Lecture 39-
Distribution of Laser Beam

Hello and welcome to NPTEL's MOOC on electromagnetic waves in guided and wireless media. In the previous module, we stopped our, you know, we started our discussions on Fraunhofer diffraction and we stopped at, the point where we were looking at, the fields, of a circular aperture that is you have a source, far away from, the aperture and then you have observing the aperture from a far away distance, so that the person essentially, looks like a circle, of certain radius a. In that case, we found that, there is a bright central spot, which sometimes is called as, 'Arago Spot' or sometimes called as, 'Poison's Spot' and around that, central bright band, you will see additional bright and dark bands. Right? So, you will see that, concentric circles, except for the central bright spot and the central bright spot, does not have the same aperture, area or the same radius, as does the aperture, in fact it will be slightly, bigger and this is what we actually, call as, 'Point Spread Function' this can be thought of as an impulse, response for those who are, familiar with signals and systems, so this can be thought of the impulse response, of an aperture, order of a point source. The point source being equivalent to that of a Delta function. Right? So, we will not dwell, on most of these concepts, but, these ideas, are widely used, in the context of Fourier optics, which essentially concerns itself, with understanding the spatial frequencies and how to filter them and how to modify them, this area is also quite important for digital image processing where many of these concepts, of images and image formation, require an understanding of how diffraction effects and how this point spread function affects the resolution of the you know, instruments which, such as cameras and other medical instruments such as microscopes and optical instruments such as, astronomy, I mean astronomical instruments such as telescopes and so on. While we could say lot of things about diffraction, let us not, you know, because the scope of the course is not, to deal with diffraction itself, but, to see, what kind of electromagnetic waves are, you know, are possible the modes that are there, in free space and guided media, with that in mind, let us look at, what happens? To a very, special type of a beam: that is emitted by a device called as, 'Laser'. Laser as you already know, is you know, used for emitting light at frequencies, which are not just at the visible range, but, also at different ranges. Of course it is not just like any other light source, such as a lamp or a you know, wood-burning, which emits wavelengths and other things, light is very, special in the sense that it, has a very high degree of coherence, which I will not be able to talk about it, but for, for uninitiated you can think of coherent as correlated, meaning that the beam, essentially reminds, in a very stable phase, relationship with itself, over long distances and over long times. Okay? So, this lasers actually, emit a beam, which is not like a spherical beam or it's not like a beam which is say, a uniform plane wave. Right? And you know, you know, what is important when you study lasers is that, the beam that is emitted by the laser, will be used for further processing. Right? So, you for example, want to cut, a metal using laser or you could cut some other metal using laser, then it is important for you to know, if diffraction affects the beam, if it affects, then, how does it affect? Right? So, you have laser output: that is light output, coming out from a very, small aperture and over the distance, it kind of widens out, because of the diffraction. Right? So, because this is an aperture and then, it kind of widens out. Okay? The beam that is emitted by the laser in its you know, most stable configuration, is what is called as a transverse electromagnetic wave 10, 0 ,0 mode and it has a very, special distribution in the aperture plane. So, if you examine the laser output in the opera plane, you will actually, see something like this. Right?

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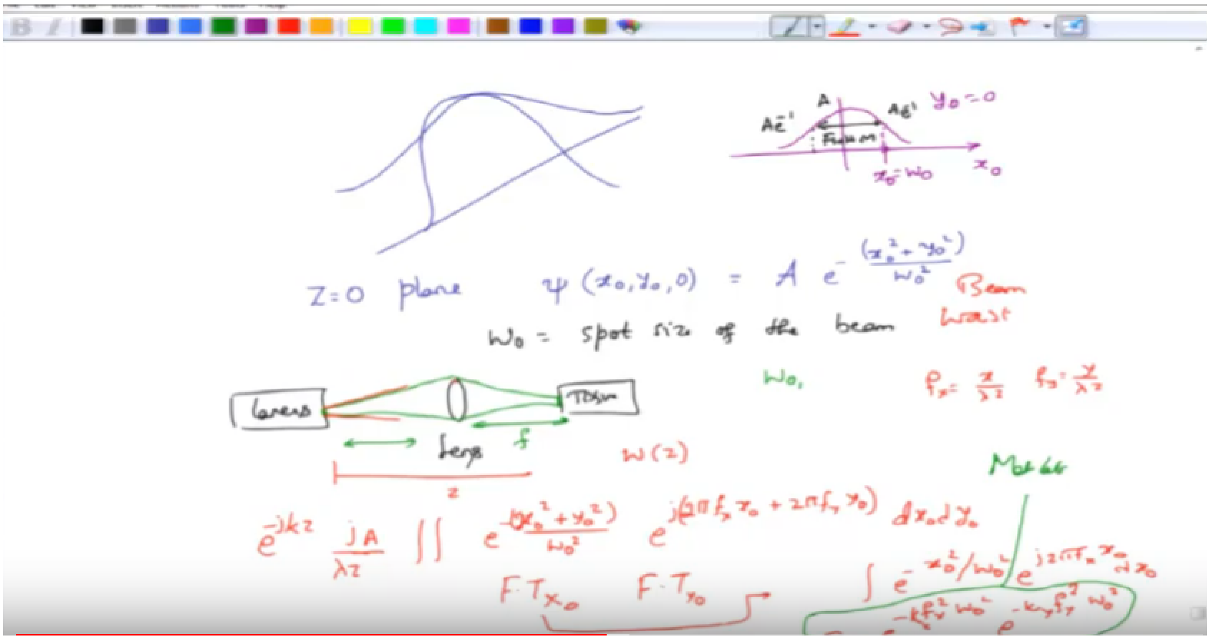

So, if you cut, along the x axis then you will see, a Gaussian, if you cut along the Y axis, you will see another Gaussian. Okay? So, it's basically, going to be a two dimensional Gaussian function, whose main you know or whose intensity, will be peaking, at the center and then gradually, will be decreasing as you move away from the center of course, it never, kind of I mean it will not, go all the way to minus infinity, plus infinity but, these Gaussian functions, are very good approximations, for the temp beam: that is emitted by the laser. Now, please understand that, what we are talking about is the beam that is, coming out of the laser. Okay? And we are talking about its distribution in the transverse plane, please note that, the wave is still, propagating around along this black direction for example that, could be thought of as the z axis, however, in the XY plane or in the transverse plane if you look at it: that would be the, Gaussian two-dimensional, Gaussian function that I talked about. Okay? So, that plane is called as the, ‘Aperture Plane’. Now, for beam: that is emitted at say, Z equal to zero plane. Okay? And there is some interesting fact about the Z equal to 0 plane that I will talk to you about after I have introduced this diffraction thing, we can represent, the field: that is coming out of this one, remember our x0, y0 from the last module, these are on the aperture plane coordinates, x0 and y0 here, we have certain amplitude let us say a and then, the distribution is supposed to be a two-dimensional distribution. So, you have X 0 square plus y 0 square, divided by W 0 square. Now, sometimes instead of W 0 square here, in the denominator for this exponential function, you will also see people using 2, this won't change anything, in terms of the understanding or the concept, with only changes a numerical value slightly. Okay? We will not worry about that factor of 2 here, we will simply write this as W 0 Square and what is this W 0 square, is a very interesting thing, if for example, I consider the, if I plot this exponential function or the Gaussian function, as a function of X, let us say or X naught let us say, while keeping Y naught is equal to 0. Right? So, I am basically taking a cut, along the x axis, so then this w 0 square, is the value of X naught, okay, when X naught is equal to w0, then at that point, the amplitude of the wave, would have dropped from a, to a, e bar, minus one. Okay? So, if you again locate at X naught equal to W, minus W naught, even you will again see that the amplitude has dropped, to about a / E minus and this width, is sometimes called as, ‘Full Width’ at ‘Half Maximum’. Okay? So, it is some sort of measure, as to what is the spread of this light beam, in the x0 direction, because of symmetry, the same spread you can observe in the Y direction as well, so you can imagine that, there is a circle, whose central value is kind of bright or center is bright. But, then as the edges go around, then the amplitude kind of decreases and you can kind of put a circle, in that you know, at the points X 0 equal to W 0, minus W 0, y equals, y naught equals W 0 and minus W 0 and that circle, would essentially tell you the size or the spot size of the beam. So, accordingly we call this, 'W 0' as the spot size of the beam. Okay. If you want to understand this one, what you can do is to just take a simple laser pointer. Okay? And darken the room a little bit, so that you can see the, light correctly and then you take a white sheet of paper, paste it on some cardboard and keep it, at different distances. Right? Or you can actually keep the laser pointer fixed, maybe you can pull up a chair and then put the laser pointer fixed and then, in the same you know, in the direction of that laser light, going around you place a beam. Okay? Or rather sorry, you place a screen and then you can move the screen away and then you can observe: that the spot size would be bright: that is there will be a central bright, bright spot and then there will be, small you know, like kind of the you know, light is kind of smearing out at the edges, but then, as you take the screen far and far away, depending on how good or bad your laser is, at some point. Right? You will start to see that, the central bright, the brightness of the central spot, starts to diminish and then everything starts to kind of expand out. Okay? Which means that, the size of the beam, in this aperture plane or in this transverse pair, is kind of increasing? Right? And it is very important for us to know, this increase. Okay? Why because, in most cases you would have not just going to use lasers, there will be some optical system, for example, a simple optical system such as a lens. Okay? The idea of using a lens and then maybe you have some kind of a tissue here, which you are trying to illuminate, I'm just giving an example, you could illuminate it with anything else. So, if the lace if the, the position of the lens is very critical, because if you start off with a spot size: that looks something like this. Okay? This is the laser beam and if this distance happens to be in the focal plane of this lens, the so called back focal plane of this lens, then it is possible for by choosing this distance appropriately, not exactly equal to F, but, choosing this distance appropriately, it is possible to make sure: that the beam essentially, retains or goes back to its original spot size here, at the image or the object point. Right? So, these kind of calculations, require you to know, what is W 0? And what is the rate at which, this is spreading, so for example, if for you write down two horizontal straight lines, then this would be an approximation, for the beam divergence, as we would call it and it is important for you to know, what the slope of these lines are, so that you can, predict or you can actually place this lens at the appropriate position, in order to bring the, light beam back into its original, spot size. Okay? So, with that in mind and with the additional assumption that we are not really in the near field, zone for any of these waves, we are far away, from the near field, what we want to understand, is what will be the spot size, as a function of Zed. Right? So, z-axis being measured, from this minimum, waste point. Okay? So, this is spot size of the beam and sometimes also, called as the, 'Beam Waist'. Okay? For obvious reasons this is called as a, 'Beam Waist' and then from there it kind of spreads. Okay? The solution is not very difficult, it's suitable tedious because you are going to work with Gaussian, integrals, but, if you look at a good handbook, then what your s n? Then you can and you can those, relationships of there are given or tables of integrals that are that I've known and Fourier transforms that are known, what you're essentially trying to do? Is to take the two dimensional Fourier transform, remember the field at any point, would be something like J by lambda Z. Right? And you will have, say e power minus, x0 square plus, y 0 square, divided by, so I will also, have an amplitude a: that I am going to pull, this out divided by W 0 square, correct. So, we have this X 0 square plus, y 0 square by W 0 square and then usually, what you would have had is, e bar J 2 pi, F X, X naught plus 2 pi F why, Y naught. Okay? So, this is what you had and then, of course this is an integral over, D X naught and d y naught, what I am going to do? Because this can also be done you know, the expressions for the Gaussian, so and you also know the idea that, Gaussian input will have a same Gaussian, kind of a Fourier transform. So, you can use the tables, of Fourier transform and then get, the output, which would also be, kind of a Fourier transform. Right? So, this is basically and you can also, split this Fourier transform over, x0 separately and a Fourier transform over Y 0 separately. Right? So, what would be the Fourier transform over x0? This would be the Fourier transform, of the one sided

Gaussian or single variable Gaussian, a e bar minus X 0 square, by W 0 square, e bar J 2 pi, F X, x naught, DX naught is, what you have and this would turn out to be something like e bar minus, F x0 square W 0 square, which some constant, which I will call as say, Kappa X. Okay. This constant needs to be multiplied, in the or put into the numerator, because you know, of the Fourier transform properties, I don't exactly remember, the Fourier transforms, but, these are available in a lot of textbooks and online and you can figure out, what would be this K X out there? Okay. Similarly the Fourier transform over Y naught, would give you something like minus e bar kappa y, this is just a constant. Right? That needs to be multiplied here; the factor of 2 pi and something like that. Okay? So, other than that, the concept is very simple, it is still the, same thing as you know, it's a Gaussian here or rather, what I would say is that, the Fourier transform is also Gaussian and then, this would be the overall distribution and you can of course remember that, FX is basically X by lambda Z and FY is equal to Y by lambda Z. Okay? So, if you fix the screen at Z equal to constant, then oh, yeah! We also have a phase factor e bar minus JK Z, which I forgot right, but now, I'm going to write that one. So, yeah! This is the, final expression that you are looking for and as before, one may try to understand this analytically, but I would say the best way to understand these equations, is to after you have looked at the tables and then found out that, this is essentially, the same form as, what you're going to get? Then I would say that, you take you know, you write a MATLAB script and plot, this expression as a function of x and y remember, you can write this as a function of x and y, simply because for a given Z, FX and FY correspond to the spatial frequencies, which are related in terms of x and y, observation points. Right? So, I will leave this, as a MATLAB exercise, for you to figure out, what would be the beam size, as you move away from this one. Okay? However, in many cases, you are not just interested in the far field; you are also interested in the near field. Okay. And what should we do about the near field thing? Well, we did some approximations to get to the far field or the front offer diffraction, what we have to do is to now? Not do those approximations,

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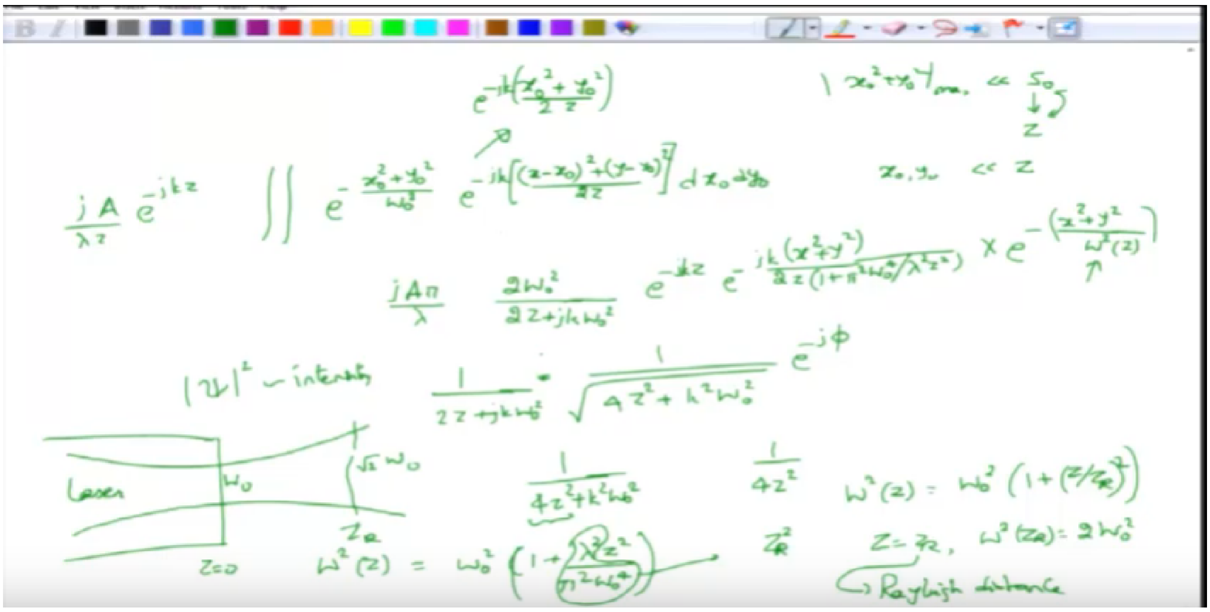

remember what approximations we did? Yes, we neglected this X 0 square, plus y0 square term, in the numerator, so there was something like divided by 2, s 0, s 0 we wrote it as, Z itself, so this was the term that you had and you also, so this is the term that we actually neglected. Right? So, you had this e bar minus JK, of course, so this term, we neglected we said that, we could neglect this because, x0 square plus, y 0 square maximum, would be very, very small, compared to s 0 and we replaced s 0 by Z, because a 0 and Z, were approximately of the same order. Right? So, this is the term, but we actually neglected, in the previous equation and therefore, we had a front half a diffraction. Now, what we do? Is not neglected. So, which means that, we will go back to the initial distribution of the aperture distribution, which is given by this expression and then, we write so, e bar minus JK Z, would still be present, some J, by lambda Z is also present, an amplitude a is also present. But, inside here, what we will have is the, know the other term that, we had you know, forgotten. Right? So that, would be this e bar minus JK, we will have X minus x0 square, plus y minus y0 square, divided by 2 Z. Okay? So, this integration, is what would give us, the fields or you know, the near-field expression and of course you can get to the far field expression, by neglecting, x0 and y0 in relationship to z. Okay? But, if you don't or in fear interested in knowing, what happens to the beam you know, growth initially, then you simply put, this you don't make this approximation, retain this part, the quadratic phase factor part and then proceed to solve this integral. Okay? This integral is also, not very difficult to solve, this can be done by completing the square. Okay? I will not go to the details here and as before you can split, this integral in terms of the integral over X and integral over Y and after you do all that, what you would essentially end up, is something like this. Okay? J a PI by lambda, don't worry about this part, you know, this is something that's going to come from, the constant PI and this one will come from the Fourier transform of the integrals for values of this one. But, what you would actually get, which is interesting, is this quantity. Okay? For a given Z this fellow will be constant, but, as Z increases, what you can see is that the amplitude here is decreasing. Right? So, you can of course take this amplitude itself and then take the magnitude of this one, as well as the face of this, so if you take the magnitude, you can see that, it would be something like say 4, Z square plus, K Square W 0 square, in the denominator, under root. Right? So, the magnitude of this fellow, would be this one and there would be a certain face, this face is called as Goa face. Okay? And this will be important when you start doing this manipulating the near fields of this caution beam using lenses and other elements. But, we will not worry about this Goa face. Okay? We will simply look at this amplitude and then, clearly you know, that Si, magnitude square is the one that is going to give you intensity or the power, so if I take the magnitude square here, the rest of it will simply be face factors, which will go away, when you take the magnitude square, but, this amplitude part if you look at it, it's actually like 1 over 4 Z square, plus K square W 0 square correct? So, when Z is very, small then the beam waist would be almost constant or the amplitude of this one would almost be constant here, K square W 0 square. However, as that increases the amplitude kind of decreases, because this term for Z0 square starts to overtake everything right. So, this quadratic decay, I mean for very large values of Z, the amplitude could be or the intensity, essentially, know goes as 1 over Z square, which of course is expected, out of this type of beams. Right? I mean these are the beams that, would have their power fall-off as, 1 by Z square, the inverse, relationship and when you integrate over a certain spear, then the power would essentially, look at this I mean this would be a constant that suppose that's, what we have seen. Right? But, coming back to the face park here, what you would see is? E bar minus JK Z, which of course was present earlier also, but the extra face that you are going to get, because of this quadratic, term X minus x0 square, plus y minus y0 square, is a very interesting term, which is given by 2Z , 1 plus pi square W 0 to the bar 4, divided by lambda square, Z square. Okay? So, this one, is the expression that you have, for the face part and of course this is, what was actually, you know, interesting for us, because you see, this is still Gaussian, but then, the effective W, has actually changed. Right? Oh, sorry, this is, this is not just the completeness, complete thing, this would be into E bar minus X square, plus y square divided by 2 or rather 2 is not there, because we didn't start with 2, so this will be W square off Z. So, this expression, is naught W square of Z, but this expression is different, so what we want to know is, this W square, square of Z which you can actually show: that it is given by W 0 square, 1 plus lambda square, Z square, divided by PI square, W 0 4. Right? Yes, this is related to this, but, the basic motivation that the field is still Gaussian, comes from this particular expression. So, the field is Gaussian, in the Z equal to constant plane away from every point, every plane away from the beam, the beam is essentially the Gaussian beam, is essentially, a the beam essentially, retains its Gaussian its characteristic. But, it's spot size W, will actually depend on Z and starts to increase. Okay? So, at set equal to 0 this fellow, will be equal to W 0 and that is alright. But, you can also define, this entire thing. Right? In this bracket, you can define this one, by some other name or you can denote this by some other name, called, ‘Z R Square’ and then you have a very, simple expression for W square of Z, which is W 0 square, into 1 plus, Z by Z R whole square, So, clearly at Z equal to Z R, the beam size, at that ZR plane, will be twice, of W 0 square or the beam size will be square root 2 times, W 0 which was the original spot size, times square root of 2 and this Z R, is called as, ‘Rayleigh Distance or ‘Rayleigh Length’ or sometimes also, called as, ‘Rayleigh Range’. Okay. So, this is how, beams that are emitted by laser. Okay. In the transverse plane behave and this is a very, interesting thing we have seen beams, in the slabs, we have seen beams in the radio you know, in the free space, because of the antennas radiating the fields, we have seen these beams or essentially electromagnetic waves or guided modes, in wave guides, we have seen them, in, in a in a free space in the context of uniform plane waves. So, this one, is another type of guided more, the guidance is now, being the free space material and the initial conditions of this beam, is the laser. So, you can think of the laser as an antenna. Okay? Which emits, a very special class of beam, called as a, ‘Gaussian Beam’ and this beam, is the one that is propagating in the free space and by because of diffraction, what happens is that? The beam size may be small, at some Z equal to 0 plane or we take this as Z equal to 0 plane, this entire thing maybe we will put in the black box and call it as a, ‘Laser’ and at Z equal to ZR, what you would find is that, the beam size is square root 2 times, the minimum value. The minimum value is called as the, ‘Beam Waist’ and it is usually the convenient location, where we put this beam up there. Okay.

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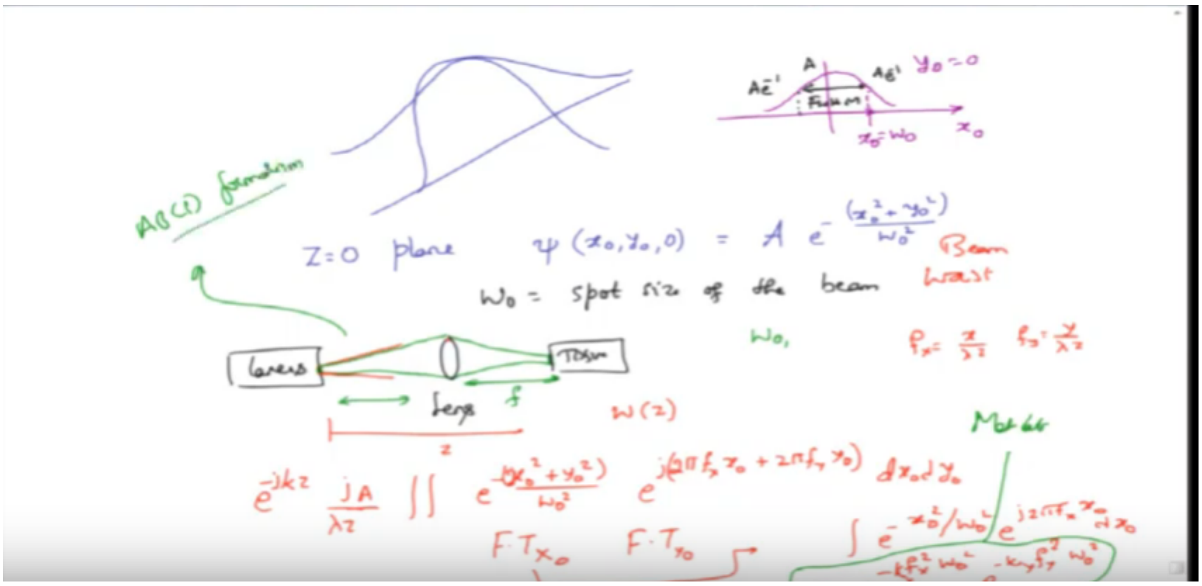

Now, coming back to the situation, where we had a lens here and laser beam and then we wanted to look at what happens to this you know, light as it goes and hits a lens and then whether it gets focused or it doesn't get focused, you can understand this propagation, by going to what is called as ABCD formalism? Okay. ABCD formalism is you know, two-port, type of a formalism, wherein you have certain parameters, which is essentially the slope, as well as the distance and then you have a matrix, which connects the slope and the height, at a different plane. Okay? So, you can think of this as one plane, you can think of this as another plane, this is the input, plane and as this input you know, at this input you define two parameters, for a Gaussian beam you define its waist, as well as you define its divergence angle and then, at another plane, you have this no relationship of type you know, waist or the spot size, as well as the divergence angle, in the free space we know, how these two are connected, this is w 0 and the divergence angle and this is w of Z and the corresponding divergence angle, which you need to take the derivative of that. However, when you put an optical element in between, what this optical element does, is to modify this W 0 or no the waist as well as the slope and how exactly, it gets modified is captured by, this ABCD matrix, ABCD or its elements and this matrix will be different, for different elements, it will be different for a simple slab, it we'll be different for a mirror, it would be different for a lens and so on. But, the point is if you are given those, matrices. Okay? Even if you don't understand the theory, if you are given those matrices. Right? And then you know, the beam waist, as well as the divergence angle to begin with, you know, at the laser, then you can manipulate, these matrices in order to get whatever the beam size and the divergence that you want. So, let's say there is certain microscope here, which accepts only, a certain beam size and a certain divergence angle, whereas my laser, is incapable of giving that directly out. Okay? The laser beam size and this one is not matching, then you can make a match, by putting these optical elements and then manipulating them or no moving these elements around and ensuring that the overall ABCD matrix is, is such that, the beam sizes on both sides of this optical system, are matched. Okay? So, there's lot of things that one can talk about, unfortunately we will not be able to talk about it, anymore, because we want to move on to, other interesting aspects in this course. Okay? So, one other interesting aspect in this in this context of diffraction, is the close connection between interference and diffraction. Okay? Interference as you know, is a phenomenon, where in two waves. Okay? They talk to each other and sometimes if they talk, nicely then there will be constructive interference and when they talk you know, know in a manner that we will soon see, there will be destructive interference. Constructive interference will always give rise to an increase in the illumination, whereas destructive interference will destroy the, illumination at that position. Okay? The most classic case of interference or interferometer that you have and that interferometer is a device to observe interference and the most classic, device to observe interference is, what is called as the Young's double slit experiment? This is something that you can do at home and this is something that has been done countlessly, in many, many laboratories and in fact it's being done every day. Okay? What I want to do is? To give us, touch which is slightly different. Okay?

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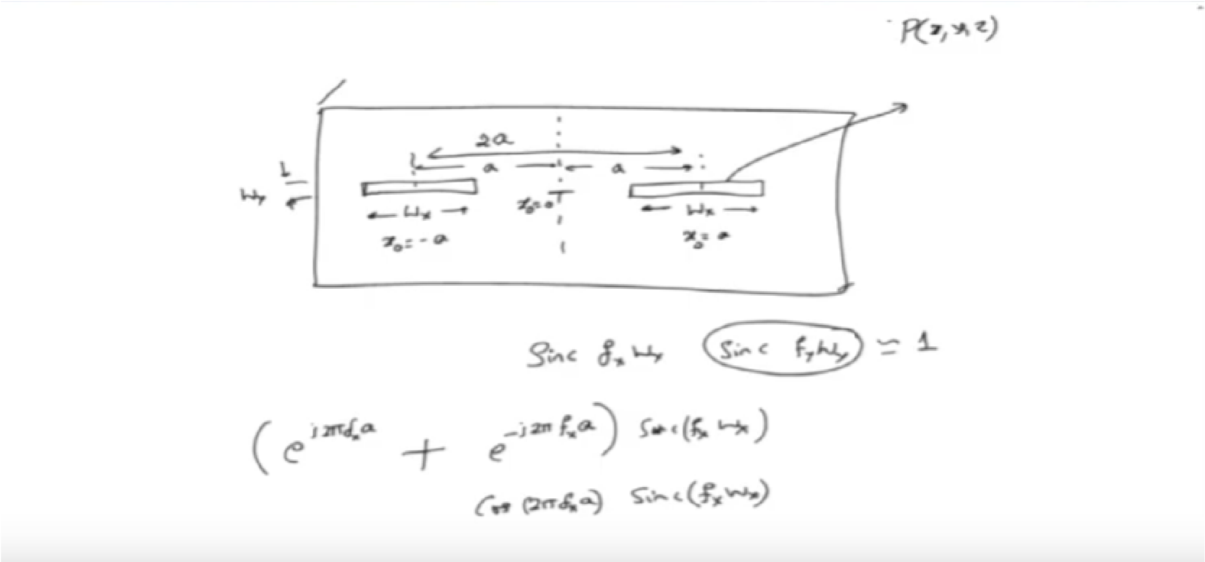

Normally this in the context of optical systems you first encounter, interference and then encounter diffraction. But, you can actually think of diffraction, integral, I mean interference in the other way around, in fact you can use this for half a diffraction idea, to actually talk about, interference itself. For that, let's carve, two slits. Okay? These slits are, of equal size WX, in a width and then we will assume: that the width W Y is very, very small. Okay? The reason why we want to assume this WY to be very small is that, we know these rectangular apertures, are going to produce a diffraction pattern, which would be sinc FxWx, times sinc FyWy. Right? So, when I make W Y to be very small, then the sinc function, would be approximately 1 and I don't need to worry about it. Okay? So, these are very narrow slits, as in that we are considering. The only stipulation is that, in this plane, these centres are separated, by a distance of 2 X naught. Okay? So, or maybe we will have to use a different this one. So, we will separate them, by say 2a meaning that, from this point, which is X 0 equal to 0, to this point will be a and this will be a similar a, distance onto this one. So, the center here is X equal to a or X not equal to a, the center here is X not equal to minus a. And then we are looking at, the field, in a faraway, point P. Okay. Which is X Y and Z and what we want to do is to understand, what would be the field at that point. Okay. Now the basic idea, you already know, you know, what is the field of a single aperture when it is illuminated by a light beam and then you also know this aperture, the only thing that is, I know, different in the previous cases, to this case is that you have to aperture, of course light is super in no superposition principle holds, so you can add the two fields, but then, you have to simply be mindful of the fact, of the phase difference that comes in because of the separation in the centres. Right? So, the phase shift from this one, will be e bar minus J 2 pi F X times a and the phase shift because of this fellow will be, e bar J 2 pi F X a and field essentially is the same. So, if when you add, the fields outside, this would be sinc of F x WX please, note that, I've already made this WY almost equal to zero and this would be the overall field. Okay? So, the field will be Cos 2 pi F X a and x sinc of F x WX and what they're going to do? Is to look at, what happens or what should be the relationship between a and W X, in order for us to observe the classic Young's double-slit interference, in the next module. Thank you very much.