Hello and welcome to NPTEL mooc on electromagnetic waves in guided and wireless media. This is module number 23, where we will discuss another type of wave guide, which will allow electromagnetic waves to be propagated from one point to another point. In fact this is such an important wave guide that it actually forms the backbone of the telecom infrastructure, okay. Connecting people across continents and providing very high data rates, which will be used you know, for various purposes, but this is the only technology today… in today's world, that can provide very long distance propagation, distances in the order of few… thousands of kilometers and at very high data rates, each channel running more than 100 GBPS or at least close to 100 GBPS for a combined data rate of more than terabytes per second. So just to give you a magnitude of what this terabytes per second is, if you actually think of you know, 1 gigabyte as the required 1 GB for non HD movie, then you can actually fit 1 trillion such movies, okay, it's in fact equivalent of streaming 3 movies per day continuously. So the global traffic actually runs based on this particular wave guide and this wave guide is called as optical fiber. Optical fibers, you may see outside, you know, or you may have seen in pictures, they are very thin strands, okay. If you look deeply into that, these thin strands have two to three layers around them, so there is what is called as an inner layer called as core, surrounding this core will be a layer called as cladding, and then you have some sort of a coating, okay, which is usually a plastic coating or an acrylic coating and this one is usually embedded in the form of a or inside… inside a jacket, okay. So this is all that an optical fiber will have. If you look closer, the dimensions, you will be surprised that the dimension here of the core ranges anywhere from say 8 to 100 micron, whereas, oh sorry, 8 to 50 micron, 50 micrometers, and the cladding is usually at 125 micron or 250 micron, okay the diameter of that one, the thickness of the coating is somewhere around 100 microns or 125 microns, and the jacket is slightly you know, bigger to give the mechanical strength, but if you, I don't exactly remember the value here, but if you look at the overall optical fiber as you can find from commercial, this would be no more than just about a millimeter, or about you know, yeah, just about a mm in width, okay. So such small dimensions, small form factor, very light, and moreover the best thing about this optical fiber is that it is immune to most of the 50 Hz lines that are going around, the power line fluctuations, because this is made out of, the base material is of the fibers, is made out of silica and then you dope, that is you introduce this external atoms into the silica in order to either lower the refractive index or increase the refractive index to form the cladding, okay. And then you coat it, put it into a jacket, and it will actually be very thin kind of a thing, right. Of course, in practice you don't just take this jacket, I mean jacket optical fiber and then lay them across, you would have seen those optical fiber laying tunnels, you would have a big tunnel in this manner, but this is basically to provide, you know, kind of a mechanical protection to this one, so on top of the tunnel, the tunnel is usually dug underground, so there could be vehicles moving, animals moving, people moving, there could be construction. So to protect the fibers

inside, we just make out a mechanical tunnel in this manner and within each that, there will be this huge pipes and huge pipes in turn consists of many many many such pipes, okay. The exact geometry is dependent on the company which is laying this fiber and in each of these ducts, there will be further cables again as decided by the company, okay, how much of the fiber they want to lay and if you look at one strands cable of this optical fiber, the cable can contain about 8 and some times more than that, okay. Each of these itself consists of multiple optical strands, so this is kind of a structure inside a structure, but the cable itself will consist of roughly each duct, and the cable itself will consist of about six or eight optical fibers, all jacketed optical fibers with some filler materials or without any filler material. So this is how mechanical and optical fiber would look like, and of course as I told you it is made out of silica and silica is essentially sand. So these are basically like thin strands of sand actually, right and it is amazing that they can actually carry the data rates. The reason why they can carry such high data rates is because the operated typical wavelength of 1550 nanometer, okay. And this 1550 nanometer is a sweet spot for silica in the sense that if you were to look at its attenuation, which is measured in alpha, whatever, dB per km for example, as a function of wave length you will see that it actually goes through a minima here at the point 1550. There are additional minima, local minima somewhere else, but this is the point where it actually goes through a minima and over this region, if you look at what is the wavelength at 1550, this will be around 10 to the power 14 Hz. And even if you were to occupy like you know 10% of this 10 to the power 14 Hz, you are actually potentially dealing with about 10 to the power 12 and 10 to the power 13 Hz of bandwidth available to you.
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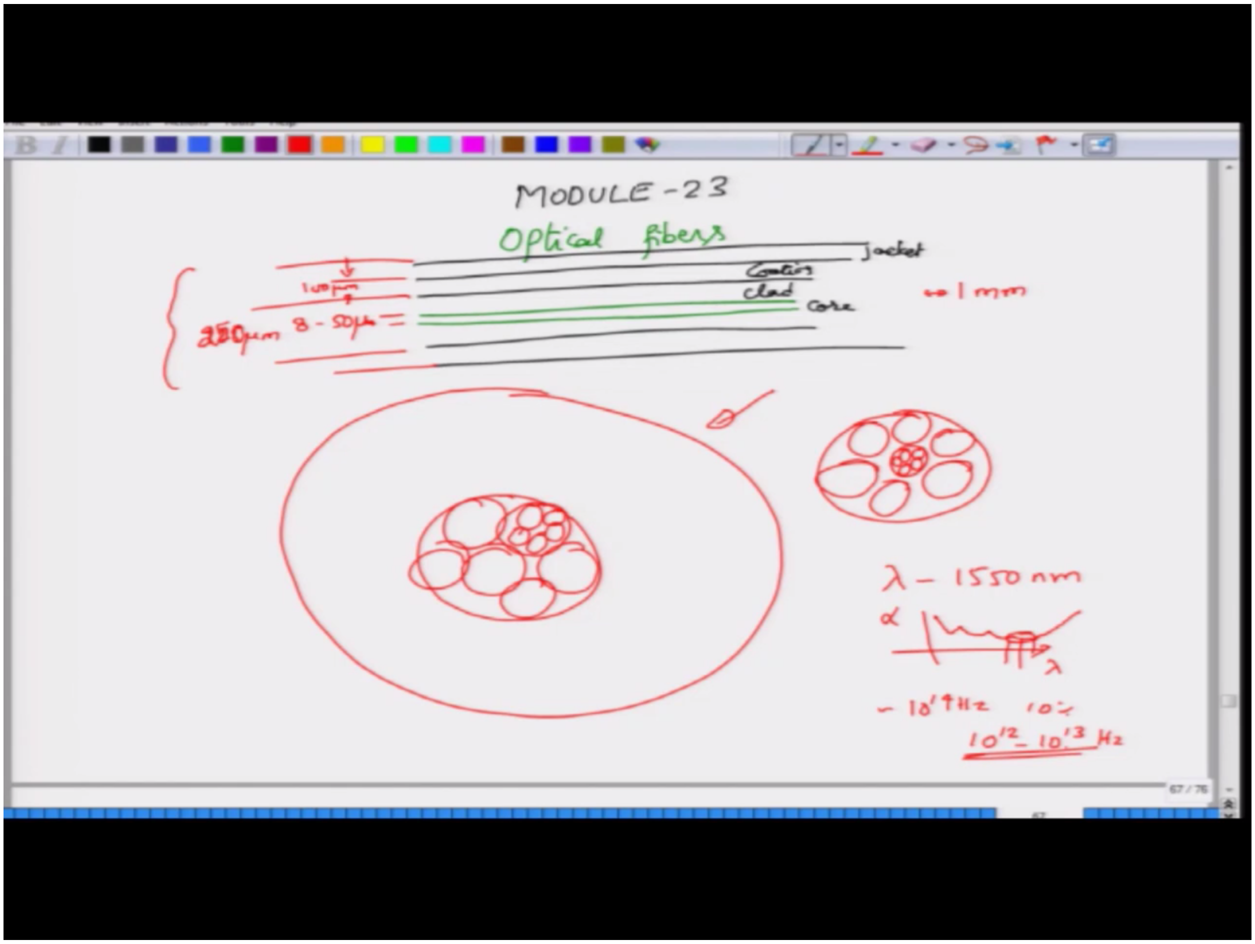

and using this sophisticated communication technology of modulation technology, then you can actually obtain an aggregate data rate of close to 1 terabytes/sec, okay, which is what recently has been done and people are moving on terabyte/sec kind of a communication. And this is all made possible again in a very simple manner of guiding light through what is called as total internal reflection, right. We have already seen and understood many things about total internal reflection. You could go back and write down the geometric optics approach for this fiber and then try to understand it. But again this geometric optics approach will not tell us what is happening in the cladding and it will also not tell us the fact that each ray corresponds to a specific electric and magnetic fields and the way these electric and magnetic fields vary across the fiber, which constitutes some more, can actually be put to good use, okay. You can end code information or rather you can put information and modulate information on each of those modes and this modes carry independently, can carry information, thus increasing the capacity of the optical fibers, okay.

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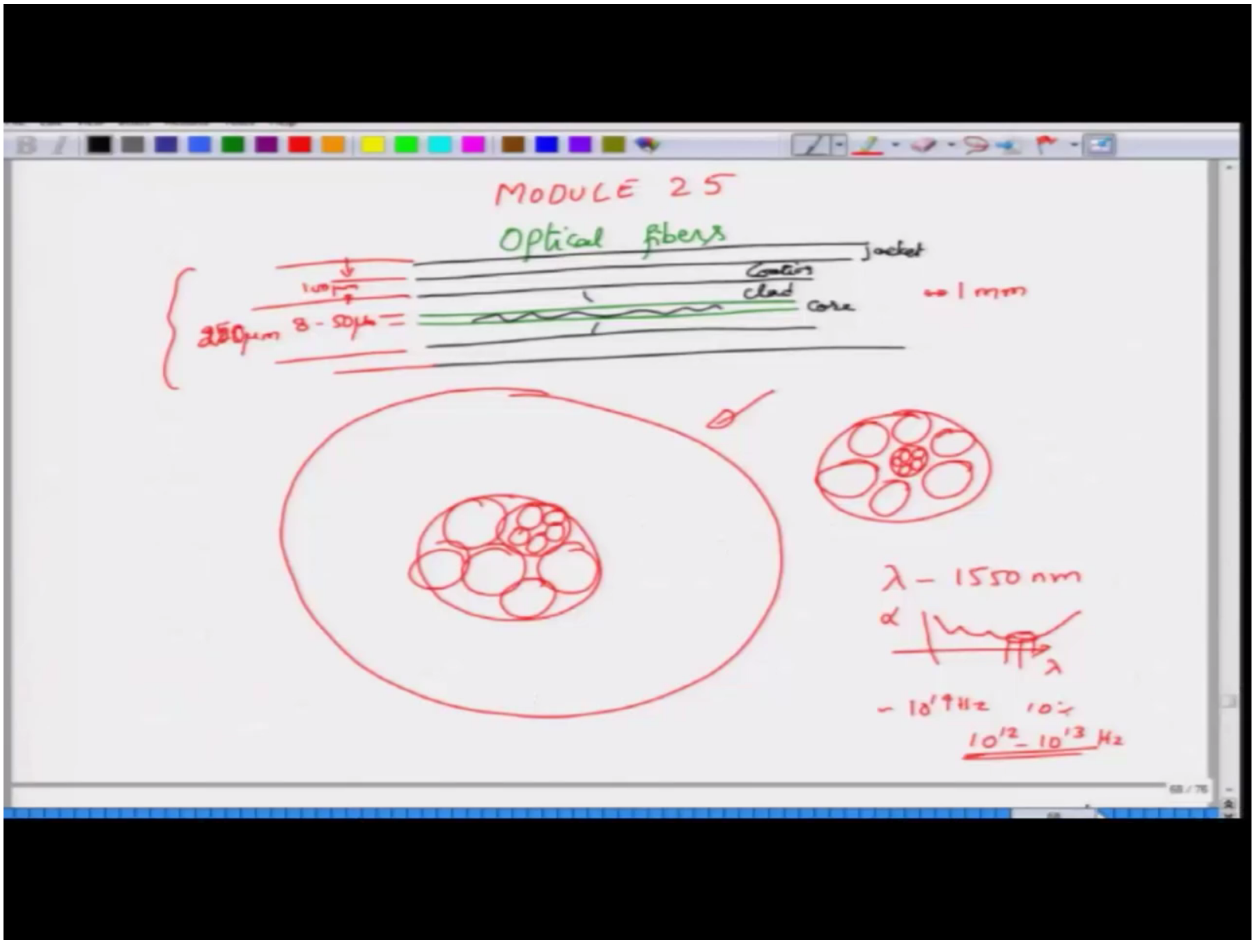

So we normally identify two types of fibers depending on the dimensions as well as the propagation. If a fiber supports only a single mode, whose dimension or whose radius is just about 4 micron. The cladding is of course at 60 micron or something like that. So the total thing is about 125 micron, okay. This is a standardized value. So if you operate this one at say 1550 nanometer, this will be called as what is called as a single mode fiber, okay. And we will use this abbreviation to refer to the single mode fiber as SMF. If you increase while keeping the same wavelength, okay, if you increase the core diameter to roughly 100 micron, okay then you will see that there will be multiple modes carried by this fiber and you get what is called as a multimode fiber, okay. So you have a single mode fiber and a multimode fiber and we will not make this assumption that we are dealing with a single mode fiber or multimode fiber, we will simply, to understand the mode structure we will go with what is called as a two layer approximation in which we will have a core, the core has a certain radius A, it is filled homogenously with the material refractive index N1 and outside is an infinitely long cladding, who refractive index N2 is actually less than N1. Now in many practical optical fibers, the refractive index profile, so if you plot the refractive index profile down here, you would see something in this manner, ideally so this would be N2 and this is N1. However, for various applications the refractive index profile as we would call it has also been optimized, okay. You don't of course get this sudden step discontinuity in the refractive index. It is not practically possible. Apart from that one, in order to better adjust the mode properties, you will also grade the profile, so you will actually see the refractive index profile changing over the cross section in this particular manner, okay. Some specialized optical fibers have a mode structure that would look in this manner, and that would be in this way. There are this what are called as triangular shape profiles, then you have profiles with what is called as trench, okay so this is one ranched step index finger, you will also have a trenched, but graded index finger. These different fiber refractive index profiles which corresponds to different types of fibers are all used at different applications, okay. When you deal with… actually in fibers you deal with a property called as dispersion. Dispersion means that as the pulses propagate, the pulses which carry information, they actually start to elongate or they start to broaden. Sometimes they contract, but in most cases they actually broaden, especially when you propagate over longer distances. And as the pulses broaden, they will start to talk to the other pulses, neighboring pulses, which are also broadening, right so in time, you are sending one pulse, next pulse, one pulse, next pulse, but if the pulse after propagating certain distance, actually it becomes you know, broaden, then it kind of one edge of the pulse, the tail of the pulse intersects the head of the next pulse, the tail of next pulse will intersect the head of the next pulse, and so on and so forth, okay. Effectively reducing the rate at which you can transmit information and to deal with the… and to moreover the properties of this dispersion itself is dependent on the type of the more that is propagating and therefore it has different values for different modes, okay. So these are actually real headaches in fiberoptic communication, which of course many smart people are working on it to reduce this one and to reduce dispersion we tend to use different types of optimized refractive index profiles of the fiber, okay. So that is how you actually get variety of optical fibers, suited for very different applications, okay.

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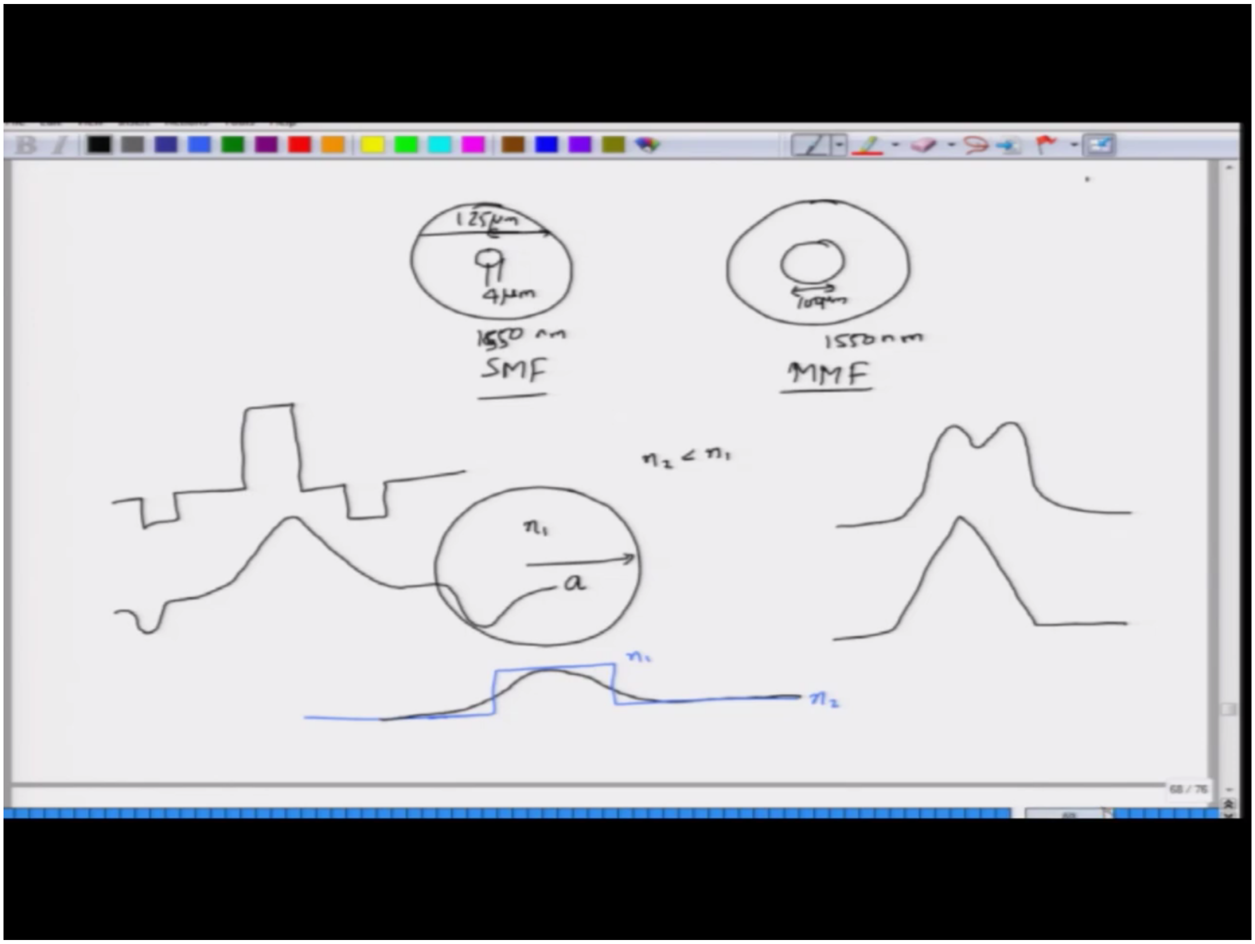

However, our analysis will include only what is called a single mode, it will include only what is called a step index profile, which means the refractive index is assume to be constant having a value of N1 inside the core and outside the core that is in the cladding will have a value of N2. This is very similar to the optical wave guide that we consider, the slab waveguide. Here in the slab waveguide we took N1 and we took N2 to be less than N1, this is precisely the same scenario in terms of the refractive index profile that we have taken. Unfortunately the geometry that we have chose in this fiber is not the same as the planar geometry that we had for the slab wave guide. The geometry in fact is a circle, okay, of course surrounded by this one, this in fact extends all the way along the fiber itself, right. So this is what the structure would be and because this distance is quite large, we will assume that this cladding boundary actually goes all the way up to infinity, okay, and this of course is the core, refractive index is N1 here, refractive index outside is N2, and this geometry that we need to consider Maxwell's equation is a cylindrical geometry, right. So we are dealing with cylindrical geometries and our equations unfortunately are not simple and they in fact are little complicated because you are dealing with the cylindrical geometry. But they are well documented, the equations and you know, the simplifications are well documented in the literature. So hopefully once you get the basic idea of the how the light propagates, how to solve for the modes, you can go into de… greater detail to understand the more propagation in different types of fibers, okay.

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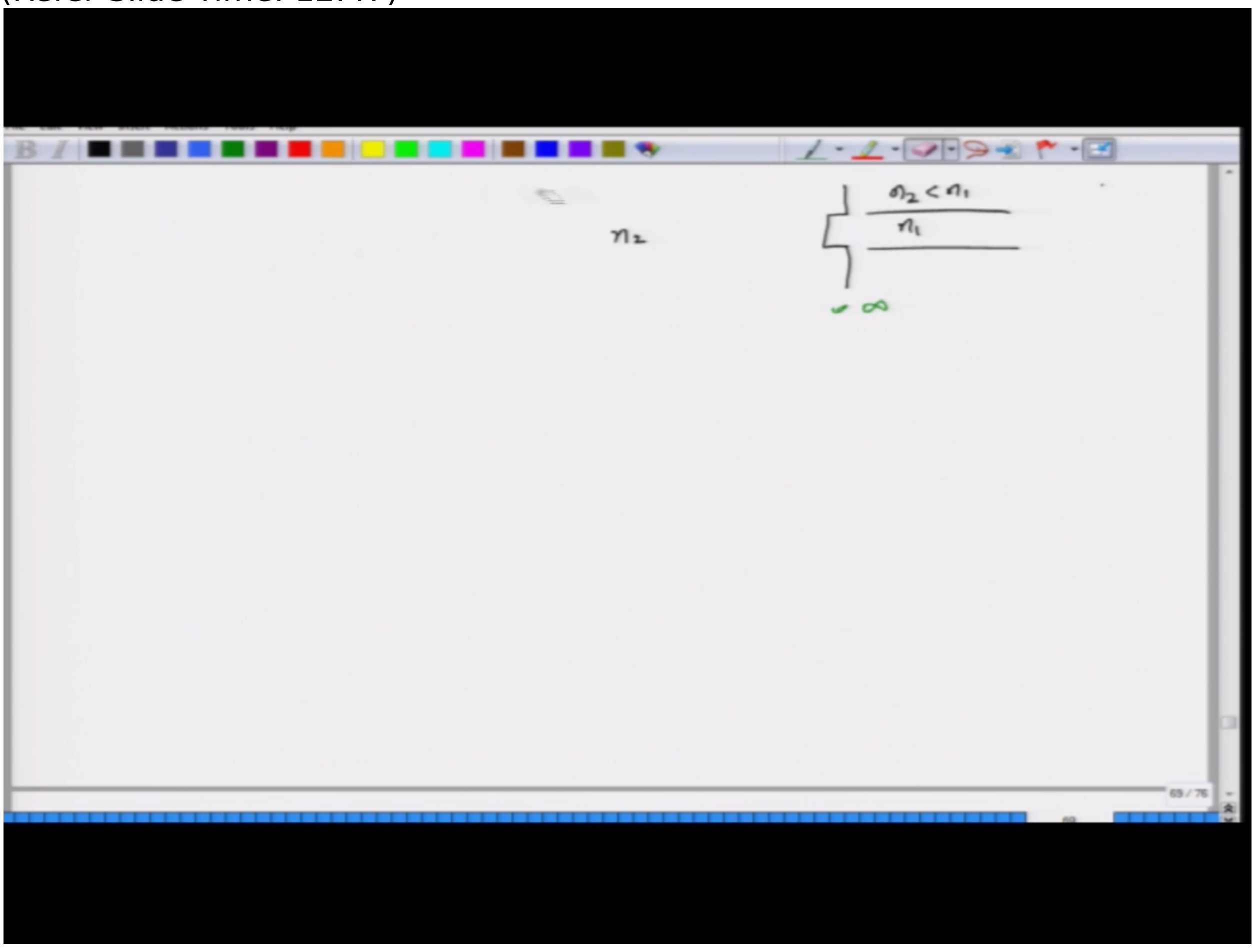

With that let us begin our model analysis. We assume core. We have refractive index N1, here we have N2. Now in cylindrical coordinate system any point is actually specified by specifying the radial distance R, the angle which it makes with respect to the X axis, which is called as the Phi or you know the angle Phi and then the height above a certain reference plain. So if this is my reference XY plane, then the height above this point will be Z, okay. So you simply imagine the cylinder, cut the cylinder, or take the base of the cylinder, and then imagine that any point here can be specified by specifying the radial distance and the angle. So you draw a perpendicular. You see that line where it connects and then that would be the angle and then height over which your point is located will be the Z. Accordingly the electric field components will also be ER, E Phi, and EZ, only EZ is the rectangular component and similarly magnetic field components will be HR, HY, and HZ, these all will be functions of R Phi and Z in general, okay. However, we want propagation along Z axis, because we took the fiber and then we take the ang… you know, the height of the cylinder along the Z axis, so we want propagation along the Z. And this means that we are going to assume that all field components are going to look like E Par – J beta Z. So luckily at least wherever in the Laplasian or any other equation, you get Del by Del Z or Del square by Del Z square terms. You can replace them by – beta square terms. We still have a problem with R and Phi in the sense that I cannot concentrate only on one particular component in a cylindrical coordinate system, the one that we are solving, you will have all three of them to be non 0. So basically I do not have TE or TM scenario, although this TE and TM modes exist as a very special condition in optical fibers, which we are really not going to look at.

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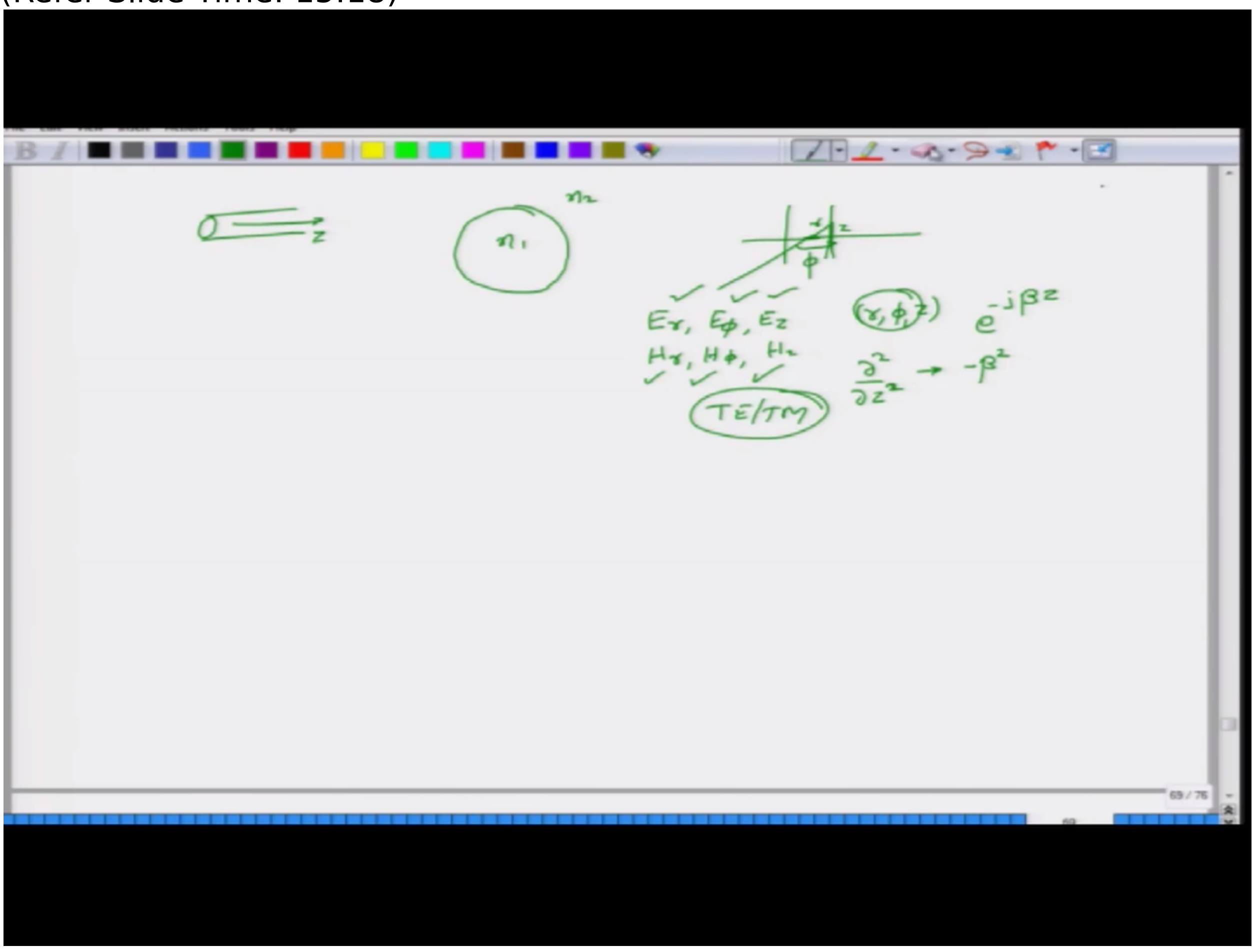

But in general just assume that I cannot deal with DE and DM, okay. Rather I have to deal with all six components being non 0 and moreover this R and Phi components, whether it is electric field or magnetic field, they actually are tightly coupled in the sense that I cannot separate the equations, the wave equations separately solved for ER and HR, E Phi and H Phi, because in the wave equation for ER, there will be an E Phi term and in the wave equation for HR, there will be a H Phi term. So my option luckily is to actually go to the EZ and HZ component and then solve this Helmholtz equation either for HZ or for H… EZ, of course we have to solve it for both, but because both satisfy a similar equation, the form of the solutions will be the same, right. So I can solve for Del square EZ, because EZ does not have ER and E Phi component coupled into it, okay. So you have Del square EZ + K0 square N square, EZ must be equal to 0, you can write down the similar equation for HZ as well. And again as in the previous lab wave guide case N can be N1 or N2 depending whether you are dealing in core or cladding, of course the method remains the same. I solve the wave equation in the core separately. I solve the wave equation in the cladding separately and this stitch the two solutions at the boundary by applying the usual boundary conditions of tangential components. Now do you want to know what the tangential components at this boundary are. Well you are looking at a boundary say R=A. The tangential components are going to be E Phi component H Phi component, okay, because they will be moving along the interface on both sides or they will be EZ and HZ components, okay. So for, if you imagine that you know you have some kind of a cylinder, then there will also be an EZ and HZ components. I have not shown it in exactly the same way that I have plotted. So you imagine that this is a cylinder and as you move along this would be the E Phi component or you move along the cylinder that would be a EZ component, but it is still parallel to the interface itself. However, the radial component would go from one medium to another medium and actually be perpendicular to this surface, right, so this would be the R=A surface that I am looking at and clearly this ER component here and ER component on this side, they actually are normal components. So once we solve for EZ and HZ, we somehow need to solve for E Phi and H Phi and this solution of course comes from relating them using Maxwell's equations. Unlike the case of Planar wave guides, this relation is quite complicated, so I won't deal with it, I will leave it for you as an exercise. However, I will go back to this wave equation and then expands Del square out, okay. In the slab wave guide, this was easily expanded into rectangular coordinate system, but the Laplasian Del square in the cylindrical coordinate system is much more complicated. It is in fact 1/R Del/Del R times R Del/Del R, okay + 1/R square Del square/Del Phi square + Del square/Del Z square. Luckily Del square/Del Z square can be replaced by – beta square, but you don’t have any such luck for R and Phi, okay. So you have to go back and put the entire equation here. You write this as 1/R Del/Del R R Del/Del R EZ, okay. So this is what you are going to get + 1/R square Del square EZ/Del Phi square + K0 N1 square – beta square times EZ=0. Now this is a complicated equation. I don’t want to solve the equation, but I want to make two points, okay. One if I want propagation inside the core, I better have K0 N1 be greater than beta, okay. When I take this same equation and apply it to the cladding I know that in the cladding this fellow will be K0 square N2 square – beta square, but I want an exponentially decaying solution. So as I move away from the fiber my light power should be you know, decaying down. I want all of the light to be confined in the core region itself. Therefore beta will be greater than K0 N2. So as long as beta is within these two ranges, of course the actual value is obtained by applying the equations and then applying the boundary conditions, but as long as beta is there, between these two values you are okay. You have modes, which are confined within the care. In fact if you device this entire expression by K0 and simply ask for this quantity called beta over K0 to be between these ranges N1 and N2, this beta over K0 is called a as an effective, it is called as the effective index of the mode, okay. And again as in the case of slab waveguide you will have multiple solutions.

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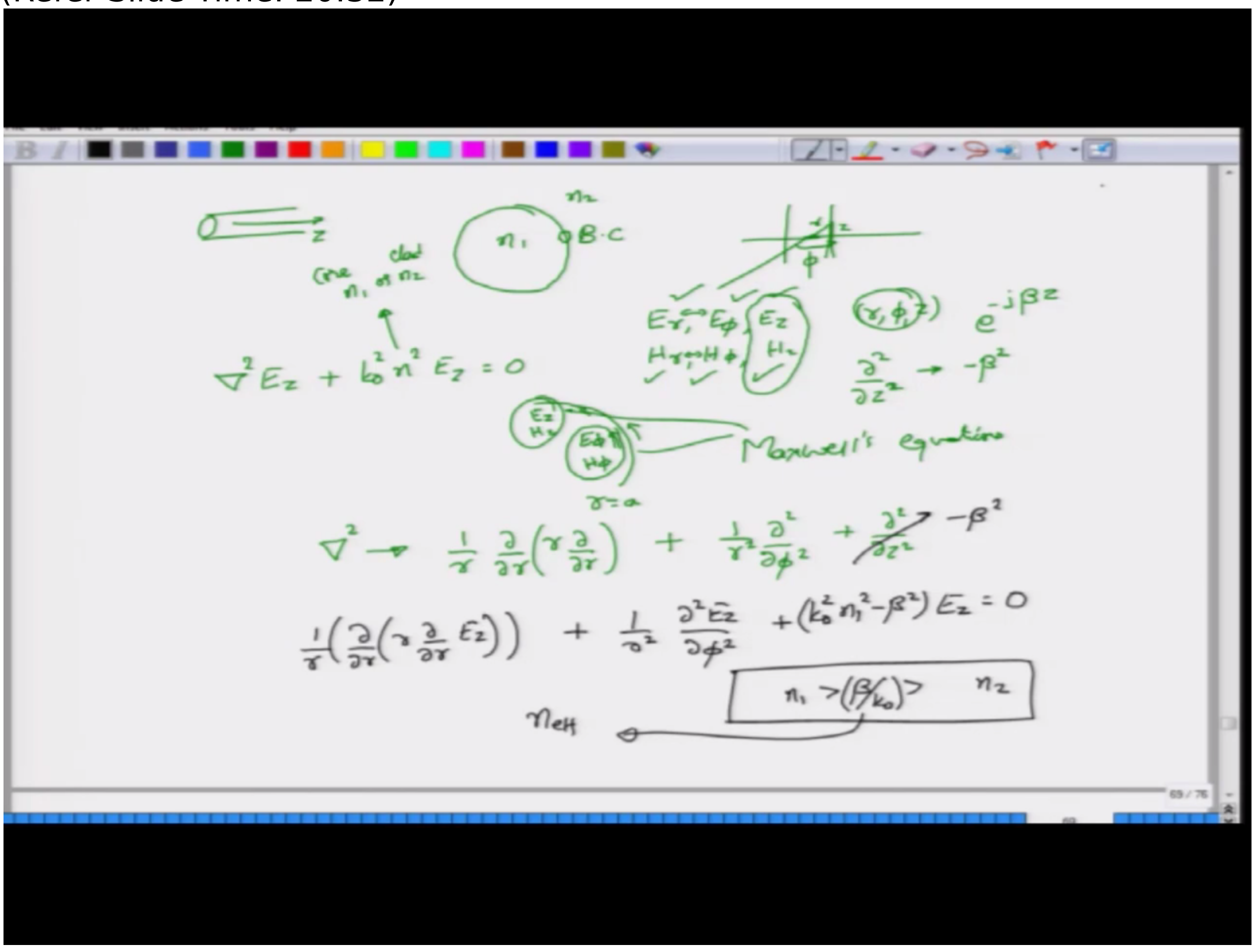

and this time the solutions can be obtained by method, by solving that partial differential equation. There are very, many techniques to solve that equation, we won't deal with that, but in terms of Phi your solutions will be of the form E power J Nu Phi or E power – J Nu Phi, where Nu is just a integer, at least in this case, it will be an integer. Alternatively you can think of this as cos Nu Phi and sin Nu Phi. We will also add some extra theta to this, you know, so that it can be used later on for boundary matching. We won't need it here, because we are not solving it, but in general it’s a good idea to just take this as E Par J Nu Phi + theta and adjust the value of theta to obtain different modes, okay. Theta of course is a constant for a given mode and you can adjust that value of theta to obtain whether a certain mode exist or doesn’t exist. Nu is an integer here for us, because if you move over one rotation, you were moving around E Par J Nu Phi, when you moved over one

rotation or when you performed one rotation, your field values should remain the same, you've come back to the same point. You don’t want the field value to just change suddenly there, right. So that is why we take Nu to be an integer. In terms of R, without solving I will claim that the solutions are going to be J Nu kappa and K Nu kappa, sorry, K Nu gamma R, where Kappa is usual K0 square N1 square – beta square under root and gamma is the usual beta square – K0 square N2 square, okay. you can of course even define kappa A as U and then Gamma A as W and then you still have U square + W square to be equal to V square where V is called as the V parameter of the fiber, okay. But what are these J Nu and K Nu. They are like cosin and like exponentials, meaning that if you look at J0, right, it would actually have a solution, which would be in this manner, right. It looks like a sinusoid, but it's actually a damped sinusoid, okay. if you look at what would be, I am plotting as a function of J Nu of X for example, right, And this fellow is J Nu = 0, so now if you look at what would happen for Nu = 1, it would look something like this, so it’s a little difficult for me to draw, but this is what J1 would look like. So if I expand it out the equation, then I can write clearly, so and then this fellow will be on this side and so on, okay. So these different functions, except for J0, which is actually non 0 at the center, all the other functions, Bessel functions as they are called, they will have 0 at R=0. So the right solutions for us in the core are the solutions, which are of this particular form, damped sinusoidal kind of a form, but they are formally called as Bessel functions of the first kind and order Nu, okay, and K Nu are a series of exponential functions, so you actually have series of exponential functions, all of them going after 0, but this is how the increasing order Nu would look, that is to say, this would be for Nu = 0, this is when Nu = 1, and this is when Nu = 2. You can clearly notice that these exponentials are going off to infinity at R=0, so clearly they cannot be the solutions inside the core, but outside the core, they are very good solutions, because you can then match a Bessel function with an exponentially decaying type of function.
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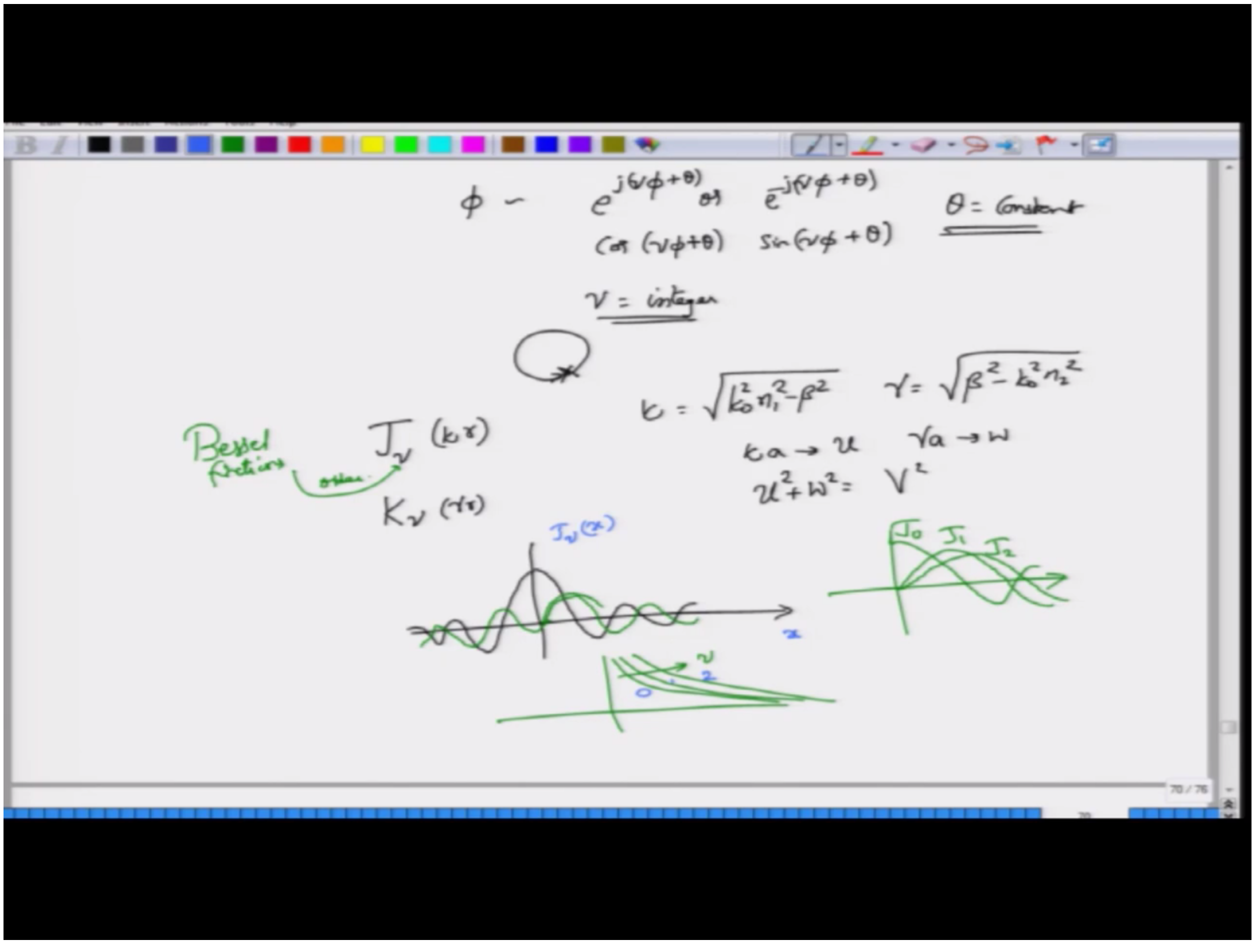

So the boundary condition would essentially become something like matching. So I am again drawing this. So assuming that I am dealing with the fundamental mode, which would be of the form of J0 function. So you can come up to this part, which is A and then from here take off into the exponential thing. So this will be K0 of say gamma R and this fellow bellow be J0 of kappa R, okay. So his fellow gamma, so gamma times R. So at R=A this amplitude should be equal to each other and in fact you cannot only deal with, you know expression with just EZ to match. To obtain the values of beta, we have to deal with EZ, HZ, ER, and sorry, E Phi and H Phi. So four equations will actually give you four sets of, I mean, four electric fields and magnetic field components, tangential components at the boundary will give you four sets of solutions, okay, and or... four sets of equations and the solution of that turns out to be beta. And we usually denote the solutions as beta Nu M, where Nu will tell you how the function is varying with respect to azimuthal angle and then M will tell you how the function is varying with respect to radial, okay. Under certain approximations you can go to the general solutions of the, you know fibers are called as HE Knew M where H is or EH Nu M, where this HE stands for hybrid electric modes and EH stands for electric hybrid modes. Here EZ dominates HZ, here HZ dominates EZ. Only in the special case of Nu=0, you are going to get T0M or TM0M modes, that is you will get the T and TM modes only when Nu=0, okay. Apart from this in general the solutions are HE modes and you may have normally seen these curves, okay you can actually generate these curves of what is called as propagation, normalize propagation constant B and the parameter V and these solutions are actually obtained, I mean, the curves that you obtain will actually look something like this, okay. This would be the curve 1 1, which is also called as LP01 mode, okay, this is called as a linearly polarized modes and these are the actual true modes. She o this is kind of an approximate mode and this approximate modes are obtained by what is called as weekly guided approximate, okay. We will not go into details, but this is the mode curve, so its proposition constant keeps on increasing and what you observe is that even when V=0, remember V is inversely proportional to lambda, because V was K0 square root of N1 square – N2 square times A and K0 itself is 2 Pi by lambda, therefore V is inversely proportional to lambda or directly proportional to frequency. What this means is that the fundamental mode does not have any cut off frequency. It does not require a minimum frequency to begin propagation. It begins to propagate right at DC value itself. Of course getting a laser to work at DC is very hard. Lasers usually work, we know very well when you are in the range of terahertz. So at pitch point the B value will be something like this, but if you increase the frequency further, keeping all the other parameters constant, you will actually see that as V increases, the allowed value V increases, you are going to see different modes, right whereas you had only one such mode when V was less than or approximately less than 2.405, we get many modes as you move beyond V, right. So this number V=2.405 is called as the cutoff, you know, number, and if your V exceeds this cutoff, only then you will actually start seeing higher order modes, okay. Those higher order modes could be LP11, LP… LP02 and so on, okay.

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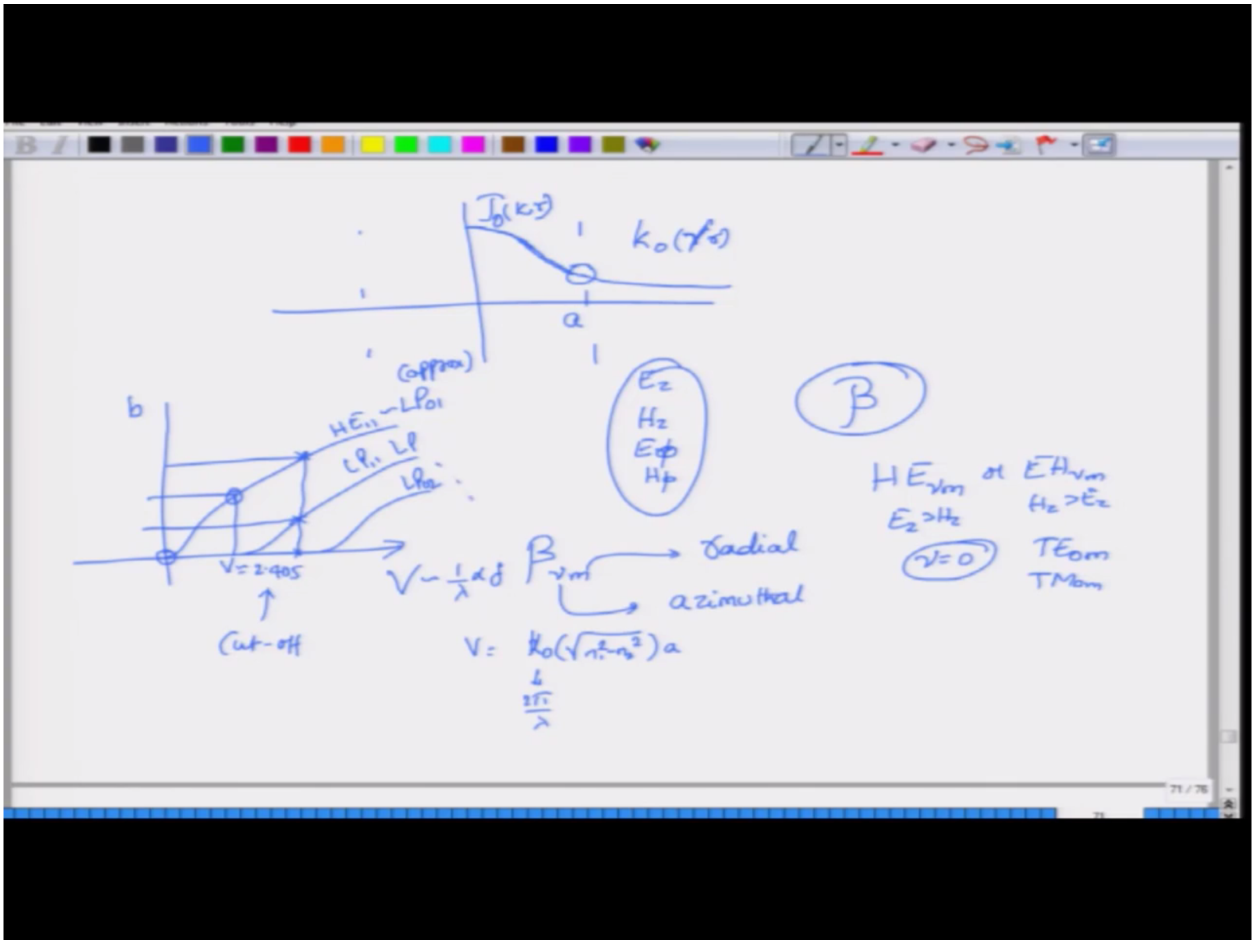

We won't deal with this modes, I will instead upload short presentation for you, wherein we have actually solved this equations using this commercial software console, and you can see how the modes actually look like. But this is the summary of optical fibers. Optical fibers also guide light, except that light is now guided by this geometry, which is in the cylindrical form and light propagation depends, you know, actually in general involves all the six components, but the fundamental mode is the so called HE11 or the LP01 mode. Thank you very much.