Module -19

Lecture -18

Normal Incidence of Plane Waves

Hello and welcome to NPTEL MOOC, on Electromagnetic Waves and Guided and wireless medium. This is module 18 and in this module, we consider the following problem. We already have seen uniform plane wave, propagating along, say plus Z direction and oriented along either X direction or Y direction and this wave, was propagating in, what is called as, 'Unbounded Medium'. That if there was no other medium in, in anywhere near the wave. And this medium was characterized by, whatever the permittivity and permeability of that particular medium itself. Now, what happens as this wave, which is propagating

along the z direction? Okay? Now, hits upon a plane. Okay? Beyond which, the medium actually changes. So, you have a medium whose properties say R epsilon R 1 and mu R 1 in this medium, which we will call as medium 1 or incident medium .and then I have, a medium which is medium number 2, which has a different permittivity epsilon R and different permeability, however we will take permeability to be the same, in both first medium as well as second medium, therefore both are characterized by same mu naught, whereas epsilon R the permittivity can actually change. So, if you imagine that this is the plane, at which abruptly the junction properties change from, being mu naught epsilon R 1, in medium 1 to MU naught epsilon R 2, in the second medium. and you have a way which is propagating along the z axis, meaning that, this plane that I have the Right hand side that, I am holding is the x and y plane. Right? Because, that plane would be perpendicular to the, z axis so as, as this wave, which is now also polarized along the x direction, what happens as this wave hits this second medium? Right? Its it, hits this particular plane. This is what the problem of reflection that, we are going to consider. And because, our direction of the perpendicular to the plane, which is the z axis, actual coincided with the direction of the propagation as well, what we have is what is called as, 'Normal Incidence'. in the next module, we will see what happens when a wave, which is propagating at an angle to this interface plane, the plane that actually separates medium 1 and medium 2, what happens when this wave in, you know, is incident on this interface at an angle, that is different from this normal. Okay? So, this is called as, 'Oblique Incidence'. And we will see, what happens to the oblique incidence in the next model. But, for this module, we will assume what is called as, 'Normal Incidence'. I will describe more, about the angle of incidence as shortly, but, what exactly happens. Right? Remember this wave, which has been incident and propagating in the medium one, actually carries some amount of energy. Right? Or it has an energy density.
So, if at all there has to be some changes, will there be some energy that is, transmitted into the second medium, will all of the energy be transmitted into the second medium or some energy will be reflected back, because the medium properties are different from the two media, these are the question that we would like to an answer. Okay? Now, this problem is not very new, in the sense that, if you imagine instead of these electromagnetic waves, take them to be light waves or light rays, as you would have studied in the high school, physics courses, you would have realized that, when light is incident from one medium, to another medium, there is usually two kinds of things, that would happen. One is refraction, refraction corresponds to, partial transmission of the wave into the second medium .and there is what is called as, 'Reflection'. Reflection is partial reflectance or reflection from the interface, back into the first medium itself. Okay? So, these two phenomenon, you would have also seen in the context of an oblique incidence, in the sense that, if the incidence angle is some Theta one, you know, how to calculate the angle of transmission or angle of refraction, as we can call it, as well as angle of reflection, these two are usually calculated by the help of, what is called as, 'Snell's laws'. Right? However Snell's law, by itself will not tell you, how much of energy is actually being carried by the second wave. And is it just that energy transport that is, going to happen when the live wave is reflected or refracted or is there more to it, what happens to the amplitudes? These are the questions. That can be addressed by looking at the electromagnetic perspective of light. Okay? So, in of course, I am not specifying, whether we are dealing with light waves or in general different frequency electromagnetic waves, these waves could, for example, be microwaves, as well there is waves, who's electric and magnetic fields are oscillating at microwave frequencies. So, we don't specify that, all we ask for is that, we have a uniform plane wave, which is polarized along the X direction, propagating along the Z direction, hitting, a medium whose permittivity is different, from the medium of incidence. Okay? For now, we will assume that, both media are lossless, we will later on see, what happens when the second medium becomes, very loss that is it has

more conductivity, than it is a insulator. Right? So, in that case what will happen, we are going to look at that. Here, is a basic sketch that I have drawn. Okay?

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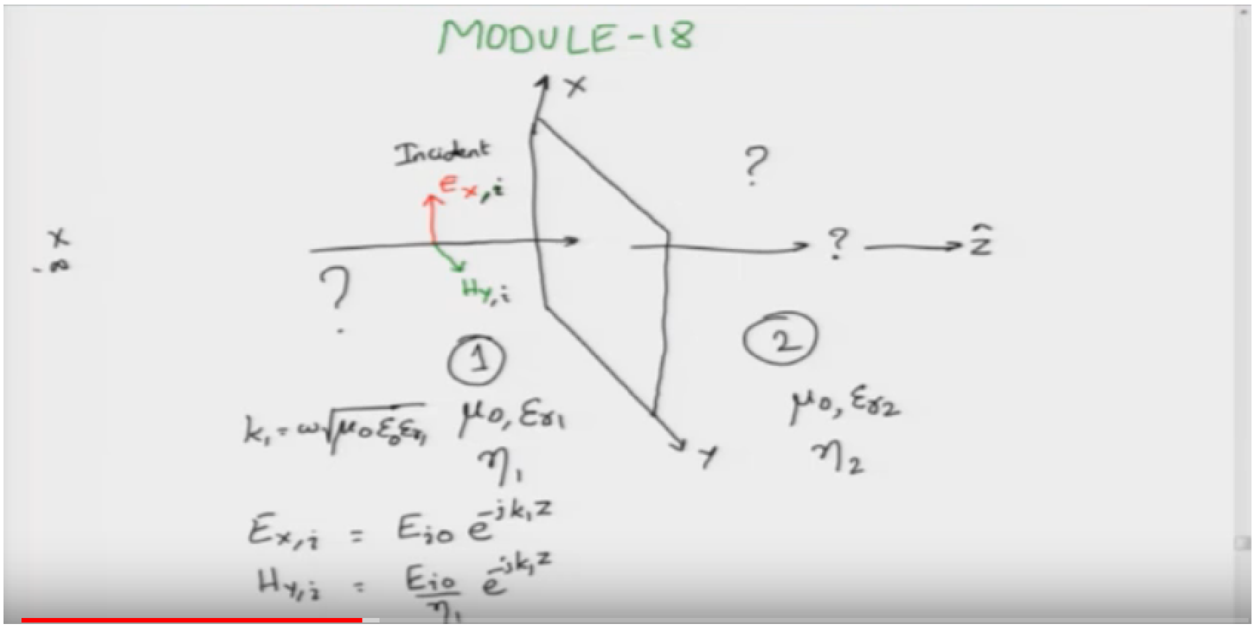

I have tried to, put in diagram, whatever I showed you on screen, just a few minutes ago. I have this axis, horizontal axis, which I have written as the axis Z. and that is the direction of wave propagation, as referred to the first wave itself. Right? Or the first medium, itself. In that first medium, so let us call this, this is medium one, this is medium 2. Medium one has mu 0, epsilon R 1. Medium 2 has mu 0, epsilon R 2. Okay? So, we have these two, permeability and permittivity values. Okay? Equivalently, you could characterize the first media, having an impedance of ETA 1 and the second medium having an impedance of ETA2. Where you know the expression for ETA as well. So, here is my incident wave, so this is called as the, 'Incident Wave'. Because, this is the wave that is, coming far away from infinity, minus infinity let us say. And this wave, which is you know, coming in is polarized along the X direction. And because, it is incident wave, I put in a comma and under I subscript, to denote that, this is an incident wave. Okay? Associated with this X polarized electric field, there will be a Y polarized magnetic component, which I have written as H, Y comma I. and now, I ask, what will happen into this medium in the second region? And what in fact? What will happen in the medium one itself? Okay? To understand what is going to happen. You need to supplement, Maxwell's equations with what is called as, 'Boundary Conditions'. Okay? before we go to the boundary condition, I would like to stress that, these are actually waves, meaning that, EXI can be written as, say EXI 0 or maybe I will remove, X because it is kind of known to us. What I will write instead is? Ei0 indicating that, this is the incident medium, amplitude of the incident medium and propagating along, the propagating along Z direction, with a propagation constant beta 1 or k1. Okay? I will use k1, for simplicity. So, this is the wave that corresponds to, the incident I mean, this is the electric field that corresponds to the, incident wave. This is X directed wave and you can of course, write down, what would be H Y of I? Okay? At any Z I know, in the medium one, as Ei0 divided by ETA 1, e power minus JK 1 Z, both of course have the same value of k1, becausek1 is basically, related to frequency and related to the permittivity and permeability. Right? So this is what you actually have for? k1. So, we have this prior information with us. Now, what we need to know is how would the electric field? Which is oriented along the x direction? When it goes and hits a medium? Whose property is different behaves. Right? This is what is called as, ‘Boundary Condition’. Or this is determined by, what is called as, ‘Boundary Conditions’.

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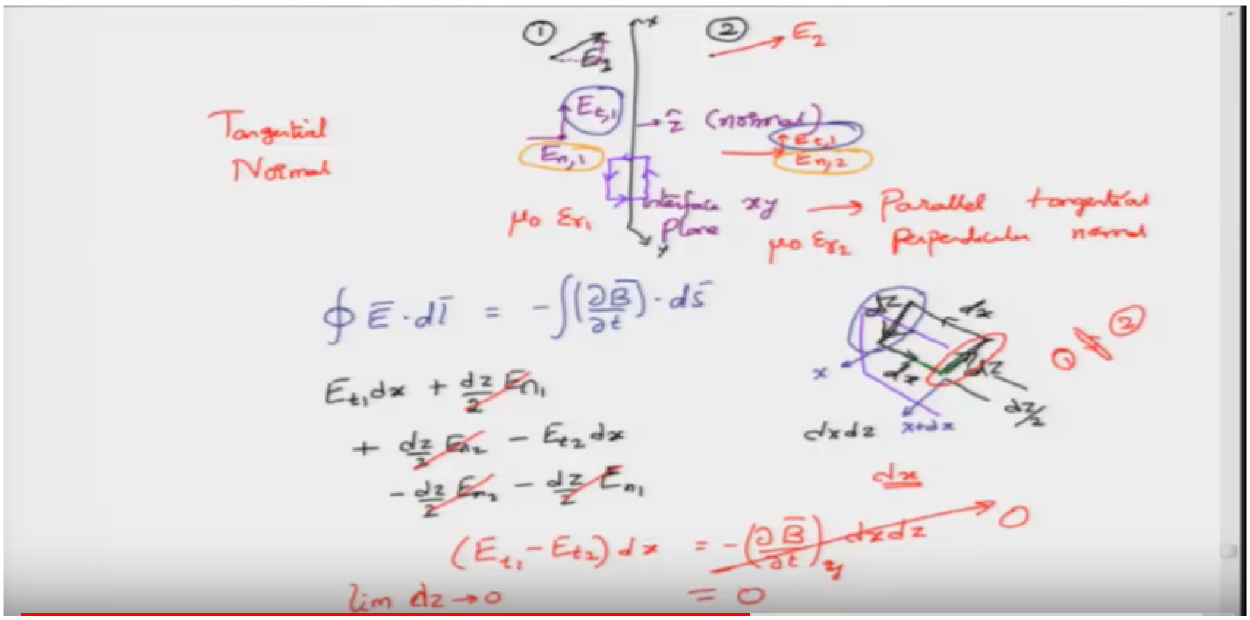

And what we have to understand is that? You can take two points in medium 1 and medium 2. And then, you have a bisector between the two, which of course, in this case the bisecting plane happens to be XY plane, as we have already seen that one. and now, let's say at this point, which is very close to the interface, my electric field has in general a direction, that is given by, say E one. Okay? This, you know, very close to the interface point that I am considering, so I have this like, general electric field direction to be E1. I can always split this general electric field, you know, which is the incident field into, two components, one component will be along Z direction or along the direction that is what is called as, ‘Normal to the Interface Plane’. So this is called as, ‘Interface Plane’. The XY plane is called as, ‘Interface, Interface Plane’. And z axis is called as normal, which actually points from, medium one to, medium two. And it is normal or perpendicular to the interface plane. Okay? So, I have not taken the general electric field that, has been incident and decompose it into two components ,one would be the tangential component and the other one would be the, normal component. Right? So, this is the tangential component that I have and this is of course, the normal component, which I will write it as E and one, maybe instead of E tan, I can write this as, ET just to simplify the notation. Okay? So this is the electric field that I have. Now, let us imagine that in this medium 2, the electric field direction would suddenly be different. Okay? So and we will also call that, ‘Electric Field to E2’. Which further can be decomposed into, two components? One would be the normal component, which we will call as, ‘E and two’. And then, we will also have, what is called as the tangential component it Et1? Tangential meaning, any component that is parallel to the interface plane, would be called as, Tangential Component’. So parallel to this plane is, the tangential component, perpendicular to this plane is called the, ‘Normal Component’.

Right? So, perpendicular is called as the, ‘Normal Component’, for us. So, you have these two electric fields; don't ask right away, how did you get this electric field e 2? We will simply assume that, that electric field E two exists. Okay? Of course, this medium is characterized by mu naught epsilon R one, the second medium is characterized by mu naught epsilon R two, for simplicity. Right? We are only dealing with, lossless medium as I have told you. Now, what boundary conditions will tell you is that? How the tangential opponents, across the two boundaries will behave? And how the normal components across the interface will behave? Meaning that, we are going to try and find a relationship between E n1 and E n2 and similarly we are going to find a relationship between E t1 and E t2. Okay? And these relationships are actually obtained, by going to Maxwell's equations in the integral form. So let us say, we have Maxwell's equations in the integral form, which is given by integral of E dot DL. Okay? This would be given by, minus say Del b by del t and i just pull this del by del t into the expression, doesn't really matter and then i have a open surface integral on this one, Correct. So this is what I have? Now, let us see, if we can write down the actual electric, you know, the expressions by looking at the electric field e n1 and e t1, e n2 and et2 here. Okay? So, evaluating this expression on the left hand side requires us to actually go through a particular contour. Okay? What is the contour? I am trying to find a color here, so let us say, this color, so what I am actually trying to, do the integration is to pick a contour, that would actually be in this manner. So this is the contour that I have, so to give your imagination as, to what the contour is, imagine that this is the plane that, I am considering. Okay? So, in this plane, I first. So let us say, this is the plane I first, take a line along the tangential part or the interface plane and then, penetrate through the interface, to get to the other side.
So, I have penetrated to the other side, as well and then I take, you know you can't see here, but I'm going to draw, an imaginary, I mean, a line here, so I'm going to draw another line, which would be parallel to the, original line that I have drawn. Okay? So, you simply have to imagine that I drew a line here, then went to the second medium, drew another line and then came back to the medium to complete this particular loop. Okay? and when I have done that loop, I have some line or one part of the line, that is lying in the medium one, then there is a normal, that is line, that is purple along the Z direction that would inter from medium one to medium 2, this there is another line that would be along the interface plane, in the direction opposite to this one and then I am coming back, to the same point to complete the loop here. So this is what the complete loop is? And this contour will of course, have a certain area as well. Right? So, if you imagine this is the plane, then the path that I have chosen, will be something like this. Okay? So this is the path that I have chosen. and this left hand side equation, can therefore be written as, so these are the tangential components, let's assume that, the length of this tangential component is say, DX and then this is say, D Z, this is DX and this is DZ. So, the area that I have, will be DX times TZ. Okay? That would be the total area that I have, but evaluating the left-hand side contour integral will give me, e t1. Okay? DX that would be the electric field, tangential component integrated along this DX piece of line and then I have DZ component, but not the complete DZ. Right? So, in this region only that DZ by 2component would be in medium one. Right? So, I have DZ by 2 times E and 1. Okay? That completes the line that I have considered from DX 2, DX Z by 2 and you can now see that, the remaining part is DZ by 2, e n - Right? In the second medium- E t2 DX. Why it should it be minus? Because, this direction is opposite to the initial DX direction. Right? And then I have, minus DZ by 2 – again, because this line is in the direction opposite to this other line. Right? So, minus DZ - e n2 and then I have, minus D n1 sorry, whatever writing, this should be minus DZ by 2, e t1. Okay? there's a small change here, that I should mention this not that it is very important, it will anyway, be sorted out very easily, please note that, if this plane happens to be X, then this plane will be at X plus DX. Okay? The normal components that I am

referring to here, on this side of the contour, which I have written as minus DZ by 2, E n2, this should have been E n1.Right? So, this should have been en 1.Right? So the 2 minus DZ by 2n 2 and minus DZ by 2 en1, that I have written, actually are being evaluated at the plane X, whereas, corresponding to this particular part of the normal contour, they are actually being evaluated at X plus DX. So, the electric field would have changed slightly, but I am assuming that, this DX is small and therefore, that change is not very abrupt. Okay? So this actually allows me to cancel off this e DZ by 2 en2 with DZ by 2 en2, DZ by 2 e n1with DZ by 2 en 1, leaving behind, the left-hand side to be, e t1 minus e t2, times DX that should be, on the Right hand side, I have del B by Del T. Okay? whatever the vector that would be there, that, that vector would only be along the Z Direction vector that I have to take, times DX, DZ sorry, Y Direction that I have to take. And then, I will write this as, so this is the Y direction, so they have DX and DZ that I have neglected the minus sign. But, if you wish you can also put the minus sign. So this completes the left-hand side and the Right hand side. Now, what we do is? we take the limit of DZ go to 0 that is I am going to shrink this, contour in such a way that, these two lines basically light very, very close to each other, the tangential lines live very close to each other, across medium 1 and medium 2 and their separation, along that Z Direction DZ, it actually goes to 0. So, I just take this contour in this particular manner and then, I'm going to squeeze the contour in this way. Right? So, I'm actually going to squeeze the contour in such a way that, along the, the height of the contour actually reduces. So, when that happens the Right hand side actually goes off to 0, unless Del B by, Del T actually goes off to infinity. But, del B by Del T you know, time derivative of the magnetic field and magnetic field is a nice physical, quantity its derivative cannot go to Z, I mean, infinity so this Right hand side actually goes to 0, as DZ goes to zero. Right? The left-hand side is anyway independent of that, but, left-hand side remains, E t1 minus e t2 times DX. But, the rest of everything should actually go to 0. Right?

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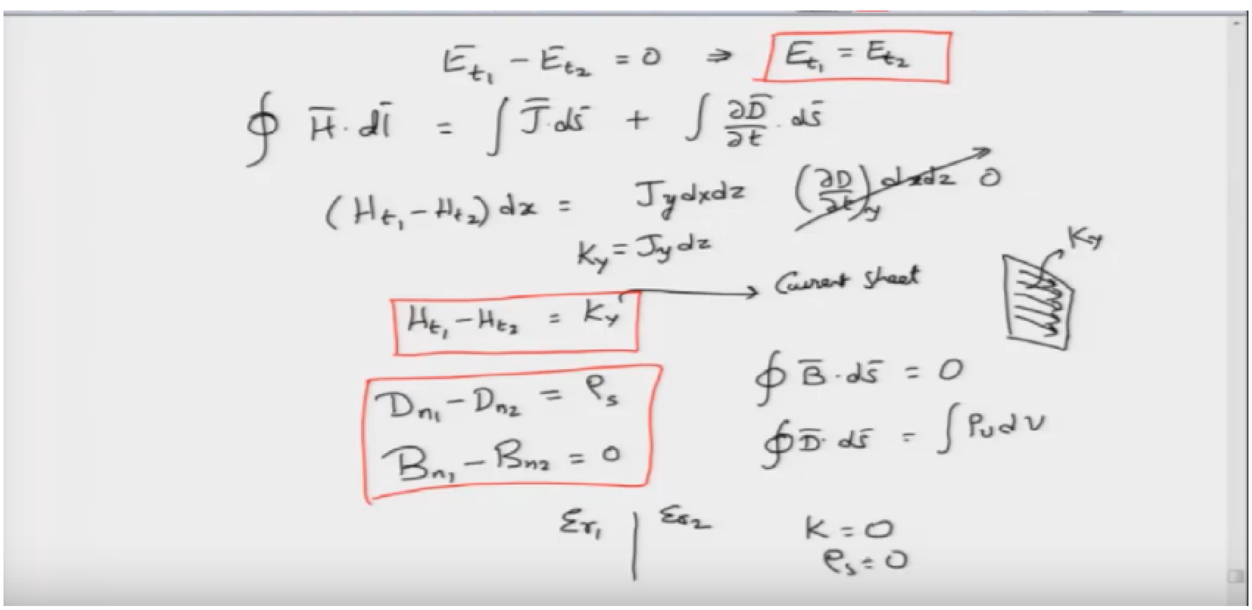

So, we have the first boundary condition, which tells you that, E t1 minus E t2, should be equal to 0, implying that, on both regions the tangential component must have equal amplitude. So, if the second electric field is present that is, electric field is present in the second medium ,its tangential component, must be exactly equal to the tangential component of the, first one. I mean electric field in the medium one. Okay? going to the same argument and writing the second Maxwell's equation, H dot DL equals, J dot d s integral plus, you have this integral of del D by Del T dot DS, you can show that, the left hand side simply becomes H t1 minus h t2, times DX, whereas this part will be, del D by Del T you know, the Y component times DX, DZ which anyway goes to 0, as DZ is shrunk to zero. However this part J dot DS, which is basically JY, d x, DZ, this actually does not go to 0. Because, it is possible that as d Z goes to 0, jy can go to infinity. Okay? Together, this can be written as K Y and what you actually get is, h t1 minus h t2 is equal to KY. And this K is called as, 'Current Sheet'. Okay? current sheet is a very, very thin layer of current that actually flows, in the interface, along say the Y direction, so this is the current sheet that I have considered, the current sheet is specified by, so and so amperes per meter. Okay? Because, J is specified as ampere per meter square and you are now, multiplying that one but dimensionally with the quantity that is having a units of meters, so the current sheet KY that you have, is actually a per meter quantity, ampere per meter quantity. So, this is the second boundary condition that I have. What it says is? The tangential magnetic fields can be discontinuous, whereas tangential electric field can be, I mean, have to be continuous, the discontinuity in the tangential magnetic field is taken up by the current sheet. Okay? I am NOT going to, go to the other two boundary conditions, but, I you know, you can refer to the books, for that or you can refer to earlier notes or on a different NPTEL course, what you will find essentially is that, the normal component will be discontinuous, the normal component of the vector field D will be discontinuous and this discontinuity is given by, surface charge that is going to be present. Okay? So this is another boundary condition, however the normal B component, will always be equal to zero. Okay? these two come from, going to the other equation so B dot d s equal to 0 and integral of D dot d s is equal to integration of Rho V, DV and instead of considering the contour, you have to consider a patches of, so it's like two patch, two patch and then you stitch the patches together, by making this Delta Z go to 0. Okay? when you look at the total magnetic flux that is coming out of this, area that would be constituted by the bn times D s, similarly that would be on other side B and times D s, but these two patches must actually carry out, equal surface I mean, equal flux and that essentially means that, B n1 must be equal to B n2. Okay? So, I won't prove this, but these two are also very important relationships and for the case, where we considered earlier, the plane waves propagating with epsilon R 1and epsilon R 2, there are no free charges, I have not put any charges on the interface, nor there are any charges in the second or the first medium. So, for that case, K will be equal to 0, Rho s will also be equal to 0. Meaning that, all fields in the previously considered scenario, will be continuous. So the ET 1component will be equal to ET 2, en 1component will, of course, not be equal but it would be continuous, DN 1 will be equal to DN 2. Okay? Right?

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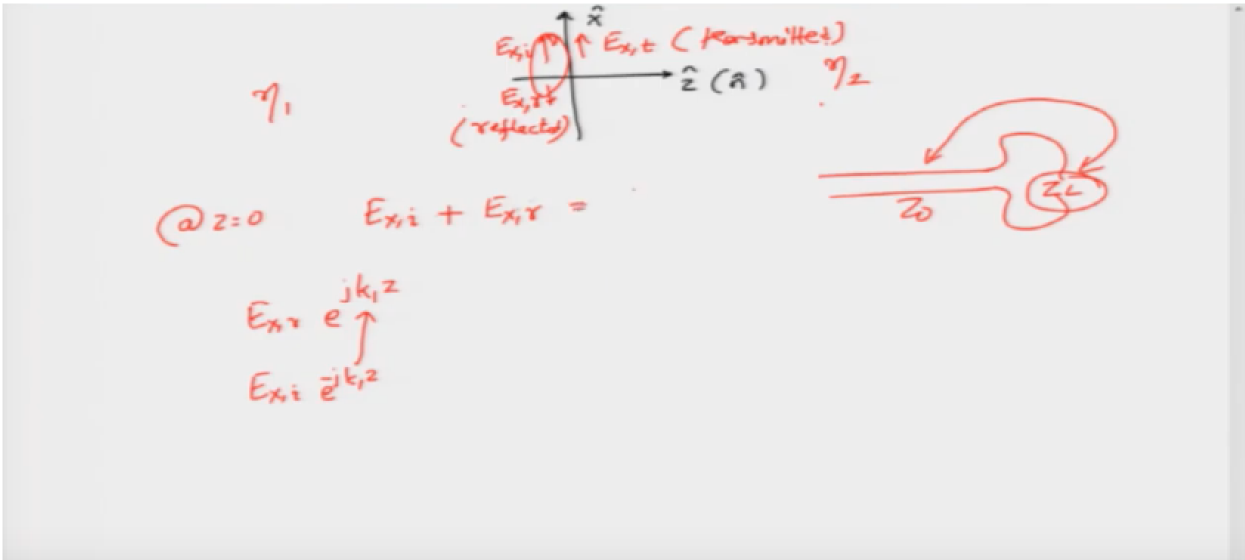

Now, how does this boundary condition help us, solve for the questions that, we raised in the beginning of the module. Right? So, I consider this, diagram again, this is the z axis, this is the x axis, this is the normal direction as well, which is going from medium one to, medium2 , I have an incident electric field, which is e X comma I. Okay? And then, E X comma I. Right? We will also have an electric field, which I mean; we will also have a wave in the second medium, whose electric field let us call it as, 'Ex Comma T'. And there will be another electric field, in the first medium, which will be e X comma R. Okay? Why should this e X comma R be present? It turns out that, the ratio of e x to H Y in this medium is given by ETA 1, the ratio of EX to H Y in the second medium is given by ETA2. Okay? And unless you introduce is reflected, so this is the reflected wave. Okay? Unless you introduce this reflected wave, you cannot satisfy the boundary conditions, you can't say that, incident electric field exactly equal to the transmitted electric field. Because, these two impedances are not exactly same, now this situation is very similar to the transmission line. Right? So, you had a transmission line, terminated in a certain Z L, unless the ZL was actually equal to Z naught, you always had a reflected wave. Right? And that reflected wave would account for, the energy that is being reflected off, from the mismatch that occurs between ZL and Z naught. In the manner that is very similar to that, we have an incident electric field uniform plane wave and a partial reflected uniform plane wave and a partial transmitted uniform plane wave, all of them together, you know, in the lossless case conserving the total energy. Okay? Or total power, across per unit area. Anyway how do we calculate, how much reflection we have? How much transmission we have? That is actually very simple; follow steps which are very similar to the transmission line itself. So you have an incident field EX comma I, in the ex directed way that will beaded to the reflected field. Okay? And both of these will occur at Z equal to zero, interface planes. So, our interface plane is located at Z equal to 0, of course the reflected wave would be propagating along, the minus Z direction, so it can be written as E Bar J k1 z, why should this be k1 as the same as the transmitted or the incident one? This is same because K One is a material property. Right? K1 is dependent on the value of mu naught and epsilon R. It is not dependent on the direction of the electric field .Okay? At least, in this uniform plane wave context, k1 is determined by the material properties; therefore the reflected wave will also have the same cable. Okay? a different k1 cannot have, cannot exist. Okay? This is the reflected wave; the incident wave of course is given by ex comma I, E Bar minus JK 1z, but the total field in this first region, which is actually the tangential electric field. Right? The sum of these two fields will be the tangential electric field in the first medium and these two, should be actually equal to the tangential electric field in the second medium, at Z equal to zero. Okay? And writing Z equal to zero and equating the amplitudes, you are going to get, this first expression for the amplitude.

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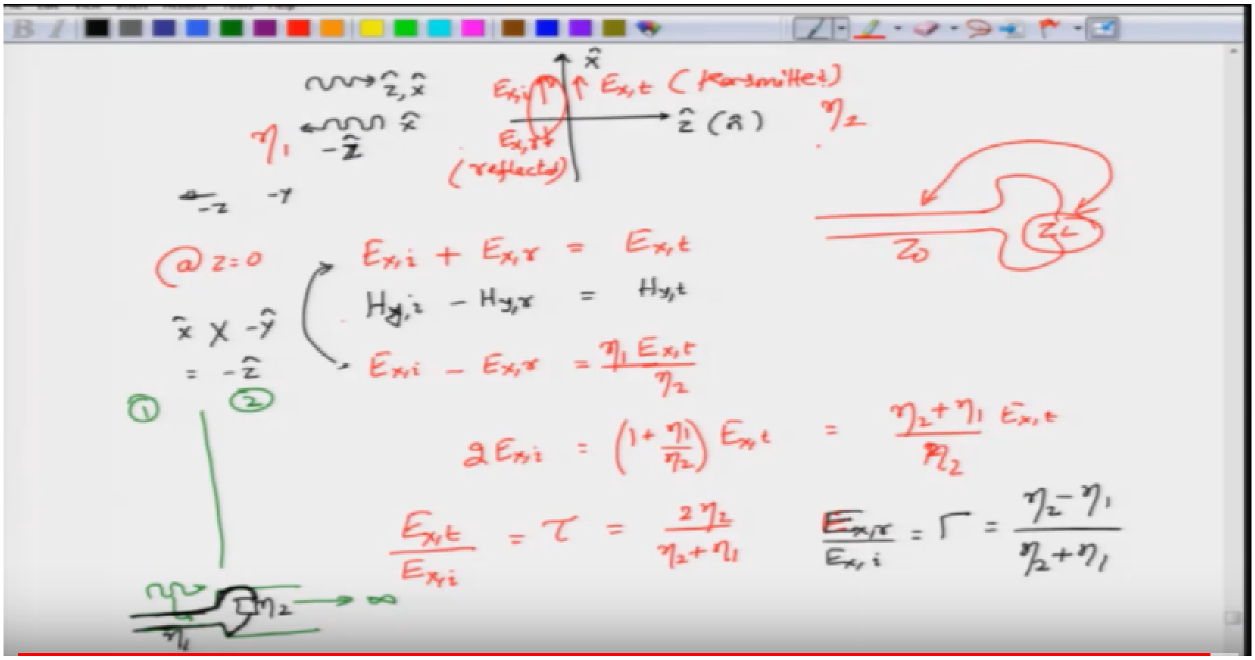

Now, the wave was propagating, this incident wave was propagating along plus Z direction, it's X, its magnate F sorry, it's electric field was oriented along the x direction. Okay? Now, the reflected wave is propagating along the Y direction, sorry, it is propagating in the minus Z direction and it's electric field ,we will assume it to be still oriented along the X direction, that is the reason, why I had this e X comma R, plus EX comma R. Right? what should the magnetic field Direction be, you know that, for the wave to go back, on to minus Z direction, the magnetic field should be along minus Y, only then X hat cross, minus y hat will give you minus Z hat which would be the direction of wave propagation of the reflected field. Okay? So, that if you remember that tell you is, I mean, what would the second equation that would tell you is, the total tangential magnetic field ,would be H x I minus sorry, H y I minus H y r that should be equal to H y comma T. however the incident wave amplitude, can be replaced in terms of, electric field amplitude, so this would be e X comma I divided by ETA1 minus e X comma R divided by ETA 1,that should be equal to e X comma T, divided by ETA 2. Okay? You can combine these two equations and you can already see that, what you are going to get? if you are going to inter if you are interested in the Ex comma R, let's first add these two equations, after multiplying the second equation by ETA 1throughout, so that I transport this off onto the Right hand side, so I will have ETA 1 here, so when I add these two I will get, 2 times e X comma I that would be 1 plus ETA 1 by ETA 2, times e X comma T this is basically, eta2 plus ETA 1 by ETA 2, E X comma T. So, the ratio of the transmitted electric field, to the incident electric field which we will call as, 'Tau', is given by 2 ETA 2 by ETA 2plus ETA 1. Okay?

Now, you can subtract the two equations and then show that, I will leave this as an exercise for you to show, that e X comma R divided by e X comma I, that is the ratio of the reflected electric field, to the incident electric field is, given by, which we will call as gamma is given by ETA 2 minus ETA 1 divided by ETA 2 plus ETA 1. Okay? To get this one, you simply subtract these two equations and then, substitute for ext in terms of EXI and then take the ratios and you are going to get this one. Now, it's very, very interesting that you actually landed up, with exactly the same equations, as we have already seen in the transmission line case. The reflection coefficient, there in the transmission line case was ZL minus Z naught by ZL plus Z naught .Okay? Here it is, ETA 2 minus ETA 1 by ETA 2 plus I ETA1. Okay? So, conceptually what you have is the medium one. So, let's say, this is medium 1 and medium 2 ,conceptually for the plane wave, what it would appear is that, this is a transmission line. Okay? And there is one more transmission line here, but because the second transmission line goes off to infinity, this is the direction of incidence; the second transmission line simply appears, as the load, with an impedance of ETA 2, across the first transmission line, termination. Okay? So this is exactly what, the wave is actually seeing. So this is the analogous thing that the wave sees and you know, because eta 2 is different from ETA 1, ETA 1would be the characteristic impedance, you see that, ETA 2 minus ETA 1 by ETA 2 plus ETA 1, corresponds to the reflection coefficient. Thank you very much.