Hello and welcome to NPTEL mooc on electromagnetic waves in guided and wireless media and in this module we study very simplified element of many integrated optical circuits called a slab waveguide. We have already introduced the idea behind a wave guide in the previous module, where we imagined that we take a glass slab, okay, and then somehow couple light into this slab. We also saw hot to couple light into that. And then if you think of this light propagating inside, light of course being an electromagnetic wave, if it is propagating inside this slab, provided that the slab refractive index, which we will take it to be N1 is actually greater than the refractive index outside, which we will take it as N2, which is less than N1, then this rays will undergo total internal reflection and as they bounce back and forth between these walls, right, we see that effectively information or energy or the wave is actually propagating along the length of the slab, so along the slab, which we will take that axis to be Z axis. So this wave would be propagating in this direction. However, we also saw that the ray picture that we have seen does not really tell us more information, especially no information is given or obtained in terms of what the electric field component would look like and what the magnetic field components or the magnetic field would look like corresponding to these rays. So if you want to obtain this information, you need to use Maxwell's equation. Moreover the geometric optics approach that we looked at internal rotation previous module does not tell us even the existence of what is called as the evanescent wave, which we saw that will actually result when you go and approach this problem in the electromagnetic terms, right, that is to say apply Maxwell's equations and you get a fuller picture than what is possible with the geometric optics. Sure in some cases geometric optics approach, such as the one that we discussed is actually quite you know, simplified and it is easy to use that picture to… or qualitatively understand what is going on, but if you need quantitative answers, you have to you know, like usually use electromagnetic equations that is Maxwell's equations and that is what we are going to do. Now before we study the propagation in this slab wave guide, let me point out that this slab, which we have taken is usually made out of, in the case of an integrated optical circuit, it is usually made out of substrates, which are silica or other type of substrate, for example it could be a lithium niobate and you don’t actually get only a slab, you get multi layer structure, but probation usually happens in only one layer, okay so the other layers for many different reasons, but propagation layer will actually happen in, typically in one of the ones. I mean you try to make light confined into one particular channel or one particular waveguide layer itself. Okay, so I am simplifying the problem. I don’t want to deal with many layers, rather than that we will simplify the problem to a single layer structure, in which we have channel formed by a material, whose refractive index is higher than the material refractive index outside. Now in practice, it is quite difficult to, you know, or it's not that difficult, but it's… in practice you won't normally see such a symmetric slabs, especially in the integrated circuits, optical integrated circuits. What you actually see is another substrate or another material with a refractive index N3, which again is less than N1, but N3 can be greater than n2 or it can be less than N2, okay. So depending on this, the mode structures will vary slightly, but again this will introduce an additional layer of pro… you know, complication to our understanding of simple wave guides. So I am going to give you only the principle behind the simplest of these wave guides called as the symmetric slab wave guides, okay. So in… in our module, we will assume that outside medium is characterized by N2 itself and you can think of a slab surrounded by these two, by a medium, or this slab actually being carved out of in a medium, whose refractive index is N2 and N2 is less than N1. Now with Maxwell's equations, we need to first determine what kind of coordinate systems are we going to deal with. So we are going to use rectangular coordinate system, that seems very natural in this case. So in that case we take this, anyway we have already taken this to be the Z axis. What we will assume is that this axis over which these two, you know the slab boundaries are defined are taken to be at X= +A and at X= -A. So we center the coordinate system at this point, clearly Y will also be the coordinate, but that Y will be coming out of this particular board, so you have X Y and said as the coordinate system. Now immediately we will have to deal with two things, right. It is clear that even if you go to the ray picture, it is clear that what we are dealing with is an obliquely incident wave, right. So if we are dealing with obliquely incident, we have choices for transverse electric polarization or transverse magnetic polarization, meaning that light of course can be polarized in anyway, but you can split that polarization into transverse electric and transverse magnetic, analyze them separately and then put the results back together. So this polarization decoupling can be done because of the properties that any general polarized light can be split into two orthogonal polarizations of which TE and TM are and then analyze them individually and then put the results back together. So we are going to make a choice between TE or TM and to keep the mathematic simple, but still essential in showing you all the steps, I will assume that I am dealing with TE polarization, okay. Note that at this time we have not said anything more about what would be the electric field and what would be the magnetic field for these polarized light and how would they behave in this region, the slab region or the region outside and for later consistency with the fiber, we will call this region as core and we will call this as cladding, okay. Although this is not really core and cladding in the fiber sense, but because our next model is going to deal with fiber propagation that is propagation of light in fibers, we will keep the terms similar to that and we call that densely, dense refractive index region as core and the light refractive index region as the cladding, which surrounds the core, okay. So accordingly the core width in our case is 2 times A where A is the half core width, okay. So to summarize we are going to deal with transverse electric polarized light. So far we have not said anything about the electric field and the magnetic field form or the solutions of the electric and magnetic fields in this scenario and we have assumed a symmetric slab wave guide. Now for transverse electric polarization, if you take this as a the K vector that I have already drawn, then the magnetic field should be present, right. So the magnetic field if I take to be going along this particular direction, don't worry what the actual form of magnetic field is, we are going to derive this as the module progresses. However, for this case the electric field has to lie in, you know, it is to come out of this page, right it has to lie along Y. So electric field is polarized along Y and then the magnetic field will have two components, one is HX and the other one is HZ. So if you look at HZ component, that HZ component would go something like this, it would be along the Z component and then HX component would be in this manner, right. Clearly on to this interface that we have consider, X=A and X= -A, at these two boundaries HZ will be tangential to the boundary or tangential to the interface at X=A or X= -A, whereas HX will be the normal component. And electric field because it is already in the Y direction and Y direction is parallel to this X=A plane or X= -A plane, it is already tangential in this case. So when you apply boundary conditions at the interface X=+A and at X=-A, you are going to apply boundary conditions on the tangential components, namely you are going to apply boundary conditions on EY and HZ, okay so we are going to see that one later on, but we will start with a simplest case. We will still assume uniform plane waves, because well you know, in this case we can still assume it to be uniform plane waves or rather we will assume it to be plane waves, they are not really uniform in this coordinate system, because there is a phase E par – JK X, which would be different, right Anyway, so disregarding that short remark, let us write down what is the wave equation and we already know the wave equation for plane waves, right, and that would be del square + K0 square N square E=0. Clearly I am dealing with only a single polarized case that is E is basically EY along the Y direction, therefore I can drop this vector and then make this into a scalar equation, okay. So I am going to get a scalar equation with EY. Of course you will recognize this as Helmholtz equation, okay, which was actually possible, I mean for us it was possible to write this equation, because we assumed waves to be plan wave kind of a thing, right. So with the wave vector K given by K0 x10. N is of course refractive index. Now here we have to think a little bit. Does this N refer to N1 or N2, in fact you have to have two such equations, one for core and the other one for cladding, okay so you will have another equation for the cladding. So you will have K0 square N2 square EY, okay. And this equation has to be solved separately and this equation has to be solved separately and you meet both equations or match both equations at the boundary by applying appropriate boundary conditions, okay.

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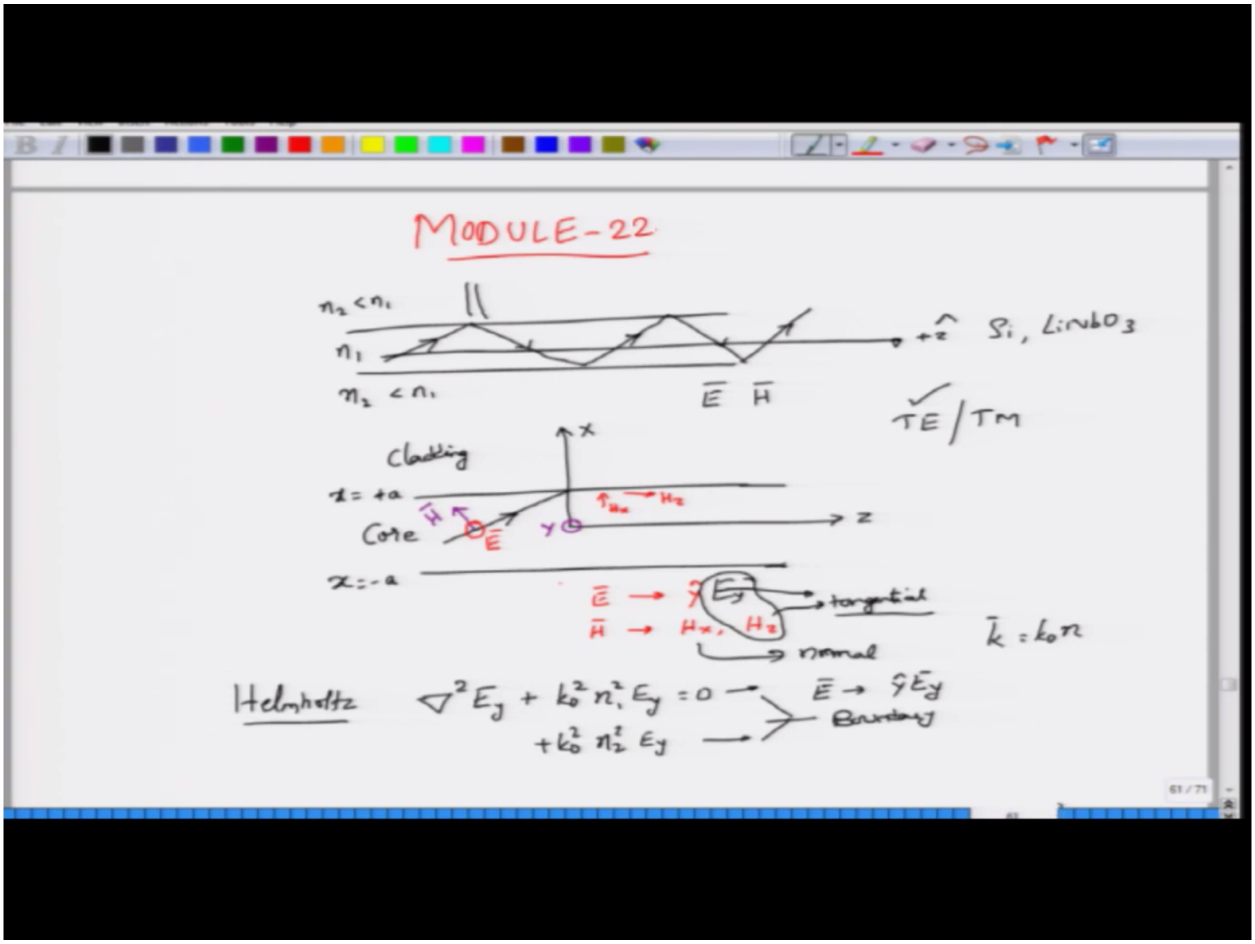

What would be the solution for the equation in N1, sorry in the core region. Well first look at del square in rectangular coordinate system is del square by del X square + del square by del Y square + del square by del Z square. Now we are dealing with plane waves. We want the waves to propagate along Z direction. So I will assume that whether I am dealing with core or cladding, it does not matter, the electric and magnetic fields will always have an E to the Par – J beta Z type of a propagation, right. So we have all, you know, we are propagating along the +Z direction and we know this very well from our earlier discussion that a plane wave propagating along Z will be described in this manner E Par – J beta Z type of a solution. What it means is that this del square by del Z square can be replaced by – beta square, okay. Because our structure actually is, you know, taken to be kind of infinite in the X and Y plane and none of the components are actually dependent on Y, where you can make this Del square by Del Y square term go to 0, okay so I am going to make this Del square by Del Y square term go to 0 and what I am now left with this Del square operator is Del square by Del X square – beta square, okay. I can put this operator on to EY, so what do I get, and because now EY is going to be functioned only of X, okay of course it will also be function of Z, but that Z function is already taken to be in the form of E Par – J beta Z, okay. So I can replace all these partial derivatives with the knowledge that along Z is this behavior, however, along X we still don't know what is the behavior, so we are going to find that out, right. So I can write down the equation which says D square EY/DX square + K0 square. For the moment I am going to consider the solutions in core region, so I can write this as K0 square N1 square – beta square EY = 0, okay. This is your ordinary second order differential equation, nothing fancy about this, the solutions of course can be in the form of an exponential signals, right, or exponential functions. So if you go back to your high school, or rather your differential equations' course, you will see this that you can write this as –S square EY or you can, you can write this is as – S square EY, where S square = K0 square and 1 square – beta square, okay. And the solutions for this one will be E to the power + or – JSX, right or you have solutions in the form of cos SX or sin SX, right. If S square were to be negative, that is when beta square is actually greater than K0 square N1 square than this would be a positive quantity, which we will call as some gamma square, okay, and the solutions would have been E to the power + or – gamma x, okay. The nature of these two solutions are very clear, one of them is an oscillatory solution. So a cosin X would go in this manner, a sin X would go in this manner, a sin SX, where as E Par + or – gamma X one would be decaying, the other one would be increasing, right. Of course we don't have any gain, so normally we can rule these out in our, you know, solution set, but these three seem to be the solutions that we are looking for, but exactly which solution needs to be used depends on the sin of this term S, okay. In the core region, we don't want any decay along X, meaning that we want in the core a solution to be in the form of cosin or a sin wave. That actually means that K0 square N1 square must be greater than beta square and instead of calling this S square as K0 square N1 square – beta square, we will go with the more conventional notation that is used in the optics, we will call this as kappa F square or we will call this simply as kappa square. So we will call this as kappa square and we demand that this kappa square be actually a positive quantity, okay, which means that the solutions for EY in the core will be of the form A cosin kappa X + B sin Kappa X, we don't know what the A and B constants are, we can find this out or we are going to make one more simplification, we will assume arbitrarily that B = 0. I am ruling out this solution because only A cos kappa X is also a solution of this equation, okay, with an unknown value of A. And if we retain only the cosin type of solutions with respect to EY as a function of X, these are called as E1 mode solutions, obviously because cosin is an even function and because I have ruled out this sin function, they are called as odd mode solutions. However, I can, I mean, I can obtain odd mode solutions by ruling out the even mode solution arbitrarily, okay. In practice you can have both modes and the overall structure can be of a combination of these two, but we will soon see that these two modes don't exactly propagate at the same propagation constant and therefore they can be discriminated against, okay. I am taking the even mode only for simplicity and I will leave this as an exercise for you to repeat the entire calculation assuming that we are dealing with a odd mode functions or an odd function, okay. So yes we have now shown that the solution in the core would be of this form.

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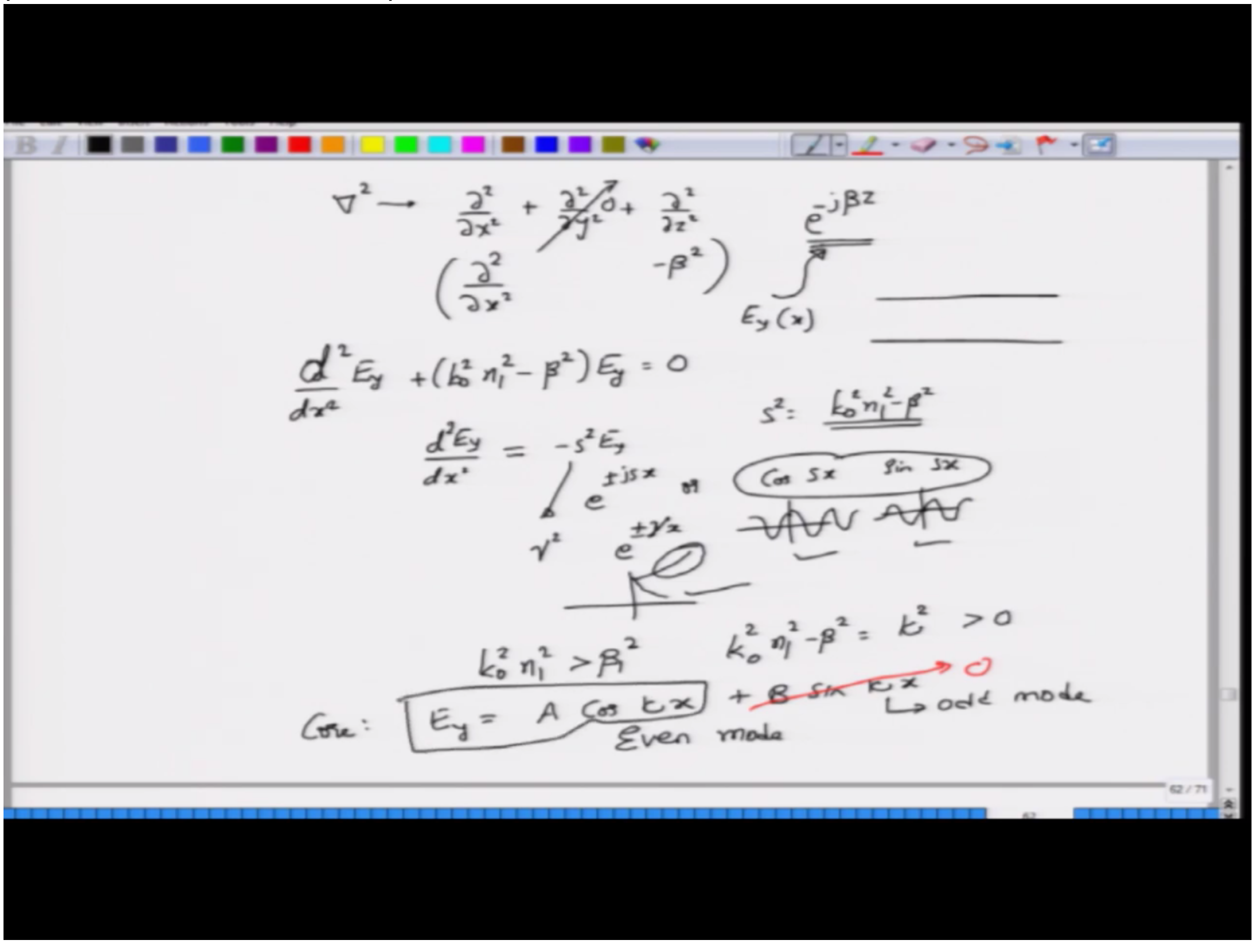

What would be the solution for cladding, what would be the solution that you would expect for cladding? Well I know that if at all this is in the form of a sinusoidal function or a cosin function, right, then outside we must have an evanescent wave, which of course decays as you move away from the interface, right so that is what we have already seen. And the same behavior should be expected in the down, you know, in this side of the cladding as well. So what we hope for the solutions in the cladding would be of the forms C E Par – gamma X –A, I will tell you what gamma is in a minute, and then you have D E Par – gamma, E Par gamma + X + A. The reason why we have taken X – A is just to simply the exponential function, cos at X=A this will be just C and at X=-A this will be equal to D. Of course from symmetric conditions, you would expect C to be = D and that is actually true, okay. So the solution will DK exponentially on both sides with the same DK coefficient and with the same starting points at the boundary. Now what is gamma. Remember the solutions in the cladding are actually the solutions of Helmholtz coil equation Dr. square EY/DX square + K0 square and 2 square – beta square, okay. If you ask or if you demand that beta is greater than K0 square and 2 square, okay, then you see that, sorry, beta is greater than K0 N2 not N2 square. When beta is greater than K0 N2, clearly that you are going to, it is clear that you are going to get a decaying solutions for DY, which is what you have actually gotten here, okay. I have shifted the coordinate system on the exponential by making sort of E Par – gamma X, I have made it into E Par – gamma X – A. If I don't shift the solution then that extra term will also be present in the solution, but that would be a constant, okay. So it doesn't matter if you just take E Par - gamma X, but to simplify the boundary conditions, I have just moved the coordinate system for the exponential to be along or to be starting at H=+A or at –A. Now we have the full set of solution. We can now go to the next step. What is my next step? Next step, graphically I have already shown. Basically what you are trying to do is to find out what would be the values of A and C for a given kappa and gamma or you are even adjusting kappa and gamma in such a way that the solution smoothly changes over from this cosin wave in the core on to an exponentially decaying wave in the cladding, okay. So that is what graphically we are doing, but mathematically what that would mean is at X=A boundary, you have A cos kappa A, which would be equal to C, which is obtained by making X= taking X=A in this equations. So what you have is A cos kappa A=C. At this point I don't know what A and C are, but that is fine, we will leave this expression here. Now I won't get much by applying the boundary condition at X=-A instead what I want is a boundary condition on HZ, right or I need form for Hz. But how do I find HZ? Well I know Maxwell's equation, it tells me that Del cross E=-J omega Mu knot times Hospital, correct. And I know E is along EY, expanding this Del cross E and equating the terms on the left hand and the right hand side, which again I will leave as an exercise, because we don't have much of a time, you can show that HZ will be equal to -1/J omega Mu know, DEY/DX, okay. So you can show that this is what you are going to get. And once you know what is, I mean, once you know the expression for HZ, and you know the expression for EY in the core and cladding separately, so you can differentiate it. So for the magnetic field component HZ, you will get a solution, which would be kappa by J omega Mu know A sin Kappa X and on the cladding side you get gamma C by J omega Mu knot time E to the power – gamma X-E, okay. So these are the solutions. Here I will leave it as an exercise for you to show that the solution will be –gamma C by J omega Mu knot E to the power gamma X+A. Please note that this region is basically X greater than –A, okay. And this region is basically X greater than A and of course this region is where –A is less than X less than A. Now you can apply boundary condition at X=A for the tangential magnetic field, that is HZ component and once you apply the boundary condition for HZ component what you get is kappa A sin kappa A from this expression, should be equal to gamma times C, okay. Because at X=A this exponential will be 1 and J omega Mu knot on both sides will actually cancer off, so you will get this kappa A sin kappa A = gamma C. We already know that C is given by cos kappa A, so I can replace in this expression C/A, so I have gamma A cos kappa A, indeed arranging the terms you get tan kappa A to be = to, so A will cancel on both sides, sin by cosin is tan kappa A, you get gamma by kappa, okay. So this is an interesting expression, unfortunately it does not have straight forward solutions.

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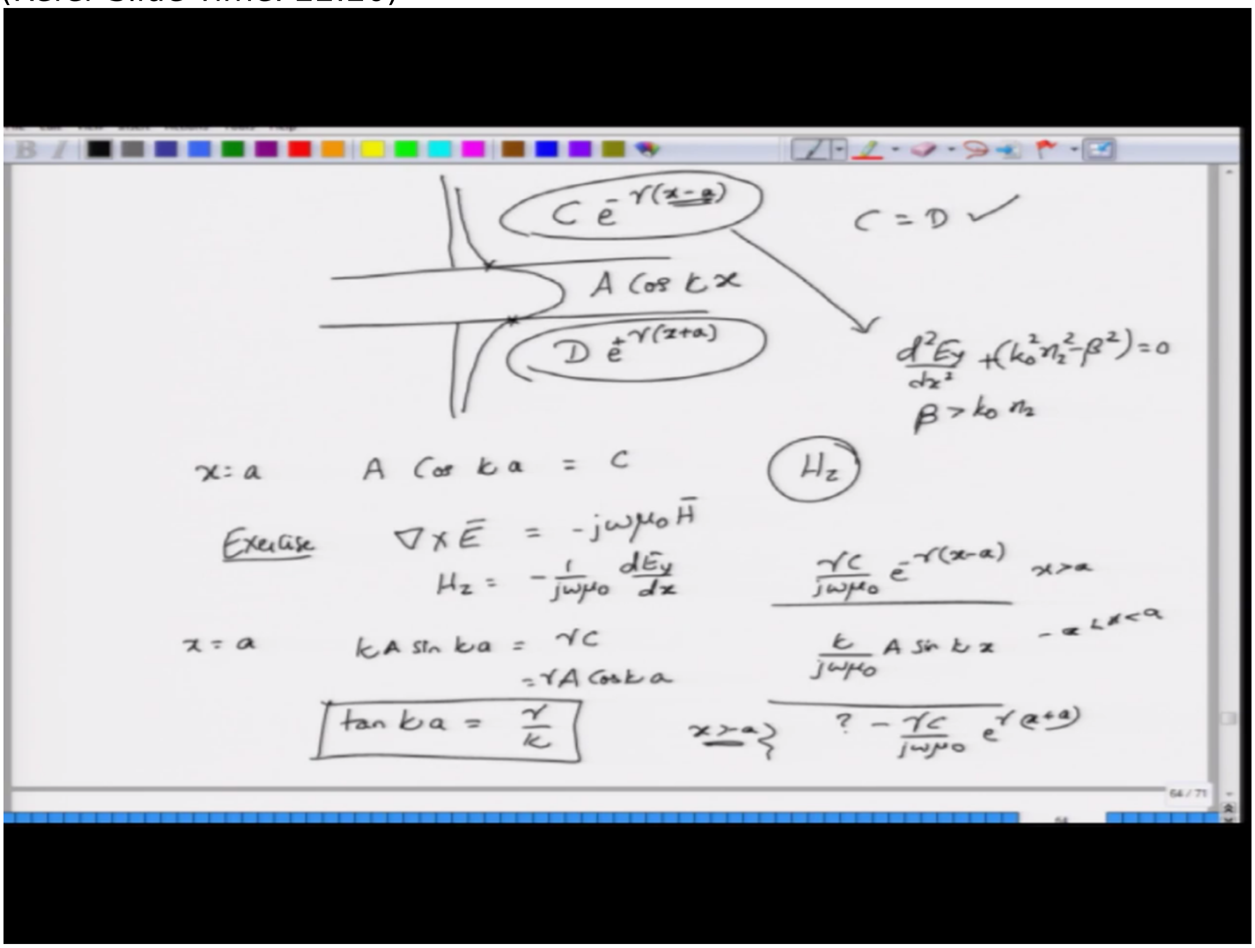

The solutions that you are going to get are all what is called as numerical solutions, because you have a function of the form tan X=X and this solution is rather you know easily found using graphical methods or numerical methods. Graphically what this is telling you is that you have X in this manner and then you have say tan kappa X, so you have this expression and then again this one, these are the solutions that you are going to get for tan, right. So if you plot this as a function of X, these are the solutions and all the intersections that you are going to get will be the solutions. And of course what you have observed is that there are multiple solutions for this tan X = X kind of a solution, right. In our case what we have is tan kappa A = gamma by kappa, okay. If I multiply on both sides by A and then define kappa A as equal to U and gamma A as W okay then I can even rewrite this equation as tan U=W/U, okay. And what is U? U is basically kappa times A and what is kappa is square root of K0 square N1 square – beta square, okay x1. And now W of course is given by square root of beta square – K0 square N2 square XA. And what you observe here is that this equation that we have written tan U=W/U implicitly contains beta. So the solution method that we normally adopt is to express this W in terms of U and you can actually do that by writing U square + W square as what is called as V square. What is this U square + W square, that is basically K0 square N1 square – N2 square times A square, which we will define as parameter V and V is an important parameter that is determined only by the wave guide geometry and materials. In this case it is only determined by the wave guide, material property, and the operation wave length lambda, right. Wave guide has a radius A, which is known lambda is the operating wave length, which is also N1 and N2 of the wave guide are also known or usually can be designed knowing certain other properties of this V. But the important point is that the solution that you obtain will be of two steps, okay. After you express W as V square – U square underfoot. So on the left hand side you have a function of U, right hand side also you have a function only of U, and you can plot the solution as a function of U, or rather plot the left hand side and the right hand side as a function of U, okay. So on the left hand side it's just a tan function, it will have different values in this manner, so this will be 0, this will be say, so this is a pi by 2, this is at pi and so on it is a periodic solution, right. And on the left hand side what will happen at, when VU… U is very small, when U goes to 0, this numerator is finite, denominator is infinite, so you will start somewhere of the infinity value and as you start to increase, it will go to 0 at U=V. So if you know what is the value of V, as determined by let's say your properties of the material or properties of the wave guide, you would actually get a value of V here, which is determined only by the wave guide and now your solution has to be until this point, right. So you start off with infinity and then you go to the solution until at this point, where it kind of decays off. How many intersections do you see, you see two intersections one intersection here that corresponds to the fundamental mode that would be propagating and this next one is called as the higher order mode that would be propagating. So in fact you get multiple modes which are propagating. The first intersection of this one will be denoted as TE one more, the next one will be denoted as TE 2 mode and so and these are all the even modes that we have considered, okay. In the odd mode scenario things are slightly different, there the relation on the right hand side or the left hand side actually changes slightly and without going too much into details, you can plot the odd mode cases, and what you observe is an intersection, which would be present in this manner as well until this value of V here and what you notice is that you will first get an even mode, then you will get an odd mode, then you will get an even mode. Odd modes are basically sin functions, okay and the higher order modes, that cosin functions are the even order functions.

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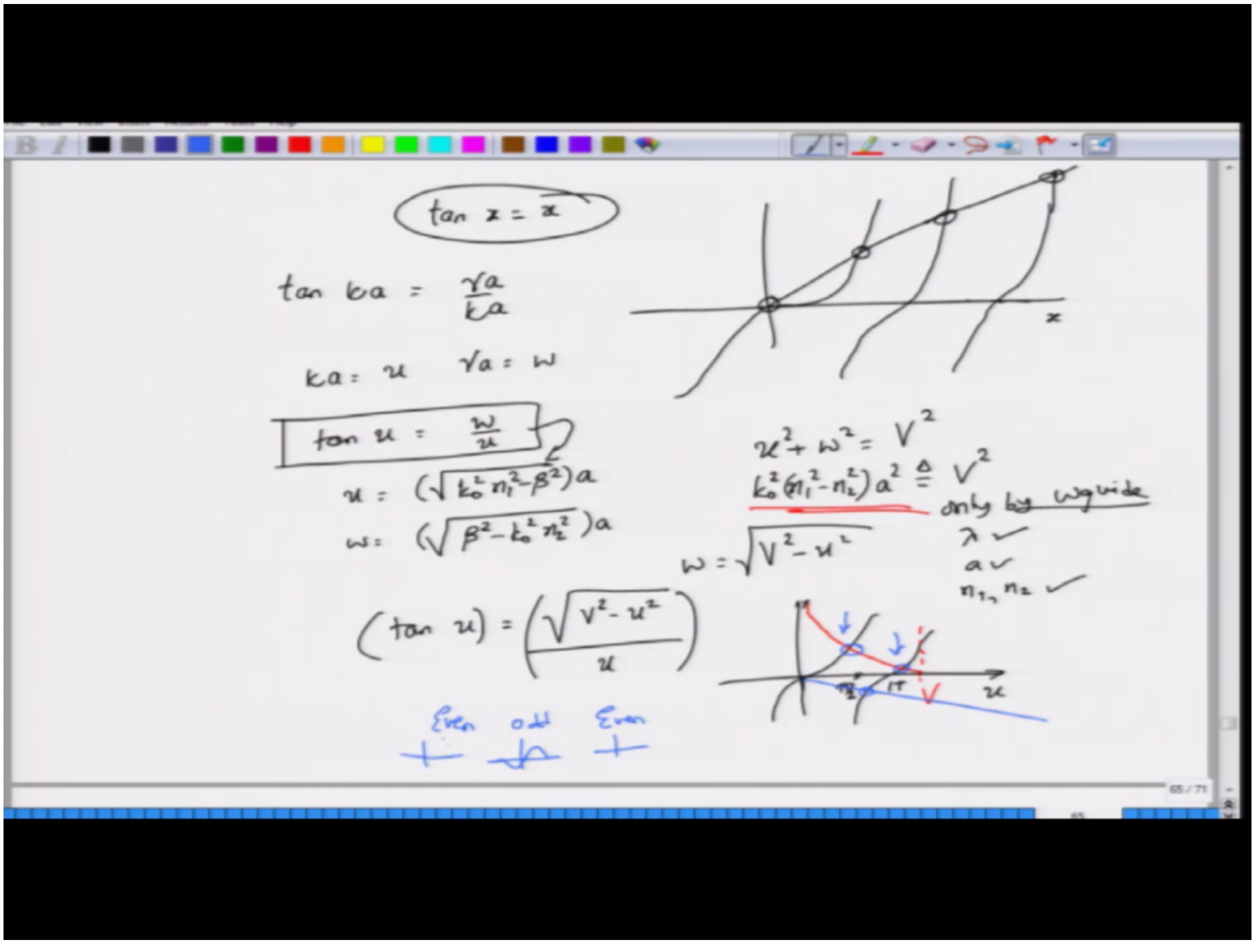

So if you were to look at the fundamental mode assuming that this your boundary for the core end cladding, the odd mode would look, sorry even mode would look something like this. This is a cosin wave and then it's exponentially decaying everywhere, and the even mode would look something like this, okay. So these are the, even I have not drawn the pictures nicely, but this is how the even and odd modes would look like and these are the fundamental modes. The higher order modes will be, you know will have more maxima in between. So this completes our basic understanding of wave guide mode. So to... take... the take away message from this module is that mode is essentially solution of wave equation or equivalently Maxwell's equation, meaning.. solution meaning the E and H patterns that I am actually interested in or the E and H solution that I am interested in and this should not only be the solution of Maxwell's equations, but they should also satisfy boundary conditions, okay. In the free space case, these modes were essentially plane waves, they were actually in the form of E Par – J beta Z for a Z propagating wave or E Par plus J beta Z for a Z propagating wave or E par + J beta Z for a Z propagating waves, okay. There were no boundaries. However, the modes you can think of, the free space modes were the plane waves, okay. There were no boundaries, however, the modes you can think of, the free space modes were the plane waves.

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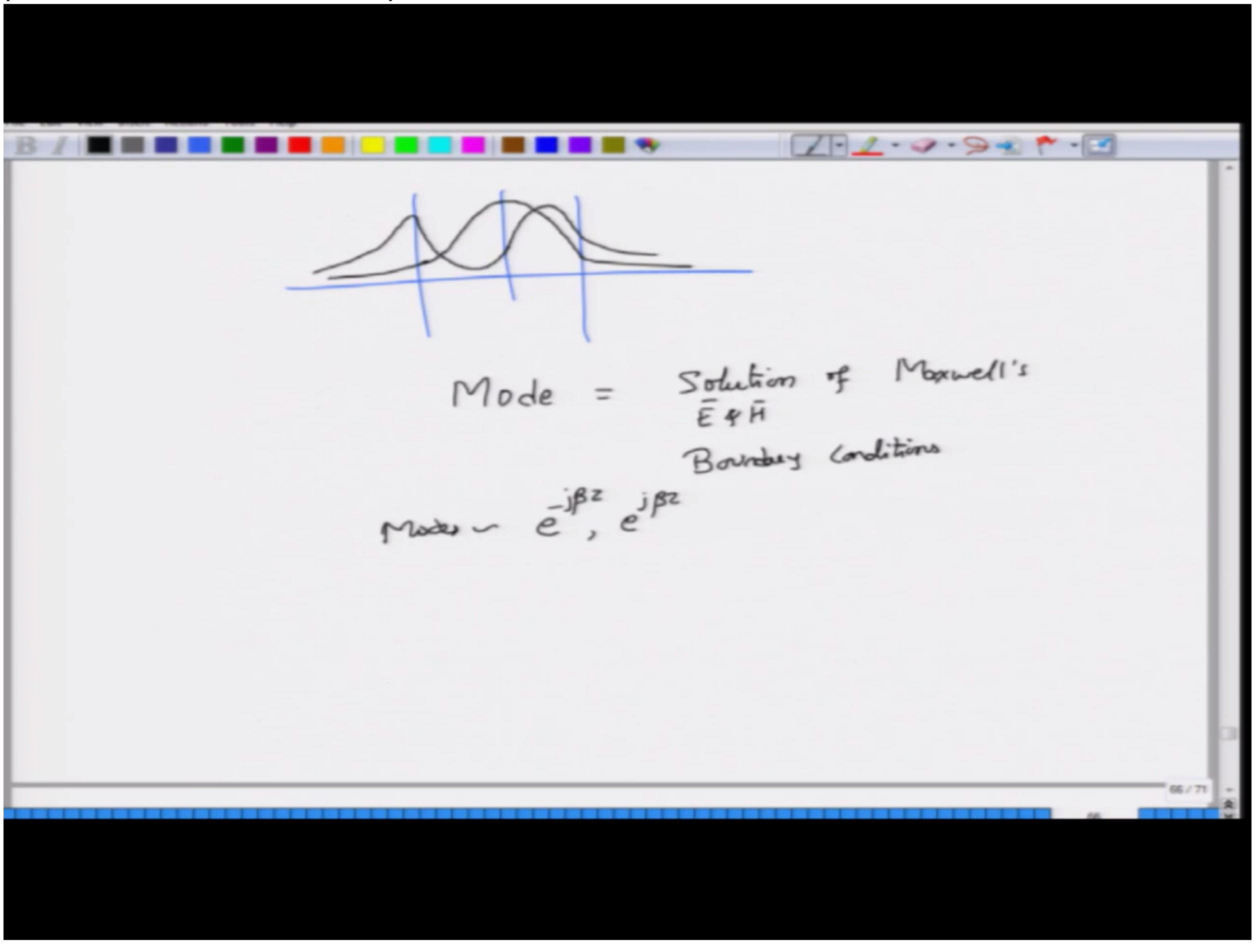

However, the mode structure in the case of a guided thing is actually very different. So in the guided media, the mode structure depends on the geometry. For the case of a planar wave guide that we have considered or what is called as slab wave guide, the mode structure is either cosin or sin with many-many periods within the core and decaying exponentially in the cladding, of course propagating along the wave guide itself, right. So what you have to understand is that this wave guide mode or mode in general is a property of geometry. When there is no geometer, when there is no constraints, we get a plane wave, but when we start putting constraints, the solution changes over, and depending on the type of the constraint that you put in, that is, if… if you send in parallel plates, the solutions inside will be given by sinusoidal functions, it will be cosin or sinusoidal. However, if you instead of shrinking this, I mean, putting this one in this manner, if you make cylinders, the functions will still look like sin and cosin, but they will be mathematically actually bethel functions, okay. So the type of geometry determines the form of the function or form of the electric and magnetic fields that can propagate within the given wave guide and the solution of Maxwell's equation, which also satisfies boundary conditions is called as a mode and we have just successfully solves for modes of a symmetric slab wave guide. Thank you very much.