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National Programme on Technology Enhanced Learning (NPTEL)

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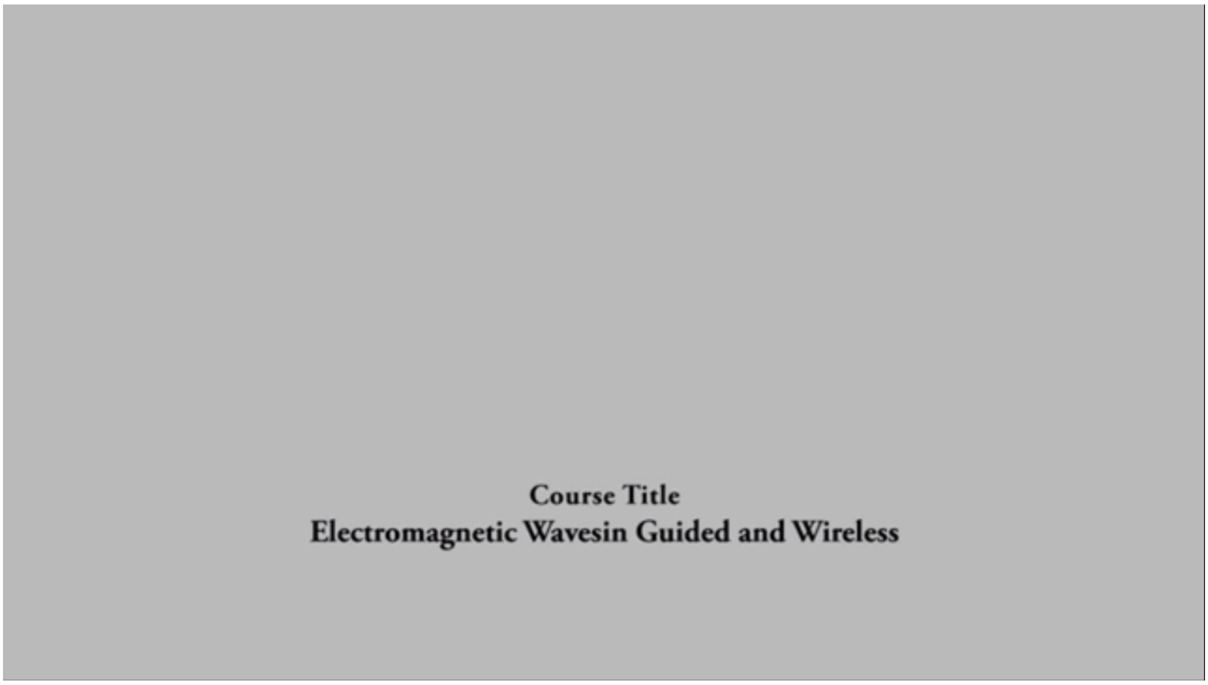

Course Title
Electromagnetic Waves in Guided and Wireless

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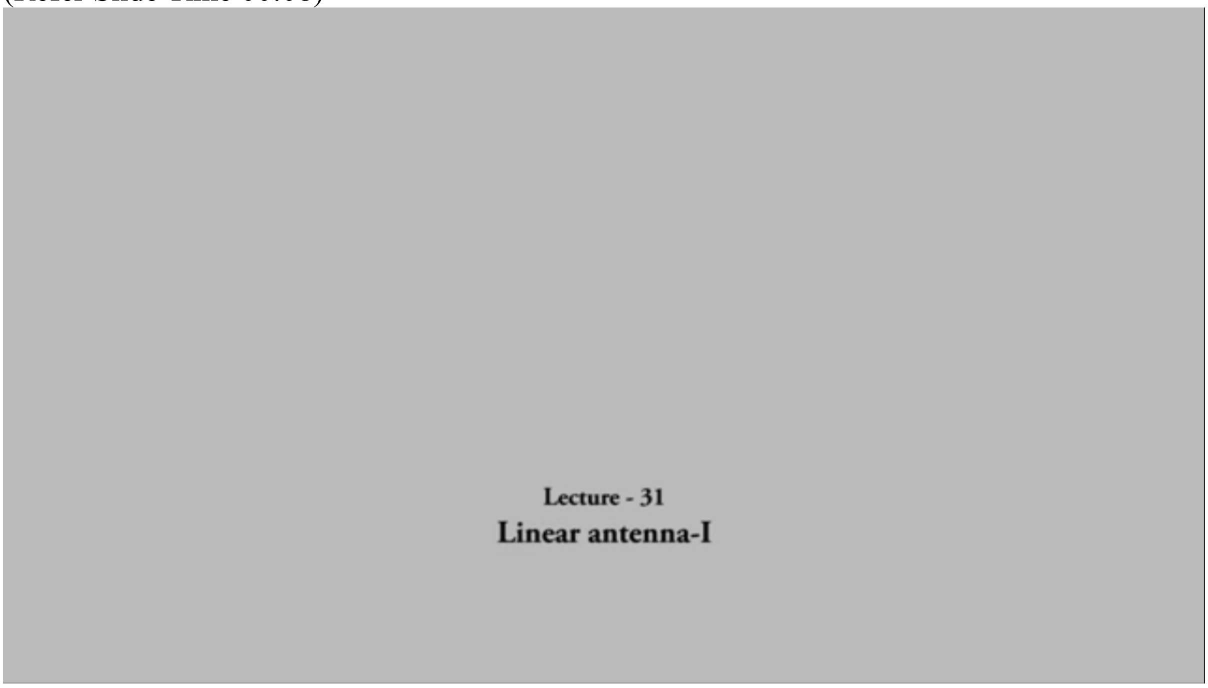

Lecture - 31
Linear antenna-I

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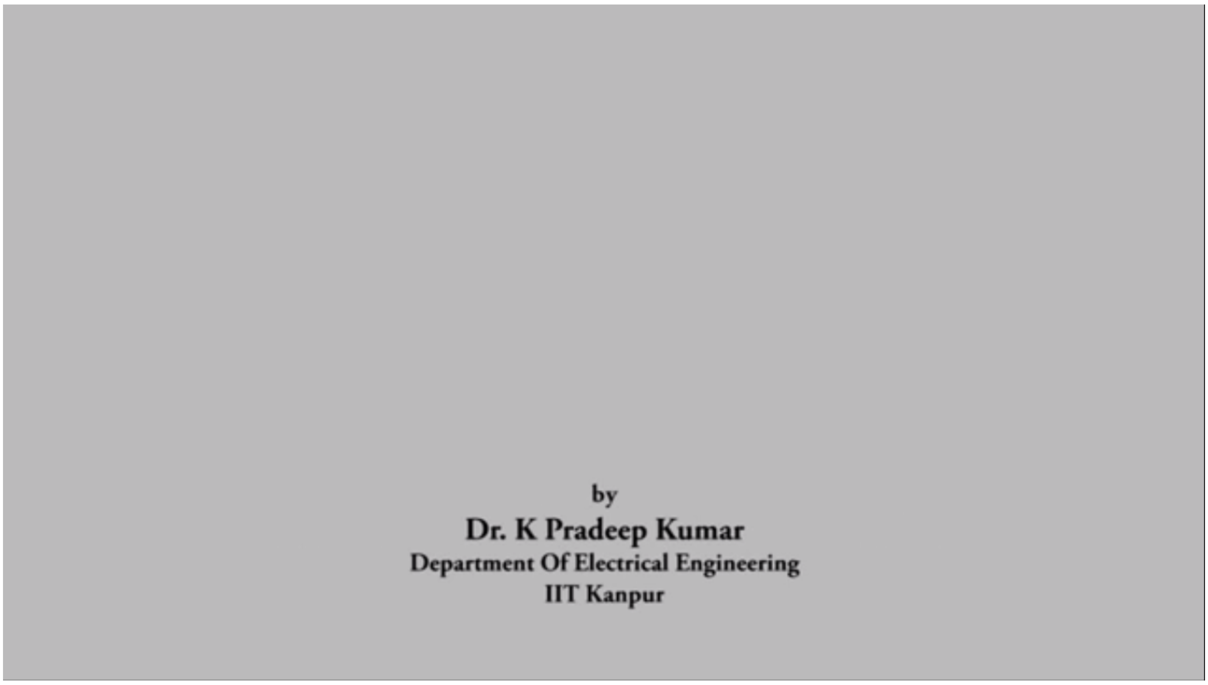

by
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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. We will continue our discussion of antennas. I would like to just motivate this module by bringing up one of the results from the last module, namely, that of the radiation resistance.

Now recall that radiation resistance is how a circuit would look or how the circuit would view the antenna as such, right, because the antenna is radiating power and that power will be supplied by some source and the amount of power that is radiated would, something that the source would never get back, right? So it is a power that is being lost, you know, as far as the source of the, you know, that power is concerned.

So the voltage generator that you would have kept in order to feed the antenna, as far as that voltage generator is concerned, this power that is lost or radiated by the antenna is something that is lost from the perspective of the voltage generator. Okay. And that is the reason why we represent that power radiated by an equivalent resistance and call that as radiation resistance, right?

So that resistance is how the voltage generator would look at an antenna. So regardless of what antenna type that you consider, you can, of course, with some approximations that we've been, that we have not talked about, but there are a lot of approximations here, but you can with those approximations, you know, replace the antenna as per the circuit is concerned and then represent that one by a equivalent radiation resistance.

Now one of the things that you would like when you're using an antenna for transmitting information from one point to, you know, another point, you know, by radiation would be to maximise this amount of power that is being radiated.

So suppose I send in 1 Watts of power from the voltage generator or the power generator, feed it through the antenna and if only like 1 mW of the power is radiated, then most of the power is either lost or rather most of the power is not being sufficiently utilised in order to transmit information. Yes, that power would be remaining within the source, but that is not what we want. What we want is to maximise the radiation that is because unless I maximise the radiation, then the receiving antenna cannot be kept at a very far distance.

We will see the relationship between the transmit antenna power or rather power density and the receiving antenna and the distance between these two in order to get appreciable, you know, signal at the receiving antenna later on, but it is kind of obvious from physical intuition that if you only transfer a very small amount of power that is being fed to the antenna and make it radiation, then that is not a very good antenna, right?

So what you want is a situation where the antenna radiation resistance should be larger, right? So one of the ways in which you can do this is what we are going to consider in this module, and we call this as a linear antenna. Sometimes it is also called as a thin wire antenna or sometimes called as a linear wire antenna, and the length of this antenna will not be very small. So in the case of a short elementary dipole that we considered, the length of that dipole dz was considered to be very, very small compared with the wavelength.

Now we do not make that approximation, and we will see that if the length is made appreciably close to the wavelength of the wave that is being radiated by the antenna, then the radiation resistance can be improved significantly. Okay. So with that in mind, let us look at what antenna that we are going to consider.

This is one of the more practical antennas, okay, and this antenna is usually centre fed meaning that the voltage generator that you have with appropriate, you know, internal impedance would be connected via a transmission line to the antenna terminals, okay, and then connected, connected to the antenna at the middle of this antenna terminals.

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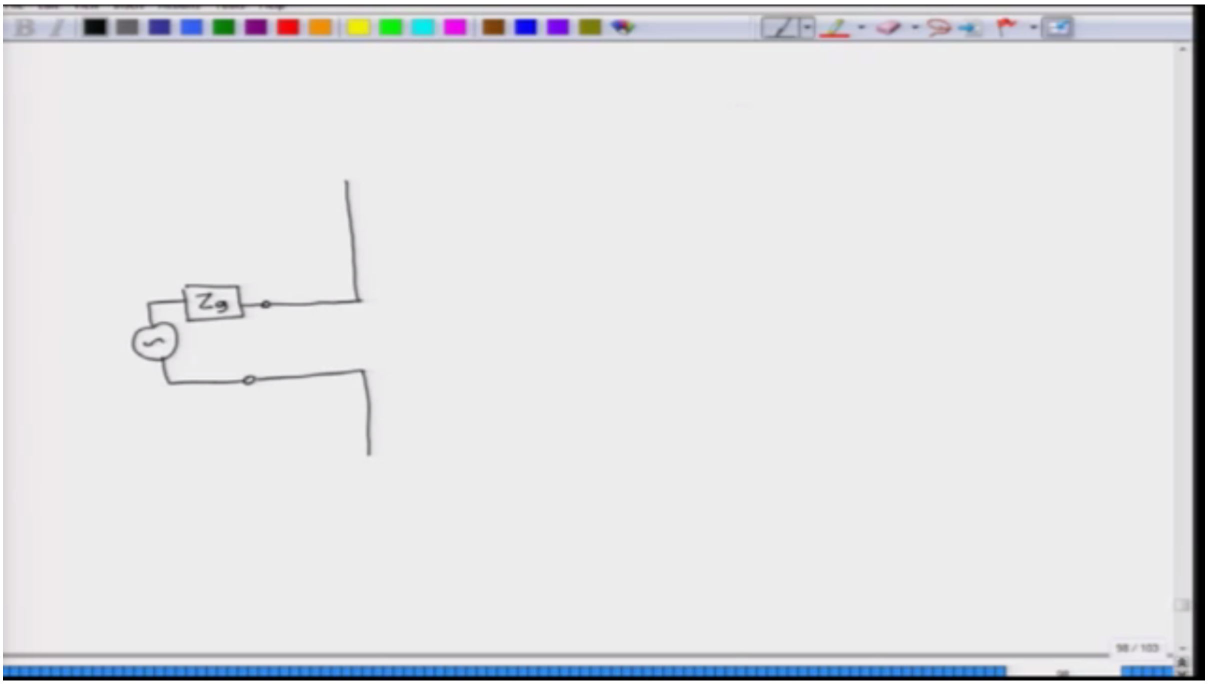

Of course, I have shown this gap to be very large, but in general this gap would be very, very small. So a proper representation would have been something like this with the transmission line connecting the antenna up here. Okay. The gap should be there, but the gap is considered to be very, very small. Okay.

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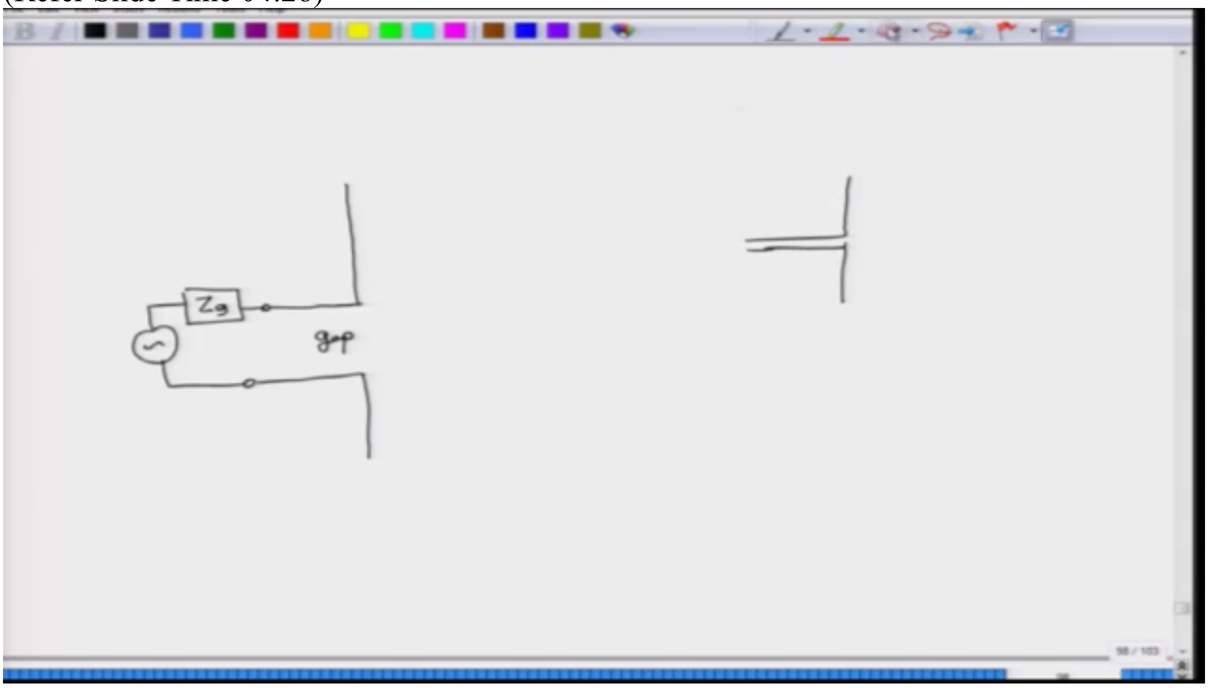

And we will put some length here. So if you consider this as the z = 0 point, then this fellow at the top portion will be z = +l and this one would be z = -l, and this is the antenna part, and this is the transmission line feed to the antenna and this of course is the voltage generator that will be, you know, generating whatever the power that is necessary to feed to the antenna. Okay.

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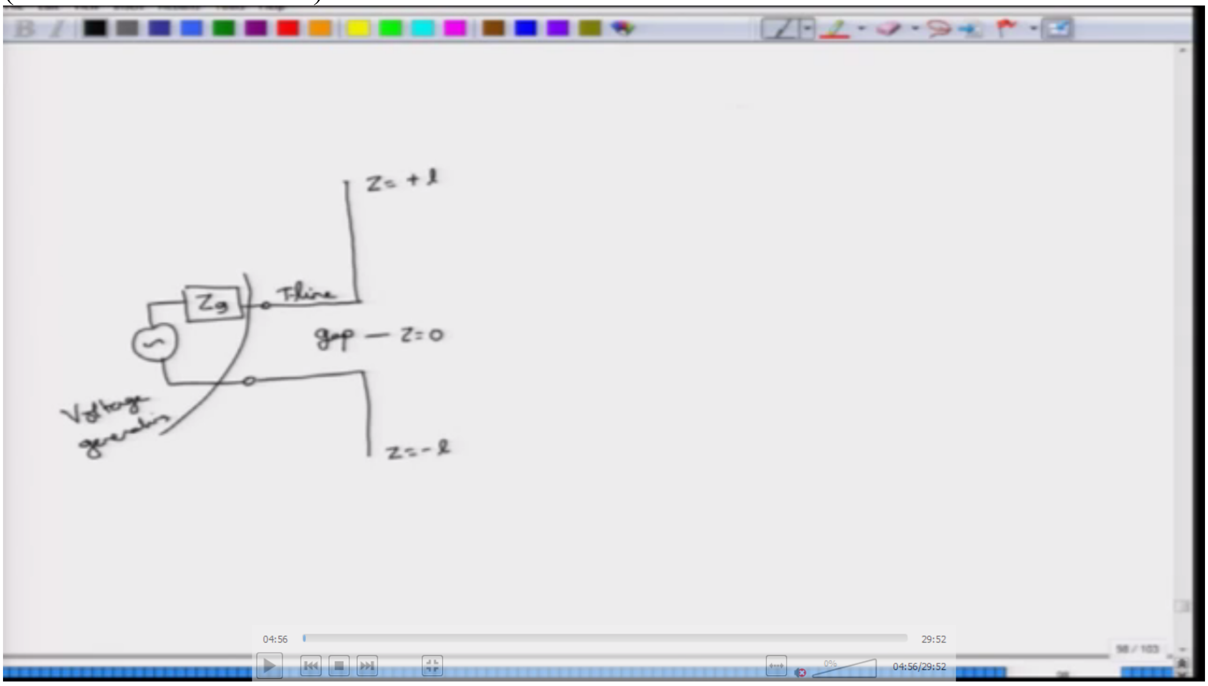

Of course, information will be residing in this voltage and it would be time varying so that the current that the antenna would, you know, have, the antenna transmit, the antenna current or the current on the antenna terminals will be dependent on what voltage and transmission line feed that we are using. Okay.

And because of this, you know, current which is varying with respect to time, you will have this A field that is vector potential and by calculating the vector potential, you can then calculate what would be the electric field and the magnetic fields of this antenna as radiated away, right? So that is where we would like to go.

So in the elementary short dipole or the short dipole that we considered, the elementary dipole that we considered, we were particularly not worried about the current distribution on the antenna terminals, right? Because we had this antenna itself to be very, very small compared to the wavelength, we took the current to be constant over the entire antenna terminal.

So you had this antenna which was fed, but then, you know, the current over which we had considered was considered to be constant. So no, no matter at what point on the antenna that you took, the current was essentially the same.

You could make this approximation because the length of the antenna was very, very small compared to wavelength. But now the situation is not like that. The situation is that you have your length, the total length of the antenna is 2l as per what we have considered here is actually comparable to wavelength of the fields that are, that it is radiating.

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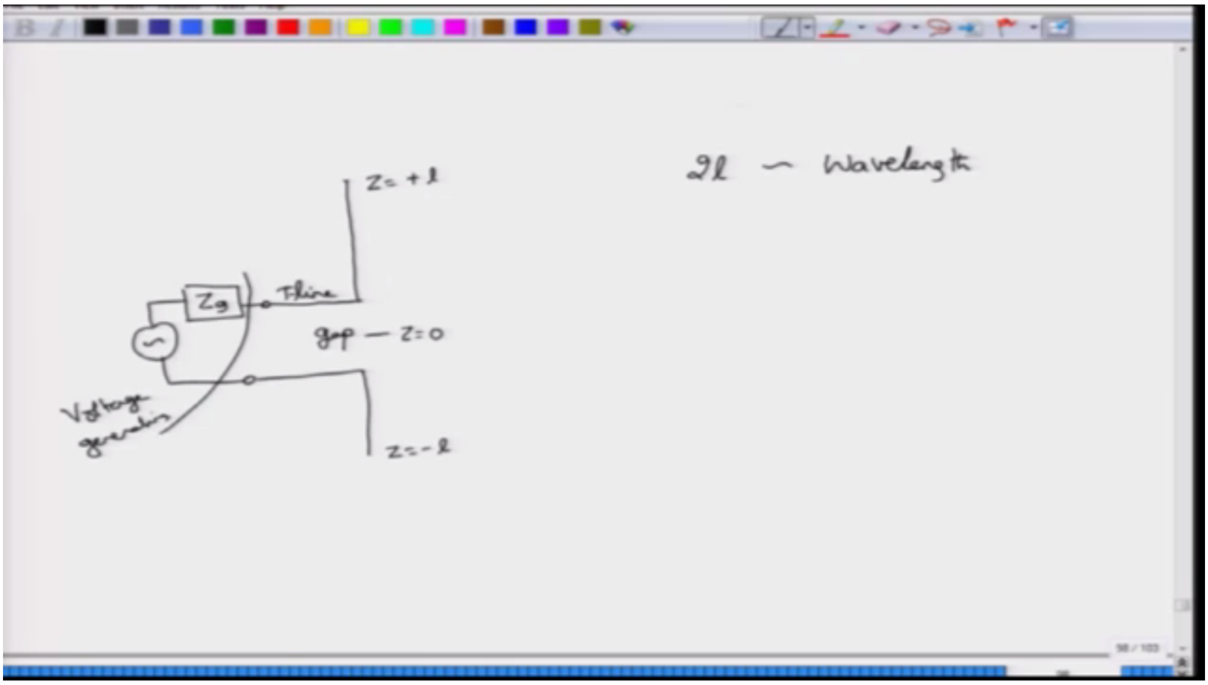

So because of that, you cannot simply make the antenna current to be a constant. So then that gives us one big question. What is the current distribution? Okay. What is the current distribution on the antenna?

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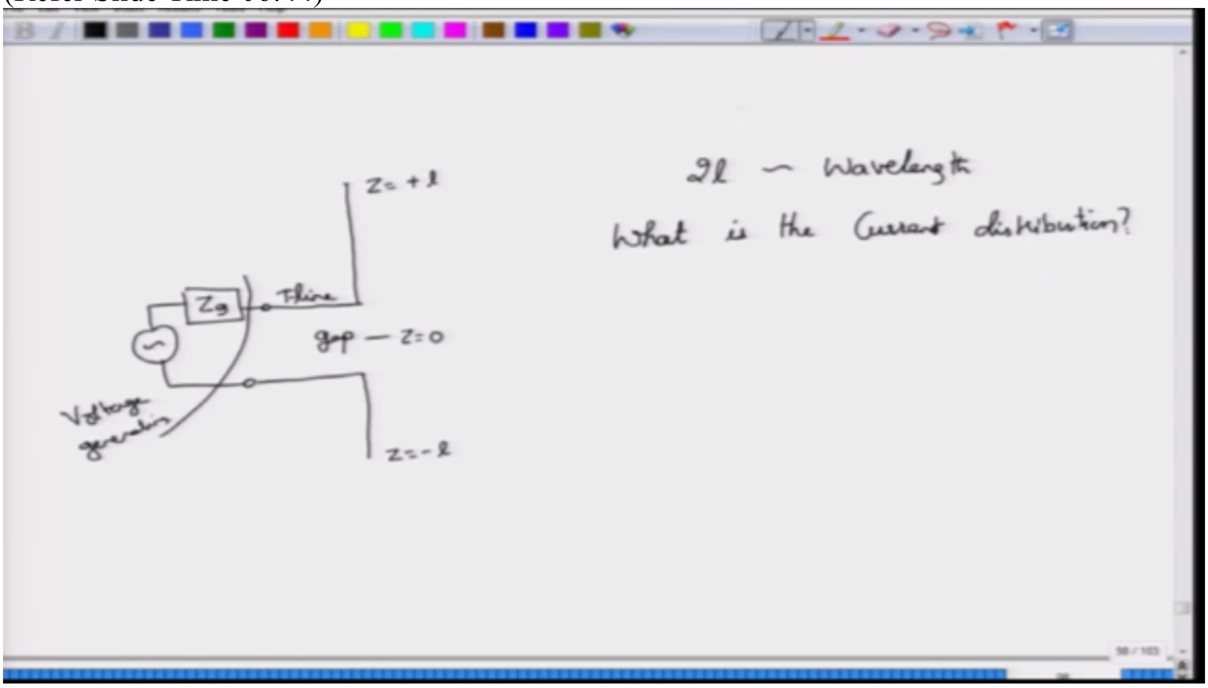

Now, unfortunately, for us, there is no specific answer because this problem is so difficult that we have so far not been able to calculate or determine the exact current distribution on the antenna terminals.

If you make this into an even more practical antenna and instead of assuming them to be thin wires, you assume them to have some finite thickness because any material that you construct will actually have, a copper wire will have some amount of finite thickness and the copper will also not be a perfect conductor. There will be some losses in that one, and if you consider that more practical scenario, then finding the current distribution is almost impossible. In fact, it has not been possible for us to determine what the current distribution is.

So unlike that elementary dipole discussion where we straightaway started with J and then we wrote down A and then we found out E and H, of course, with lot of tedious mathematics, that procedure unfortunately breaks down unless we make some approximations to the current distribution.

In practice, what we do is we use lot of numerical techniques to obtain a better approximation, okay, of the current distribution on the antenna terminal. Why this current distribution is very difficult to determine, which is one of the central problems in antenna theory is something that you will have to learn in a different course on antennas itself, but for now you can kind of motivate yourself to see that this particular antenna that we have, you know, considered can be thought of as being, you know, starting off with a simple tapered transmission line and eventually flaring up over, right?

So you can think of this antenna as being starting off with a straight transmission line and then as you start bending the two wires of the transmission line at different angles and eventually reach 90° bend, then that would be the antenna that you would obtain, which we have called as a linear antenna, right? A linear or a thin wire antenna.

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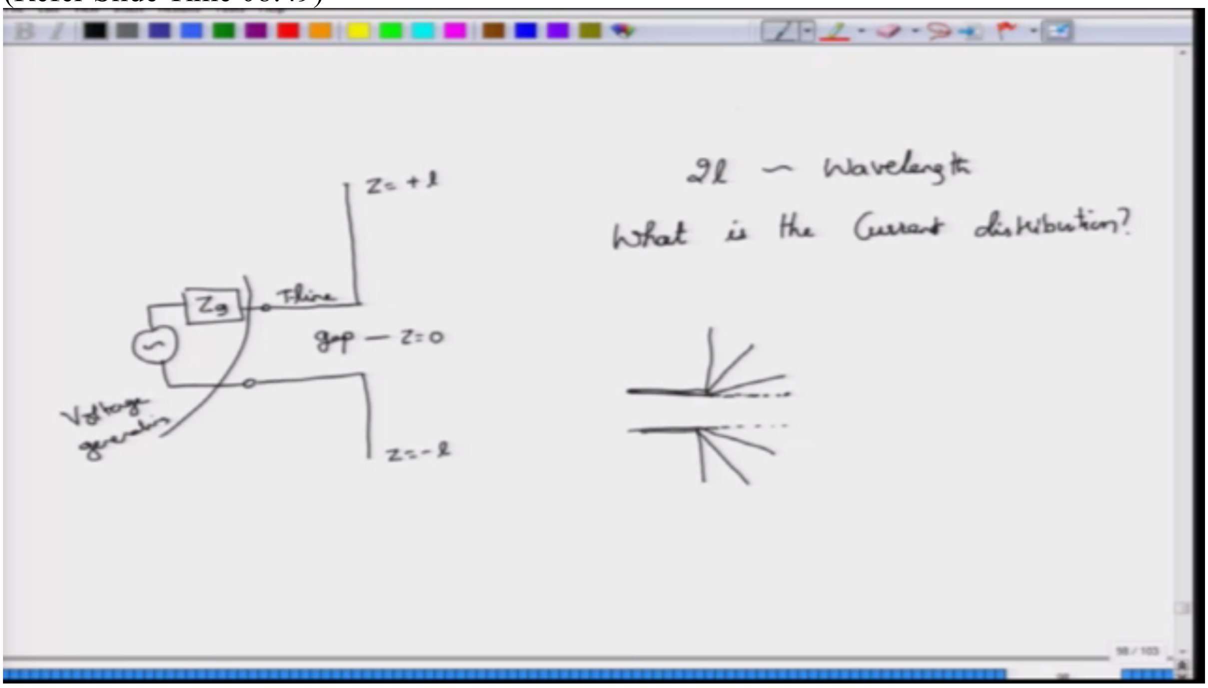

And we know that because there is an open circuit at the ends, right, the current distribution has to be in such a way that you will have, so, of course, it also depends on the length that you have. So, for example, if the flaring length happens to be say λ/4 here and λ/4, then this situation is actually originally corresponding to a transmission line whose length here is about λ/4, right? And that is what we have taken this length and then bend it in this particular fashion.

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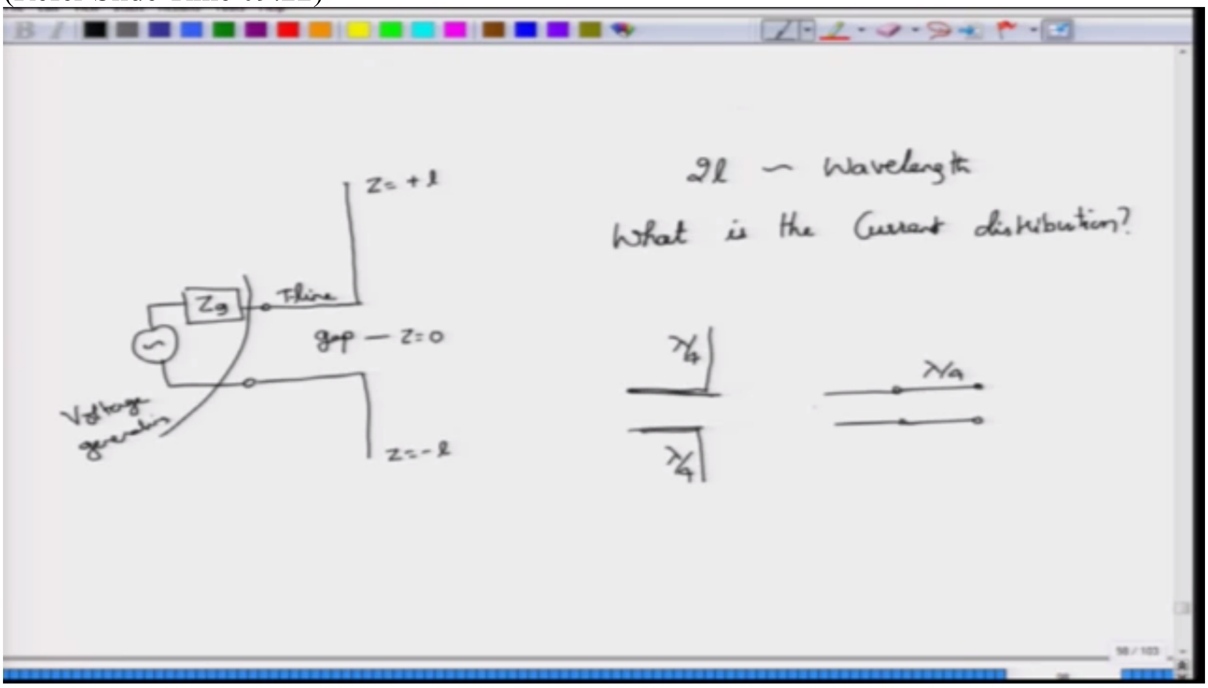

So on an open circuit and you connect this transmission line, what would be the current distribution? The current distribution would be something like or the voltage distribution let us say, the voltage distribution will be maximum at the open circuit, right? So on the open circuit, yes, it would be maximum and over λ/4 it would have reached the minimum, right? So this could be the current, voltage distribution.

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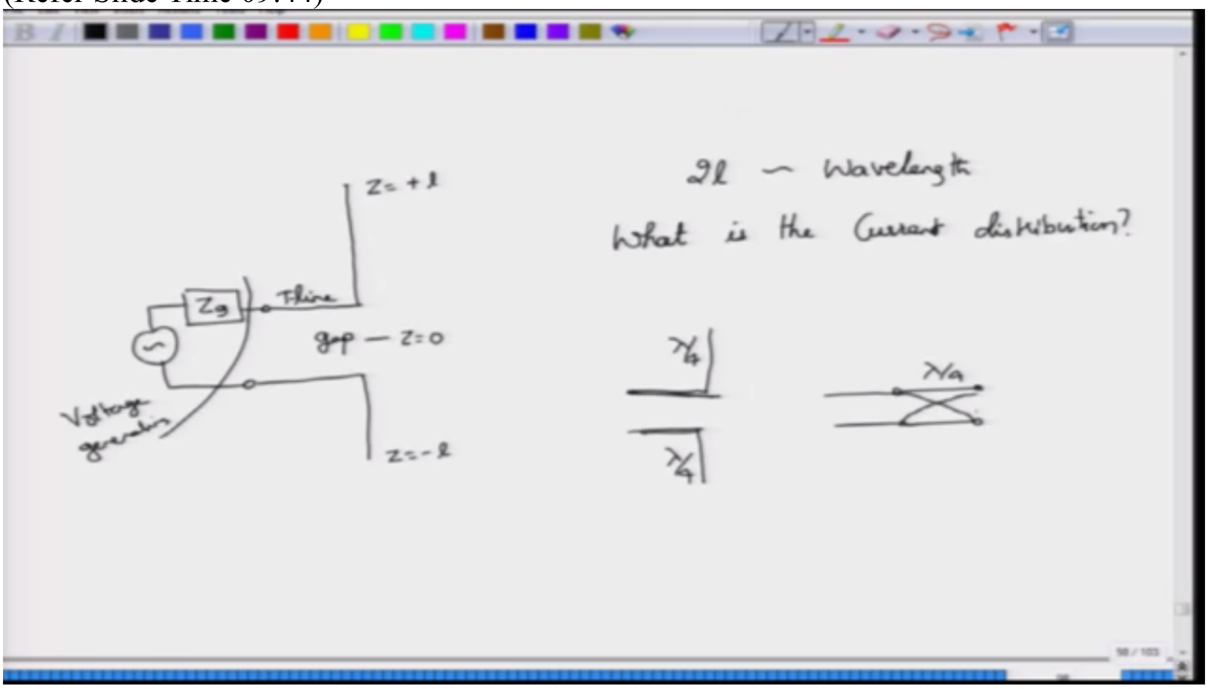

The current distribution, of course, would be minimum at the gap, but then it would reach the maximum. So this could be the current distribution, right? So this is the current distribution that you have considered.

So, essentially, if you think of now the two wires being bent and imagine that the length is about λ/4 and λ/4, then the current distribution could be well approximated in this particular manner. Okay. So it could be...

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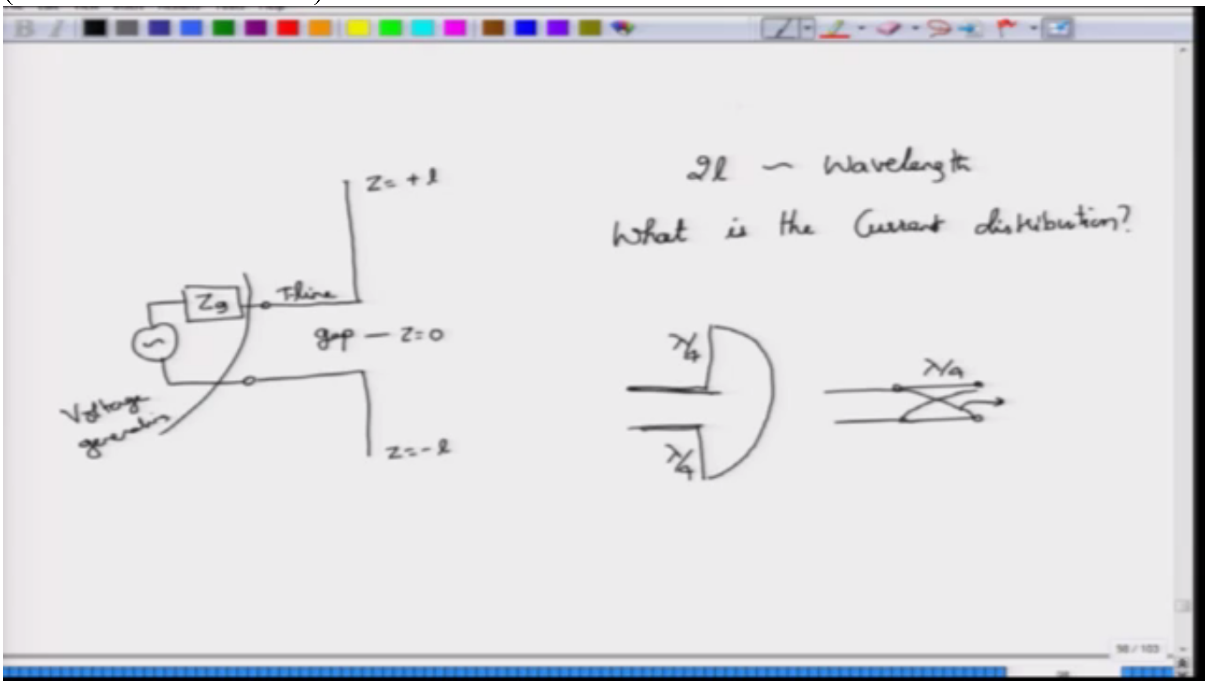

Now if instead of λ/4, you make it λ/2, then what happens? Then you should actually consider the fact that the current distribution would look something like this.

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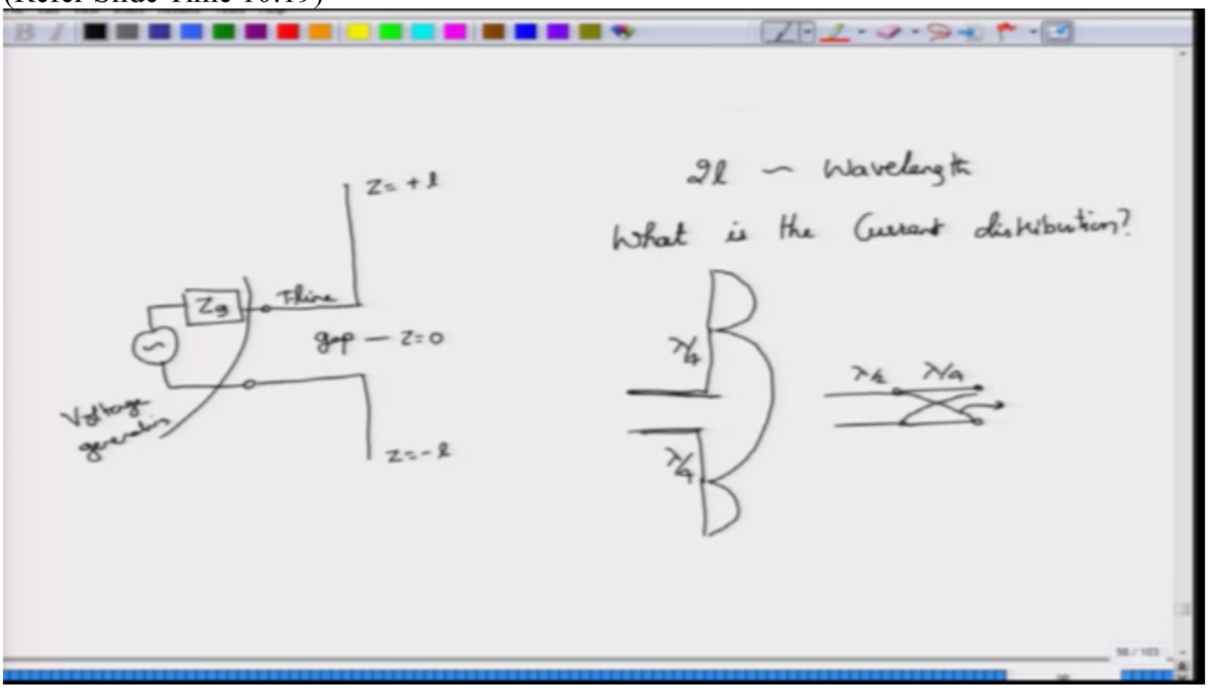

Of course, is this the correct answer? Not really, but this is a very good approximation. So as the length of the antenna increases, you will actually see minima, maxima, but this motivation, which we have done it from the Transmission Line Theory is only a motivation, okay, because it has been found that the current distribution is not exactly a cosinusoidal distribution, but in some cases may even take a triangular distribution, okay, and you can even in some very low approximation consider it to be a step or a pulse like approximation, okay, so or a square wave kind of an approximation.

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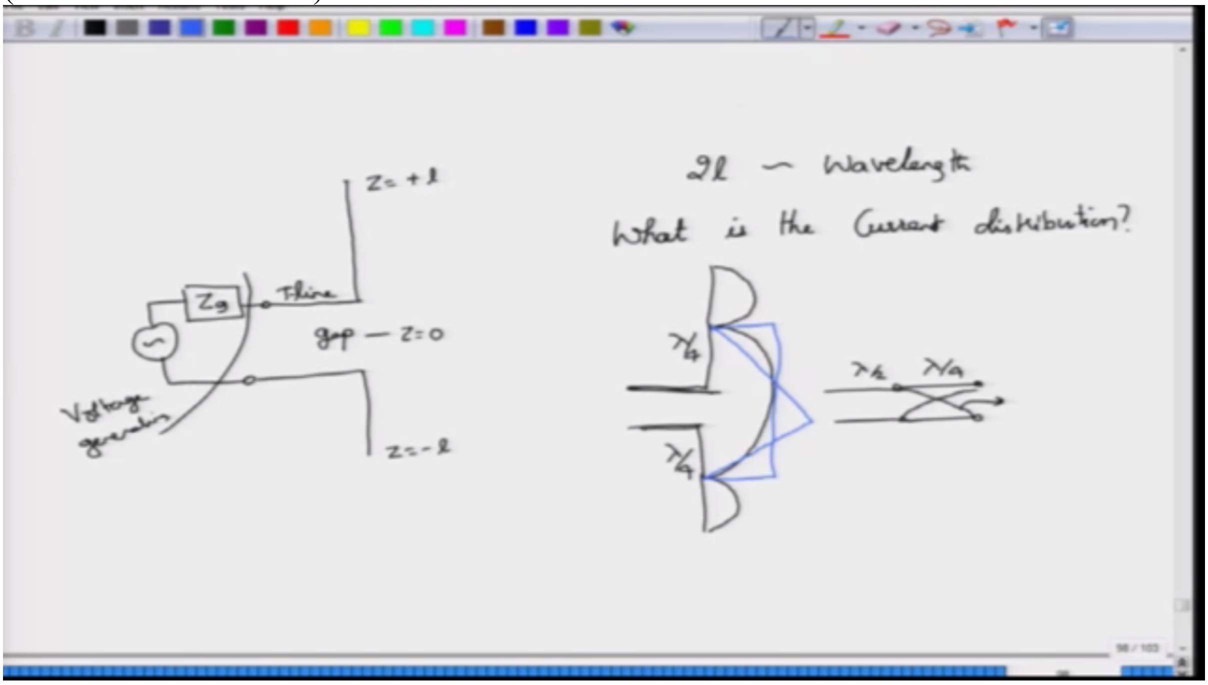

But for analytical purposes, we either consider it to be a, you know, triangular wave or we consider it to be a cosinusoidal waveform. Okay. And we will assume that is the scenario over here. We will, of course, not write this as λ/4. We will keep it slightly general and say that this is l and l, but then we assume that the current to be cosinusoidal. Okay.

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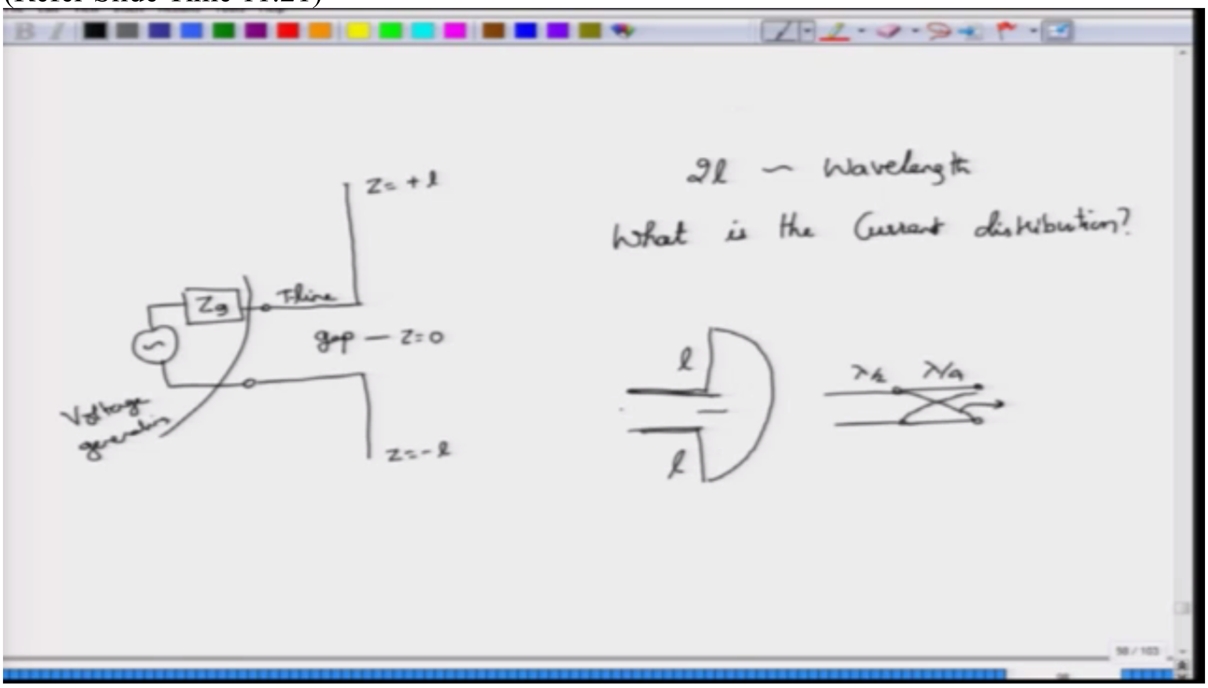

So since we took this as the z-axis rather because this is the source coordinates, we need to actually put a prime on to these coordinates, which is correct. So the distribution has to be maximum at the centre and then it has to be a minima at these two points. Okay.

So with that, let us write down the expression for the current distribution. So I(z′), which is how the current would be, you know, on this particular antenna, that would be equal to some I_0, which is the maximum of the current times sin k |l-z′|. Okay. So I will tell you the reason why I am writing this as |l-z′| and then I have sin kl. Okay.

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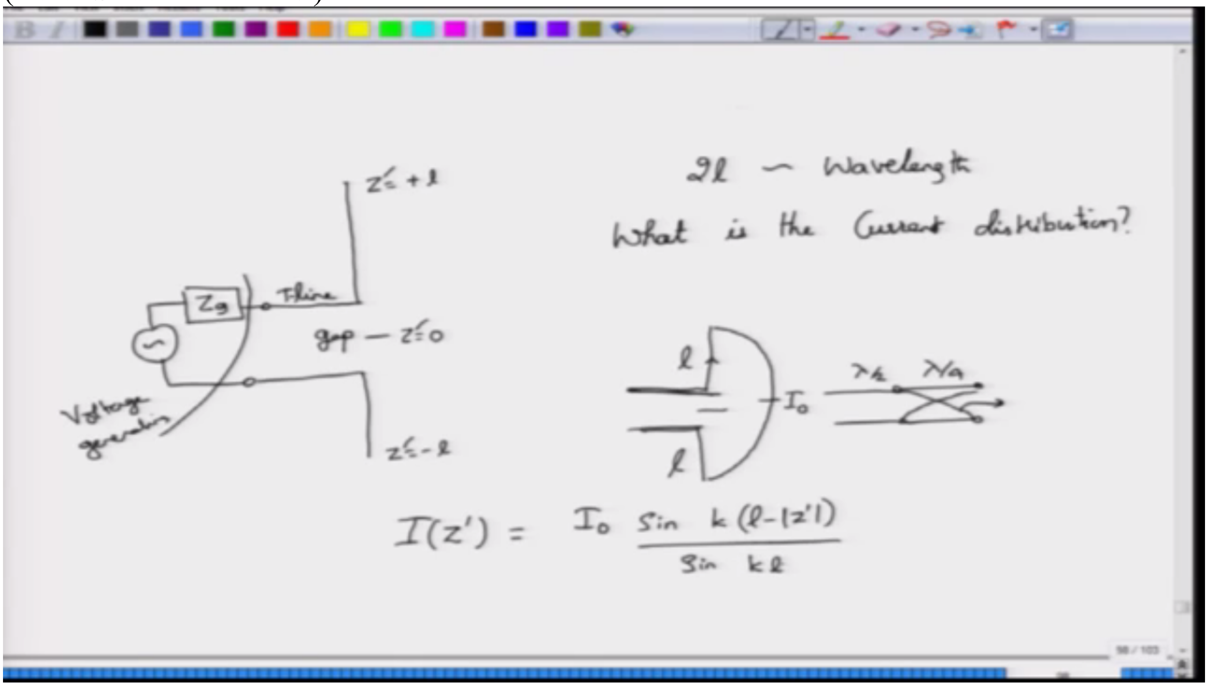

So at z′ = 0, which is at the feed point of the antenna, the current would actually be equal to I_0 and at z′ = 0, you have sin kl/sin kl. Yeah. The current will be I_0 and then it will go to 0 at the edges. Okay. Of course, k is in this particular case 2π/λ and the actual result will depend on l and λ ratio as well. Okay.

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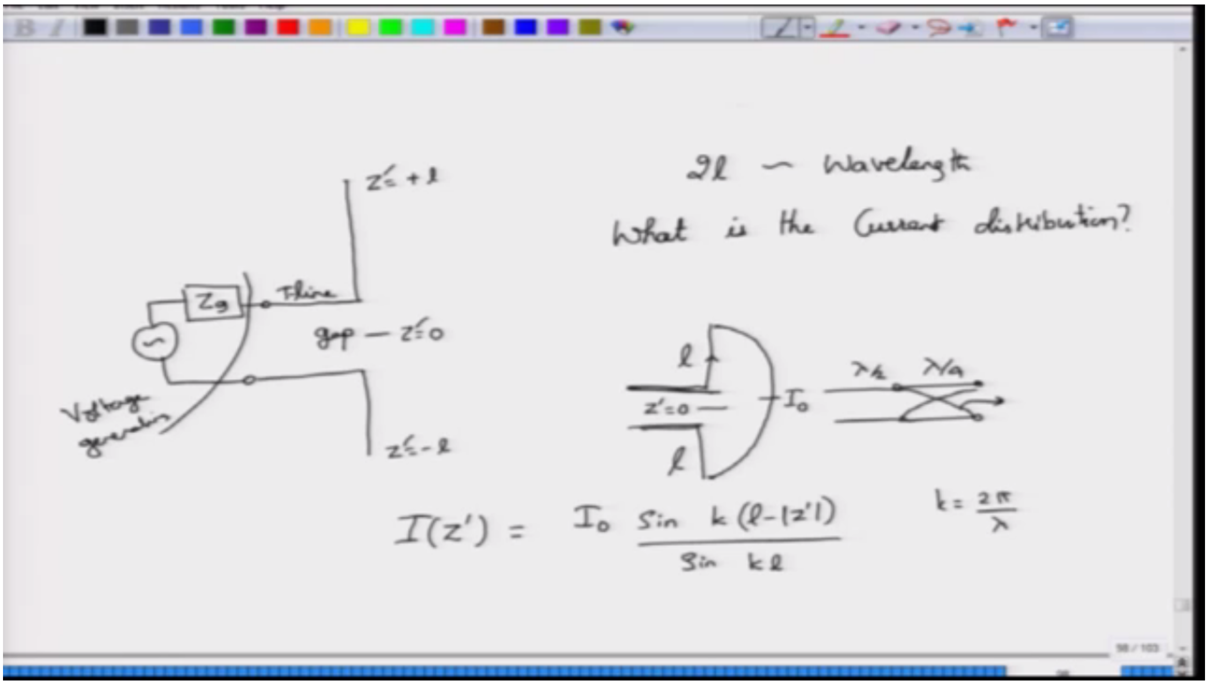

So for one of the, so I should have probably considered this to be a λ/2 antenna. So that's why I have put this cosinusoidal distribution or a sinusoidal distribution, but this is the distribution that we are going to assume. Okay.

So at the $z' = 0$, you have an antenna current of I_0, which then goes to 0 at the two edges, but please understand that this is only an approximation. This is not the true current distribution, and true current distribution is a very complicated topic and there are no solutions for the true current distribution or the exact current distribution. Okay.

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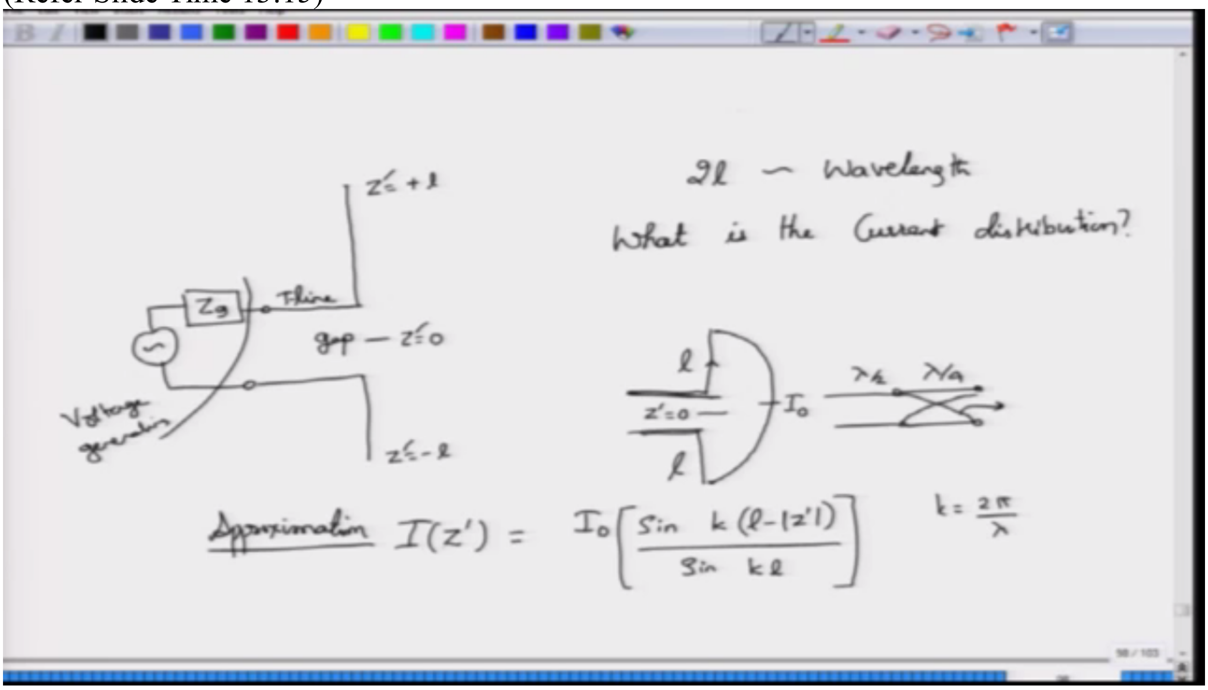

With this current distribution that is a well approximated current distribution that we have assumed, we can now proceed to the next steps.

Now I will minimise the mathematics here. I will leave most of it as an exercise for you to figure it out, but I will give you the expression for the electric field and tell you couple of approximations that we are going to make because those approximations are necessary to obtain tractable analytical expressions. Otherwise, the mathematics becomes very tedious and will be very difficult for us to, you know, look at that one. Okay. I mean, look at that in this short course. Okay.

I know the current distribution to be given as z′. The procedure next will be the same. The A field, the vector potential will actually be around, I mean, will be something like z′ A_z that is to say it has only the z component, okay, and we know the expression for A_z, right? A_z is given by $\mu_0/4\pi$ integrated the current distribution. So you had this J.ds or Jds, but now this is a linear antenna, so there is no area here. So it becomes instead of J times dv′, sorry, that is the volume, now it simply becomes current times dz′, okay, and divided by r-r′, these are of course the observation and the source point and you have a phase factor that would also multiply. This is a retarded potential part that you are looking at and this is what you have the expression for.

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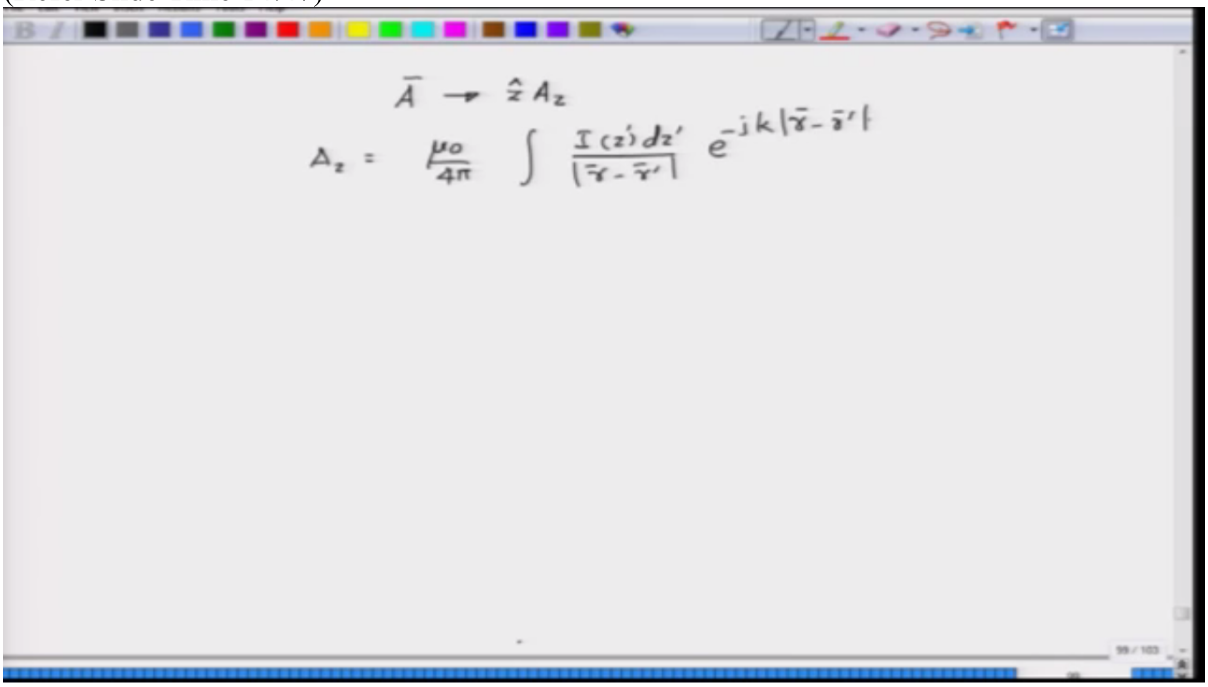

Now we know that this is the antenna that we are considering. This is the feed line of the antenna. So the source coordinate is moving along z′, but then the observation point that I have, which, you know, if I take this as the origin of the coordinates, this would be the distance from the observation, I mean, from the origin to the observation point, which we will call as point P and at any point on the source, the current is, of course, Iz′, dz′ at this junction, so which is at a length of dz′. This would be the radial distance R. R is the distance between the observation point and the source point, right? So this is the observation point, source point and of course we are working still with the spherical coordinates. Okay.

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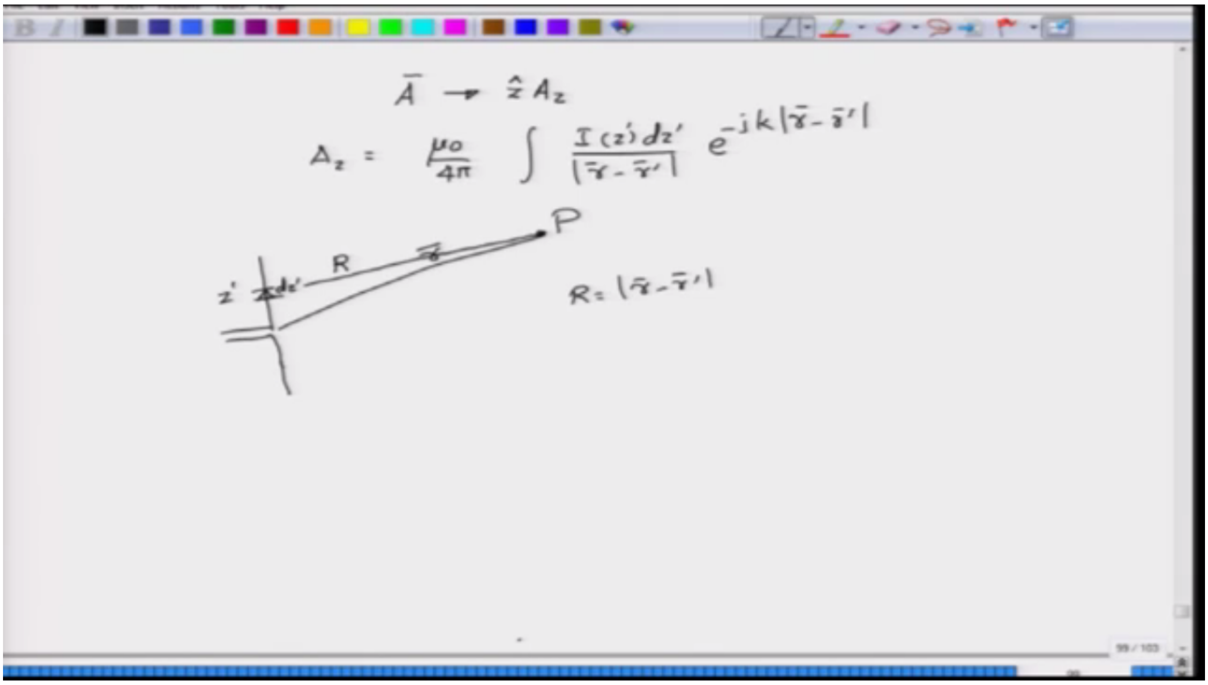

Now I have, because I have defined this R as this magnitude, that is this particular line, okay, this is the line that I am considering, I can simplify the expressions over here. Okay. So, first, I will substitute the expression for I(z′), which is the current distribution that we have assumed and that would be I_0/sin kl and then you have sin k l-z′. Okay. The magnitude of z′ because z′ can go from -l to + l covering the entire antenna region. Okay. And along with this one, there is a denominator R and then you have e^{-jkR}. Okay. So this integrated over the source element from z′, which is from -l to +l. This if you perform this, this is what you are going to get.

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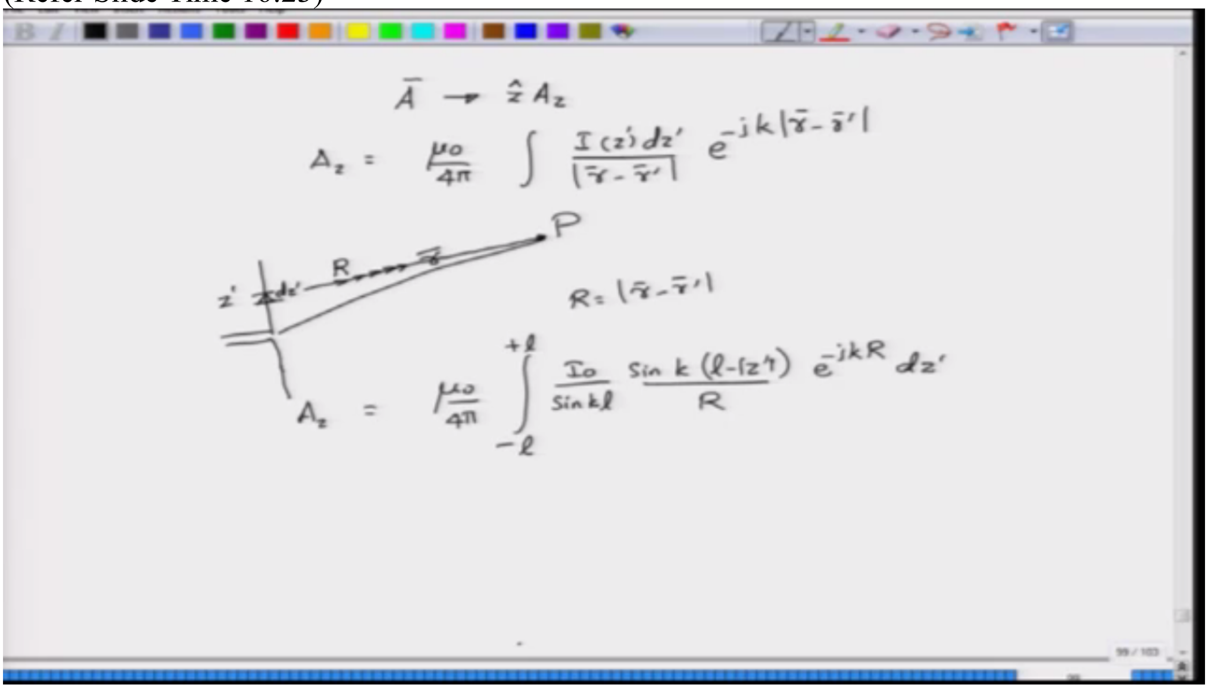

Of course, you can also see that there is some amount of symmetry into this problem in the sense that as you move around the antenna, if you leave out the gap part, if you move around the antenna, you will see that the current distribution would remain the same, right? So if this is the antenna and you move, the current distribution would remain the same.

So there is a φ invariance in this particular problem. So moving along the azimuthal will not really change the distribution and that symmetry is built into this problem because A_z is now function only of R. Okay.

Even then the problem is rather not simple to solve. So we will have to make couple of approximations and this approximation is something that you may have seen earlier in the dipole scenario, okay, but it is worthwhile to make that approximation.

See I'm considering a particular point here at, you know, with a width of dz′ at a height of z′, correct? And then I also am going to consider an, you know, similar point, which is at a distance of -z′ from the origin, but with a patch length of dz′ itself, that is the width is dz′ itself and now I'm looking at the observation point P here. Okay.

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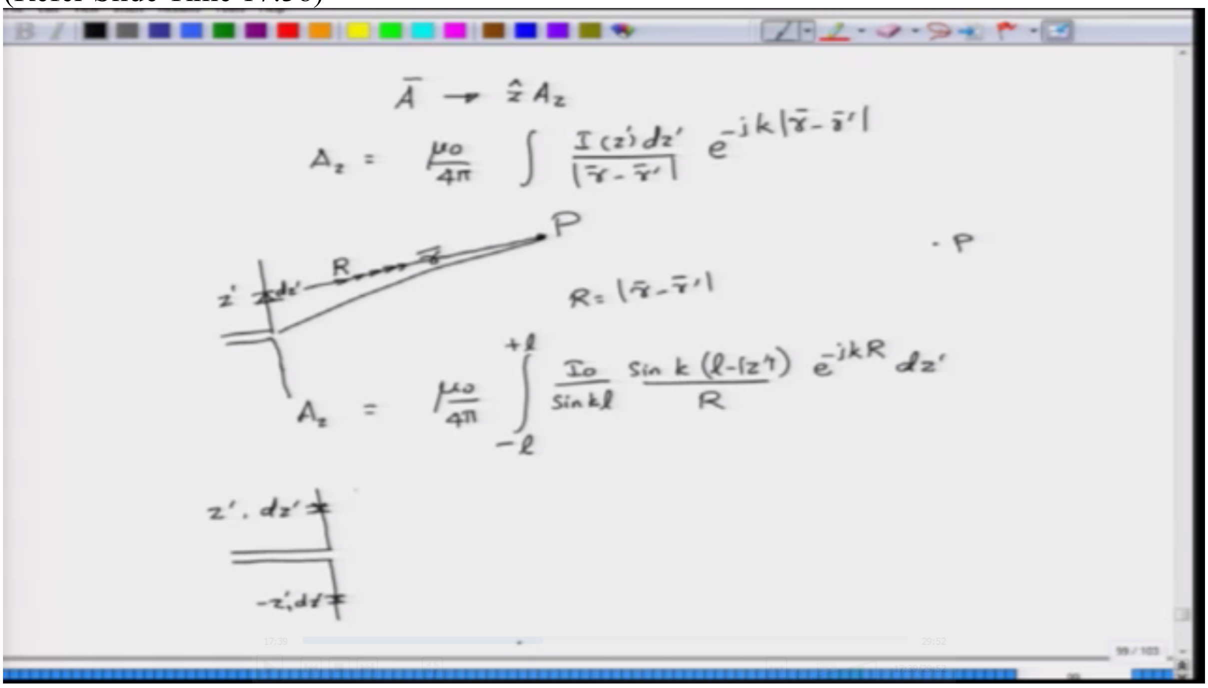

Now technically when I consider the point at z′ and dz′ width, the R value, the R that I consider, let me call this as R_+. Okay. And similarly, I will call this fellow as R_-. Okay. And whatever that is from the origin to the observation point, of course, is your distance R. Okay.

So I have three lengths and you obviously know that R_+ or you can obviously see that R_+ is kind of larger compared to the small r and small r is smaller compared to, sorry, R_+ is smaller compared to small r and r is smaller to R_- meaning that R_+ is less than r, which is less than R_-. Okay.

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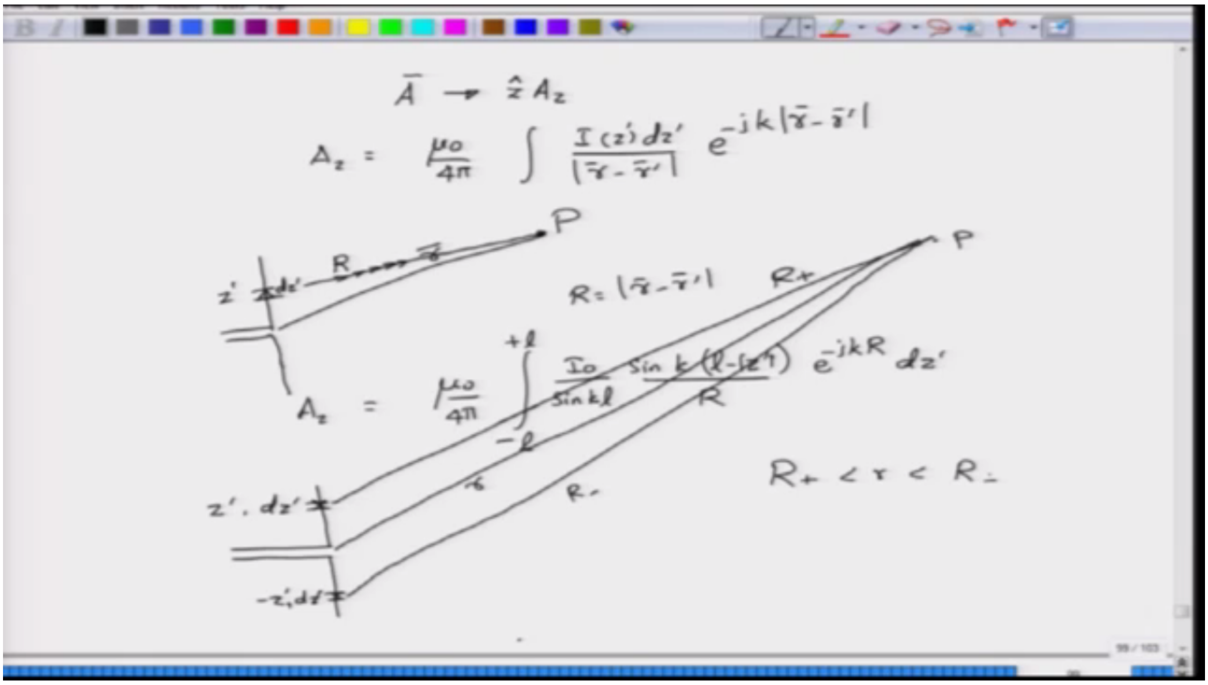

But imagine that my distance to the antenna is so large that this entire antenna would essentially, you know, appear to me as a single point. Then what happens is that these lines which kind of converge at point P can instead be considered to be three parallel lines. Okay. They are three parallel lines.

Of course, because one of them is at z′, dz′, the other one is at z′ = 0 and the other one is at -z ′, the lengths of these will be slightly different and if you move the point P, the distance as well as the orientation of these points move, right?

So if this is your antenna point, so let's say this is the r and as you move the point P, the orientation will change. The orientation with respect to z-axis, which we will call as angle θ will change. Okay.

So calling this as angle θ, which is the angle between the observation point P, the length, the radial line between the origin and the observation point P plus this z′, dz′ patch that we have considered, then if you approximate these three lines R_+, R_- and small r by three parallel lines, you get, you can simplify the analysis and what you get is something like this.

So let's first write down the small r case and of course this will work only when you have the point, observation point P to be very, very far away from the antenna so that the antenna would essentially look like a point to you. Okay.

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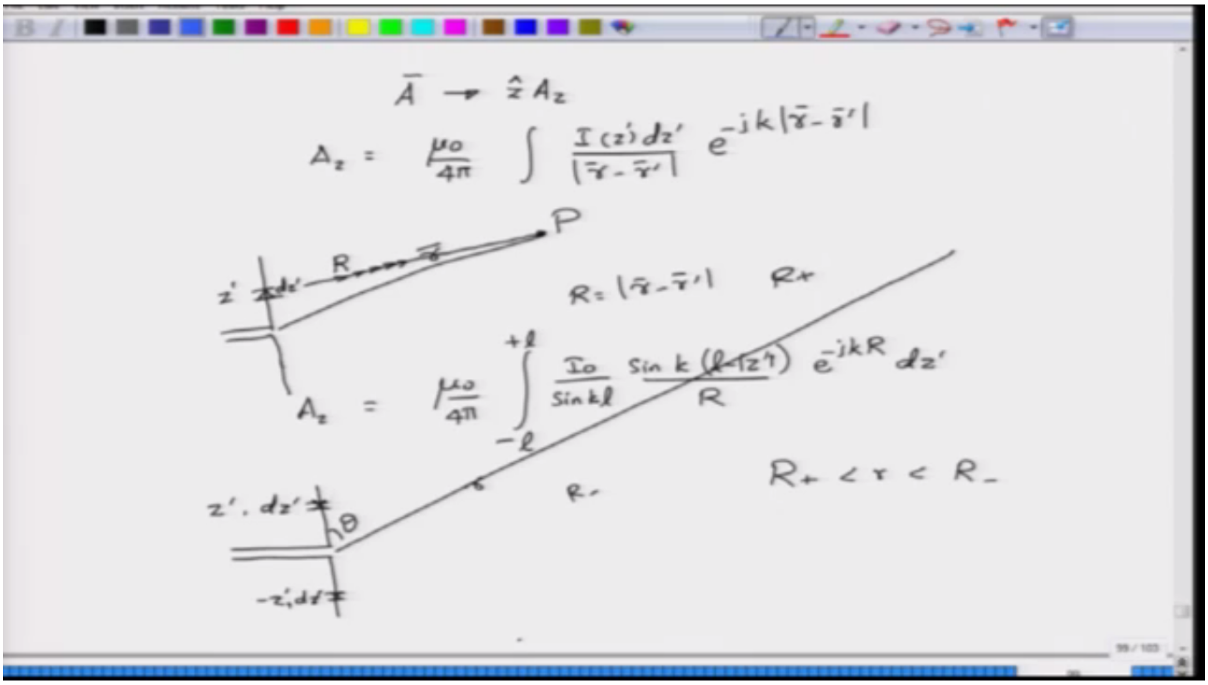

Now I am going to assume that this is, you know, parallel line and I am going to assume that this is also parallel line. Now what is the relationship between these three lines?

See if you look at R_+ and r, the only extra length that you have is this one, right? So this is the extra length that you have. What is this extra length? I know that this is at a distance of z'. I know that this length, I mean, you don't really need to know that length because you know what is this θ, and if you want to find out what is this extra length, right, so what about $\cos \theta$? $\cos \theta$ will be this adjacent side whatever the length that you want to find out. So we will call that as say Δ, Δ itself. Okay. So Δ divided by this would be the hypotenuse, right? So that would be z'. So this will give you Δ of $z' \cos \theta$. Okay. And therefore, you can write R_+ as small r, which is this length radial line minus $z' \cos \theta$.

And similarly, you can convince yourself that R_-, which is this fellow, can be written as $r + z' \cos \theta$. Okay. So I can write in this particular manner. So I'm going to get this as $r - z' \cos \theta$ and $r + z' \cos \theta$. Okay.

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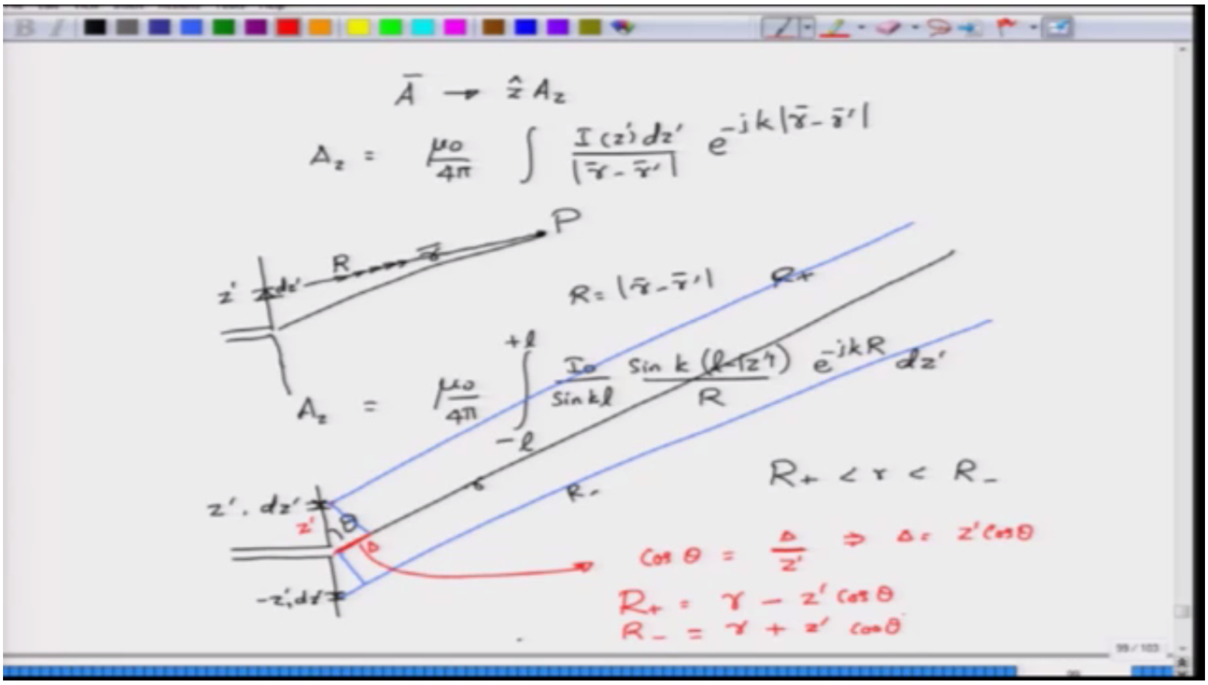

Now what is the use of this? See the integral was actually going from -l to +l, correct? So all these integrals were going from -l to +l. Now I can break this integral from 0 to l and then one integral from -l to 0. Okay.

And in this integral, I will have only the distances R_+ involved and in the second integral, I will have the distances R_- involved, but both essentially look kind of symmetric.

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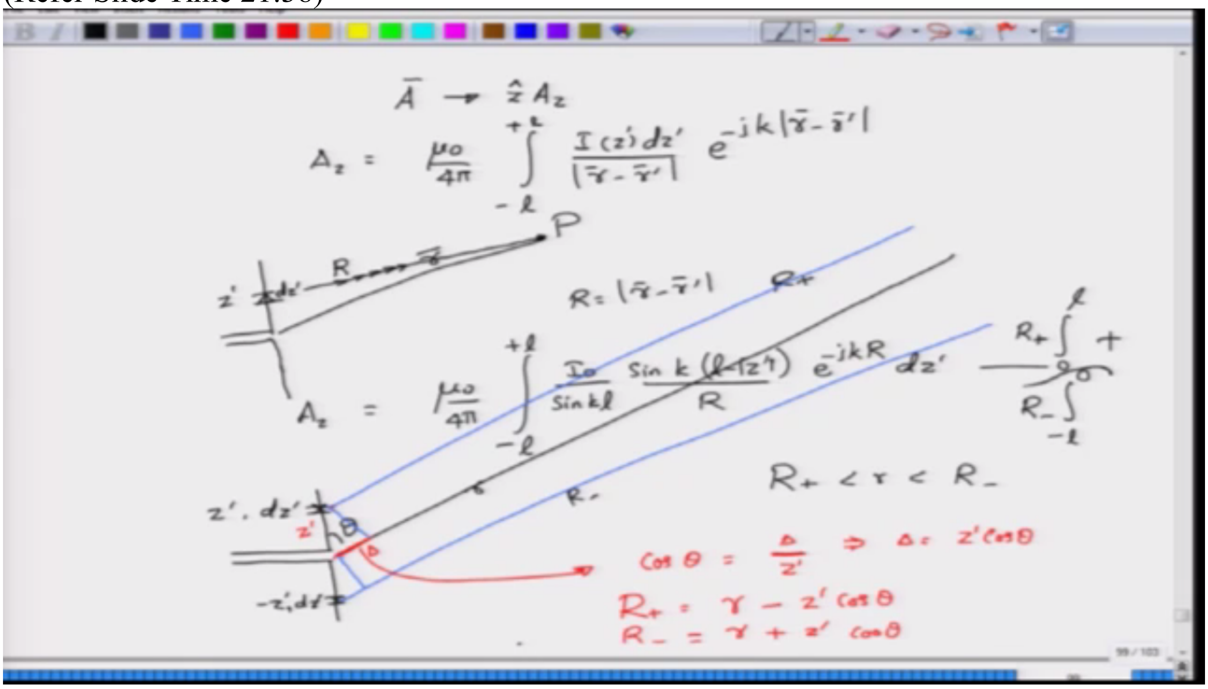

And I can convert this integral from 0, -l to 0 to o to l and then rewrite the expressions for finding A_z as, I mean, I will split the integrals. Of course, $I_0/\sin kl$ is constant. So I can pull this out. So I have sin kl here and then this integral is from 0 to l dz′.

Notice here that I still have z dependence in the form of the current distribution. So I don't have to now write magnitude z because this is basically z′ positive only, right? And most importantly, in the denominator the, this is the source point that we have, right? And the source point is basically r - z′ cos θ $e^{-jkr-z' \cos \theta}$. Okay.

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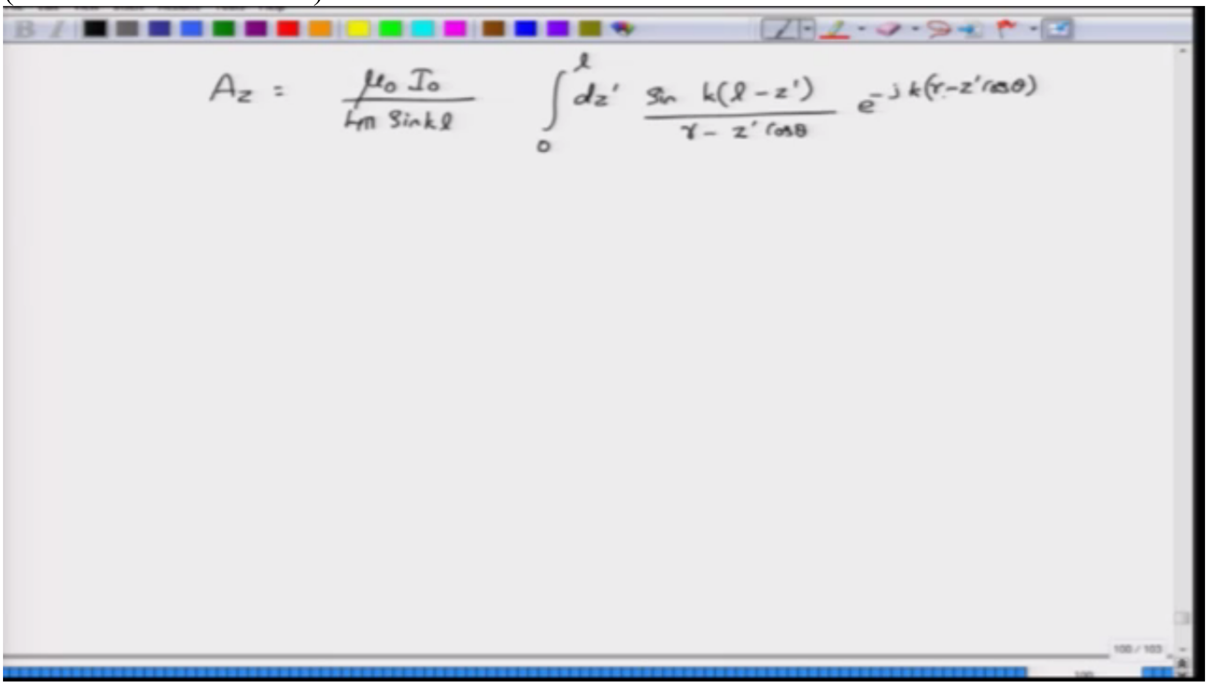

You may think that we have not really simplified the problem because z′ is present in the denominator. z′ is present in the numerator. Luckily for us if the overall length l is very, very small compared to the radial distance from the observation point, then this z′ cos θ maximum value that it can have is l and l is very small compared to r. So in the denominator, it doesn't really matter if I take this as r - z′ cos θ or I will just take it as small r. Okay.

So one of the approximation that I make is the approximation in the denominator. Therefore, I can, you know, or rather I will rewrite in the same equation here, so I can neglect this z′ cos θ because you know that is very small compared to r. Okay.

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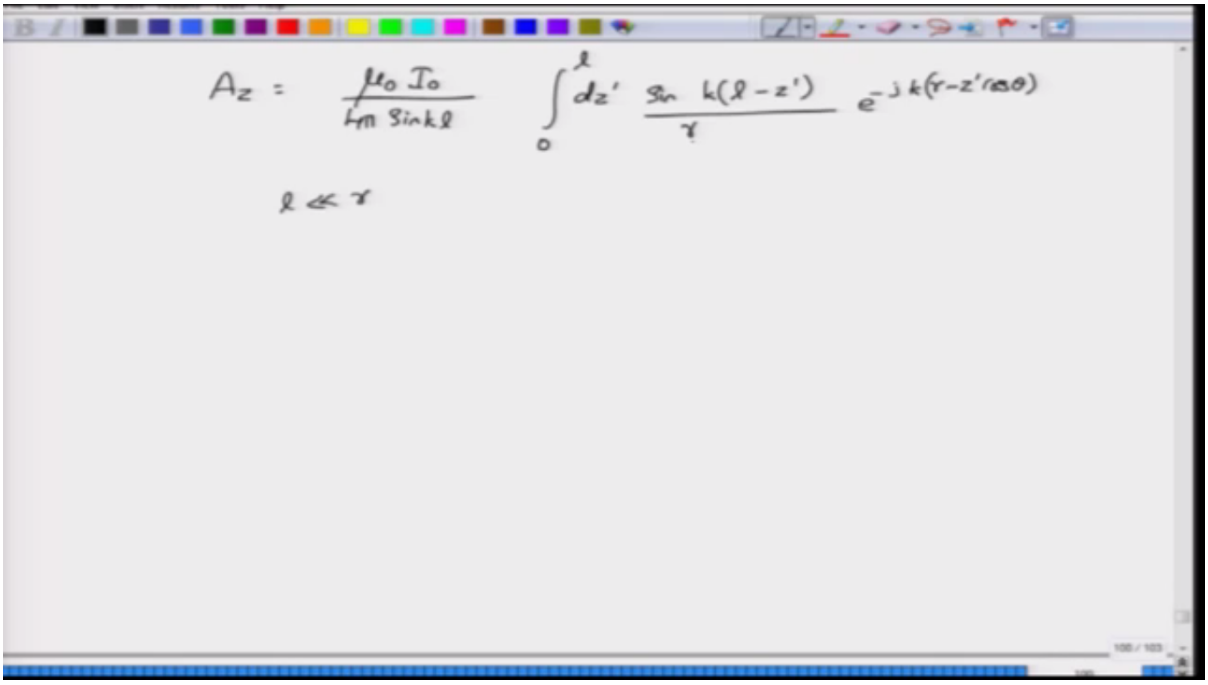

Can I do the same thing in the numerator? Unfortunately, no. Why? Because z′ whose maximum value will be l is actually comparable to the wavelength λ, correct? See you have k, which is equal to 2π/λ that gets multiplied to z′ cos θ. The maximum value of this one will be 2π/λ times l and I cannot neglect this because l is approximately λ or in the range of λ so that you have a 2π. I mean, the factors are large in the numerator. Therefore, in the numerator, I cannot neglect this. Okay.

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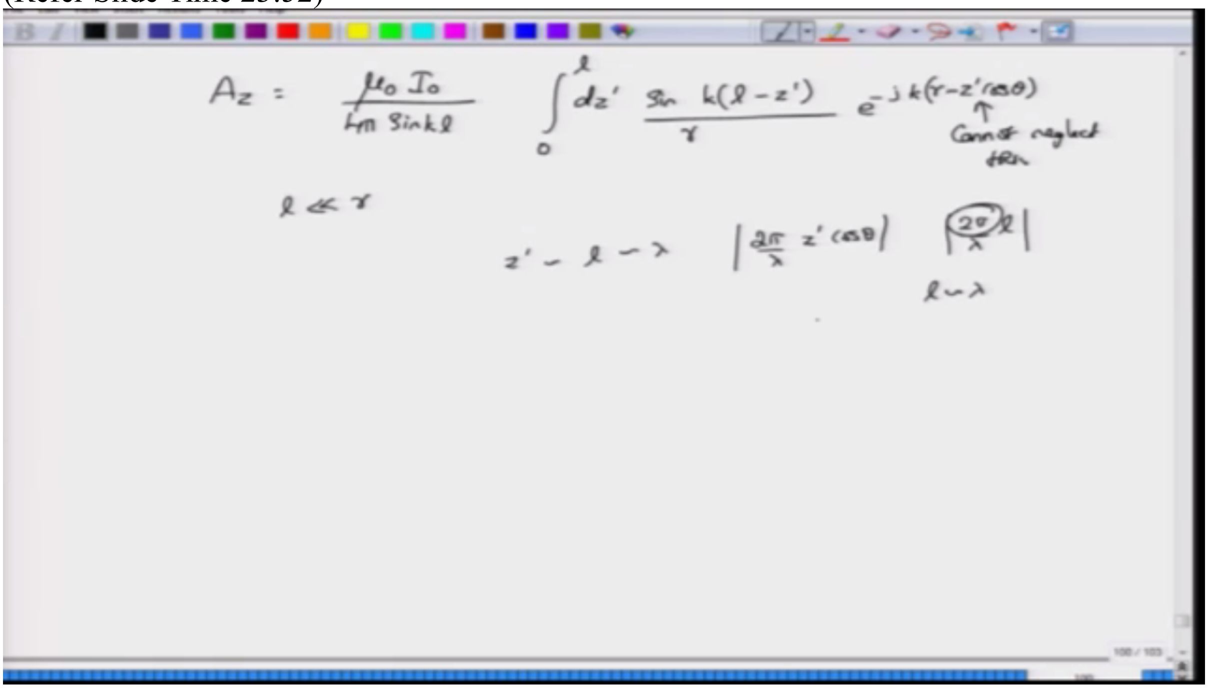

So this is an important approximation and with these two approximations, your problem is kind of solved now because of course I have written only for 0 to l on this side. You have to write down similarly for the other one also. So you will have instead of $e^{-jk(r-z' \cos \theta)}$, you will have $e^{-jk(r+z' \cos \theta)}$. So you can do that or you can just write down only over 1l and then write down the other part as well.

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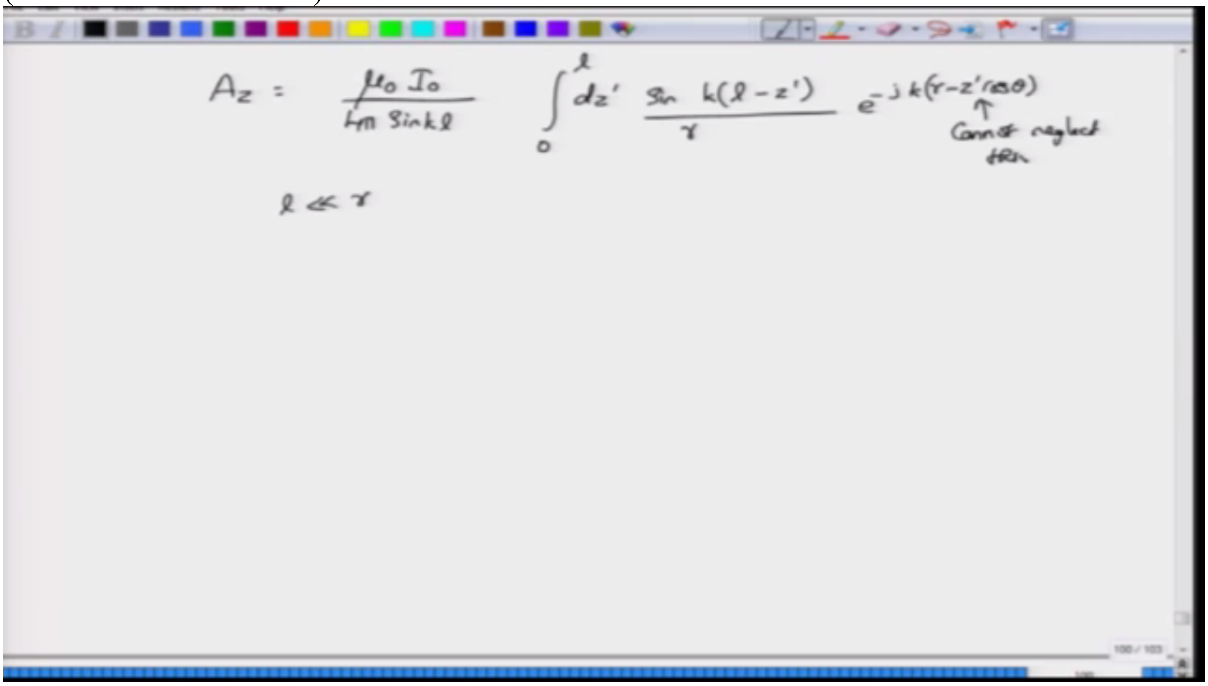

Well, I will complete the next step and after that I will leave it as an exercise for you. So at this step what I am interested is to write down in this way. I will move this r outside the integral. I can do that because r is not dependent on z and then I have sin kl. This is just a normalising factor.

And now look at this exponential. I can write the exponential by splitting it. I have e^{-jkr} and I have $e^{-jkz' \cos \theta}$. Retain this. Pull this outside the integral. So you have e-jkr and integral of 0 to l dz′, the sinusoid current distribution times this $e^{-jkz' \cos \theta}$, right? So you have $e^{-jkz' \cos \theta}$ integrated over this and this is what you are going to get for the first integral.

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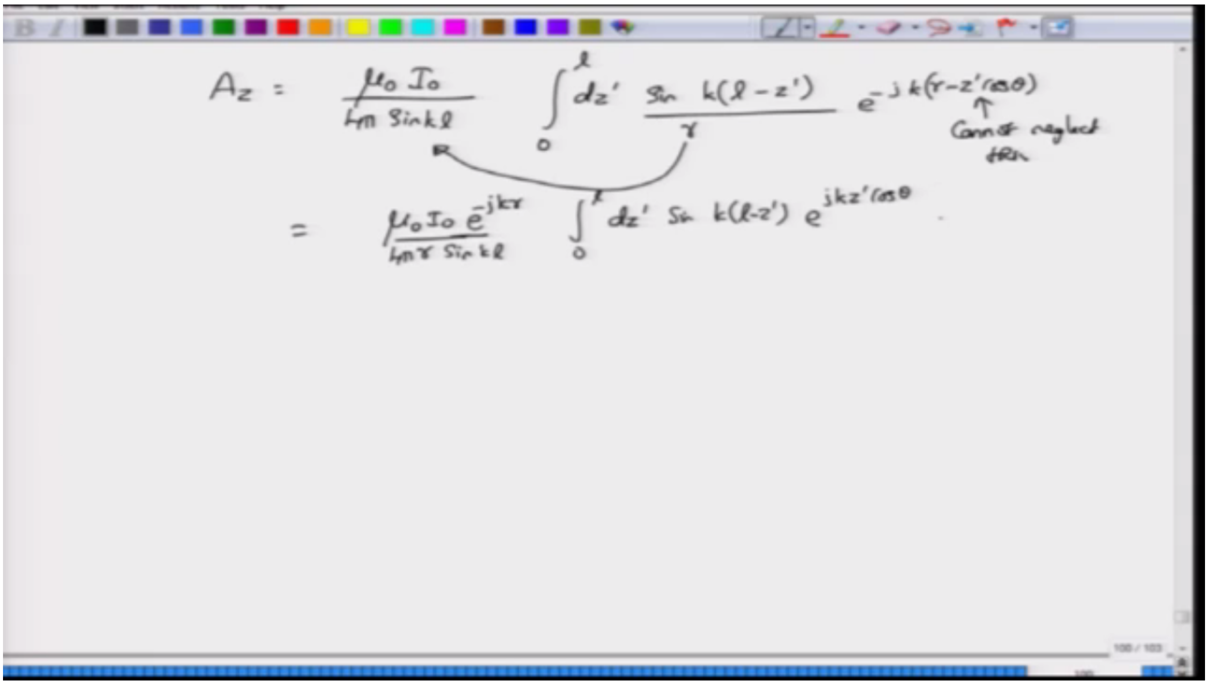

Now there will be another integral, which would be from -l to 0 and you can use your standard change of variables out there or you know interchange the orders and then get a - sign, + sign and then you can rewrite that expression, I mean, write down that expression as well and we will call some of these as constants.

What you can do is you will have that other integral from -l to 0 with appropriately kl + z′. Remember that l z′ is now negative here and then you will get $e^{-jkz' \cos\theta}$ integrated over dθ. So you can reverse this order of, you know, variables from -l to 0 to 0 to l and add it to the same integral, and then you can, you know, get a simplified expression and integrate the whole thing.

And I will leave this as an exercise to you. It's just about two, three steps to show this one as $\mu_0 I_0/2\pi$ sin kl multiplied by e^{-jkr}/kr. Okay. And you have cos kl cos θ - cos kl divided by $\sin^2\theta$. Okay.

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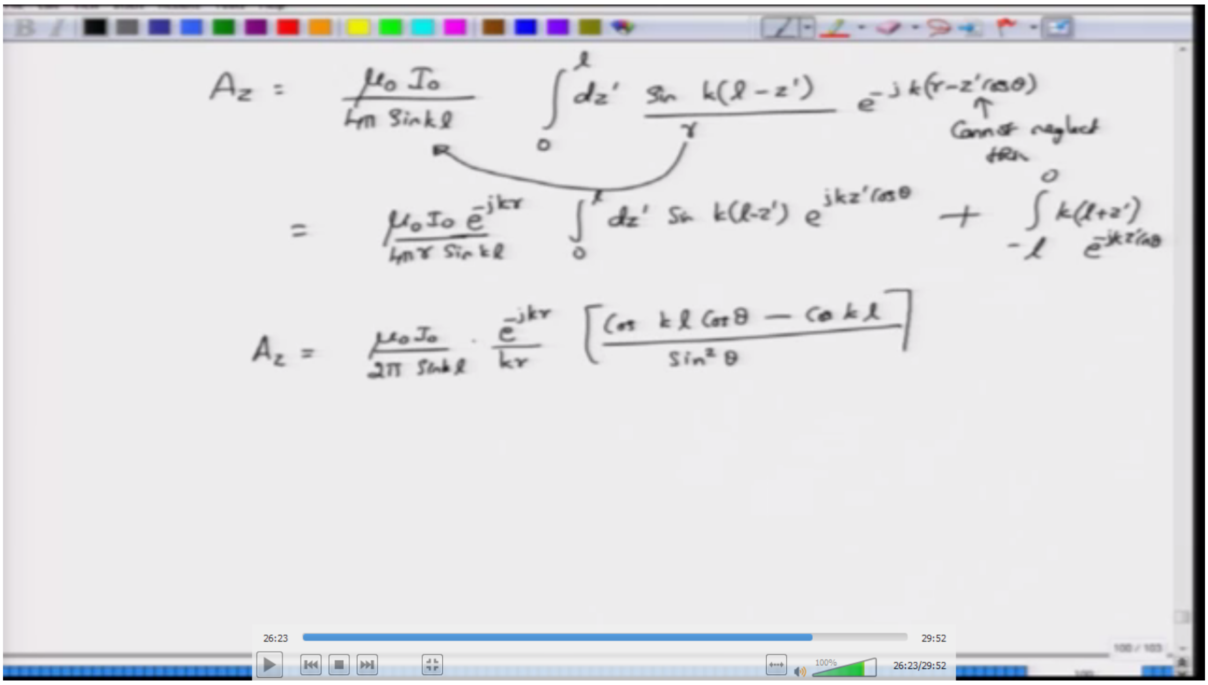

So this is the expression for A_z that you get and from this you can find out what would be H. Okay. H is ∇ x A and E is basically obtained from H by writing it as ∇ x H/jω ε_0. These are, of course, Maxwell's equations. Sorry. B was actually equal to ∇ x A. So B is basically μ_0H. So, therefore, H is basically ∇ x A/μ_0. Okay.

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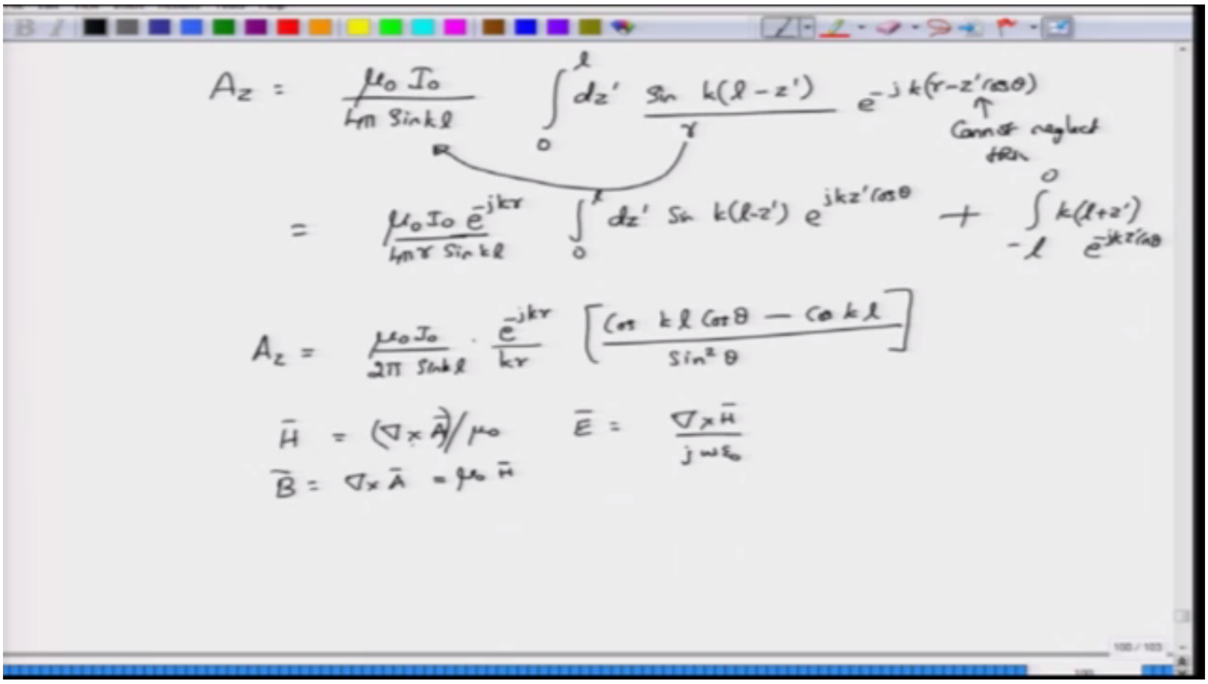

So you can find out the kernel of this A_z in the spherical coordinate systems. Again, the expressions will be slightly, you know, tedious. Not difficult, but it will be tedious, requires you to write down the curl in the spherical coordinates, which normally we don't do, but when you do all of that, that is when you carry out the calculation, subsequent steps, you can see that H will still be oriented along the φ direction and it would be function only of r and θ, no dependence on φ luckily, but it will be the direction will be φ, but the component itself will be independent of φ. Okay.

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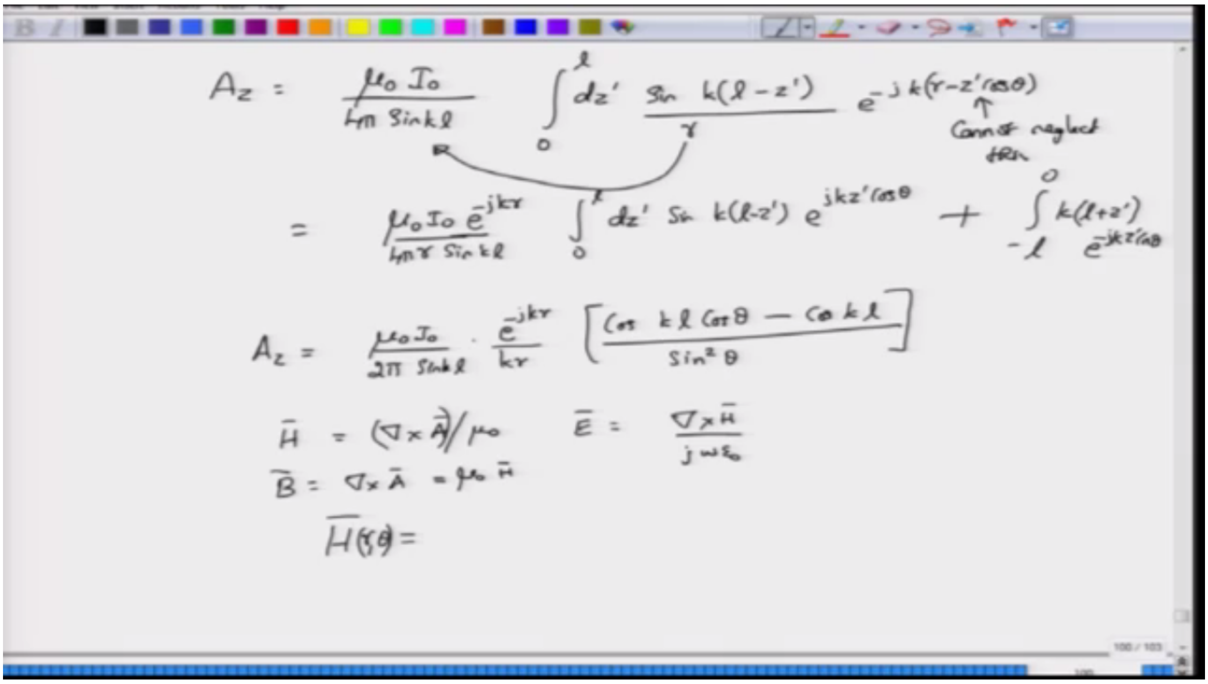

So this will be, you can show this. It would be $jI_0/2\pi$ sin kl, and then you have e^{-jk}/r and then you have cos kl cos θ. Sorry. This is not $\sin^2$ θ. This is basically just sin θ. Okay. So this sin θ and then you have kl cos θ - cos kl divided by sin θ in the φ direction. Okay. And the electric field can be shown to be function of r and θ, but this would be, you know, in terms of its magnitude it would be η times H_φ and it would be oriented along the θ direction. Okay.

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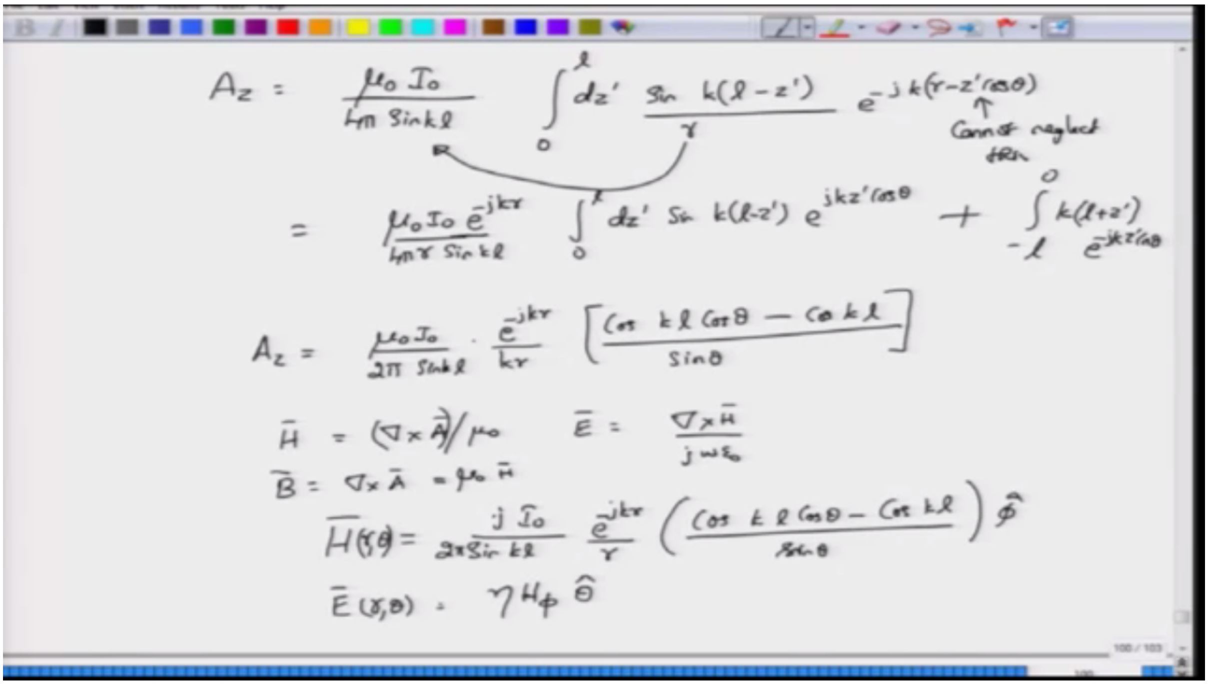

Now because H and E are both, I mean, H and E are oriented along φ and θ, E x H will be θ x φ and because θ x φ will be along the r direction. Okay. So you can show that the power will be radiating in the, you know, in the r direction, which is, or the radial direction, which is what you want from an antenna. Okay.

So these expressions I agree that are, I know I have not derived it, but the dirivation itself would take about half a module. So rather than that I have given you the basic, you know, approximation steps that you needed to make and once you have made those approximations, which are very valid because of the reasons that I have told you in the module, you can go ahead and find out the magnetic field component H as well as the electric field component.

Now what is remaining? You have to find out what is the average power that is being radiated by this antenna, which of course will be calculated first by forming the pointing vector, okay, and then you will have to find out the directivity and most importantly find out what would be the radiation resistance of this antenna. Okay. We will do all of this in the next module.

Thank you very much.

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