it is interesting to look at what would be the impedance characteristic of this transmission Line of length L. so, if I have a transmission Line of length L, with a propagation constant beta and I short-circuit this transmission Line. Ok? so, I short-circuit this transmission Line, then I know that the input impedance seen of the length L will be given by J tan beta L. now, what is interesting here, multiplied by Z naught of course so, I will erase this s J is Ed not tan beta L. what is interesting here is that? Z naught is a real Quantity, tan beta L can take on positive or negative values but the input impedance is now purely imaginary. Right? when will we have a situation where the In is pure imaginary, that is when you have an equivalent capacitor or inductor, type of a load or inductor type of an Impedance, these are called as reactive impedances ,they actually can store energy they cannot dissipate energy. So, what this equation is telling you? This is the din of L, is that you can simulate, inductor or capacitor using transmission, Right?

So, you can simulate an inductive reactance or a capacitive Reactance, simply by taking the length L. Okay? Because, you plot this impedance as a function of beta. Okay? When beta L is equal to zero, of course it is zero. And then a span beta L ,I know starts to increase up to beta L equal to PI by 2 or less than PI by 2, the impedance keeps on increasing but, the sine will remain Positive. So, in this region one may think of this as an imp no inductive reactance, which is going all the from 0 to infinity .so, this was the equivalent inductive reactance, that you have simulated with the transmission Line itself .so, whatever value you want? So, you want plus J 30.7 ohms of inductive reactance, you have to appropriately choose the length L. Okay? given Z naught and beta constants or that particular operating wave length, then the choice of design will be to choose the appropriate Length, and when you do that you can simulate this you know, reactive Impedance, pure reactive impedance, when beta L goes beyond Phi by 2, then you know that it would actually go through in this manner .so here, you can actually have an equivalent capacitive reactance of course, if the length L that you have Chosen.

So, let's say I want this length L equal to L one, in order to obtain a given inductive reactance but, if this length L 1 is very small compared to the you know, that you can handle so, This L 1 will turn out to be just about you know half a micrometer, maybe that's not a good idea Right? Or that is not a good practical Length. So, you can increase the practical length by going to the other branch here, so the same value of L 1 but which has been shifted by, whatever the value of phi? I think this is the periodicity that you are going to get Okay? So, transmission Line offers you a wonderful opportunity to actually simulate, inductances and capacitances in fact this is what we do on a high frequency printed circuit boards. Because, in a high frequencies in printed circuit boards , can't take a coil and then wire it around and then you know put it on the PCBs there, I can simulate the effect of an inductor or a capacitor when I'm doing filter designs or when I'm doing matching networks ,by simply choosing the transmission Lines with appropriate Termination, please remember this you know equivalent inductive and capacitive reactance of the length depending on whether the transmission Line has been terminated in short circuit or open Circuit. Okay?

For the short circuited case we have discussed, I believe the open circuit case as a example for you, or an exercise for you. Okay? at this point I also would Like to mention something else, gamma L that we have obtained is actually a complex quantity Z L minus Z naught by Z L plus Z naught Because, all those it not could be real most load impedances will actually be equal to Z L, I mean most load impedance Z del will actually be a complex number therefore because, of that gamma L will be complex and gamma L can therefore be expressed in terms of its magnitude and an angle. So, this is the polar form of expressing a complex number by giving its magnitude and an angle and we can actually do this. Right? So, gamma L I power J theta Omega.

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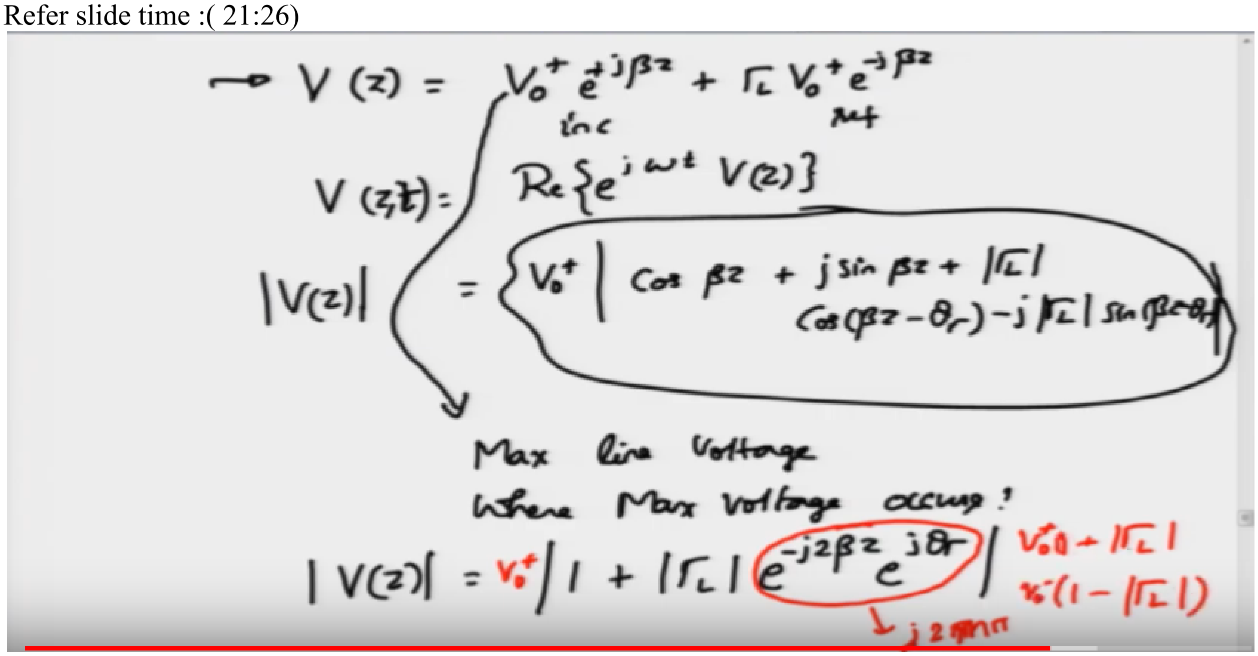

In fact, when you look at the total Line voltage going back to our expression that we had so, You have this v-0 plus, E power minus J beta Z, that we had anyway, plus gamma L v-0 plus E power plus J beta Z .Right? oh Sorry, this one will be mine so because, my Zed is actually in the other way around this would be v-0 e ,e power plus J beta Z and this would be we a power minus J beta Z so, this would be the reflected voltage, this would be the incident voltage ,the signs have changed simply because you know, you have taken the coordinate system to be set equal to zero at the load side. Okay? but doesn't really matter this one, however if you look at what is the actual expression for V or Z you have to of course go and then you know, multiply this entire expression of the phaser, with E power J Omega T and then write down and then take the real part of it, and when you do that you are going to get v-0 plus I am assuming it to be a real therefore I am NOT putting any magnitude into that, then you have cos beta Z magnitude of this fellow so, magnitude of cos beta Z plus J sine beta Z plus magnitude of gamma L ,cos of beta Z minus theta gamma minus J gamma L magnitude sine of beta Z minus theta Gamma. Okay?

I know this, writing these equations are kind of hard but you don't really have to write the equations here. Okay? You can write a simple MATLAB program or some other program could just find out what would be this V at Z and T. Okay? oh sorry, I have not written it Zed and T here yet I have to multiply this one by E power J Omega T and then take the real part of it I believe I won't really do that one we can still work with the phasers alone but, if you look at the phasor here, that I have this is the phasor. Right? This is the magnitude of the phasor and now, if I ask what would be the maximum value? What is the maximum Line voltage that I can have? And when or where will this maximum Line voltage occurs? So where wills maximum voltage on the Line occurs .Right? So these two questions, can actually be Answered, by looking at this expression that I have written and I have circled and then finding out the value of Z Where this expression will be maximum and also find out what would be the maximum value of this expression .Okay? You can also do that one by going back to this we have said itself, the first equation and then taking the magnitude.

So, you can do that by removing this V zero plus E power J beta set as a common Factor, and then you have maximizing this 1 plus gamma L max, e to the power minus J 2 beta Z e bar J theta gamma Right? Because, this comes from gamma L, and then you ask for where will this be maximum Right? This we'll be maximum, provided this term Right? this term e power minus J 2 beta Z and I power J theta gamma will be equal to some multiple of 2 phi that is e power J 2 m phi , where m is a multiple I mean when M is an integer then the maximum voltage that you are going to get will be 1 plus magnitude of gamma L, similarly when this term that I have circled in red is a multiple of R multiple of PHI then because, II power J PI will be minus 1 the voltage that you are going to get will be 1 minus Magnitude of gamma and so, clearly gamma L times V 0 plus of course so, whatever this voltage that you have 1 plus magnitude gamma L times be 0 plus or 1 minus magnitude of gamma L times V 0 minus are going to be the maximum and the minimum Line voltages on the transmission Line.

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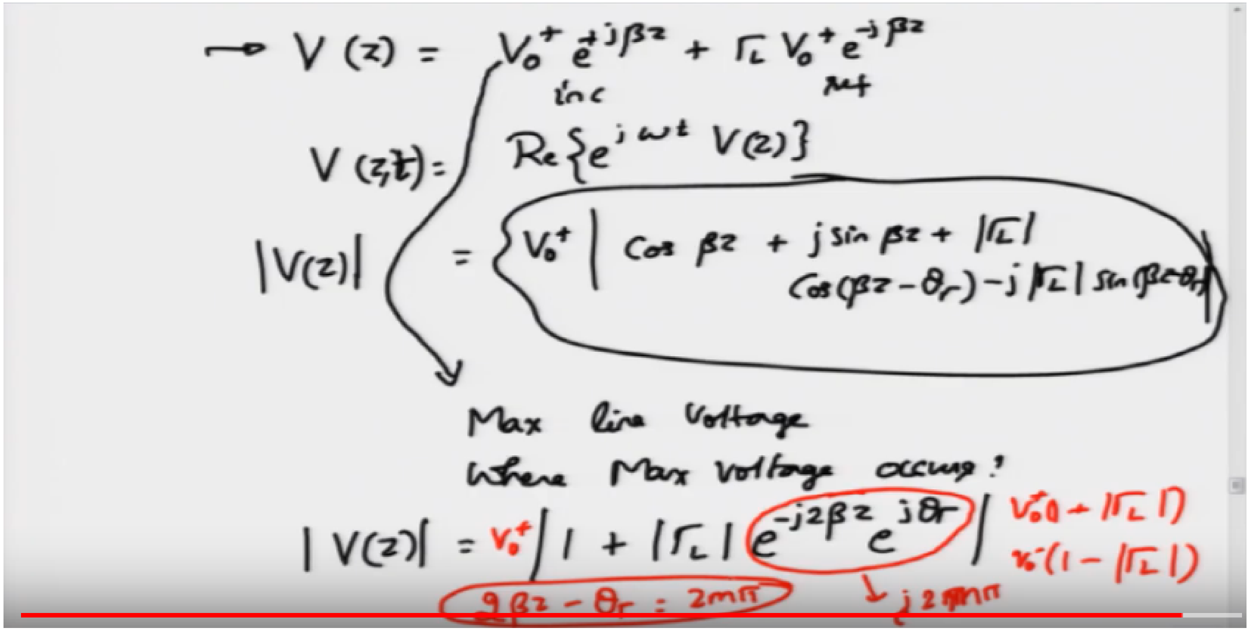

And the ratio of these Line voltages, the maximum and minimum Line voltages is called as the standing wave ratio and because, we are dealing with voltage, we will call this as voltage standing wave ratio or the SWR Sometimes, we write the term V or B most of the time just write it as SWR and this is WR is given by 1 plus magnitude gamma L divided by 1 minus magnitude o gamma L and s is what the symbol that we normally used to denote here, in fact you Can invert the relationship here, and ask for what is the magnitude of the reflection coefficient maximum of the magnitude of the reflection coefficient, this can be shown to be equal to s minus 1 by s plus 1 .Okay? And the standing wave ratio is important because, in the next module I will show you that Lines having higher VSWR, will have higher reflection coefficient which is also obvious from this and that is bad again because some of the power will be actually reflected back instead of being delivered onto the Line. Okay? So, consider the case in an ideal scenario where gamma L equal to 0 then s will be equal to plus 1 this is Ideal. So, in even in the case of ideal match s will be equal to 1, meaning that there is no maxima and minima on the Line. The Line voltage would descent I mean would essentially be the same everywhere. Okay? However, for the case of Open circuit gamma L will be equal to plus 1 so, the magnitude also is equal to plus 1 and then s will be equal to Infinity that is a Line that has been terminated with an open circuit. Okay?

An Ideal lossless transmission Line that has been terminated with an open circuit actually has a maximum mismatch, which is Obvious, Right? you mean you have an open circuit no power is being transferred so, all the power is being reflected so this is clearly the complete mismatch Scenario, you can also have a complete mismatch scenario when gamma L is equal to minus 1, in which case again s will be equal to infinity, these two are completely mismatched, most transmission Lines Lie anywhere from one to infinity, that is their s WR values will Ile anywhere from one to infinity. So, this is the range that is allowed, and of course the corresponding magnitude of gamma L range for this to happen, will be from zero to one. So, one would represent complete mismatch zero would represent, ideal scenario. So, these relationships between the reflection, coefficient, standing wave ratio, maximum voltage, minimum voltage is very important for you to, remember in fact I will leave this as a small exercise for you, to show that the maximum impedance on the Line, which is the ratio of v-max to IMAX is actually given by s

times Z naught. Okay? Where s of course is the standing wave Ratio. So, in a sense that if you know the standing wave ratio you also know what the maximum impedance on the Line is and you also know how much is the Line Mismatch, but of course what you don't know is where this Maximum would occur? If you want to know where the Maxima would Occur, well you go back to this expression and then set this term equal to 2m phi, that is you set the term to beta z minus theta gamma 2 to M PI and then adjust the value of z in such a way that Z is negative because, remember Z equal to 0 corresponds to the load and then you go all the way backwards in that direction so, you will see fine you will find the positions of maxima and Minima. These problems can actually be solved by writing these equations or by writing MATLAB programs or some other coding mechanisms but, there is a nice graphical way which tells you how these quantities can be calculated in a much simpler way and in a very intuitive way to give you understanding of the entire transmission Line theory, that most people look at and that graphical method is called a Smith Chart, and I will be using the Smith chart in the next class ,or in the next Module, to solve problems which otherwise would require a Little bit of a tedious calculation ,or you would have to write your own numerical methods or computer code to solve these problems. Okay? So until then,
Thank you very much.