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Indian Institute of Technology Kanpur

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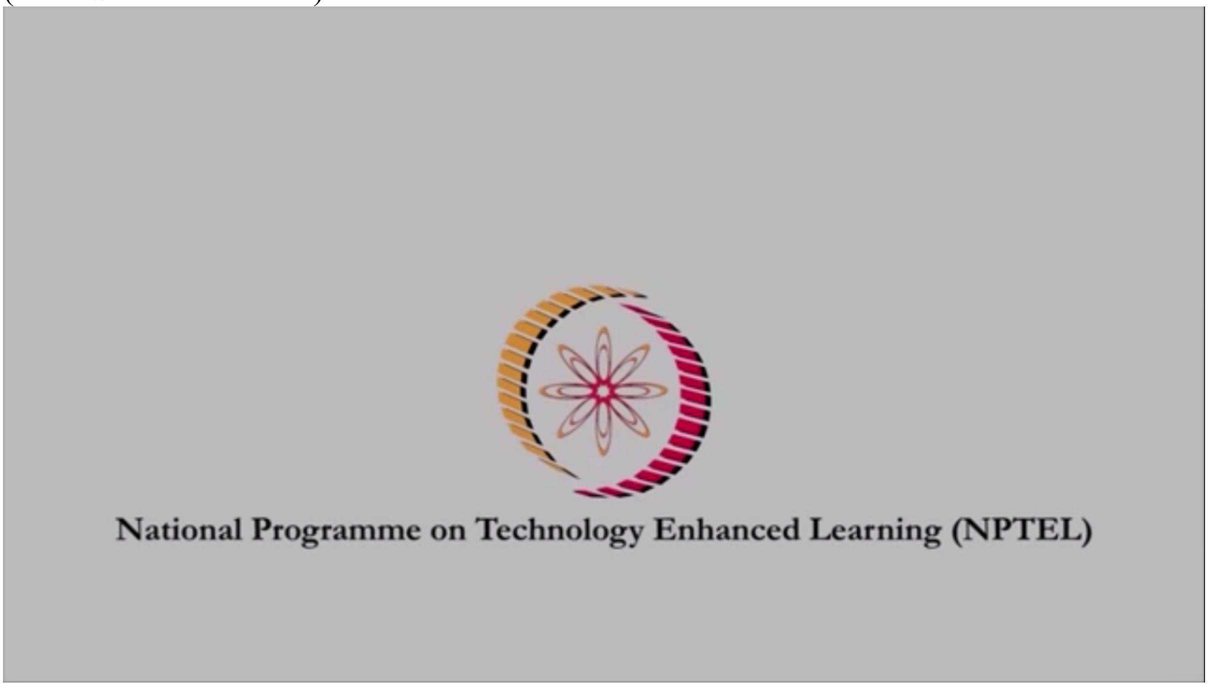

National Programme on Technology Enhanced Learning (NPTEL)

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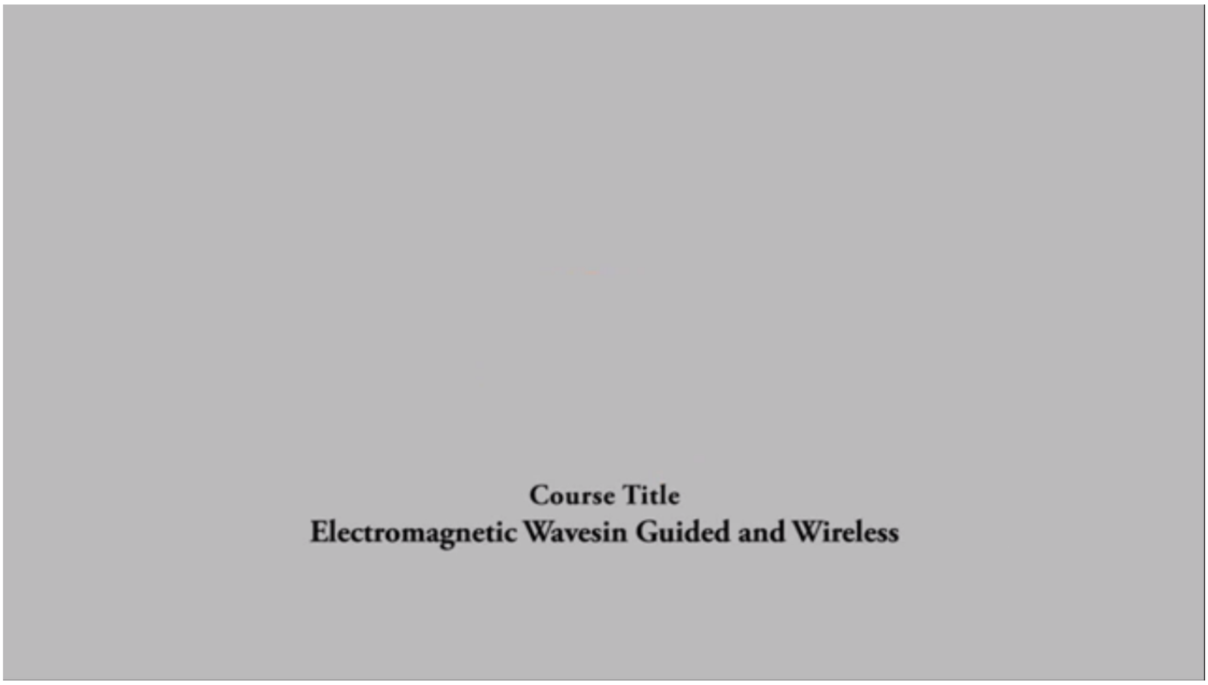

Course Title
Electromagnetic Waves in Guided and Wireless

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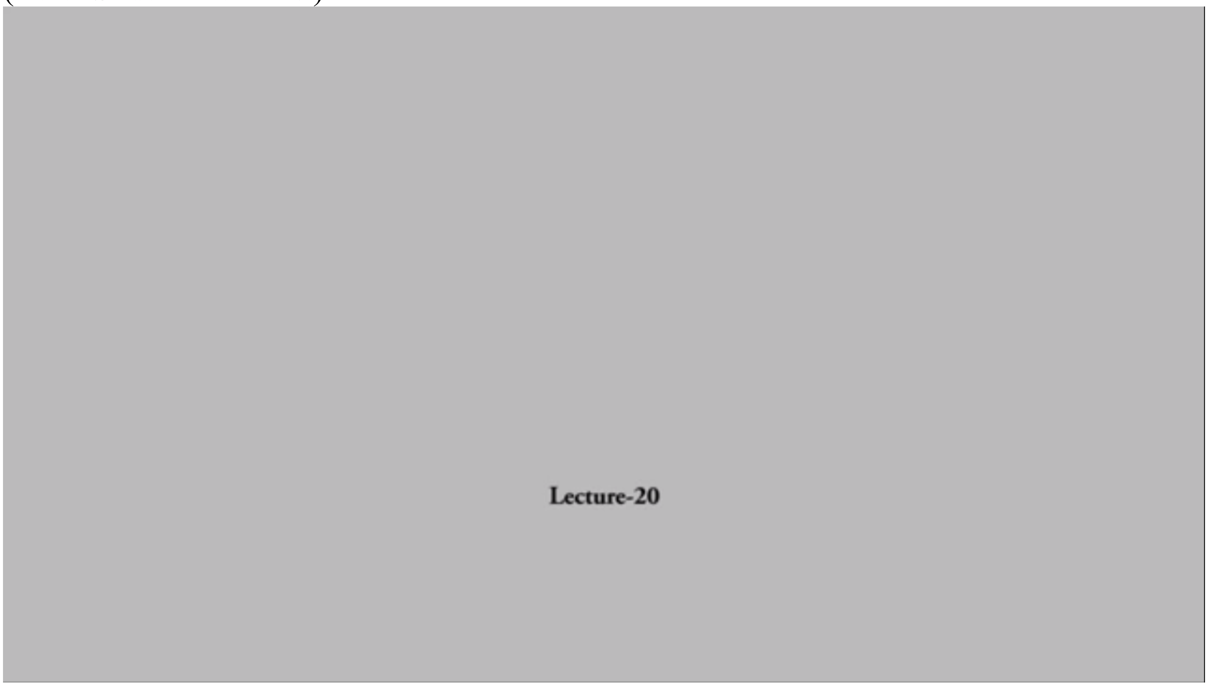

Lecture-20

by

Dr. K Pradeep Kumar
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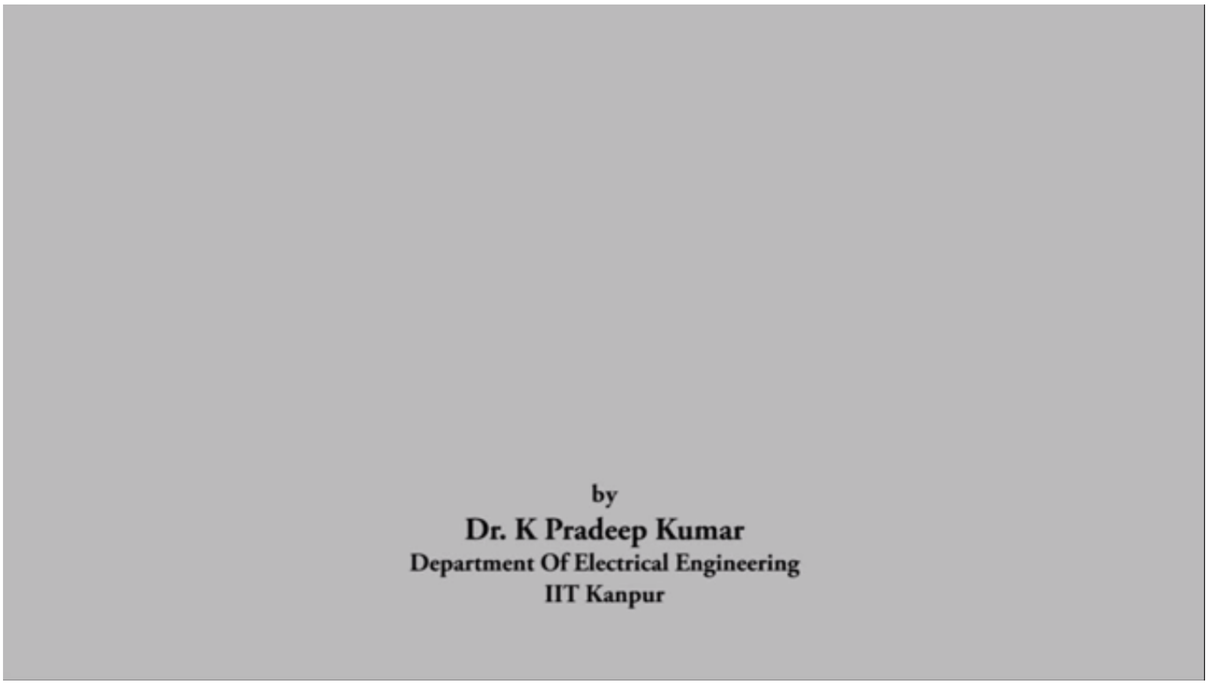

Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media.

This is our continuation of the previous module where in the previous module we, you know, discussed the oblique incidence of a uniform plane wave from medium one to medium two. Both medium one to medium two were assumed to be perfect dielectric. We will continue to assume that one, and we had set up the essential equations, but we did not derive the final expressions for reflection coefficient and transmission coefficient, which is what the goal of this module is, right?

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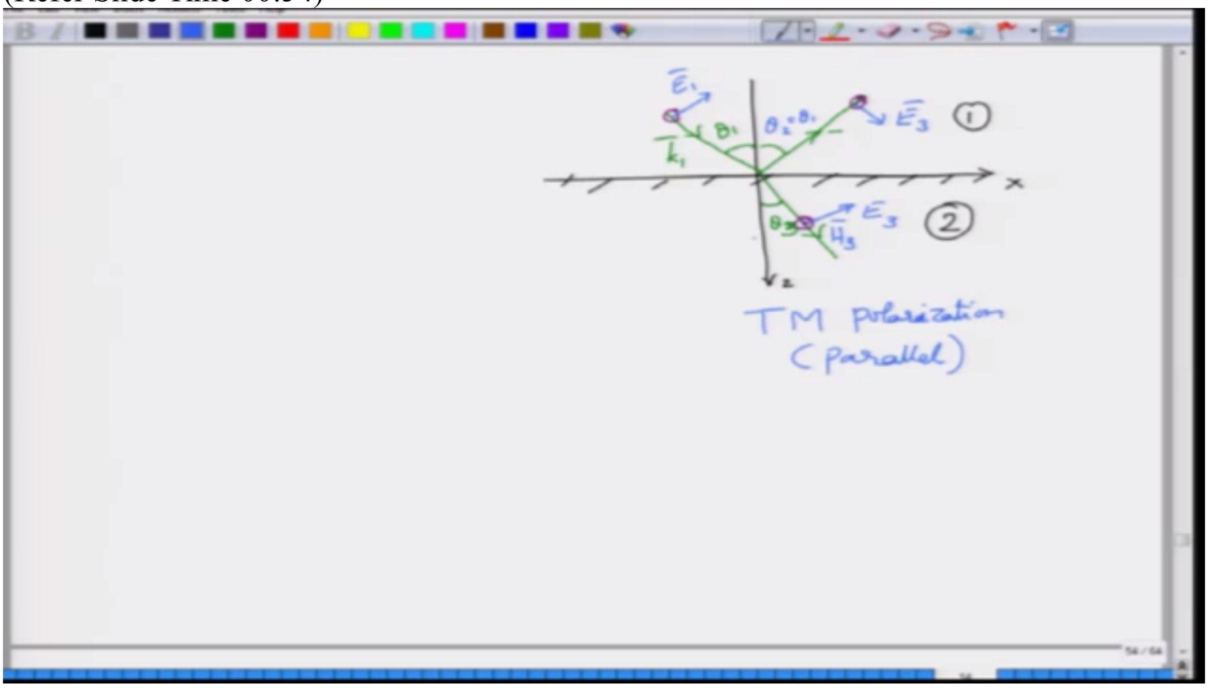

So we begin by, you know, looking or recalling this figure that we had drawn earlier. So there are two media, which is supported by $z = 0$ plane. Okay. So the normal to the interface is basically along the Z direction and we were considering what is called as transverse magnetic polarisation in which the electric fields are given in this particular manner. The magnetic field in, you know, associated with electric field E_1 would be coming out of this board perpendicularly and then magnetic field H_3 would also be coming out in the same manner perpendicularly.

However, the magnetic field for the electric field E, sorry, this is E_2. The reflected field is E_2. So for that E_2, the magnetic field H_2 would be into the board. So this would be out of board for the case of H_1 and H_3 whereas H_2 will be into the board. The directions are chosen in this particular manner so that the incident field which is associated with E_1 and H_1, the components of that particular plane wave. When you do a right angled, you know, rotation starting from E_1 along H_1 if you curl around, then the direction of the propagation of the wave is towards this particular medium, right? So you have this E_1 and H is coming out, so the curl rule, the right-hand curl rule will give you a wave which is propagating in this direction.

Similarly with the assumed E_2 direction in this manner and H going in to the wave, so you have this E_2 in this manner and H going into this one. So if you now turn the screw kind of a thing right-hand rule, it will tell you that the wave is propagating away from the interface. Okay. So the directions E_1 and E_2 as well as E_3 and H_3 were all chosen such that the wave is propagating in the incident medium. The incident wave is propagating towards the +z direction; the transmitted wave is propagating in the +z direction whereas the reflected wave is propagating in the -z direction.

Now with the help of this diagram and the fact that we know boundary conditions mean that tangential electric fields and tangential magnetic field in this case must both be continuous across the interface, you can write down two equations and with two equations you can actually obtain the ratios of the transmitted electric field amplitude to the incident electric field amplitude. This ratio we will call it as a transmission coefficient. Similarly, you can write down the ratio of the reflected electric field amplitude to the incident electric field amplitude and we will call this as a reflection coefficient. Okay.

There are three, you know, electric fields in this problem. You have $E_{1,}$ E_2 and E_3. However, E_1 and H_1 are usually fixed by the input conditions because it is coming from some source and we know what is the power that the incident wave is usually carrying, so it is necessary or it is sufficient only for us to just calculate the reflection coefficient and the transmission coefficient. The remaining components or the remaining metrics of this problem such as the reflected power, transmitted power, and if at all there is a power loss, they can all be calculated with the help of these two coefficients.

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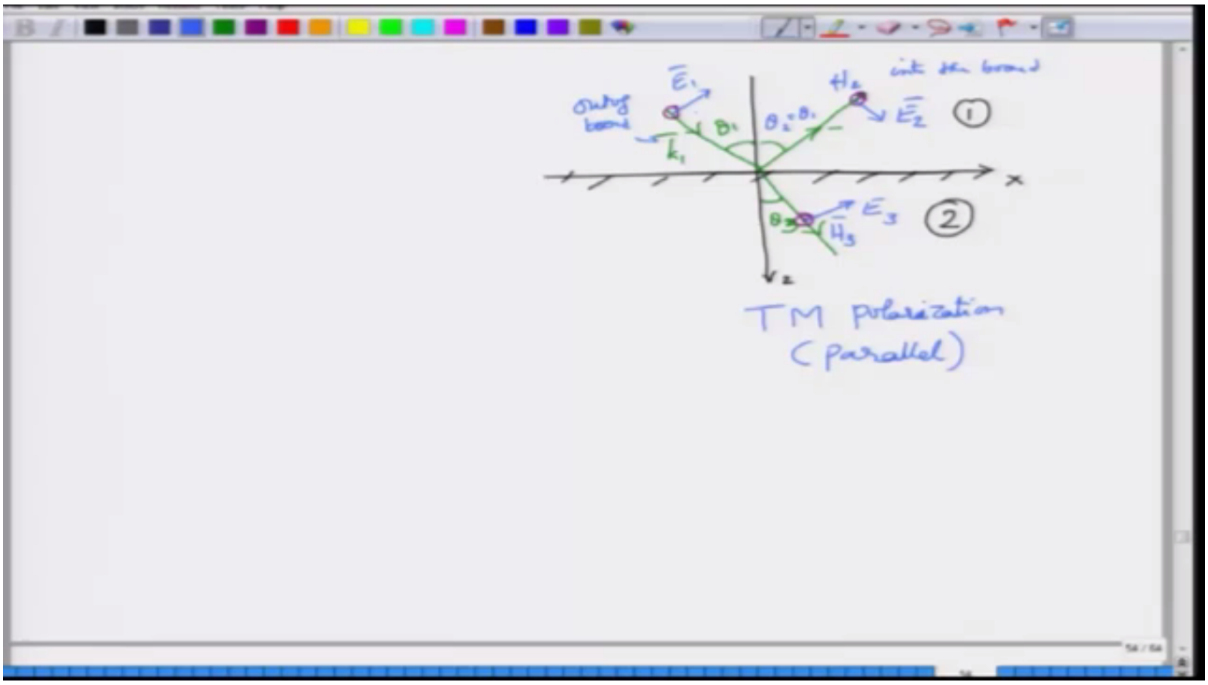

So we have shown what the components, the tangential components are going to look like. Okay. So I won't repeat the discussion there, but I'm going to write down the tangential electric field here. What I would suggest is that you observe the figure carefully with the angles θ_1, θ_2 and θ_3 taken. We also know, of course, how θ_1 and θ_3 are related. θ_2, which is the angle of reflection, is basically equal to the angle of incidence from first Snell's law, and this θ_3 and θ_1 are themselves related by the second Snell's law.

Now what we do is before we write down the set of equations, what we are going to do is to make a small change in the notation. Because we know θ_2 and θ_1 are essentially equal to each other, we are going to rename the angles. We will call this as θ_i meaning that this is an incidence angle, θ_r for the reflected angle, which of course is equal to θ_i, so we don't bother about that anymore, and this angle we will call as θ_t, t standing for transmitted wave. Okay.

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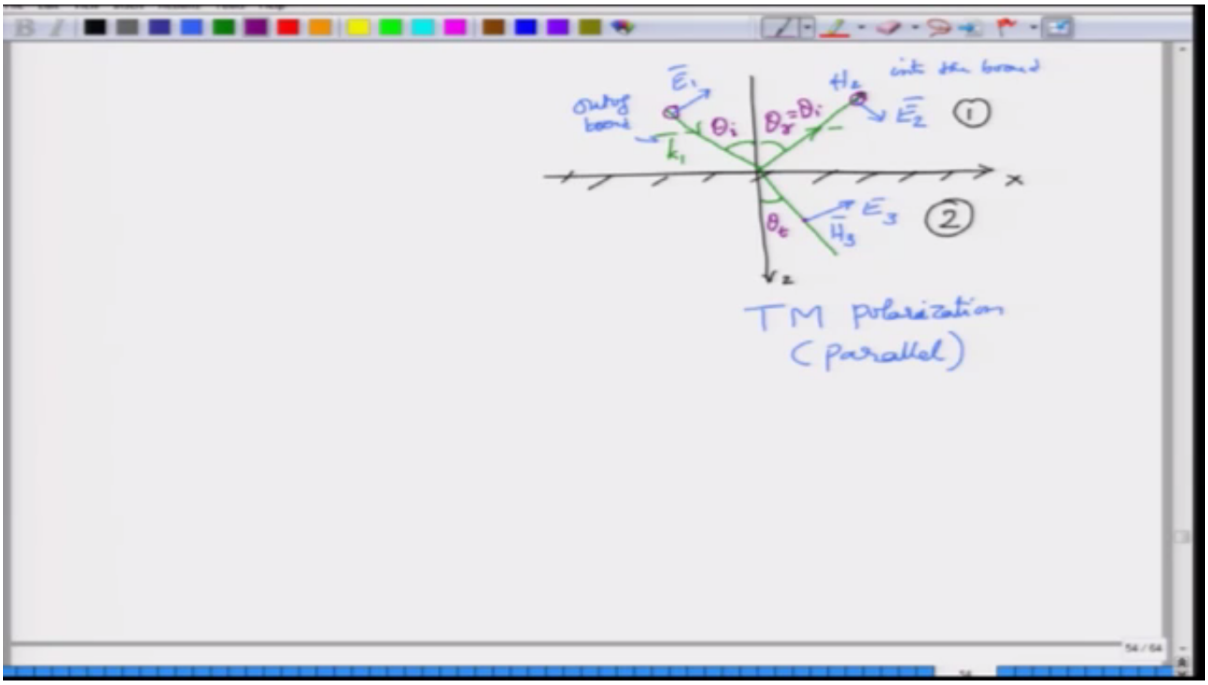

So this is with these new angles that we have written. This is just to avoid θ_1, θ_2 and θ_3 kind of this one, but that will give you the idea that this is concerned with an incident, reflected, and transmitted wave. We have relabelled the angles. Okay. With that, the set of equations, the tangential electric field on region one which must be equal to the tangential electric field in region two at the boundary surface z = 0 and for all values of x turns out to be E_1 Cos θ_1 + E_2 Cos θ_2 must be equal to or sorry, we will write this as 1 and 2, right? So we are going to write this as E_1 Cos θ_i + E_2 Cos θ_i only because θ_r would essentially be equal to θ_i and then I have E_3 Cos θ_t. Okay.

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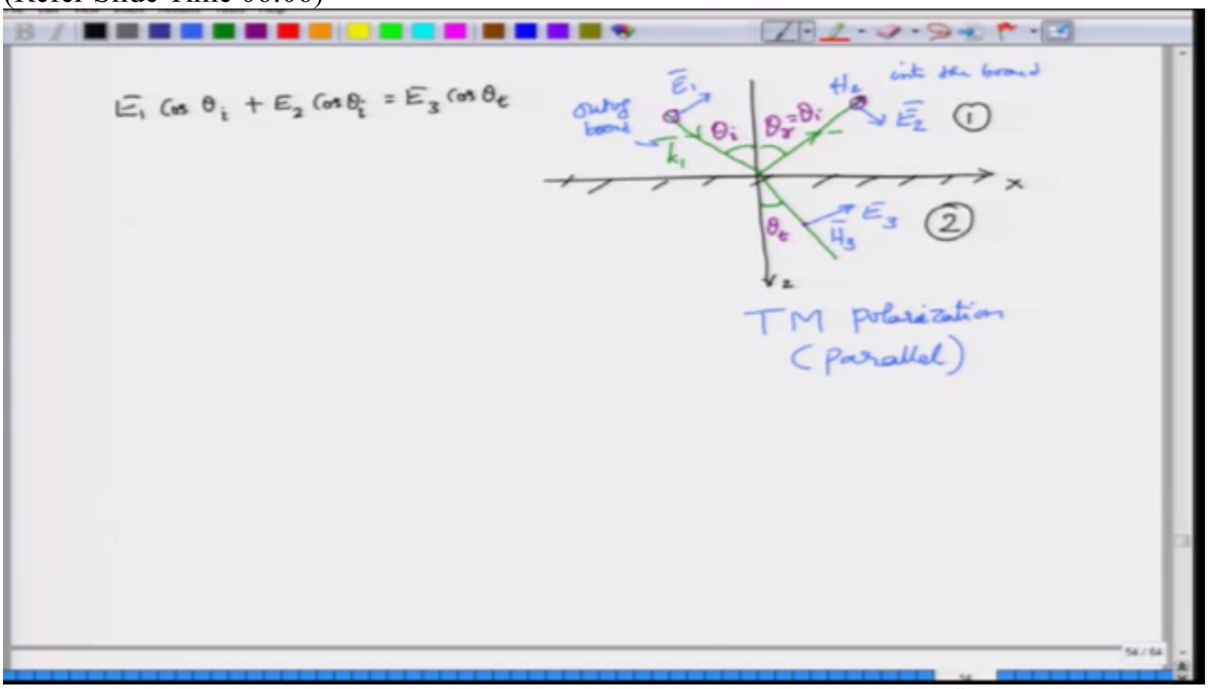

If you are worried about what happened to the phase factor namely this $e^{-jk.r}$ corresponding to the incident wave, and reflected wave, and transmitted wave, we have shown that for the boundary conditions which have to be valid at the entire z = 0 plane and at any x and y point, right, on the plane, we know that these phase factors essentially have to be equal. In fact, equating these phase factors gave us Snell's law. So I hope where this equation is coming from is very clear. Okay.

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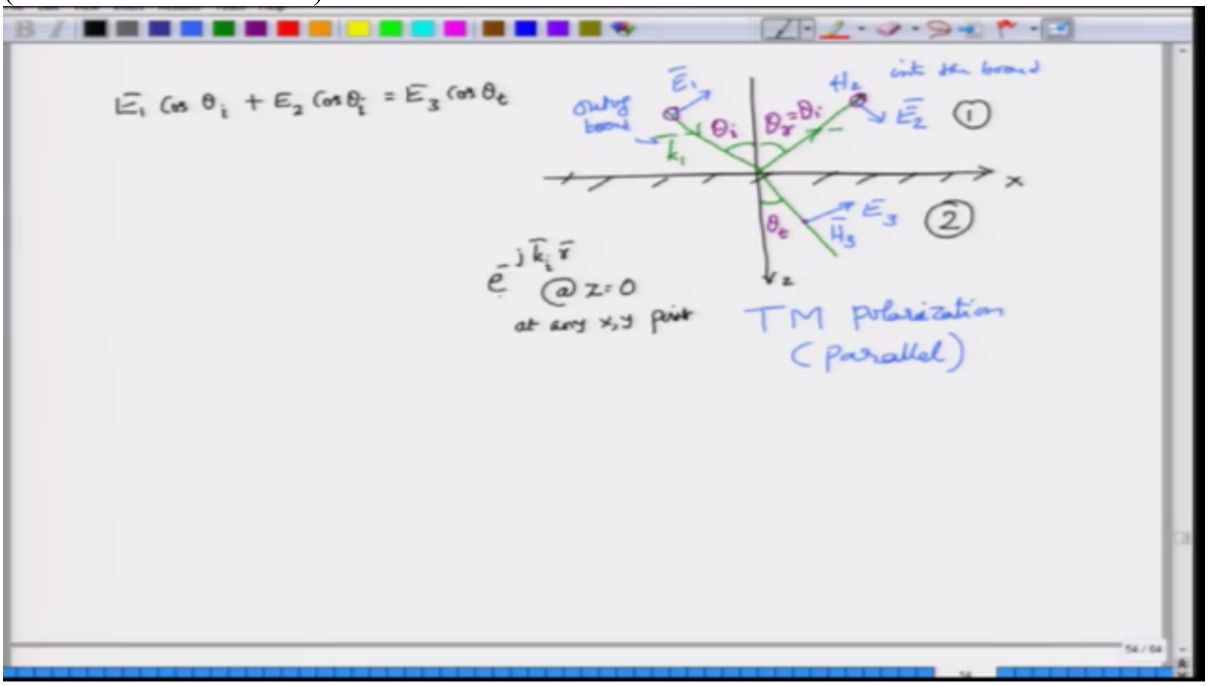

Now what about the magnetic field? Well, magnetic field will be all tangential to the interface, okay, so that you have H_1-H_2 to be equal to H_3. In terms of E, we can write H_1 as E_1 divided by η_1 where η_1 is the medium impedance. η_1 is square root of omega, sorry, square root of μ by ε. Correct? So this is square root of μ by ε, and H_2 is given E_2 divided by η_1 again. Why η_1 again? Because both incident and reflected waves are in the same plane. Okay. So this would be equal to E_3 divided by η_2. That is for the second medium. Okay.

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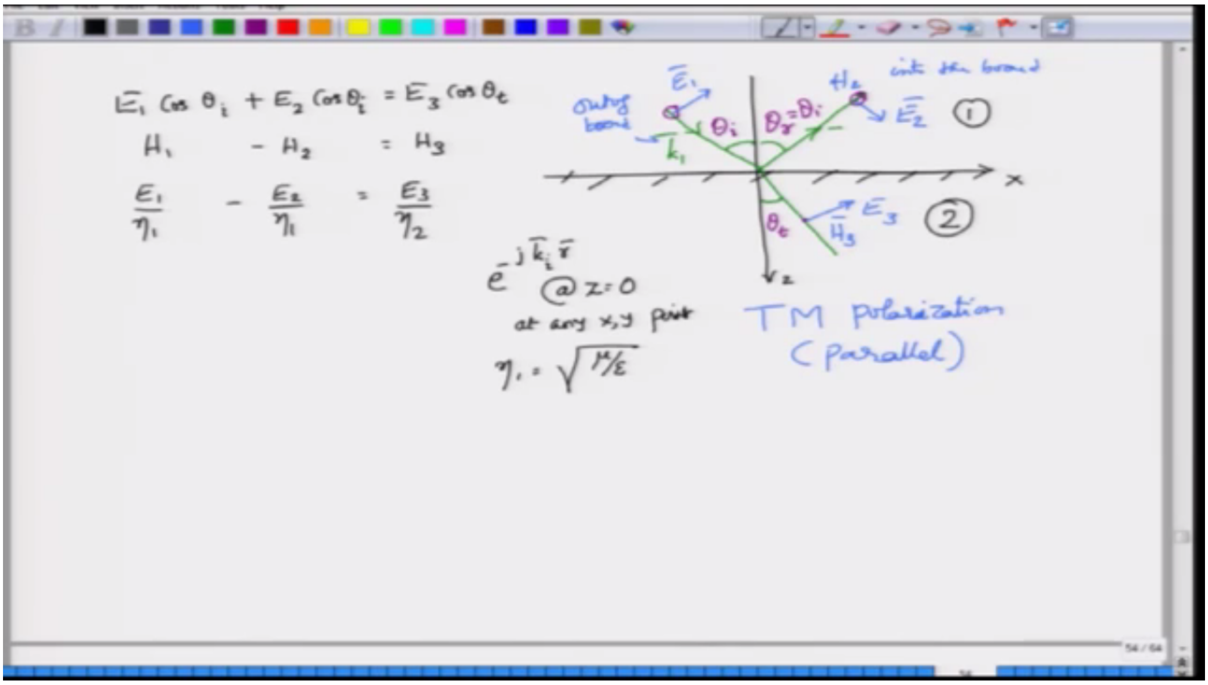

Now that we have these, you know, equations with us, I will not simplify them further, but I hope that you are able to take this as a small exercise and then show that the ratio which we will call it as T_{Tm}, okay, so this is T_{Tm} as the ratio of E_3 to E_1, okay, and call this as a transmission coefficient. So this we will call as the transmission coefficient, which is basically the amplitude ratios. Okay. So you have to keep in mind that this is amplitude ratio, not power ratio, and you can show that this expression can be written as 2 η_2 Cos θ_i divided by η_1 Cos θ_i + η_2 Cos θ_t. Okay.

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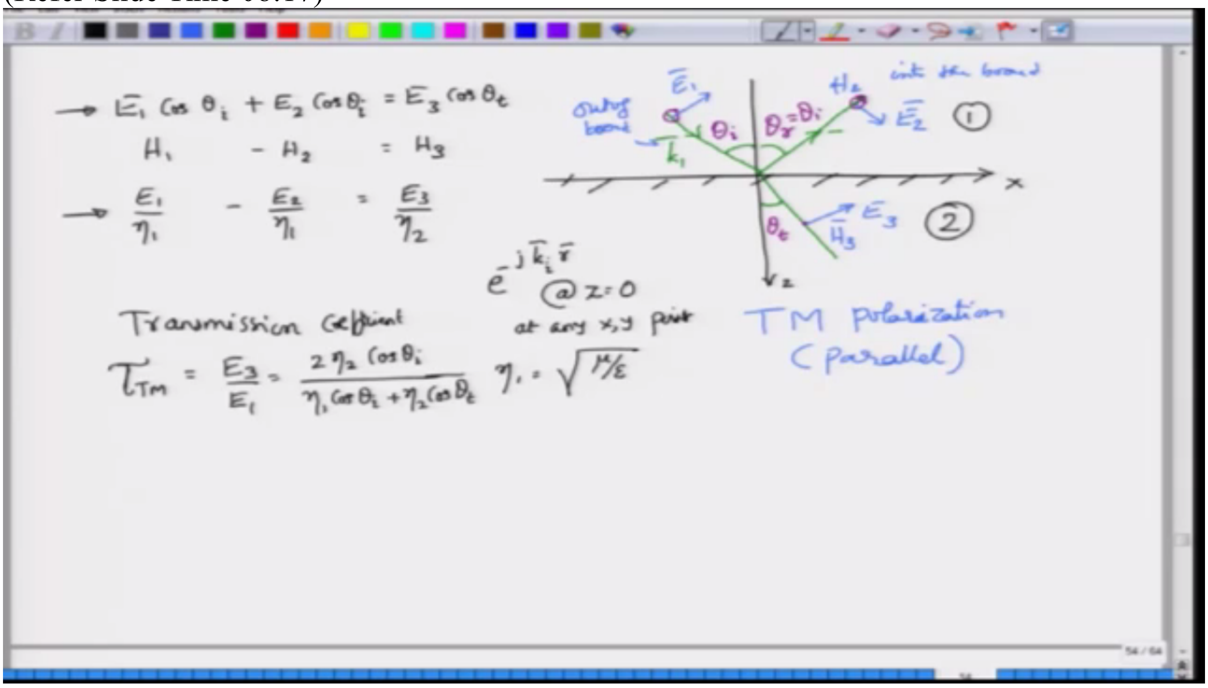

So I would encourage you to show that this equation what we have written, you know, comes out all right. So you can check that we have done our, you know, this one. So you have η_2 Cos θ_1. So hopefully everything checks out correctly and show that this is the equation. Okay.

Now you can also show that the reflection coefficient, which would be the ratio of E_2 to E_1, which further can be written as (E_2/E_3) times (E_3/E_1), and this particular thing we already know to know and you can find out E_2/E_3 by subtracting these two equations: equation 1 and equation 2 and then adjusting the ratio, I mean, adjusting the equations to get this required ratio, and we can show that this is basically given by (η_2 Cos θ_t - η_1 Cos θ_1) divided by (η_2 Cos θ_t = η_1 Cos θ_i). Okay.

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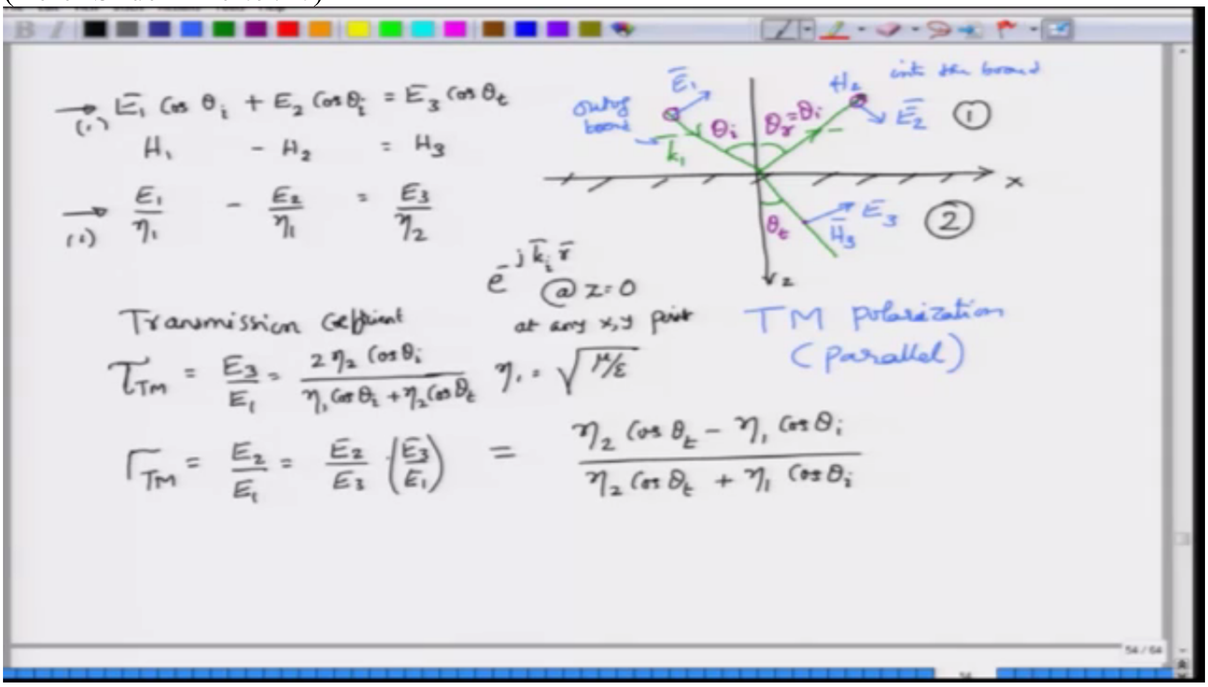

Now observe this equation very, very carefully. Okay. If I rewrite this equation by saying that this is basically like η of the second medium corresponding to Tm minus η of the first medium corresponding to Tm polarisation divided by $\eta_2^{Tm} + \eta_1^{Tm}$. What will you observe? You'll actually think that this is equivalent to a transmission line whose input characteristic impedance is η_1^{Tm} and it has been terminated in a load, okay, whose impedance is η_2^{Tm} provided you associate this η_2^{Tm} as the medium impedance η_2 times Cos θ_2 or rather Cos θ_t and you identify η^{Tm} as η_1 Cos θ_i.

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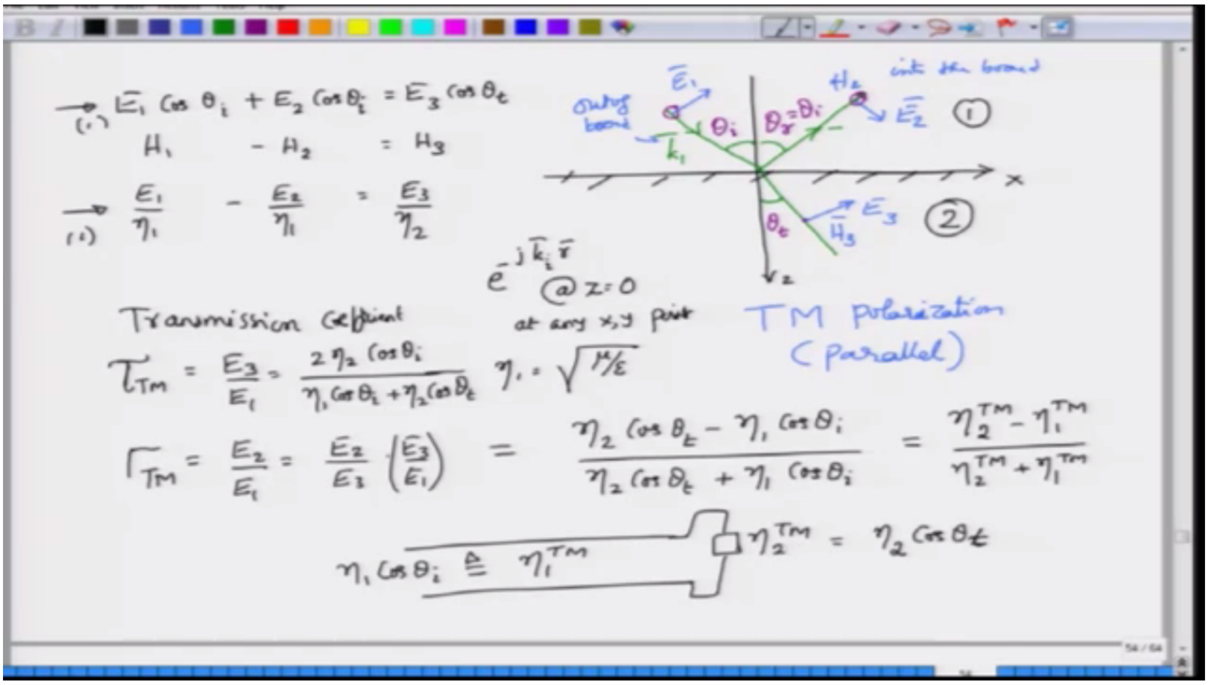

So depending on the angle of incidence, if you just identify the transverse magnetic impedance, you know, wave impedance or the intrinsic impedance of the second medium to be η_2 times Cos θ_t and η_1 times Cos θ_i, then the equation that we have written for Γ_{Tm} follow very straightforwardly. It looks like we simply have this as η_2^{Tm}. This is like reflection coefficient of a normally incident wave. Alternatively, this is equivalent of a transmission line in which the characteristic impedance of the terminating or the load impedance of a transmission line is η_2^{Tm}, which is basically η_2 Cos θ_t and connected to an infinitely long lossless transmission line having the characteristic impedance of η_1^{Tm}. Okay. So this situation that we have drawn here in the characteristic in the transmission line is exactly equivalent to the plane wave that we have been considering. So this is the one-to-one correspondence up here. Okay.

Now we will not derive the corresponding equations for the TE polarisation. TE polarisation is also called as perpendicular polarisation. Okay. The reason is very simple. You will see once I draw the picture. As before, we will assume that z = 0 separates the two interfaces, medium one and medium two, sorry, separates two medium with an interface at z = 0. This is your x-axis, z-axis.

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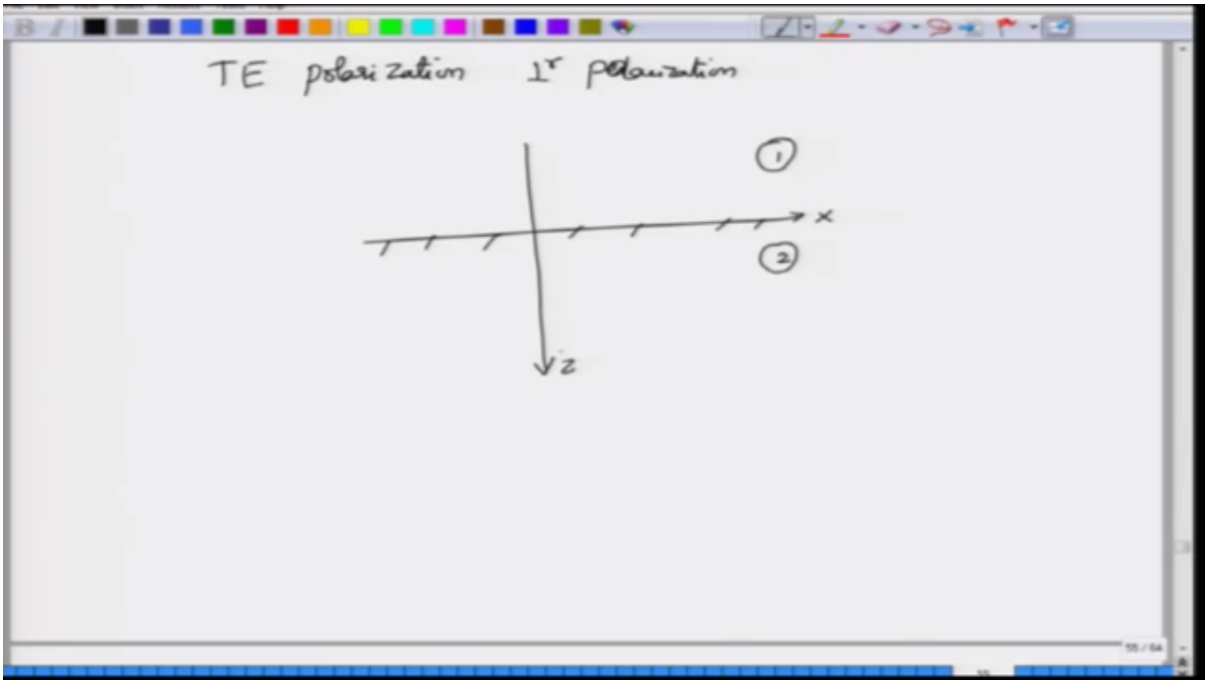

Now as before, you will have an incident wave; you will have a transmitted wave and then you will have a reflected wave, each of them having their own k_1, k_2, and k_3 vectors. Okay. And instead of the kinetic field being transverse everywhere, we will assume that it is the electric field which is coming out of plane. So this would be E_1. Somewhere here let's say this would be E_2 and here this would be E_3. And in this case, we will assume that all these electric field components associated with the incident, reflected, and transmitted waves are all actually coming out of this particular board. Okay.

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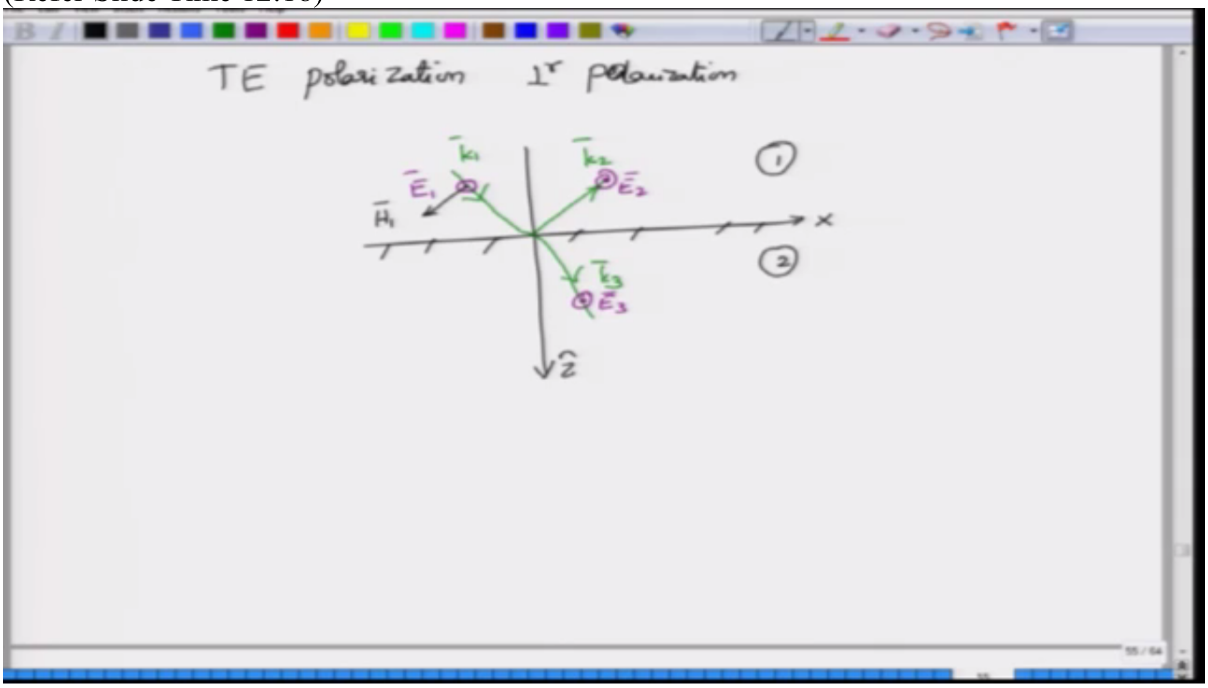

And then you have to adjust the directions of H accordingly. So if you take the direction of H to be say H_1 in this manner, you can show very clearly that if E_1 is coming out here and this is your z-axis, so you can see my thumb. Then you take this E x H. Then the corresponding vector will be perpendicular and that vector will be the incident k_1 vector.

Similarly, if my electric field E_2 is coming out and I want the wave to move away from the interface, I need to have electric field and the magnetic field in this manner so that its electric field rotated on to the magnetic field will cause the wave to go away from the interface. Okay. So the magnetic field will be located in this manner. Of course, in this case, the electric field, sorry, magnetic field associated with this medium will also be located in the same direction as H_1. Okay.

The equations, I will just write down the tangential boundary conditions or the equations resulting from the boundary conditions of tangential continuity. So I get $E_1 + E_2$ to be equal to E_3 because everywhere this is parallel to the interface and however, for the magnetic field, what we get is $(-E_1/\eta_1)$ Cos θ_1.

So this clearly, E_1/η_1 is basically H_1 and you are looking at H_1 Cos θ_1 or Cos θ_i in our angle that we were considering and because this tangential component of H_1 is actually along -x direction, you get a - sign here. Okay. Apart from that one, the rest is quite simple. So you have (E_2/η_1) Cos θ_1, which would be equal to $(-E_3/\eta_2)$ Cos θ_i and θ_t.

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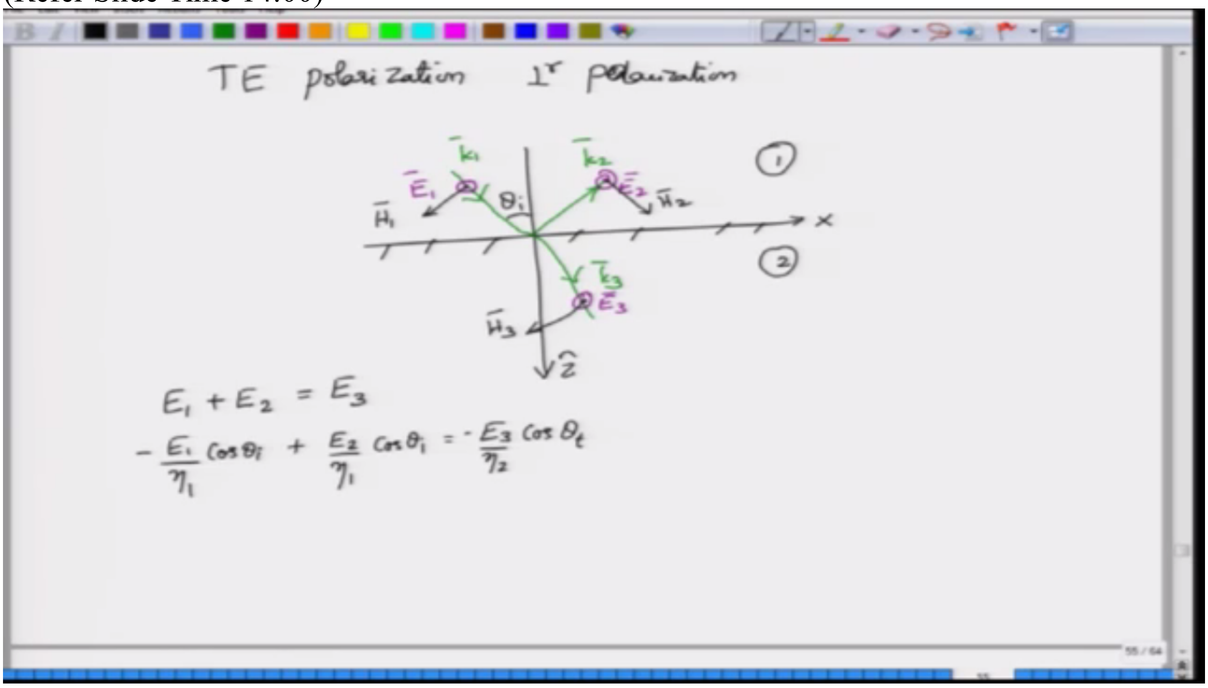

Now without, you know, solving the equations, we can already see a similarity between this set of equation and the previous equation that we wrote, right? So you had E_1 Cos θ_i there. You had E_1/η_1 here.

So if you simply, you know, rewrite this equation, instead of that way you write this as $E_1/(\eta_1/\text{Cos } \theta_i) + E_2/(\eta_2/\text{Cos } \theta_i)$, which would be equal to $-E_3/(\eta_2/\text{Cos } \theta_t)$ and associate these denominators with transmission lines with appropriate TE impedances. This is η_1^{TE}, which by definition will be equal to η_1/Cos θ_i, okay, and this will be terminated in a load impedance, which is actually the intrinsic impedance or the wave impedance of the second medium given by η_2^{TE}, which is actually equal to the impedance $\eta_2/\cos\theta_t$.

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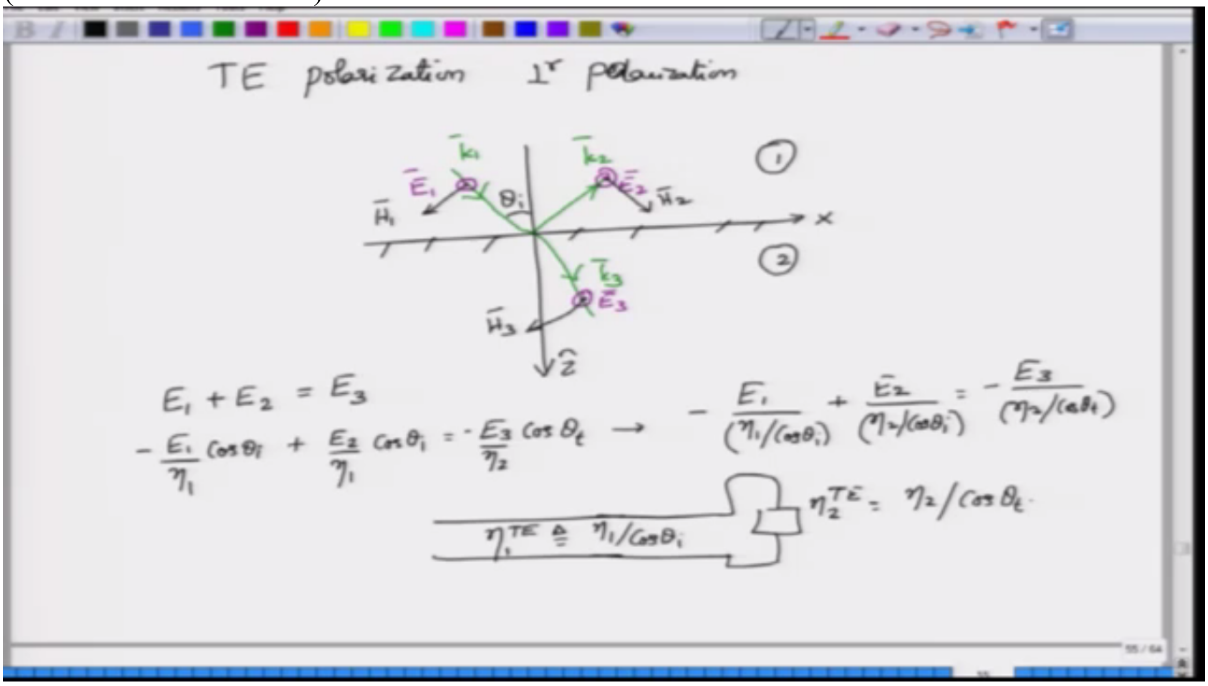

Of course, θ_i and θ_t are still related in the same manner as Snell's law, so there is no change. So what you should understand is that this scenario of TE polarisation can be made exactly analogous to a transmission line that we have already seen with the impedances being η_1^{TE} for the main transmission line. That transmission line is terminated in the load whose impedance is η_2^{TE}.

So from this it's very easy to write down the expression for for Γ^{TE}. This would be η_2^{TE} - η_2, sorry, η_1^{TE} divided by $\eta_2^{TE} + \eta_1^{TE}$. Okay. You can expand this equation and then show that this is what you actually get if you were to solve these set of equations. We will call these set of equations are 3 and 4, which basically express the boundary condition.

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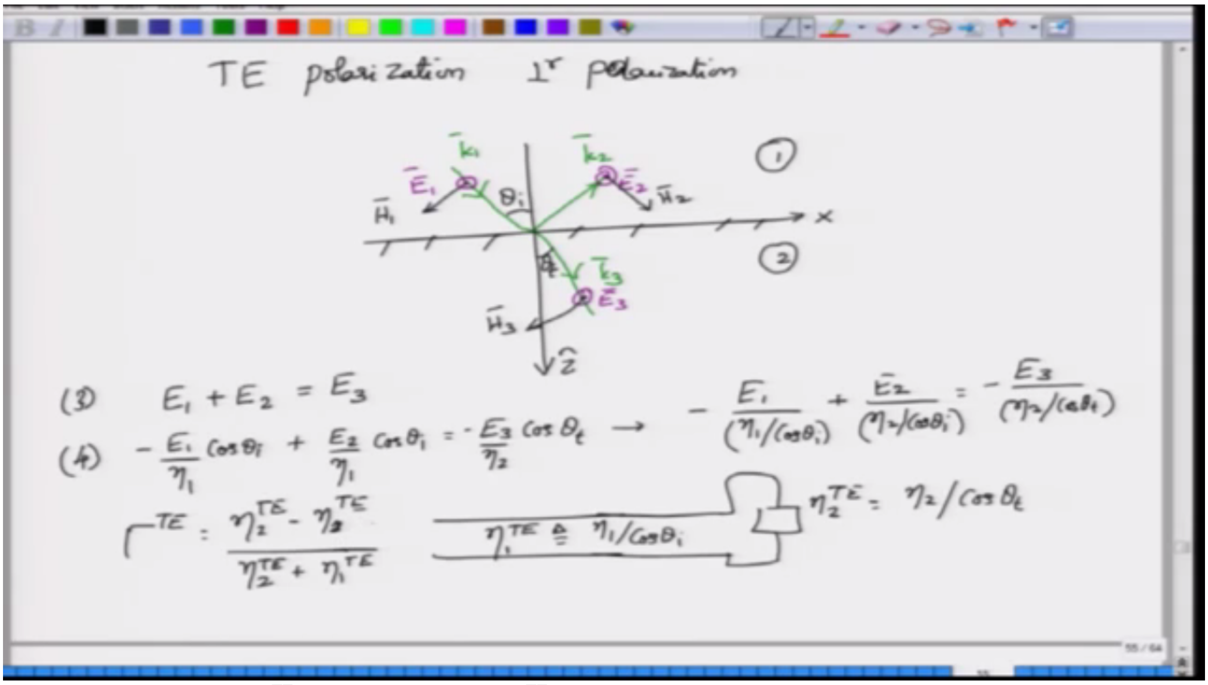

So solving for this Γ^{TE} or equivalently Γ^{TE} from these set of questions is as simple as writing down the appropriate reflection coefficient expression for the transmission line case. A longer calculation indeed confirms that what we have written is true. Okay.

We are most of the times interested in the reflection. Sometimes we are interested in the transmission, but most of the times we are interested in the reflection because what we are trying in a manner that is similar to the transmission line problems is that if I have a material and if I have another material and I will send light from say one material to the other material, obviously, if these material intrinsic impedances are mismatched or they are not equal as in, for example, air glass interface, right, so glass has a different permittivity or a medium impedance η whereas air has a different permittivity, and hence a different characteristic impedance or different wave impedance or intrinsic invaders. When these two are not same, which of course is true, there will be some amount of power that would be reflected back. Okay.

Because the reflected power or the power density is related to the transmitter power density in a straightforward manner, magnitude of gamma square will tell you the fraction of the power that is actually being reflected back. Okay. And this in many cases may not be okay. For example, in many situations, you would want most of the light power that is incident on to the glass to actually be absorbed by the material, okay, and then transmitted into the other medium. Okay. You don’t want much of a reflection coming out, coming from that one.

So this is a simple case of say, you know, antiglare glasses. In antiglare glass if you, you know, buy and then wear that one, whatever the sunlight that you are going to get would actually be, you know, reflected back. So you basically can make the reflection approach as, you know, high as possible or in some cases where you want light to actually enter in the form of a laser cavity, for example, you have to put what is called as antireflection coating.

Antireflection coating minimises the reflection. Okay. It kind of makes the reflection go to zero, so which means that most of the power is actually transmitted.

So whether you are designing for minimizing the reflection or maximising the reflection, you would deal with, you know, fact that you are going to coat materials with different type of materials of certain thickness, then your job would be essentially similar to that of matching on a transmission line. So in a transmission line, you had one main line. You had another line and then you put in what is called as a stub line, the one that we studied, which was actually forcing the reflection on the main line to actually be equal to zero, right? So a concept that is similar can be used here also and the design of antireflection coatings and corresponding, you know, anti-transmission coatings as the glare thing that we talked about, they all are very important and very practical uses of this set of equations. Okay.

However, before going to that, I would like to start off with couple of observations by actually plotting this reflection coefficient. Okay. And I would like to do that one in order to bring out certain very interesting aspects. So we are going to do that. Okay. We will take two different cases and before we actually go to that one, let me remind you that η of any medium which is given by square root of μ/ε, in our course this would actually be equal to $\mu_0/\varepsilon_0\varepsilon_r$. Okay. And we have already said that square root of μ_0/ε_0 or μ_0/ε_0 is basically the free space impedance η_0, which has a value of 377 ohms approximately, and then you have divided by square root of ε_r. If ε_r is a real medium, then this ε_r will actually, square root of ε_r will actually be equal to refractive index.

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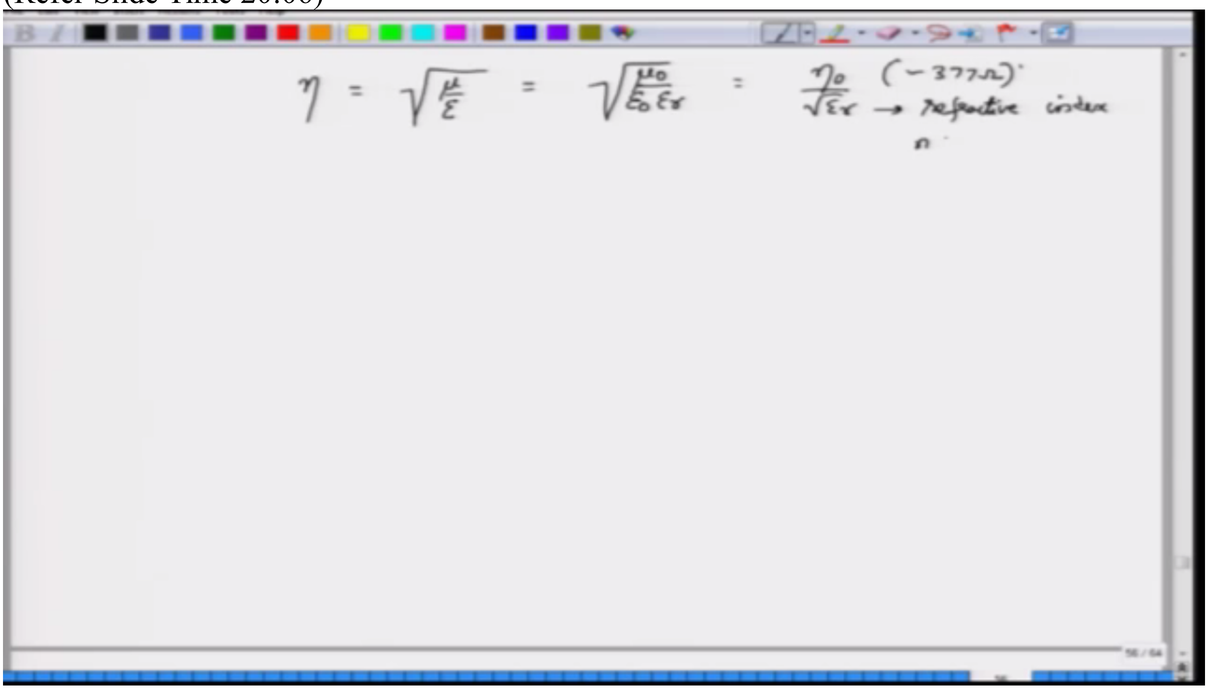

However, to confuse you, we use the letter n to devote refractive index. Okay. Unfortunately, n and η have to be distinguished. I hope the context makes it clear, and it is common practice in dealing with optical materials, okay, when you deal with optical materials or in optics to use refractive index rather than to use the wave impedance η. Okay.

So with that in mind, what I wanted to tell you was η is inversely proportional to refractive index. Okay. So any medium impedance that you want to obtain, you take the impedances of the free space divided by the appropriate refractive index. Okay.

Now let's write down the expression for Γ^{TM}, okay, which I will write instead of in the wave impedance or using wave impedance, I will use the refractive index here and when I use the refractive index, please check this out. This is what the equation is going to look like: n_1 Cos θ_t - n_2 Cos θ_i where n is in the refractive index. So I'm going to write this as refractive index for, okay, so using refractive index is what I wanted to tell you, divided by n_1 Cos θ_t + n_1, sorry, this is n_1 Cos θ_t + n_2 Cos θ_i. Okay.

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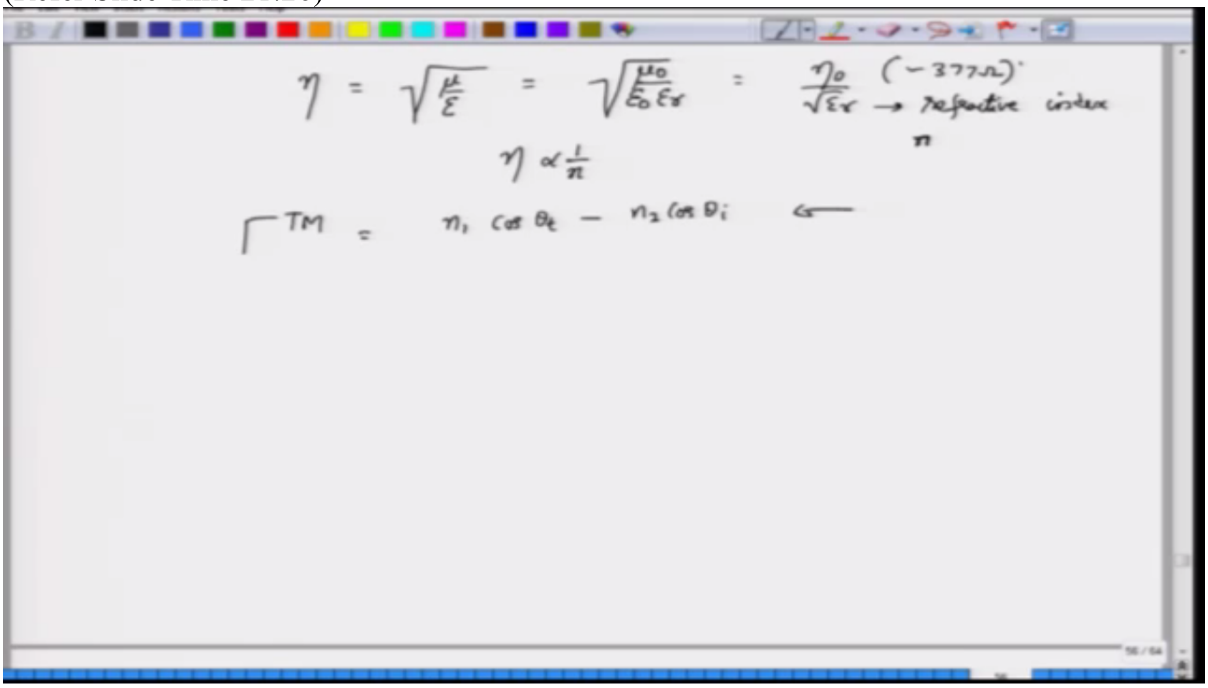

Now Snell's law tells you that n_1 Sin θ_i should be equal to n_2 Sin θ_t, which actually allows me to write Cos θ_t as $1 - (n1/n2)^2 \text{ Sin}^2\ \theta_i$. So I can rewrite in this expression Γ^{TM}, the expression can be made to depend only on the angle of incidence θ_i and given n_1 and n_2 are constants, I will have an expression for the reflection coefficient of a transverse magnetic wave purely in terms of angle of incidence.

Now what I can do is I can actually obtain what I would now kind of a plot or a graph wherein I am going to vary the angle of incidence θ_i. What is the acceptable range for θ_i? $\theta_i = 0$ corresponds to normal incidence and $\theta = \pi/2$, which is basically 90°, right? So this is 90° would correspond to what is called as grazing incidence, okay, meaning if this is your interface here, $\theta_i = 0$ would have the k vector directly in this manner. The electric field will be here and the magnetic field let's say will be along perpendicular direction. Okay.

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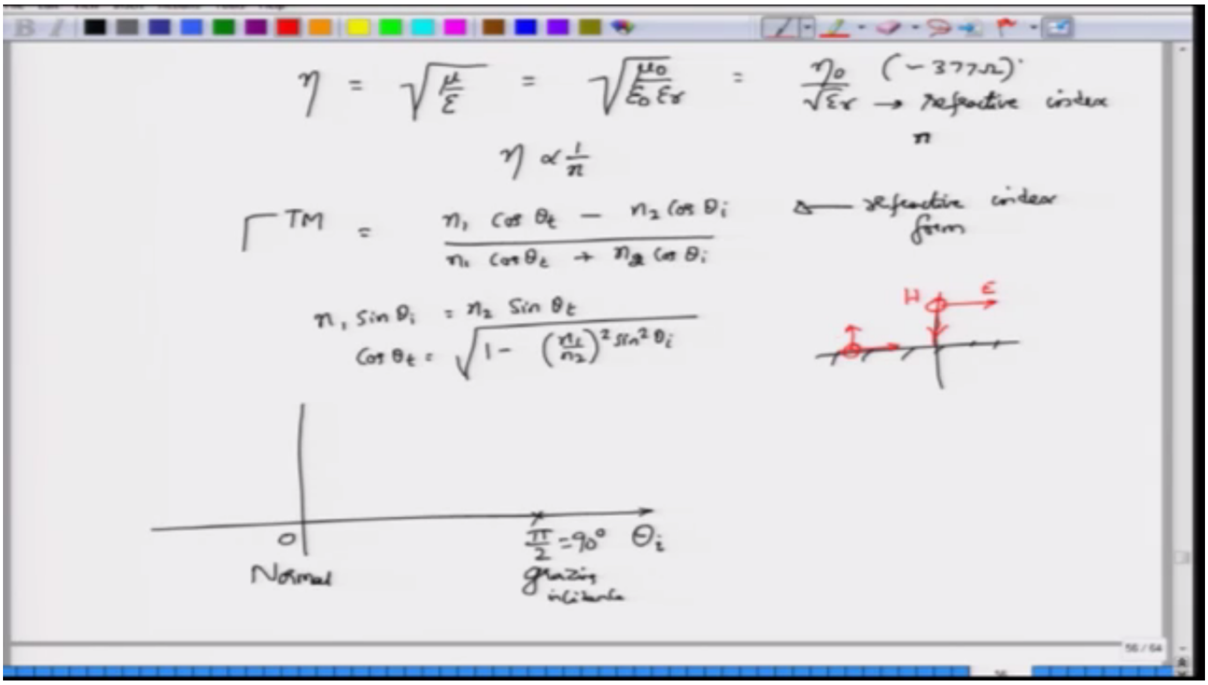

However, for the grazing angle, this is the k vector that would be almost along this direction. The electric field vector would be say perpendicular or parallel depending on which type of the wave you are considering and this is how you are going to get. So you have this interface and then grazing angle is just touching the interface in this manner, and if the electric field is perpendicular, that would correspond to the perpendicular polarisation and that is precisely what we wrote for Γ^{TM}. I hope that connection is also very clear. Okay.

So this is the range over which the angle θ_i can vary from 0 to 90° and if I plot the magnitude of the reflection coefficient, which is what I am interested, for this kind of a lossless material and having no active gain medium, the maximum reflection coefficient value will be equal to 1. Okay.

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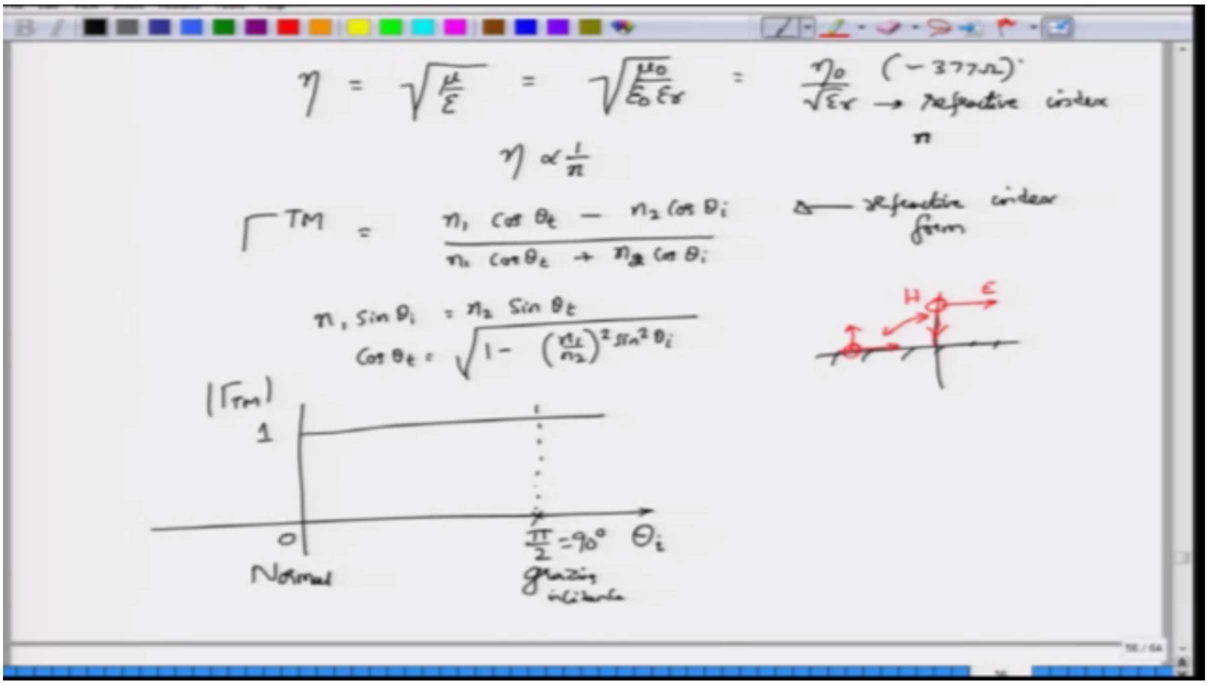

Now if you consider two cases, so let’s assume that n_1, which is the medium of, you know, this is the refractive index of the medium one and I will assume this n_1 to be equal to 1 and n_1 = 2.5, okay, and then vary the angle of incidence θ_i, evaluate the magnitude of this particular expression and then plot it. I can write a simple Matlab program with just two, three values here or write a simple Excel sheet in order to do that, do this calculation.

What you will observe is that when you plot this Γ^{TM}, at t = 0, it roughly starts around with a value of 0.43. Please check these numbers. Okay. That will actually give you some facility in using these equations as well. So it starts at 0.43.

What happens is as you start increasing the angle of incidence, at somewhere at a point of 68° approximately, okay, the reflection coefficient actually goes to 0 meaning that there is total transmission at this angle, okay, the light is basically totally transmitted into the second medium provided it is purely of TM polarisation. Okay. And we will call this angle as θ_B. This is basically called as Brewster's angle and you can will show that this Brewster's angle will actually be given by $Tan^{-1} n_2/n_1$. Okay.

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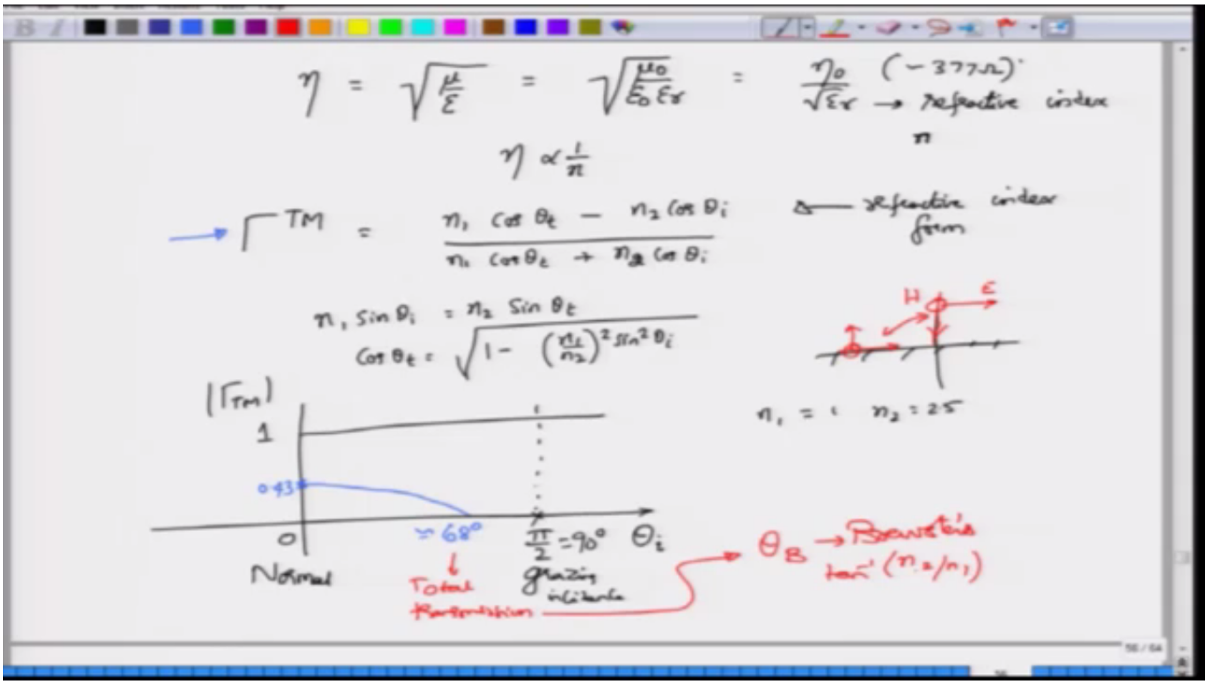

I am not deriving the expression for Brewster's angle. This is there in the references that we have taken, but please verify that for the medium cases that we have considered $n_1 = 1$ and $n_2 = 2.5$, this angle where this reflection coefficient goes to 0 is roughly 68°. Okay.

Then what happens is that the reflection coefficient magnitude keeps increasing and eventually approaches 1, okay, at 90°. Okay. So this is what you are going to get for TM = 1, I mean, Γ^{TM} equals this particular case.

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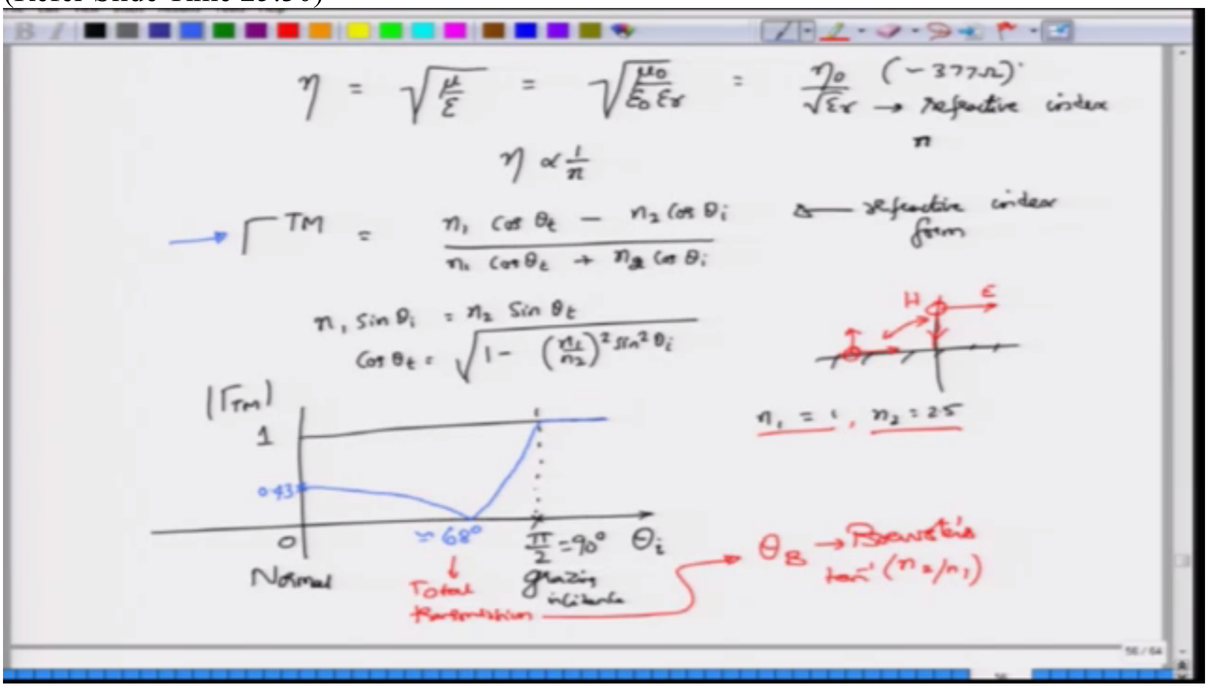

How about the transverse electric case? In the transverse electric case, things will be slightly different. Okay. We will do the transverse electric case, but this time we will assume that medium $n_1 = 1$ but $n_2 = 1.44$ so that this equation starts at 0.2, that is the amplitude will actually start at 0.2. The reflection coefficient magnitude starts at 0.2 and then very slowly goes up. So this is for the transverse. I have not drawn the equation nicely, but this is how it actually goes up. So there is it doesn't go to zero. It simply scales up in a very slow manner and then eventually reaches 1 at θ = 90°. Okay. What you have to observe here is that medium one was a lower refractive index medium and medium two was a higher refractive index medium. Okay.

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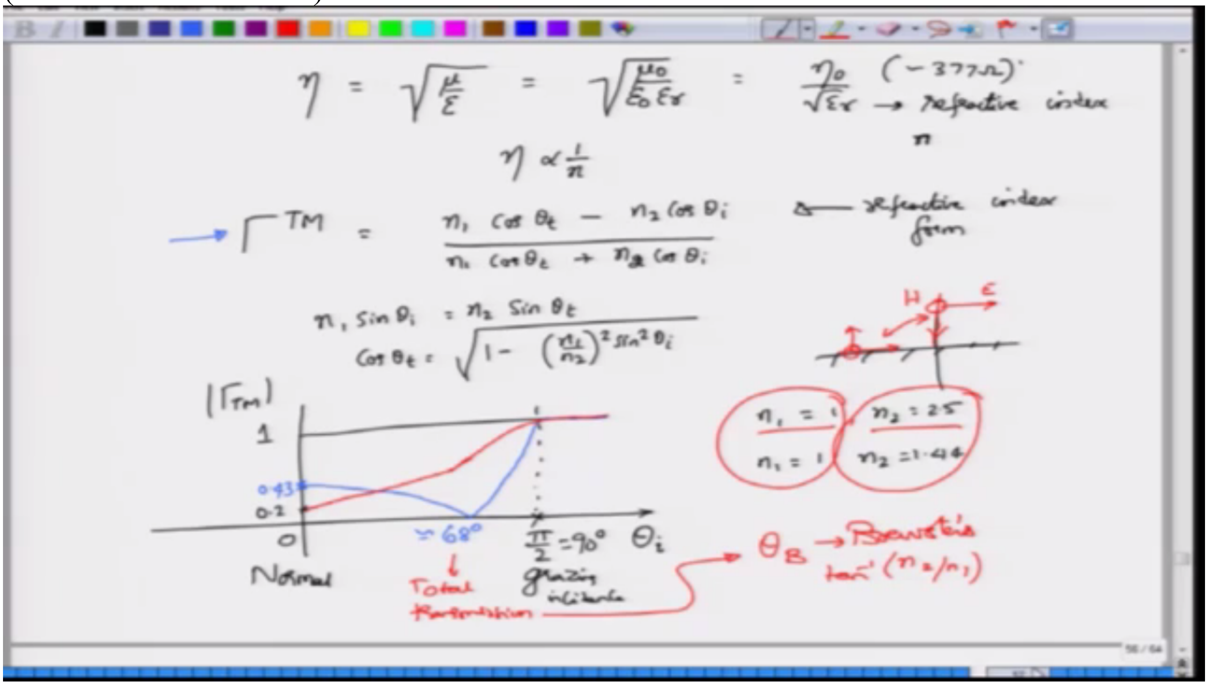

Now let's switch these two cases, okay, and then consider what would happen for TM and TE when we let the angle, so what I will do is I will have the same angle say from 0 to 90°. Okay. I know the maximum value is going to be 1. Okay. So now I'm ready, but this time I am going to assume that $n_1 = 2.5$ and $n_2 = 1$. Okay. So I am assuming that n_1 is basically equal to 2.5. I am assuming that the incident medium has a higher refractive index than the medium, second medium, which has a lower refractive index. In this case what happens?

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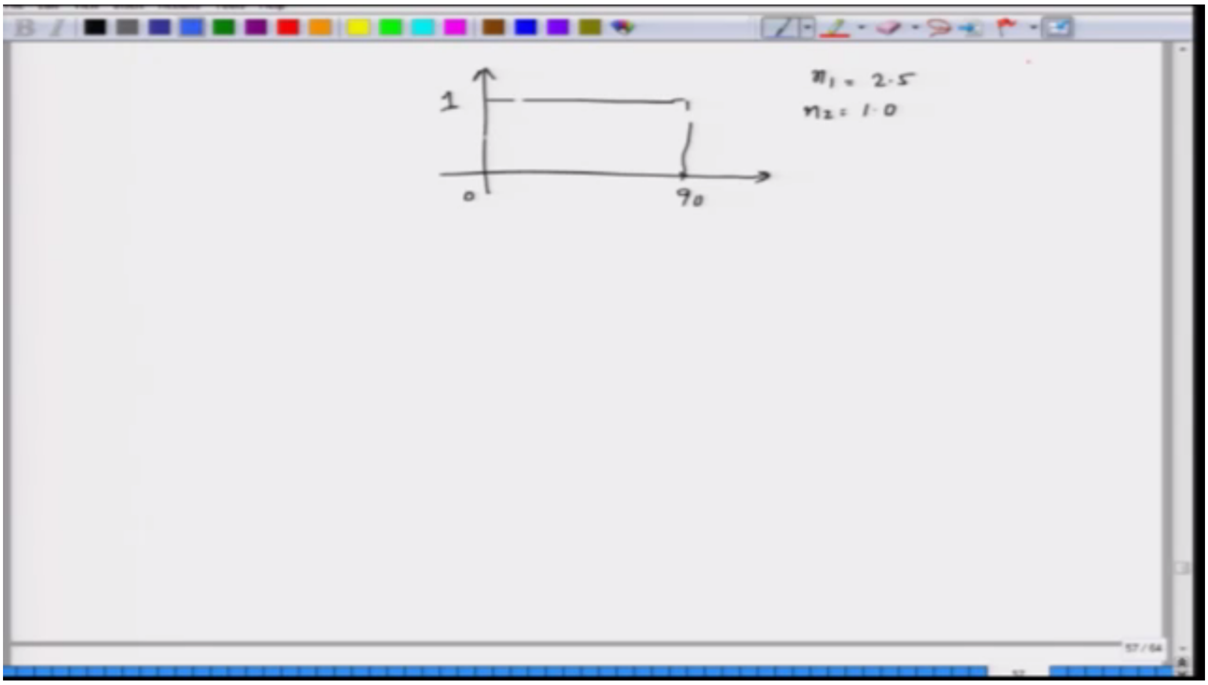

You look at this. Starts off at somewhat, you know, close to 0.5 or 0.43, maybe even slightly different than that one. Let's not worry about that. However, as you start decrease, I mean, as you start increasing the angle of incidence, it drops down at some angle 22°, which would correspond to the Brewster angle, the magnitude of TM will be equal to 0 at this Brewster angle.

However, after slight change in the angle, the reflection coefficient suddenly becomes 1 and then remains 1 throughout the rest of the incidence angle. Okay. If you wish to observe this carefully, you can actually decrease the refractive index difference. So I assume that that n_1 = 1.44 and n_2 = 1.0 and then redraw this picture, okay, with same 90, 0, and this 1 that I have.

And when you do this case, you start off with some value, which I will leave as an exercise, but you will quickly go to 0 at 33°, okay, and then as it starts to increase slightly, at some arbitrary point it is going to suddenly increase and then remain equal to 1. Okay. And this angle not very far from this 33 angle, this is actually just about 44°, we will call this as some critical angle beyond which, so when the angle of incidence is greater than the critical angle, the magnitude of the TM as well as TE, I have not shown this, but it would also be equal to the same scenario, both will be equal to unity.

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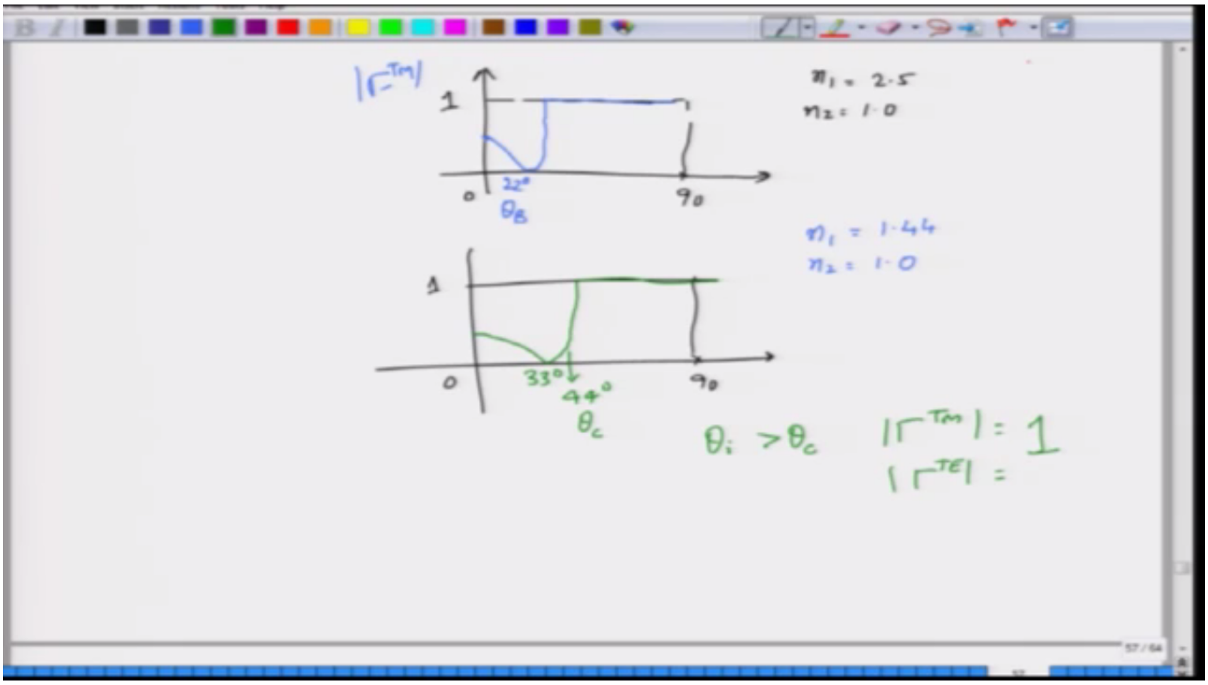

And this phenomenon is called as total internal reflection, and it figures mainly in guiding light and it is a very important topic in optics that we are going to talk about next modules. Thank you very much.