Lecture – 03

Voltage and Current Equation of the Transmission Line

Hello friends and welcome to NPTEL MOOC on electromagnetic waves in guided and wireless media. We are in the guided portion of this course, and this is our third module, wherein we are going to finally complete the circuit model of the transmission line, and then solve the resulting set of equations. Okay? So, that we can begin to study the more interesting aspects of transmission line. Okay?

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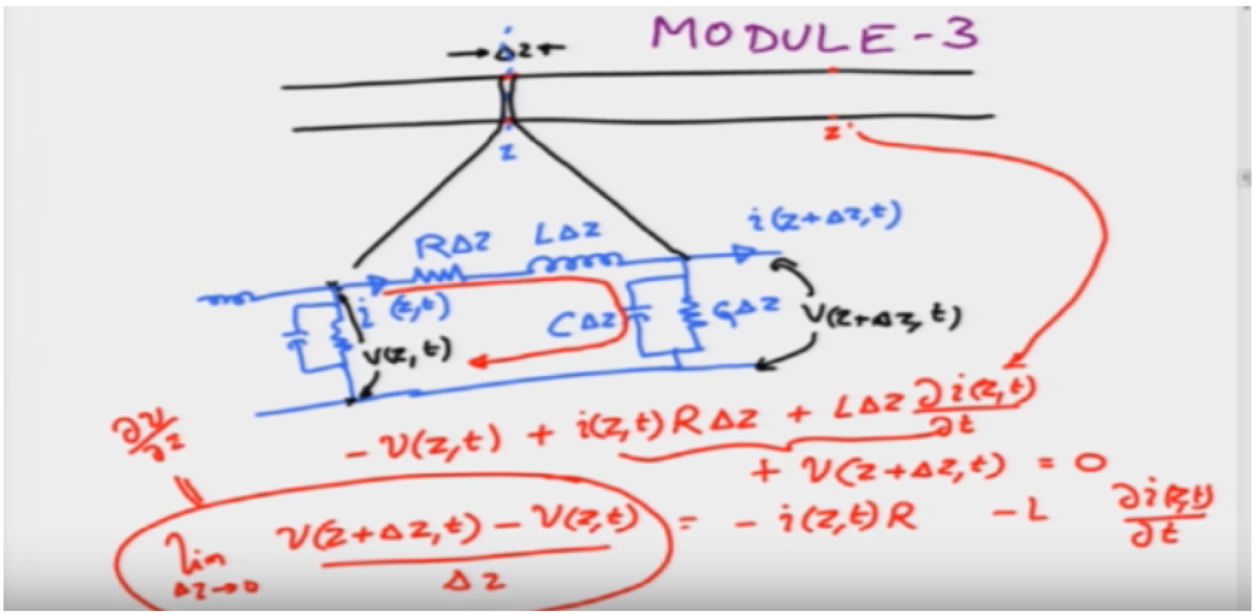

you all remember this ,model of the, circuit model of the transmission line, that we derived in the last class or motivated in the last class, someone might argue that this is not the correct model you know you, have to chain the model because it doesn't account for a different parameter and they will be Right? however the model that we considered here is fairly accurate and it will explain many of the observed experimental results, that involves transmission lines, and that is the justification for us to go about with this transmission line, and as I said over this unit cell Kirchhoff’s laws both voltage and current loss will hold and therefore, we may relate the voltage V and the current I, by applying this KVL and KCl, to apply KVL let me go around this loop. Okay? And then remember that, you know Ohm's law will tell me how? The current and voltages across each element is related. So, I start around this loop by writing this as minus V of Z, T this is my notation of KVL. So, whatever I encounter, a minus sign I will start with that sign that is encountered in fact and then I have plus I Z, t R Delta Z which is the voltage drop across the resistor, plus L Delta Z del I of Z T divided by Del T that would be the voltage across the inductor, remember the voltage and currents across the inductor are related by IR drop across the resistor, and L di by DT drop across the inductor. and you may have noticed that instead of writing this di by DT, written here as del I by Del T, or sometimes is called as dou I by dou T, and the reason why we have done this one, is also fairly obvious to you now because the current is not a function of time alone as it was in the circuit it is also a function of Z as well Okay? So, if for example I move this unit line you know, to a different location, say Z prime then in all these equations have to use instead of Z prime. Okay? Because, I have to account for the fact that my voltages and currents are functions along the transmission line as well, distance along the transmission line as well. So, this finally around this loop here I have the voltage drop of V, Z plus Delta Z Gamma T is equal to zero. So, I can I prefer to rearrange this equation slightly. So, I will have V of Z plus Delta Z Gamma t minus V of Z T so, by moving or by retaining these two terms here, and on the Right hand side by moving these current and no drops across onto, the drop across the inductance on to the Right hand side, I will have minus I Z ,T R Delta Z minus L Delta Z del I by Del T. now, you what you can do is ?you can eliminate Delta Z from these equations, by dividing both sides by delta Z. Okay? And then, by taking the limit of Delta said to zero. Right? I can recognize that this left-hand side is actually the partial derivative of the voltage with respect to Z .Right? So, this is in fact equal to Del V by Del Z, and that is what I would like to retain in this Expression?

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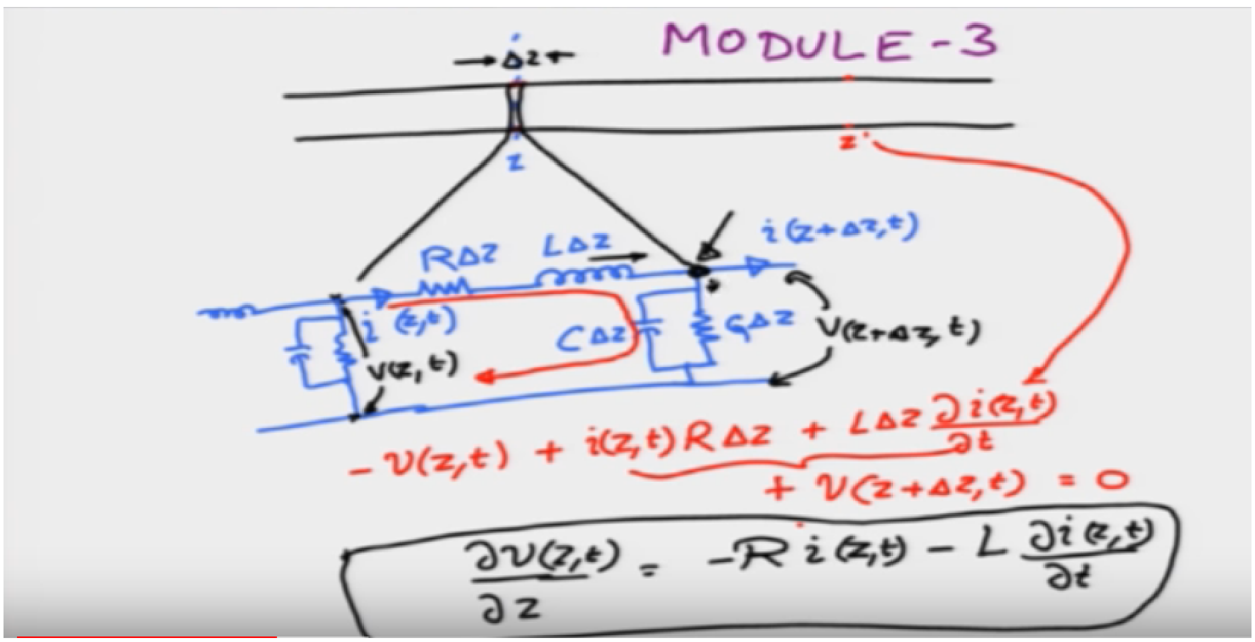

So, by doing that and then rearranging the equation, I am going to get simple equation, please do keep your pen and paper, and then continue to work on I mean continue the steps Here, I am erasing because I am running out of this page, but please ,if you are following the derivation that, I am also writing here, it will be very obvious to you ,that this left hand side can be written as del V by Del Z V of course is a function of Z and T, and on the Right hand side I have minus R I z t ,minus L del I by Del T. this is a very simple and beautiful result ,which relates the voltage changes along the length of the transmission line ,that is captured by del V by Del Z on the left hand side, to the voltage drops across the transmission line. So, you have a voltage drop because of the finite resistance of the wires, that is captured by our I drop or the IR drop, and then you have this you know the, inductance effect because of the current magnetic field, conductor linkage that I talked about, and that drop is captured by this expression L del I by Del T. so, this is one equation for us, unfortunately the left hand side is telling us how the voltage is changing with respect to set the Right hand side is telling us how the current and its current derivative with respect to time are related to this one. So, they are still not sufficient for us to solve the equations and then tell you this is how the voltage will change with respect to Z and T ?and this is how the current will Change? In order to obtain that knowledge, we need to apply KCl, and we are going to apply KCl, to this particular node here. Okay? so, in this at this node we apply KCl and clearly the incoming current will be the same current I of ZT the current that goes out is I of z plus delta z Gamma t and in between the difference between these two currents incoming and outgoing current, would obviously the current through this parallel branch, given by this one. Right? So, what would the equation look like for that case, I'm going to you know, because I don't want to draw this picture again.

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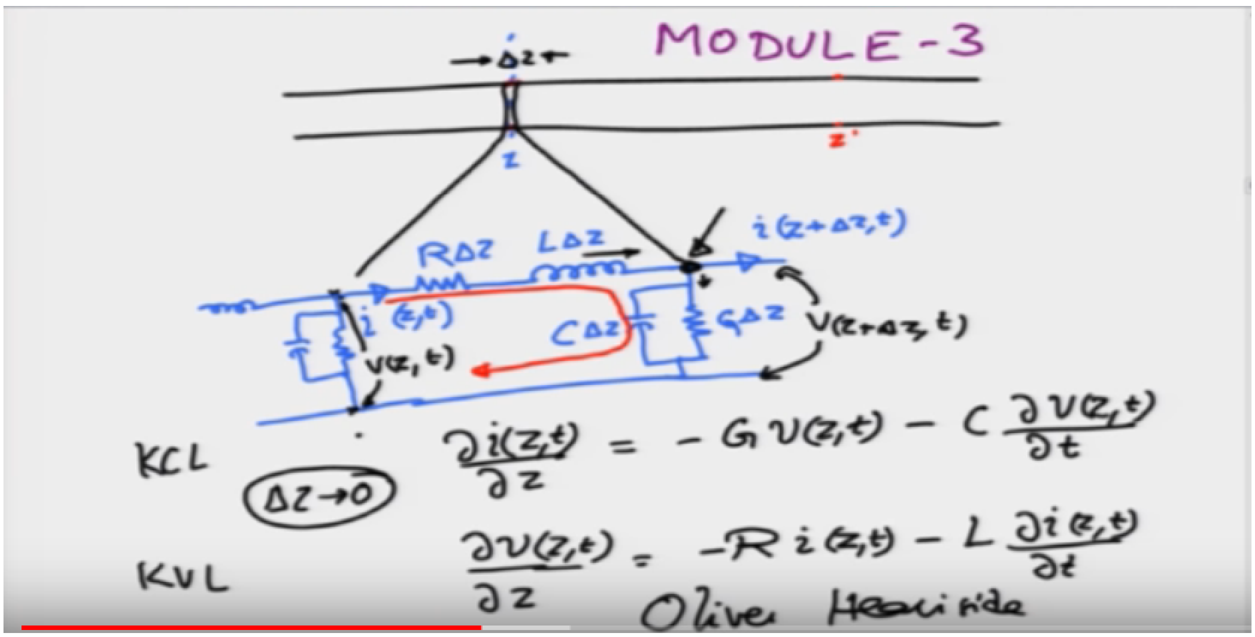

What I will do? Is to raise, I raise this voltage equation. Because, we have anyway captured, whatever that is require for us in the lower half of this you know, page. So, I am going to write down KCl here. Right? So, by arbitrary choosing the incoming currents to be negative, I will have minus I Z gamma T outgoing current is I of Z plus Delta Z gamma T and this will be the current that is outgoing so, that would be plus C Delta Z the current through the capacitor is del V Z plus Delta Z Gamma T divided by Del T. Okay? and then the last, minus G Delta Z V of Z plus Delta Z Gamma T. so, this is slightly not something that we had you know hope to get because, we would have you know, be very happy if we had del V of Z T and we have Z Gamma T but unfortunately, what we are getting is V of Z plus Delta Z? but here ,is where the power of approximation comes through, because sorry, this entire thing is equal to 0 by KCl. Right? I can, again retain the currents in in no current part on the left hand side and push the capacitance and conductance parts onto the Right hand side. Oh sorry, this is not minus, this is plus. Because, this is also current outgoing Right? So, this is the equation so I can simplify by dividing on both sides by delta Z. now, following the same steps as the voltage here I would also obtain on the left hand side the current. Okay? The partial derivative of the current so, I will have del I Z T by Del Z, this will be equal to Delta 's it has gone from both equations. So, I will have C, and then I will have G this will be minus, minus. Okay? The only thing that is now, you know interesting here is that, as Delta Z goes to 0, the voltage at Z plus Delta Z actually becomes equal to voltage at set. So, this is a small change in our expected model, I mean not change something that we did not expect this one but because the Delta Z value itself is very, very small and in the limit it is actually going to 0, it is conceivable that the voltage here, at Z plus Delta Z, approach the voltage V of Z. Okay? so, with that in mind and with the approximations that we are making I can replace this Z plus Delta Z in the equations by Z itself, and simplify the equation here, to write this as minus G V Z, T this is all in the limit of Z Delta that going to zero, and then you have minus C del V Z, T by Del T. Okay? please take a moment to see these two equations, and confirm to yourself that these equations are derived under this circuit model, are you know consistent and it is actually, correct set of equations and please remember that, what you have made the major assumption is that Delta Z goes to 0 and therefore, instead of having an infinite number of nodes you simply have a continuous value of there. So, this is how you go from discrete cases to the continuous case by making these Deltas's it go to 0. and these two equations are you know, they look very similar to each other on the left-hand side of both equations you have, the corresponding partial derivative with respect to Z which tells you how each of those components are changing along the length of the transmission line, whereas on the Right hand side, you have the quantity, which is the complementary quantity?

So, if you are looking at the change of current here, along the length, then it would be described by the voltages, as well as the derivative of the voltages with respect to time, that relates in the current changes along the transmission lines. So, this pair of equations were originally derived, by a British electrical engineer called Oliver Heaviside, and these equations are in fact called s telegraphists, equation because they were used by Heaviside to actually describe ,the propagation of voltages and currents along the transatlantic cables of Telegraph's. Okay? So, telegraph lines were quite long and they would know they would go miles and miles away, and they would actually transfer information. So, while trying to model that Heaviside came up with these you know, set of equations and he called them as telegraph it's equation because they are the kind of starting point to understand telegraphy signals, well now telegraphy doesn't exist but the equations are luckily for us the same equations can be used to describe the transmission line behavior as well. So, in fact now you may also understand the telegraphic lines are essentially transmission lines. Okay? So, how do we go further about this? I mean I have this set of equations.

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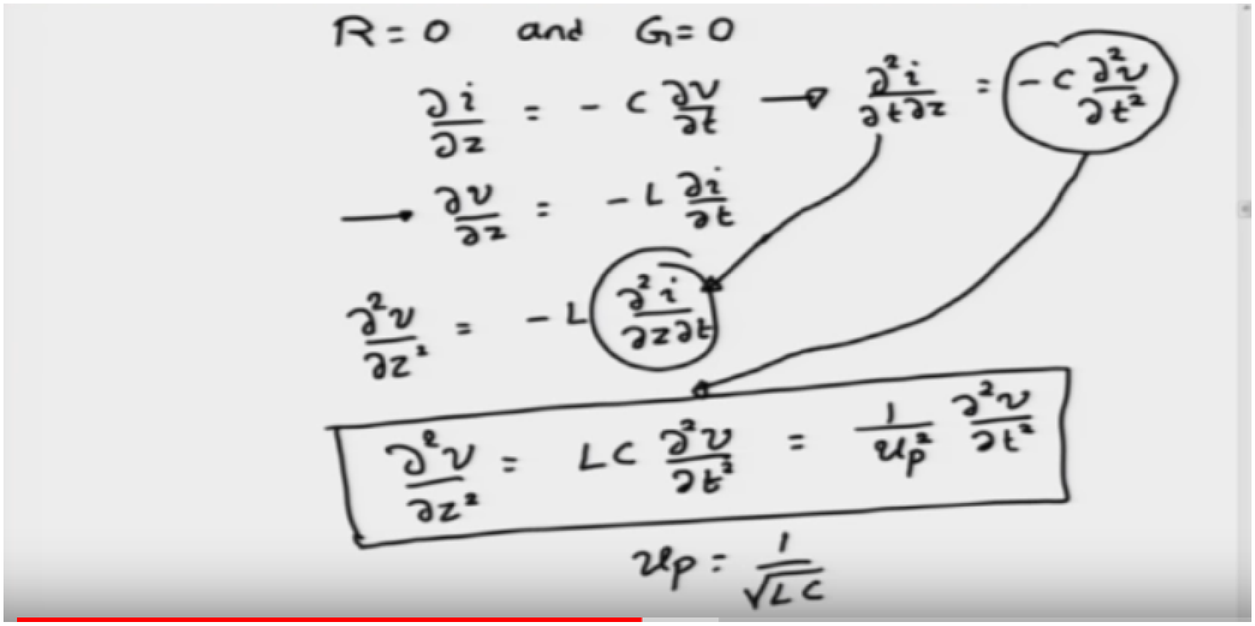

Now, what I will do is? I will consider one additional approximation; I will take R equal to zero and G equal to zero. I do this approximation because, it makes my equations slightly tractable and for the first encountering of this set of equations, it is perhaps wise to you know obtain the major behavior of voltages and currents ,without going too much into the mathematical complexity, which anyway would be present when R is non zero and G is non zero. Okay? so, we are going to make this small approximation the character is not really change, the solution character is not changed because, these two effects resistance and no the imperfect dielectric, can always be considered as a second-order effects, because you can make good-,good wires with very small resistivity and you can make very good insulators with very small leakage currents .so, while you can eliminate these two, of course you cannot eliminate the capacitive and inductive behavior, because those are related to much more fundamental electric and magnetic fields themselves Right? So, you can't make C and L equal to zero in that case there is actually nothing in these equations, but you can make G and R equal to zero because these two effects will not alter the I mean it will change you will see some small changes, but they are not so drastic changes from what you observe on the transmission line ,the capacitive and inductive effects are much more fundamental and there, they cannot be eliminated without making your model useless to describe, what is happening on the transmission line? Okay?

So, I took R equal to zero, G equal to zero ,that leads to simplification ,I am also going to drop writing the Z Gamma T every time, with respect to I and voltage, current and voltage because I do you know have emphasized many times that these currents and voltages are functions of both Z and time. Right? So, I have Del Y by Del Z equals minus C del V by Del T interesting, and then I have another equation which tells me Del V by Del Z is equal to minus L del Y by Del T. Okay? Now, observe very interesting thing what would happen, I'm going to take this equation second one, and then differentiate this one with respect to Z. Okay? So, I will have Del square V by Del Z square is equal to minus L del square I by Del Z del T I do not know this, but I have this equation. So, if I differentiate this equation with respect to T, I will get Del square I by Del T del Z, which is essentially same as this one Right? So, if the currents and voltages are well-behaved functions of Z and time then it is possible for us to, see that these two partial derivatives are going to be the same, it does not matter which order you differentiate them first, they are essentially going to be the same, but the Right hand side will be interesting so it would be minus C del square V by Del T square, Right? So, I can replace this fellow is the circled one, with the circled term here and doing that replacement or making that replacement gives me L C del square V by Del T square. Okay? and for reasons that will become very clear shortly, I am going to call this LC as 1 by u P Square and then I have del square V by Del T square. Okay? so, I have the equation del square V by Del Z square is equal to LC del square V by Del T Square and because ,I denoted u P as 1 by square root LC, LC itself will become 1 by u P Square, and this is a set of equation, that when you to solve.

So, it is still a second-order partial derivative equation, voltage is a function of Z and time, but luckily for us or you know, whatever that we have managed to achieve, is that the left-hand side talks about the same quantity as that of the Right hand side so, left hand side also you have the voltage, Right hand side also you have the Voltage.

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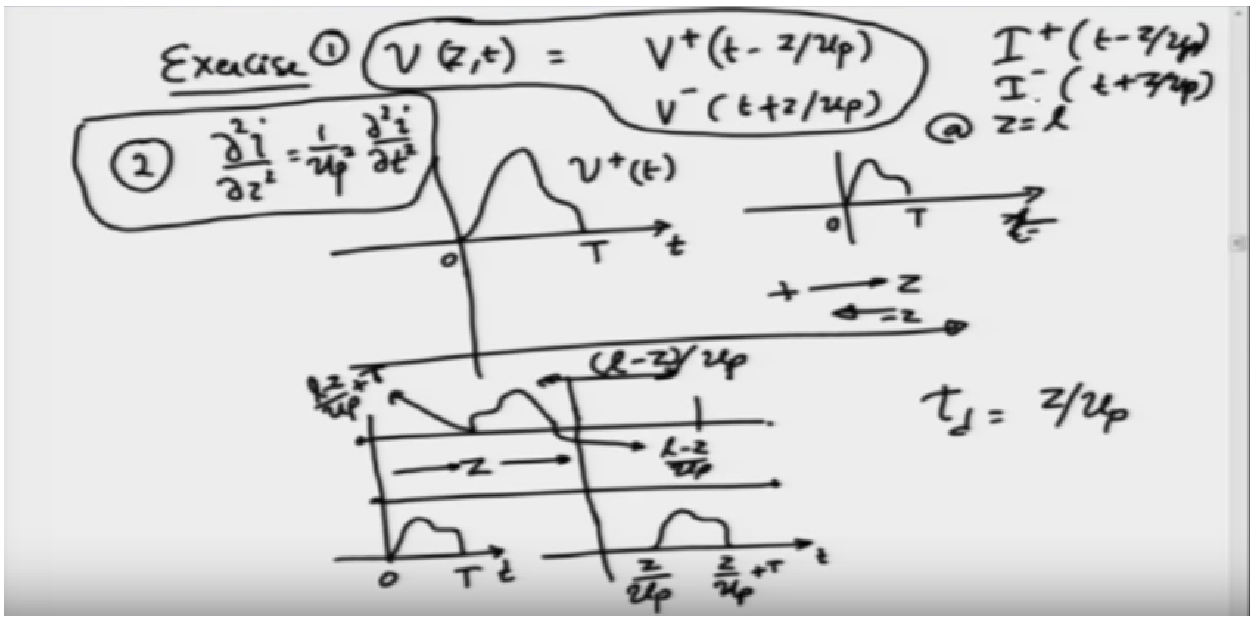

And you can show, by direct substitution, that any of these two possible solutions. So, for example, if you assume the of Z T to be equal to V Plus t minus Z by u P is one possible solution for the equation, as is this V minus of t minus Z by u P where V+ and V minus are functions actually. Okay? So, they could for example, be something like this so at Z equal to 0 as a function of time, they could be like this. Okay? So, let us say this is T, this is 0, this is as a function of time, and this is the function v plus of t, and what we are now claiming, is that at a different point on the transmission line? So, let us actually go back and write this, so this is the transmission line. So, at Z equal to 0, they will the voltage here, has some time behavior which have captured in this particular arbitrary pulse like behavior, and what we are saying now, at this point is that after a distance Z along the transmission line. Right? After a distance or at a distance Z along the transmission line, the voltage distribution, one of the voltage distributions could be simply the voltage V plus of T, that has been delayed by a value of Z by u P.

So ,that is this would be the equation .So, you now have Z by u P and z by u P plus T as a function of time here, do you see this important thing what has happened ?what has happened is that ,the voltage which was launched at the input side has now, you know has now, appeared after a suitable amount of

delay, which is given by Z by u P along the transmission line .so, the further you keep going, the further the voltage will appear after a certain amount of delay and this, delay is in fact given by, we will call this delay as TD, that will be given by Z by U P. so, the longer the distance from the source end assuming there are no losses in this transmission line and that is what we ensured by raking R equal to zero and G equal to zero, all that we are seeing is the shape of the pulse that is launched the voltage remains the same, but it considerably or it progressively gets delayed as you move along the transmission line .so, it just keeps delaying, delaying, delaying, and if there is of course a load at some particular point ,then the voltage will be transferred to the load. Okay?
However it's not so simple because you know, if the load is not match then you will have lot of problems, which is what we are going to address later on ,there's another way of looking at this one, there is another solution sorry ,this is should be t plus actually, this solution is V minus T plus Z by UPI's another solution, it's another function as well ,so V minus of T so, if you assume that you're far away along the transmission line say it Z equal to L. so, this is at Z equal to L and this is the time domain, and then you have you have a certain pulse behavior, we'll call this again as T and again as 0 ,then another solution as you move towards the source side, will be the solution, that will be advanced by a certain amount of time, as I know as you come to a certain distance here, you would have seen that this solution so it would have advanced here. Ok? And this advance would be, so although I'm saying this as advance of course nothing is actually in advance, what is actually happening? is the voltage launched at the other side, has been delayed by the appropriate length so, if the transmission line has a length of L, then the voltage launched at Z equal to L, will appear after a distance of or after a delay of L minus Z by u P along the transmission line.
So, therefore this part will be L minus Z by u P and this fellow will be L minus Z by u P plus T. ok? so, you can see that there are voltages which are going from Z equal to 0 along the way to Z equal to L and voltages which are going from Z equal to L to Z equal to 0, in case you consider a situation where there is no load ,and the transmission line simply goes all the way to infinity, then the only way you can have a voltage he is that, if you launch a voltage and set equal to zero and this voltage keeps on progressively delaying but continuously goes on to the infinity side ,and there is no return voltage here, on the other hand, if you put the voltage source at Z equal to infinity ,then there will be a voltage which is propagating along this z direction, in the reverse direction and these two voltages, with respect to the coordinate system that we consider are called as forward and backward voltages, forward voltages are those which propagate along plus z direction and those which propagate along minus z direction are called as backward waves. Okay?
So, these are the two types of waves that can exist, in a finite length of the transmission line both waves will exist, and they will also sometimes talk to each other or superimposed on each other leading to all kinds of interesting effects, I am calling them as interesting because, they can be constructive or they can be destructive. Okay? so, I will leave this as an exercise for you, please carry out this exercise, and to show that either of these two functions and in fact if these functions have, the second derivative with respect to Z and second derivative with respect to time as continuous values, that is continuous functions, then any arbitrary shaped pulses, will satisfy those set of equations ,the simplified or the lossless telegraphists equation and interestingly you can derive a similar expression for the current as well, I will leave that also as an exercise in this model so, the second exercise is to show that, del square Y by Del Z Square will be given by 1 by u P square del square I by Del T Square.
So, you can see that the form of the equation remains the same whether you are dealing with voltages or currents and the solution will also be something like, current I plus of a function t minus Z by u P ,which is propagating along the plus direction, and I minus P plus Z by u P, there's only one small point that I

wish to make here with respect to I - ,please note that, if you go back to our transmission line, you will actually see that there will be a signal current I ,which is propagating along this path, and though from the from left to Right? And there is another current I, which is called as the return current. Now, please do not confuse the return current, with the current I - Okay? In fact the return current will also have, I return plus and I return -. Okay? of course I return plus will be propagating in this direction from Right to left ,from my side of course and I return - will propagate in the same direction as from left to Right again in my direction, but there are two currents forward and backward waves ,or no pulses that are involved even with separately with the return current, and separately with the signal current. Okay? so, please keep that difference in mind, such a difference does not come for the voltage because there is nothing like a reverse or the return voltage ,there is only a voltage difference between the pair of wires. But, current there is a return current and the return current also has returned current plus and a return current - Okay? AlRight? So, we looked at these solutions, in it's no simpler case of R equal to zero and G equal to zero.

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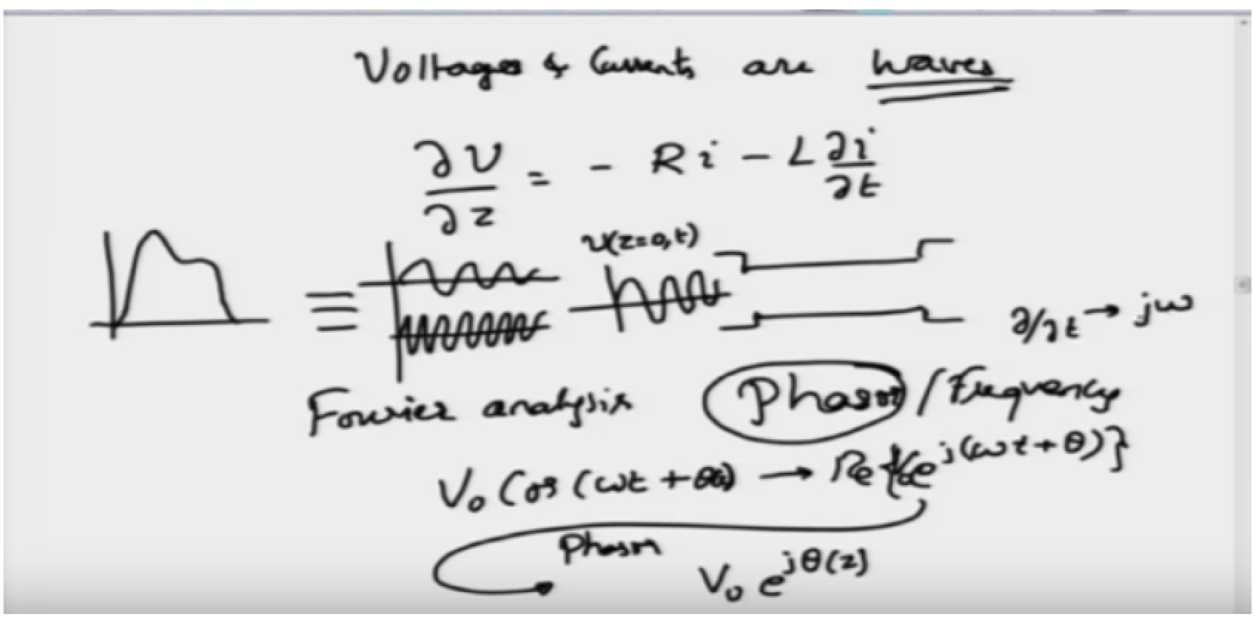

And what we figured out is that, voltages and currents, on the transmission lines are actually waves Right? they exhibit a wave-like phenomena because, these are actually getting delayed nicely and they exhibit a wave-like phenomena and therefore, they are called as voltage waves and current waves, rather than calling them as voltages and currents ,we call them as voltage waves and current waves. Okay? This is in complete contrast to the ordinary circuit theory, where you simply call it as a voltage quantity or a current quantity, they are not going to be waves in the ordinary circuit case, whereas in this transmission line regime waves. Okay? We also saw two equations Del V by Del Z equal minus R I minus L del I by Del T Right? So, this equation will look R equal to zero to simplify this equation, but the original telegraphists equation was this one and we did not mind particularly what would be the shape of the solution, I just know drew some random picture of there a pulse like behavior and I showed you that this pulse will be delayed as it propagates along the transmission line.

But, I know from Fourier analysis, that any pulse which is reasonably well behaved, can be actually described in terms of many ,many sinusoidal quantities Right? So, they will they can be written in terms of appropriately scaled up and phase shifted versions of sinusoid. Okay? In what is called as, Fourier analysis. And therefore the instead of looking at different types of pulses, arbitrary pulses, what we can actually do? is to look at the behavior of this transmission line. Okay? Behavior of the transmission line, to a single sinusoidal signal. Okay? so, I will launch a voltage at say Z equal to zero which will be a sinusoidal signal with a certain amplitude and a certain frequency, the initial phase can be taken to be zero without any loss of problems. Okay? Or loss of generality and we would like to see how the transmission line would respond to this voltage which is sinusoidal. Okay? We already kind of know the answer, but we want to put them on proper a mathematical term, no that's what we want to do? And when we are considering sinusoidal excitation of this transmission lines, it is possible for me to actually go to what is called as phasor domain? Okay? or sometimes called as frequency domain Okay? In the phasor domain any quantity, which is say cos of Omega T? usually that's what we would have so, V 0 cos Omega T was your circuit so the voltage full sinusoidal voltage you would actually plus may be theta, we will write this initial angle theta, you can express this as, real part of e power J Omega T plus theta by using Euler's identity and then go to the phasor domain, by dropping, this real and E power J Omega T .so sorry, there will also be v-0 here assuming v-0 to be real so the corresponding phasor is obtained by simply writing this as V 0 E power J theta so, showing only the magnitude and the angle. Okay?
And instead of taking this theta to be a constant, if you let this theta be a function of Z, this still will represent a phasor except that this phasor will now be dependent on the position Z, on and on the transmission line this is a very good approach I mean very good representation for voltages and currents. Right? So, on the sinusoidal signal, once the sinusoidal signal is launched, instead of working with the full time domain picture, we work in the phasor domain picture or the phasor picture .so, that Del by Del T can be replaced by the derivative J Omega. Okay? So, differentiating real quantity in time, a sinusoidal signal in time, will actually be equivalent of multiplying the phasor by J Omega .Okay?

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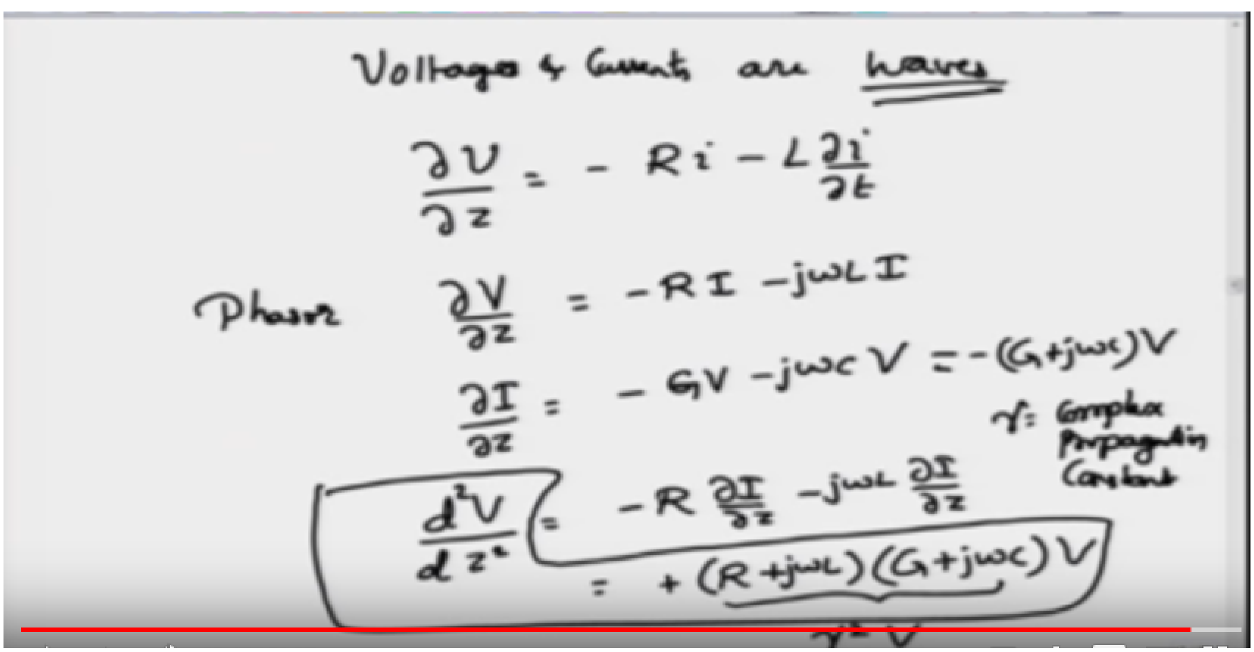

So, doing that and converting all quantities that are in time domain, into phasor domain, I can rewrite this equation. Okay? Into the equation that maybe is easier to work with Okay? so, I will write this as del V by Del Z. please note that, I have used B and put a capital v out there, so I will write this as del V by Del Z equals minus R I, again I is a capital thing, so indicating that this is the phasor domain picture that we are writing and then you have minus J Omega L I. Right? similarly please show that, del I by Del Z can also be written as minus G V and then you have minus J Omega C times V,differentiate the first equation with respect to Z. Right? So, differentiating the first equation with respect to Z, will give you minus R del I by Del Z, minus J Omega L del I by Del Z, of course this is equal to minus of R plus J Omega L, del I by Del Z but hey, del I by Del Z is nothing but, G plus J Omega C times V, minus G plus J Omega C times minus. So, there is a minus here and another minus that becomes a plus and then I have G plus J Omega C times V. So, I don't need to actually use partial anymore here, because these are only dependent on one single quantity Z, the dependence on time is implicit, all these quantities are actually sinusoidal quantities, that is they actually vary harmonically with the frequency of Omega radians per second and therefore that being, implicit assumption of the time variation, will allow us to write down this or simplify these expressions ,in terms of only a single parameter dependency onset, we call this R + J Omega L into G Plus J Omega C product as gamma square .Okay? And gamma is what is called? As complex, propagation, constant. Okay? So, this is complex, propagation, Constant, I don't like the word constant here, because it is constant only if Omega is constant, for different values of Omega this gamma is actually different. Okay? but we will see the consequences of this dependence on, gamma on, Omega later on, but this equation, where we have written del D square V by DZ square equals gamma square V is the characterization of the voltage distribution along the transmission line, in the phasor or in the frequency domain, you can show and take it as an exercise to show that a similar equation with the same value of gamma, is applicable for current as well Okay? We will now see the consequences of having these voltage equations in this manner and study more about complex propagation constant in the next module, incidentally complex propagation constant is called as the, ‘Secondary constant of the transmission line’.