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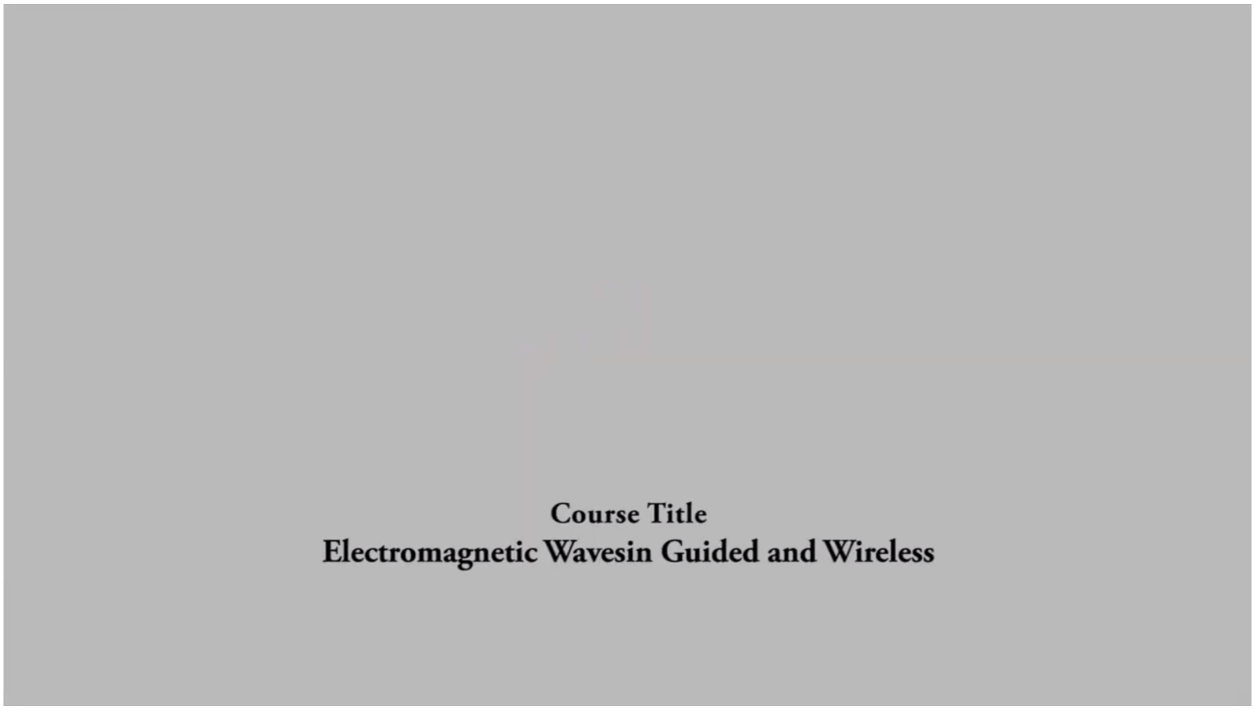
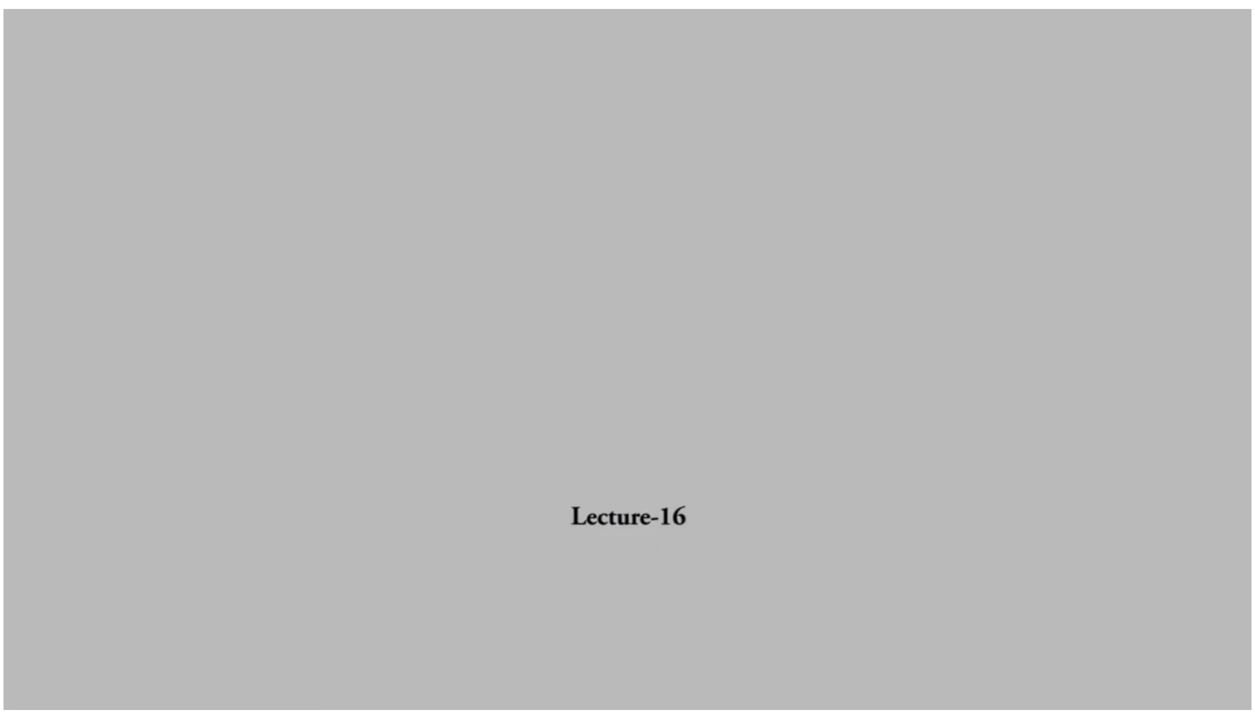
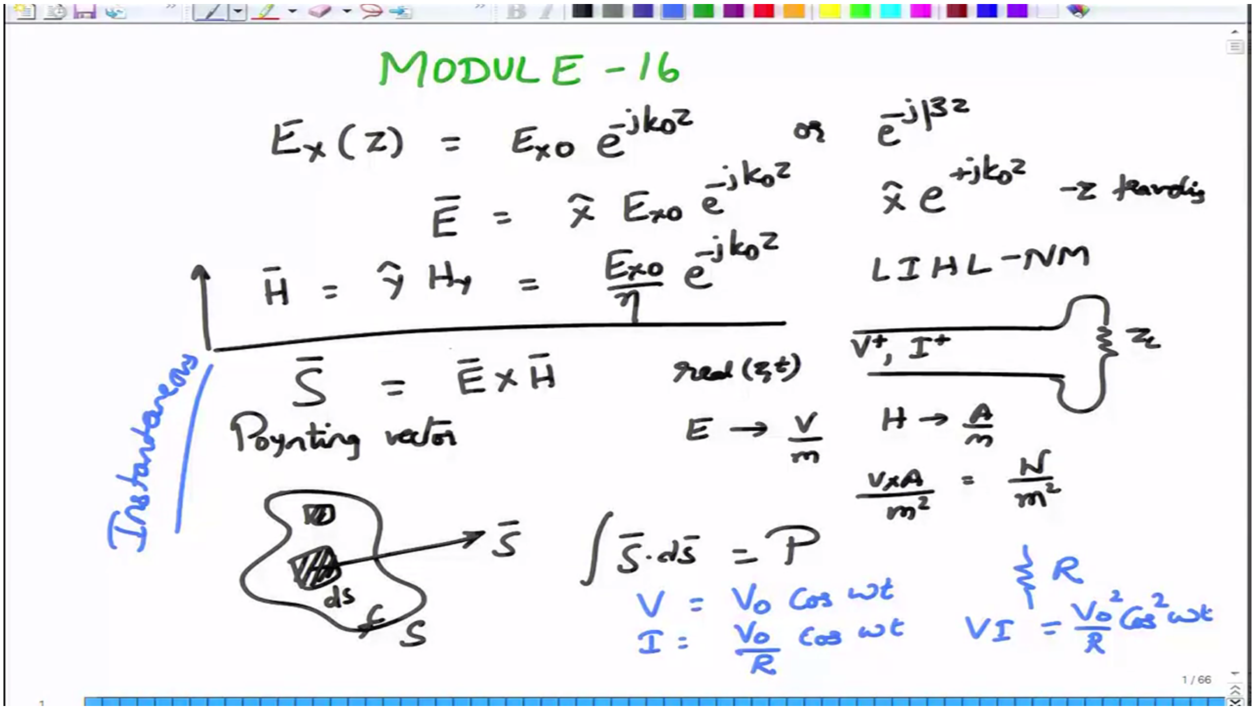




Dr. K. Pradeep Kumar: Hello and welcome to NPTEL MOOC on electromagnetic waves in guided and wireless medium. This is module 16 of the course, and in this module we continue our discussion of uniform plain waves.



As we've seen earlier for a sinusoidal propagation or sinusoidal waves, we can specify the electric field for assuming that z is direction of the propagation, +z is the direction of propagation, then you can specify the x component of the waves, being a function of z right in phaser domain as the amplitude that you had E_{x0} e^{-jk0z} or sometimes written as $e^{-j\beta z}$, okay, and we have already related this β or k0 to the frequency as well as the permittivity of the medium. So I will not go further beyond this one. In the vector domain, of course, I can write this E as x̂ indicating he orientation of the electric field and writing this as E_{x0} e^{-jk0z} for a forward or a z propagating wave. If you wish to talk about a -z propagating wave, then you replace the minus sign with the plus sign, so that this would correspond to a -z traveling wave, but please note that if I specify the polarization or the unique vector of the electric field component as x, then it would actually mean that the wave is still polarizing on the x direction, right.

You can also find the corresponding y component and you can also put that y component in terms of a vector, so that you get the H vector, which is given by ŷ H_y and this expression for H_y is very similar to the expression for x, so you can write it as, say, E_{xo}/η where η is the wave impedance that we've seen and it is also assumed to be propagating along the z direction. So this completes our plain wave description when the medium is linear, isotropic, homogenous and lossless as well as non-magnetic. So this is the medium that we have considered and we have this one.

Now in the last module, at the conclusion of the last module, we raised an important question. Now on a transmission line when I have the V+ as well I+ phasers or the voltage and current phasers, they do carry power, and in fact, they are known to deliver power to the load that you would connect at the end of the transmission line. Now I do not have a load as such in the traditional description of this one, but I do want the electromagnetic field, the electromagnetic waves to carry energy.

Well, why do we want that one? Suppose I actually have an antenna which would produce this electric and magnetic fields or the electromagnetic waves, say, E_x and H_y and then I have a receiving antenna somewhere here or receiver somewhere at this point. Only when waves carry energy and, in fact, deliver that energy to the receiving antenna, then the rest of the receiving antenna can actually work. So that is you will have some electronic circuits let's say, which would use this energy, and then if the energy is also modulated in order to carry information, then you can retrieve information, okay. So you are going to pump in some energy while generating the electromagnetic waves if the medium is lossless, then this entire energy will be carried by the electric and magnetic fields or the waves, electromagnetic waves, and then that would be delivered to the receiving antenna, okay. So for this to happen, you need to have the electromagnetic waves carrying energy or power, okay.

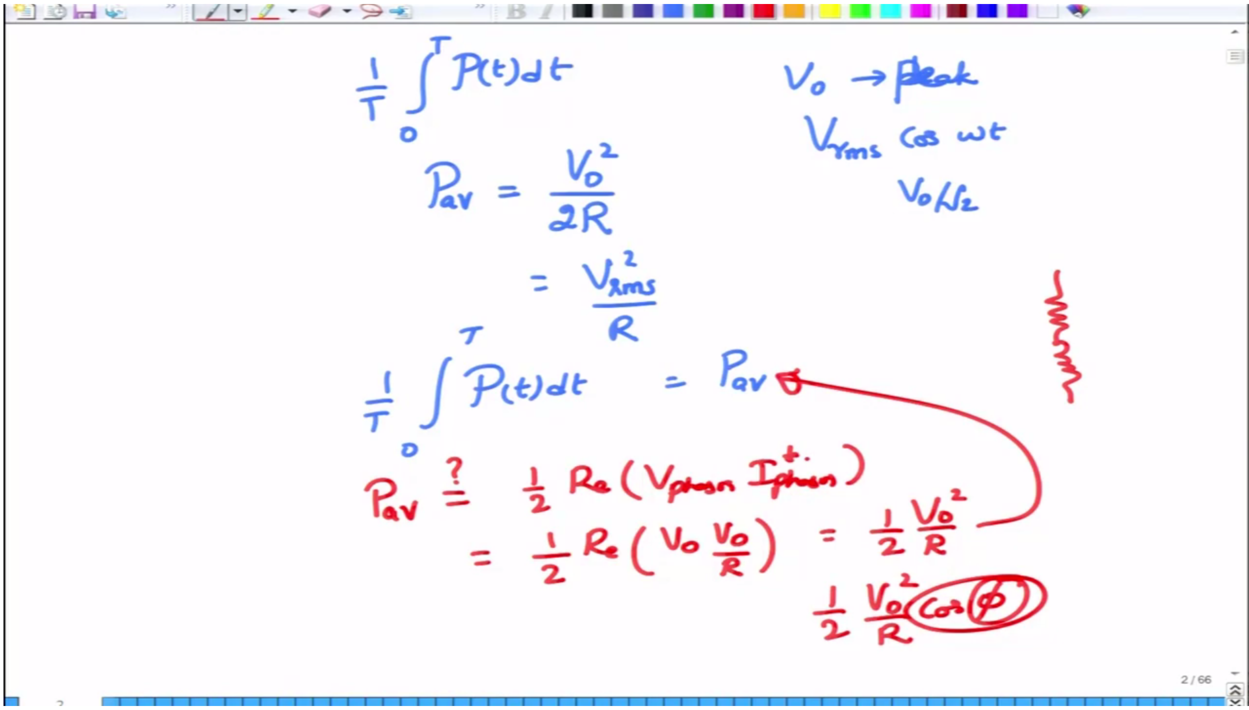
We define what is called as a Poynting vector, which we will denote it as S, some people also denote this as P, so I am going to denote this one as S, okay, and this S is defined as E x H, okay. Now we have to be little careful here. So we had assumed all phasers, okay, so all the equations that we wrote above are all corresponding to the phasers. However, this definition of S has actually given in terms of the full electric field. That is real z and time dependent electric field as well as z and time dependent magnetic field, okay. So that is the pointing vector that we have and it is important to note that this is a vector, okay.

It is also energy density. The vector S is also some sort of energy density. Why? Because I know energy density or power density. Why? Because I know that the units of E is kind of volt per meter, the units of H are ampere per meter. So then you take the cross product, okay, and we will soon simplify this one for the uniform plain wave. What you will find is that dimensions of this S or the units of S would actually turn out to be volt times ampere by meter square. Now volt times ampere is basically power and power we measure in watts. So this is basically watt per meter square. So since this is energy per cubic area kind of a thing, this is an energy density. So this is called as Poynting vector, because the nature of this one is a vector, and it is also sometimes called as Poynting power density, okay. So that is because it actually carries power or rather it is actually a quantity that represents the power density, okay.

To obtain the total power, you need to know the direction of this S vector. Suppose this is the direction of the S vector and this is the patch of the surface area that I am considering, then the total power being passing through this patched area, whose surface area is given by S is basically S.ds, okay. You can, of course, go to other patches and then essentially cover up the entire open surface. So this is actually the contour corresponding to the -- contour C corresponding to this open surface, and when you integrate over all these different patchy combinations, then you are going to get the total power, okay.

The other important thing to note here -- and this power, of course, is actually scaler, right. So this ds is the surfaced area which we will have units of meter square and that will cancel out the meter square, this one of the watts. So what you will essentially end up with is a scaler quantity which is basically the power.

However, the power that we have obtained or the Poynting vector that we have written here are all what is called is Instantaneous Poynting Vector, okay. I probably did not get that spelling correct for the instantaneous, but if I have got it fine. So this is basically instantaneous Poynting vector. This is very similar to considering, so let’s say, I consider voltage V to be some V_0 cosωt flowing across the resister of this voltage, and the corresponding current written as, say, V_0/R assuming that the resister has the resistance of R given by V_0/R cosωt. So then I multiply this V and I, I am going to get V_0^2/R. I am assuming V_0 to be a positive quantity. Similarly, R is positive quantity that I have assumed. So you have $\cos^2\omega t$. So clearly this is something that is varying with respect to time and this is an instantaneous Poynting vector.



What we are interested is not in the instantaneous Poynting vector, because many things are actually dependent on the average power that is dissipated in the resister. So how do I calculate the average power? I can look for the instantaneous power and then average it over a certain time. So I am going to assume that the time is arbitrarily chosen to be off with t starting at t=0 and going all the way up to T. This is a good expression especially when instantaneous power is periodic, okay, as it would happen when both voltages and currents are periodic. Otherwise, you can just take this as one snapshot of P(t), average it. Technically, you would also let this also go to infinity, okay, to obtain the better estimates. We will not worry about all those things.

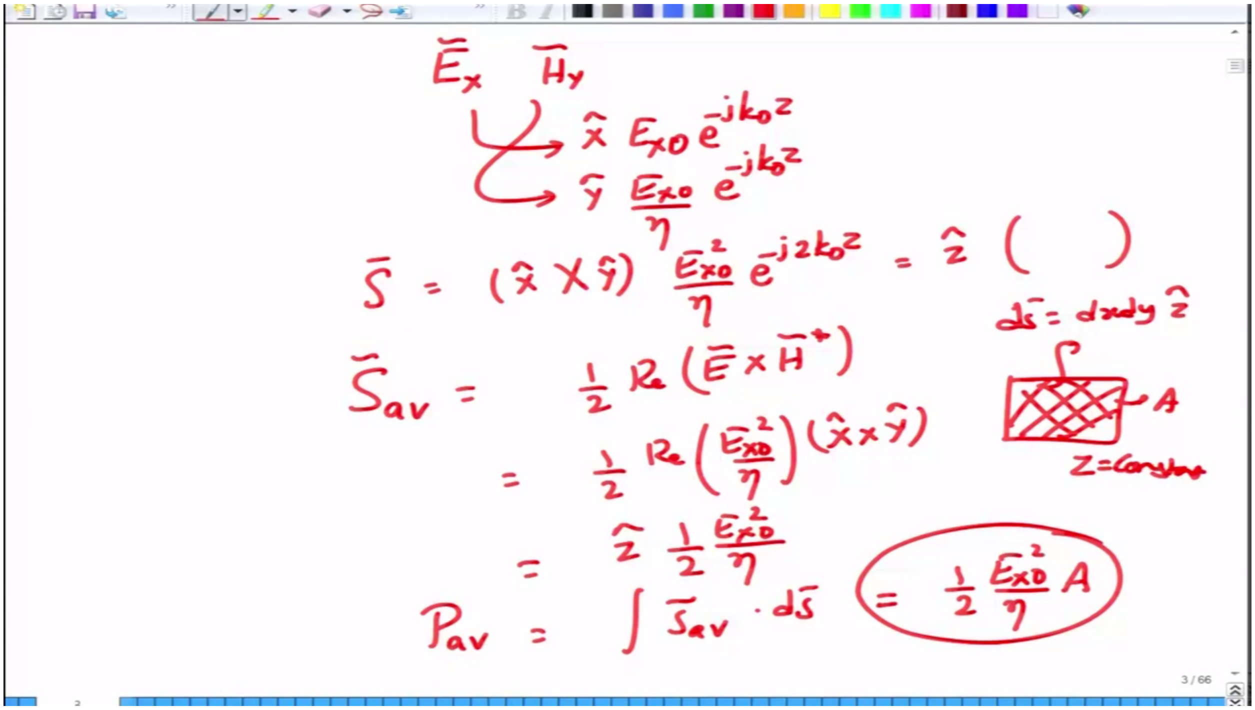
In this case, of course, when voltages and currents are both sinusoidal voltages of frequency ω and current of same frequency ω, then integrating out what you will see is that the expression basically becomes $V_0^2/2R$. So this would be the average power that is dissipated across a resister when there is a voltage V_0 cosωt and the current which is V_0/R cosωt, okay. We have used V_0 and denoted this as a peak value, sorry this is peak value, but if you were to denote or if you were to use the rms value and then write V as, say, V_{rms} cosωt, then you know that V_{rms} is given by $V_0/\sqrt{2}$. So in terms of that, this would simply be V_{rms}^2/R, okay. This also agrees well with whatever we have studied earlier.

Now a very similar thing is happening over here. This S is basically instantaneous vector, right, and then this ds integrating it is going to give you an instantaneous vector, but only when you integrate that power, which would be instantaneous power over whatever that time period that we consider, then we are going to get an average power, okay.

Now there is a slight easier way of arriving at this average power without carrying all those integrations. The idea is if I use this V and then replace that V with its corresponding phaser, right, the corresponding phaser in this case will be simply V_0, the corresponding phaser for this one is V_0/R, and then I form this quantity, which is ½ real part of V phaser, which is actually -- I'll write this as (V_{phaser} I_{phaser}) conjugate. This I will claim to be actually equal to the average power. Is it equal to average power? Yes, in this case it is exactly equal to the average power, because ½ real part of V_{phaser} corresponding to that V_0 cosω0t is basically V_0. The current is real in this particular case, so the current phaser is basically given by V_0/R. We also know that R is real in our case so this is actually going to be equal to ½ V_0^2/R, which is exactly the same expression as we have obtained earlier, okay.

There are some situations where the voltage phasers and current phasers are not going to be in phase. For example, when you have -- instead of just a resister, you add just a little bit of a reactance, maybe some amount of inductance. In that case, there will be phase difference between the two and what you would actually obtain would be something like ½ V_0^2/R cosφ or maybe $\cos^2\varphi$ I think and that particular φ or cosφ, sometimes called as power factor, okay. This power factor can go from 0 to 90, so in that case the power can actually go completely 0 to completely equal to maximum power that you can get. It will be equal to 0 when you have a pure reactance and it will be equal to maximum value when there is a pure resistive load, okay.

These basic ideas that one can define the average power as ½ real part of voltage phaser and current phaser conjugate. I need to have this conjugate, because otherwise this expression won't turn out to be correct as you can see in the assignment, okay. It can be extended for the electromagnetic waves as well, okay.



We will consider the case of Ex and Hy with the expressions that they have already written, and I am specializing this to this particular case, because the general expression is also very similar to this, okay. So if you understand the idea of average power calculation here, then you would have understood everything else. So now I have Ex phaser and Hy phaser, we already have written the expressions for E_x phaser, which is basically E_{x0} e^{-jk0z}, okay, with x̂, so this is a vector phaser, okay, and H_y is basically ŷ E_{x0}/η e^{-jk0z}, okay. The instantaneous Poynting vector, of course, will be given by E x H, in this case it would be (x̂ x ŷ) and then you have E_{x0}^2/η e^{-j2k0z}, which is okay, we don't worry about that one, and then what you get (x̂ x ŷ) will actually be pointing along z with this additional expression for E_{x0}^2/η e^{-j2k0z}. So if you fix z as a constant, that is you're evaluating this Poynting vector at a particular z, then you can still see that the Poynting vector would actually perpendicular to that z equal to plain constant, okay.

It also makes a lot of sense to us, right. We had considered a uniform plain wave propagating along the z direction and we are happy that the energy is also being carried along the z direction and not being carried by any other direction, okay. So this fact, you sometimes used to say that energy direction or the energy transport is at the same direction as the Poynting factor. There are few situations when you have an isotropic materials where this statement is not true. That is the Poynting vector as well as the direction of the energy will be different, but we will not study that one right away, we will postpone that study later on, okay.

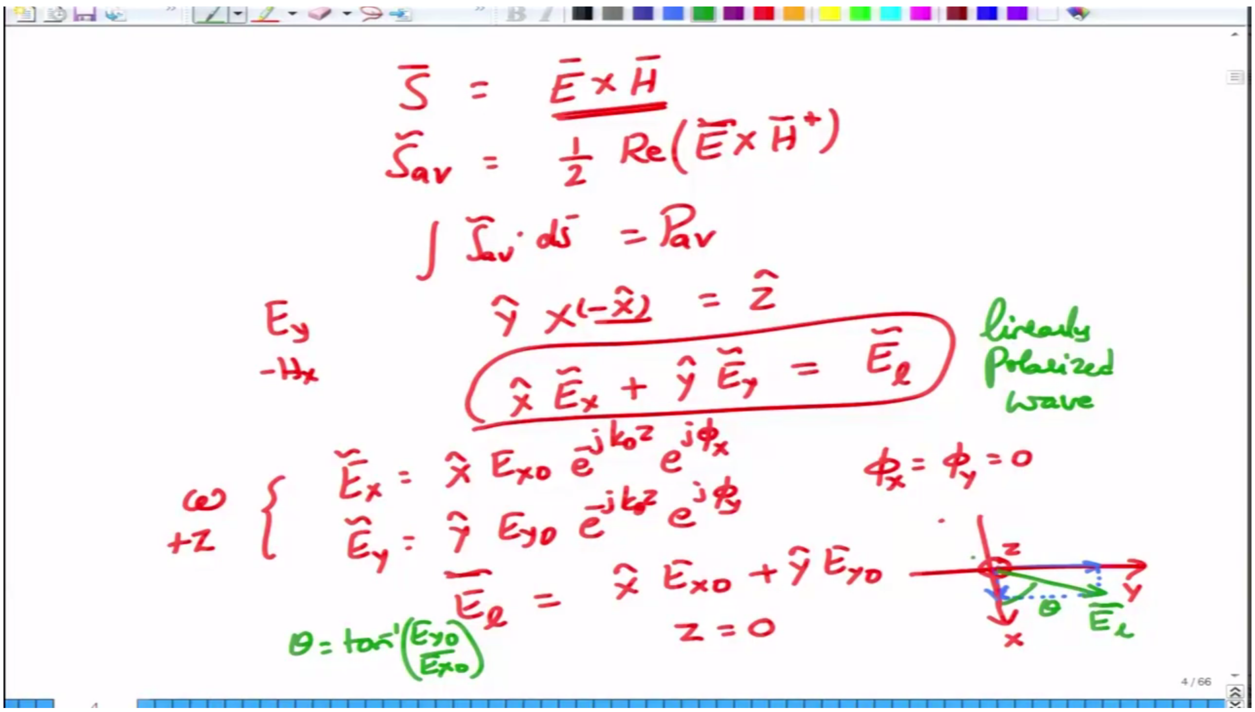
So this is the instantaneous Poynting vector. This is also a phaser quantity, but to obtain the average power or the average Poynting vector, I will write this as S average, I will write this tilde to denote that this is an average, but this is also a vector that I am denoting, is given by (x̂ x ŷ) ½ real part of E_x H_y, okay. We could not write the (x̂ x ŷ) right away. We could simply write this as ½ real part of E phaser x H phaser conjugate, okay. So you can expand by putting in the known electric field component and the known magnetic field component expressions into it, and you will get ½ real part of E_{x0}^2/η and please note that because I am taking the conjugate here, this e^{-jk0z} coming from the electric field will cancer out this e^{-jk0z} coming from the magnetic field. So I will have (E_{x0}/η) (x̂ x ŷ), right. Now (x̂ x ŷ) is basically z-hat, so indicating that the average power is also in the same direction as the z axis times ½ E_{x0}^2, which I have assumed to be real, divided by η. So this is the average power density.

Now to obtain the total power or the average power, what you have to do is take this average power and then integrate it over the corresponding open surface that you're considering, and then you will obtain the total power, so total average power. And this total average power will actually be equal to -- maximum value will be equal to ½ E_{x0}^2/η when you consider the plain to be some z equal to constant plan, okay, because in that z equal to constant plain, the unit surface area ds is given by dxdy pointing along z axis, okay, because this S average is also pointing along the same axis. dot product will be maximum in this case, and you will get this expression, okay. Please note that this actually independent of the plain area that you take, okay.

This is the importance of a plain wave, okay. Plain wave actually has electric field component not varying with respect to x or y, similarly, magnetic field component not varying with respect to x and y. And moreover, the average power density will also be independent of x and y. Therefore, when you orient your plain at some z equal to constant plain, then the Poynting vector will be pointing in the direction of increasing x, let us say, because that is what the direction of waves we have considered. Then if you integrate it over a unit area, right, and then see what is the power that is being carried by that wave, that would be the same if you double the area size, okay. So that is completely independent, because what you're actually seeing is that the average power density is independent of that area.

So if you simply take this one over -- I probably missed that one. So this is the average power that I have and then multiplying it by ds will actually give me the total power that is coming out as sorry area A. I am sorry what I meant to say is that when I increase the plain area, then the average power will also kind of increase, okay. So that is what we have written. This is E_{x0}^2/η, and then when I consider the area A of this integration, then I am going ½ E_x and H_y are independent of x and y; however, the total power is simply proportional to the area. So you can actually get a lot of power by simply increasing the area. Technically, you can get an infinite amount of power when you take this area to the infinity. So unfortunately, that is not practical and moreover, plain waves also are not realizable in practice. They an only be approximated in some sense, and in many cases, you will not be able to find -- actually in real life scenario you won't find waves which actually are independent of x and y directions, or x and y coordinates, okay.

So this uniform plain wave is a mathematical idealization, which is very good approximation for many real waves. However, you have to also keep in mind that they carry a finite power and finite average power, which of course also while it is showing that it is proportional to A, there's a limit to this particular equation, okay.



So I hope I have confused you enough about the average power. So to recap little bit, to summarize, this is the instantaneous power density, which is E x H. The average power density which you are mostly interested can be obtained by going into the phaser domain, so ½ real part of E phaser x H phaser, okay, and this would be the power densities, okay. To obtain the total power, you have to take this average power and integrate over appropriate surface area that you consider and that will give you the average power that is being carried by this electromagnetic wave.

Now this is also the reason why we paired last time E_y with $-H_x$. Why did we pair these two component? The reason is very simple. If you consider the electric field to be oriented along the y direction, and then I don't know the direction of the magnetic field. However, this is given that the wave is propagating along the z direction and it is a uniform plain wave, and you expect the power also to be carried along the z direction, okay.

So then you expect the power also to be carried the z direction, then what component I should take the cross product of y in the order. Remember, the order is E x H when you shift the order of H and E, because cross product is actually anti commutative. When you switch this and make H x E, then that would be a different result, okay. So we are not looking at that result. So it is important to note that the direction is actually fixed by E x H, okay.

So in terms of that to what components should I take the cross product of y such that the result will be along z. If I take y x y, that would be 0, if I take y x x that would be -z. So the option to me is to actually take y cross $-\hat{x}$, okay. So this is the reason why I took the electric field to be around the y direction and the magnetic field to be along the -x direction, okay. So this is the uniform plain wave, which is propagating in a lossless material and oriented along either x direction or in the y direction.

Now you can immediately ask the following question. Maxwell's equations are all linear, okay. I have an x polarized wave, I have a y polarized wave. Is it possible that I actually have a combination of these two waves? It is possible to have a combination of these two wave, because as I told you, Maxwell's equations are linear, the wave equation is linear, and inf act, if you take the x component electric field, okay, or the phaser corresponding to that, and add that to the y component phaser E_y, this will define a new electric field phaser, which we will call, say, some E_l, okay. I am just using some arbitrary dummy variable here, and I can actually have this particular case, okay.

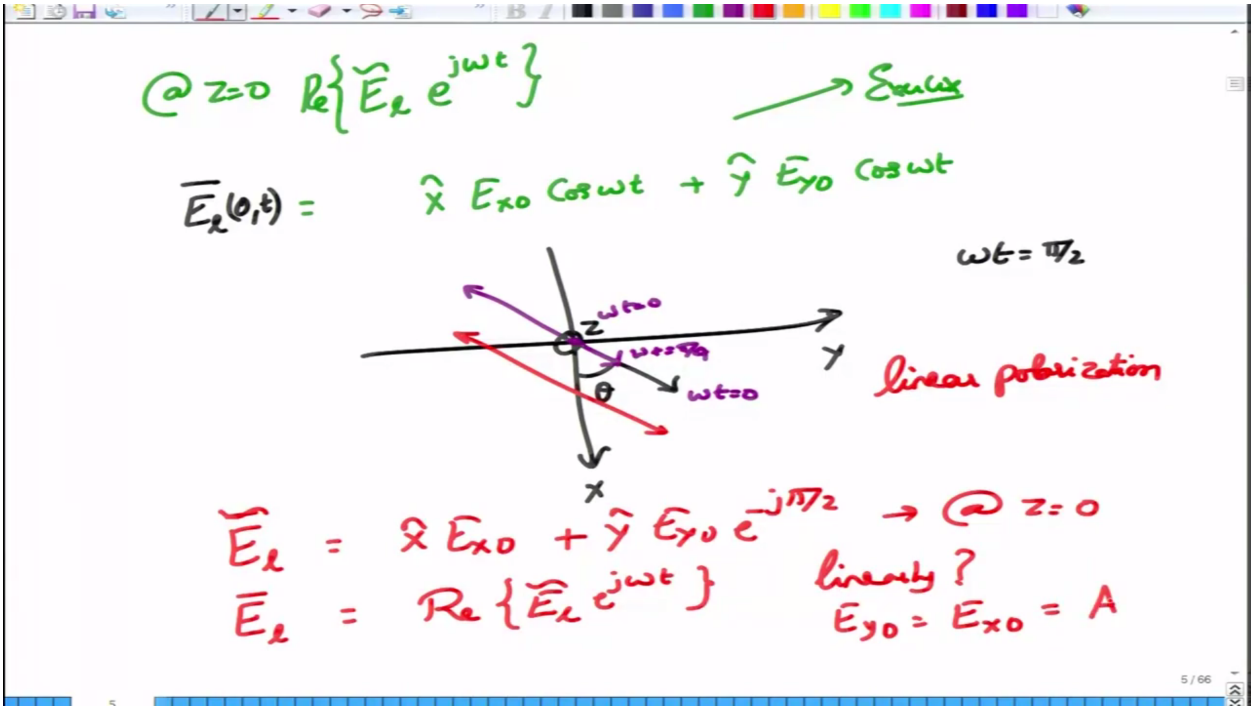
There's also another thing that I can do, right. I wrote the expression for E_x phaser as, say, $\hat{x}\ E_{x0}\ e^{-jk0z}$, okay. You could also add a phase φ_x corresponding to this expression without changing anything. All this is saying is that your phase reference or the field E_x that you've considered has some phase shirt φ_x, a constant phase shift φ_x perhaps, or even the phase shift that can vary with respect to z, but we're not going to talk about that one. This constant phase φ_x can be added without changing the solution. Of course, when you add this phase φ_x to x component, you will have to add the same phase to the y component of the magnetic as well, because E_x and H_y are the pair, okay.

You can similarly E_y phaser given by $\hat{y}\ E_{y0}\ e^{-jk0z}$, please note that both have the same frequency ω and are propagating along z direction, then you can add some phase φ_y without changing the solution, right. What it means is that x and y pair, that is E_x and H_y pair, or the x polarized electromagnetic wave, can propagate along z direction at this particular frequency, as can the y polarized wave propagate, and there can be a phase shift between these two or phase difference between these two, which will show up in some sense to define what would be the polarization of the new electric field component that is obtained by adding the x and y phaser, okay. So direction of this E_l is actually dependent on how these two are added with what phase that you need to add, okay.

To illustrate this, I will consider a simple example. I will assume that φ_x equals 0 and φ_y both are equal to 0, okay, and then I can put that expression into this and calculate what would be E_l. E_l phaser will be now ($\hat{x}\ E_{x0}$ + $\hat{y}\ E_{y0}$), okay, e^{-jak0z} is common so I am not going to worry about that one. I'll put this out, okay. Now I can also take any z equal to constant plain. I choose z=0 constant plain, okay. In that plain E_l will simply be equal to this expression, okay. Please note that this expression that I have written is actually at z=0 plain, okay.

In this plain, I have ($\hat{x}\ E^{x0}$ + $\hat{y}\ E^{y0}$) and clearly this x0 and y0 would actually correspond to, if this is my y axis and this is my x axis. There's a reason why I am writing this in this manner. The idea here is this. When you look down form the top, right, when you look down the direction that you are seeing will be along -z, okay, in that direction x will be in the direction hat I have written and y will be in the direction that I have shown, okay. So the z axis would actually be coming out in this manner. You're looking directly into the z axis. So you take x x y, that would actually come out along the z direction and you're actually looking directly onto that axis and you get this x and y axis, okay.

Now you have E_{x0} and you have E_{y0}, let's assume that E_{x0} has this length, okay, and E_{y0} length is this much, okay. So on this z=0 plain what would be the direction of this E_l phaser? This is two vectors being added, okay. So when you add these two vectors, the resultant vector will be pointing at an angle with respect to x axis. What is that angle θ? That angle θ with which this E_l phaser makes is given by tan inverse of (E_{y0}/E_{x0}). This angle θ as measured from x axis is given by tan inverse of (E_{y0}/E_{x0}). When E_{y0} amplitude equals E_{x0} amplitude, in that case the angle will be 45 degrees, okay. So this is the direction of E_l, which is now 45 degrees, making an angle θ with respect to x axis, and that is direction of the new wave that you have or the new super position of he two polarization components that you have, and because you've added these two component linearly and the resultant vector, phaser E_l, will also be in this particular plain along the line, this is called as linear polarized wave or sometimes called as linearly polarized wave. So this is very important for you to note down. So the linearly polarized wave will have its polarization direction along the particular line.



Well, not completely yet, we are not done with that one yet. I know the E_l phaser, let me convert that E_l phaser into a real -- and by the way, this real phaser is at z=0, okay. So I'll convert this real phaser into an expression which would also be a function of time, that is real expression. So I have to do that one by multiplying by $e^{-j\omega t}$ and then taking the real part of the entire expression.; when you do that, you're going to get two vectors, which are both varying with respect to time. So you have $\hat{x}$, E_{x0} cosωt + $\hat{y}$ E_{y0} cosωt. I'll leave this as an exercise for you to show that what we've written is correction, and once you have convinced yourself that this is correct, now let's go back to the axis, this is my x axis, this is my y axis. Please remember the z axis is coming in the right hand side rule according to this.

So now, let's see what would be the direction of this E_l vector, now this is a vector, the direction of this E_l vector at different times. This is all being done at a z=0 convenient plain for us. So at different times, what would be the direction. Now at time t=0 or ωt=0, cos function will be maximum, right, it will be equal to 1, and then the direction will actually be at an angle, which is given by θ and that angle is basically given by tan inverse of E_{y0}/E_{x0} as we have already seen, okay.

Now take ωt = π/2, what happens now? When I take ωt = π/2, I know that cos will be 0, cos(π/2) will be 0. So the direction is now right at the origin, okay. So somewhere between ωt = 0 to π/2, say, perhaps for example, at ωt = π/4, you would have had the vector to actually have this length, right. So this was when ωt=0, this is when ωt equal to, say, π/4, and this is when ωt = 0, okay. Now you reverse it. Suppose I take ωt = π, in that case cosine will be -1, cosine of π will be -1 and the direction actually shifts onto this axis, right. It would still lie on this same line. In fact, what it will do is that it will actually move between these two points, okay. So it will actually move along this particular line, which is making an angle of θ with respect to the x axis, and because it is moving along a particular line, we call this as linear polarization or we call the waves to be linearly polarized, okay.

As a different example, now you imagine taking this El phaser to be $\hat{x}\ E_{x0}$, and plus $\hat{y}\ E_{y0}\ e^{-j\pi/2}$. This is the φ_y that we have considered, and this is also at z=0, so this is also at z=0. What I would like you to do before I give you the answer or in the next module is that you should find out what would be this El, okay. The real version of this one that can be obtained by taking the phaser and then multiplying it be $e^{-j\omega t}$ and then drawing a diagram that looks similar to this one that I have drawn, and then tell me what is the polarization. Is it still going to be linearly polarized? If it is linearly polarized, what would be the angle of polarization? You can even assume in this case that E_{y0} amplitude is exactly equal to E_{x0} amplitude, which is equal to A.

So try this exercise before you see the answer in the next module, and what we have now done is to complete our study of uniform plain wave propagation in lossless, homogenous, isotropic and linear non-magnetic material. We will relax some of these considerations starting in the next module. Thank you very much.

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