Noc19-ee21

Lecture 15

Uniform Plane Waves-II

Hello and welcome to NPTEL MOOC on electromagnetic waves in guided and free space or wireless media. This is module15. We continue our discussion of uniform plane waves.

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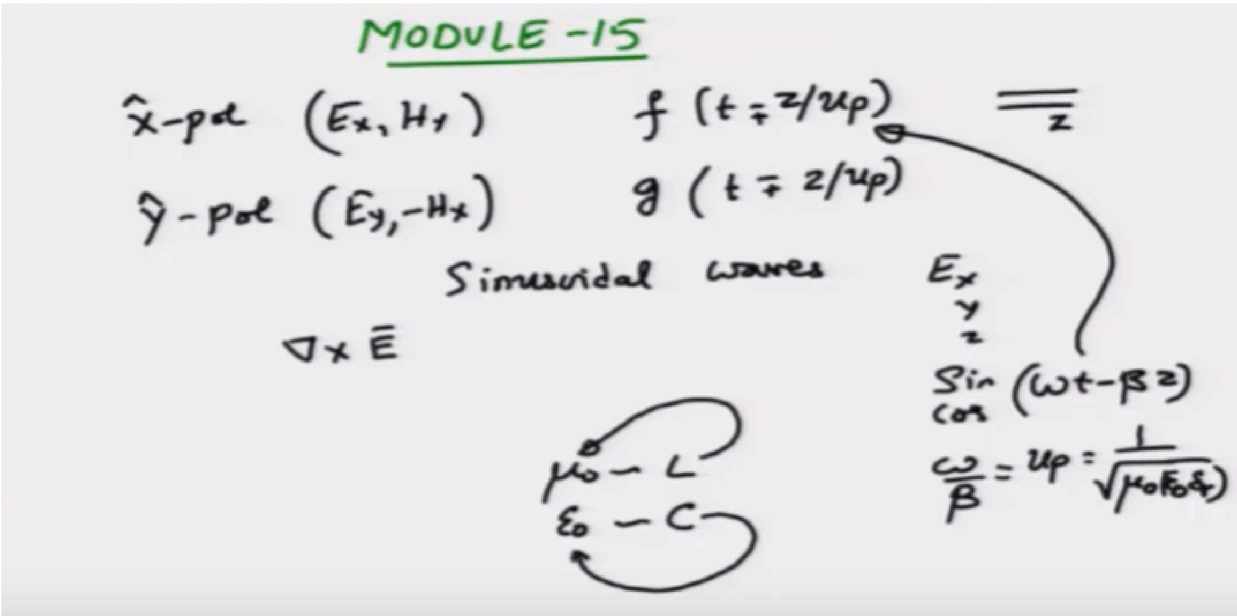

we have already seen that the electric field component Ex, the magnetic field component Hy, together satisfy the wave equation, where the waves are of the form some function of t minus or plus z by up, where up is actually given by 1 by square root mu naught, epsilon naught, epsilon R. we have already seen that, that corresponds to the velocity. Okay? And we also have seen that another solution exists, which would be Ey and minus Hx, Okay? which can be written as some function G, of minus plus Z by up and from our past experience we know that if I choose a minus sign that would correspond to a wave, which is propagating along the plus Z direction and If I choose a plus sign, in this function that would correspond to a wave propagating along the minus Z direction with the same velocity UP. Okay? This set of wave components Ex and Hy, which is analogous to the voltage and current on a positive or negative travelling transmission line along z axis is called as “X polarized waves” and this is called as Y polarized wave. Okay?

We will focus for now, only on the X polarized wave, okay? But whatever we talk about the X polarized wave would be applicable to the Y polarized wave as well, Okay? We will also specialize ourselves to, what is called as sinusoidal waves? Okay? Just as we start off with a general expression in the transmission line case and then we went on to sinusoidal You know, voltages and currents we said because any general form of a voltage or a current, can be built up using this sinusoidal functions by using Fourier series, so the same principle is applied or applicable here also, you can build up pretty much any decent electric field component shape Ex component or Hy component by, superimposing appropriately weighted sinusoidal components, appropriately weighted both in amplitude as well as the face of these sinusoidal waves and therefore, it makes quite a sense to talk about a single frequency or a sinusoidal signal or a sinusoidal wave, which of course is described by Ex and Hy components in our case. Ok? So, we are going to consider the sinusoidal waves or the phasor waves, as we can call them.

Okay? Now it is possible for us to go back to Maxwell's equations and then redo this analysis under the assumption that every component, that we are considering, whether it is Ex component, Ey component, Ez component, although in this case Ez component doesn't exist or any of the components, that we are considering all exhibit a behavior, which is like either sign or cos of Omega t minus beta Z type of a behavior. Please recall that, this is a very good, you know function in the same way as this function of t minus, Z by UP, because this UP can be related to Omega and beta right in the same manner, as you have done it in the transmission line case, You can still write the same thing for this Omega and beta okay?

Except that this Omega by beta, which is UP is given by 1 by square root of mu naught epsilon0 epsilon R. ok? So that is the only difference that you are going to see from the expression of UP in terms of the distributed inductance L or the distributed capacitance parameter C okay? In place of those L and C, you have mu naught and epsilon naught. If you consider a medium to be completely you know no material it's just a free space, then epsilon R will be equal to 1 and you can formally relate mu naught to L and Epsilon naught to C, right? in fact this relationship makes sense, because L is the inductive component that are the part of the inductive reactance, that was modeling the lossless transmission line and inductance is associated with magnetic fields and magnetic fields are associated with Mu naught. a similar arguments AC is associated with electric field or D field, electric field and E field is associated with epsilon zero.

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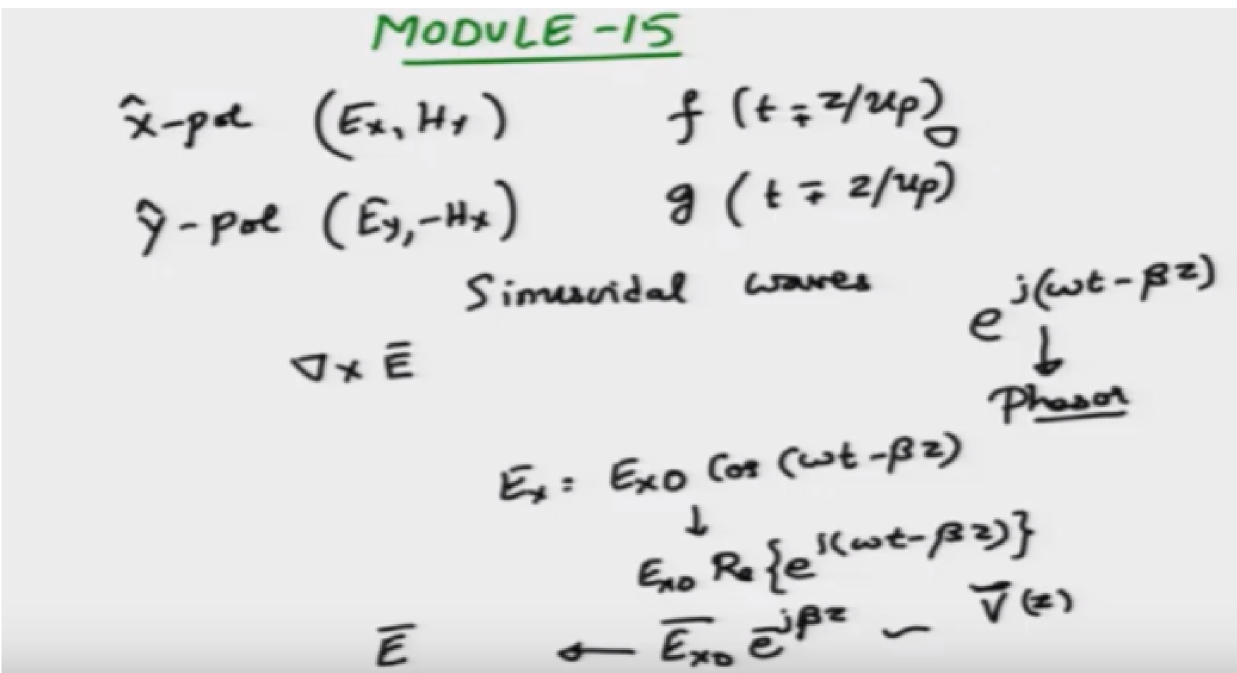

So this relationship of a transmission line can actually be made mathematically very rigorous, by identifying Mu naught and epsilon or mu and epsilon, with the distributed constants L and C of a transmission line. Okay? so that we will also have I know multiple times, where we are going to use this analogy, in order to solve even the plane wave problems but coming back to this sinusoidal wave analysis, since we assumed every wave to go either as cosine or sine of Omega t minus beta Z, you can drop the sine and cosine from the expressions and then assume that all field components move as or go as e power J Omega t minus beta Z. Okay? When you do that and further drop this power J Omega T, you will end up having what is called as a phasor? Okay? So, electric field Ex, which would have some amplitude, let's say Ex0 and a real variable Omega t minus, beta Z, can be first written as Ex0 real part o,f E power J Omega t minus, beta Z mathematically This is all fine and then drop this real e power J, Omega T and then simply write this as Ex0 e power, minus J beta Z, right? This was in fact very similar to the voltage phasor, that we wrote v of Z, right and there it was the voltage V zero times E power minus J beta Z, here it is the electric field component. Okay? So, but when you express everything in terms of this electric field component and convert this Ex0, which is an amplitude into a vector, you will end up having a vector expressed as a phasor. Okay? the vector phaser kind of a thing Okay?

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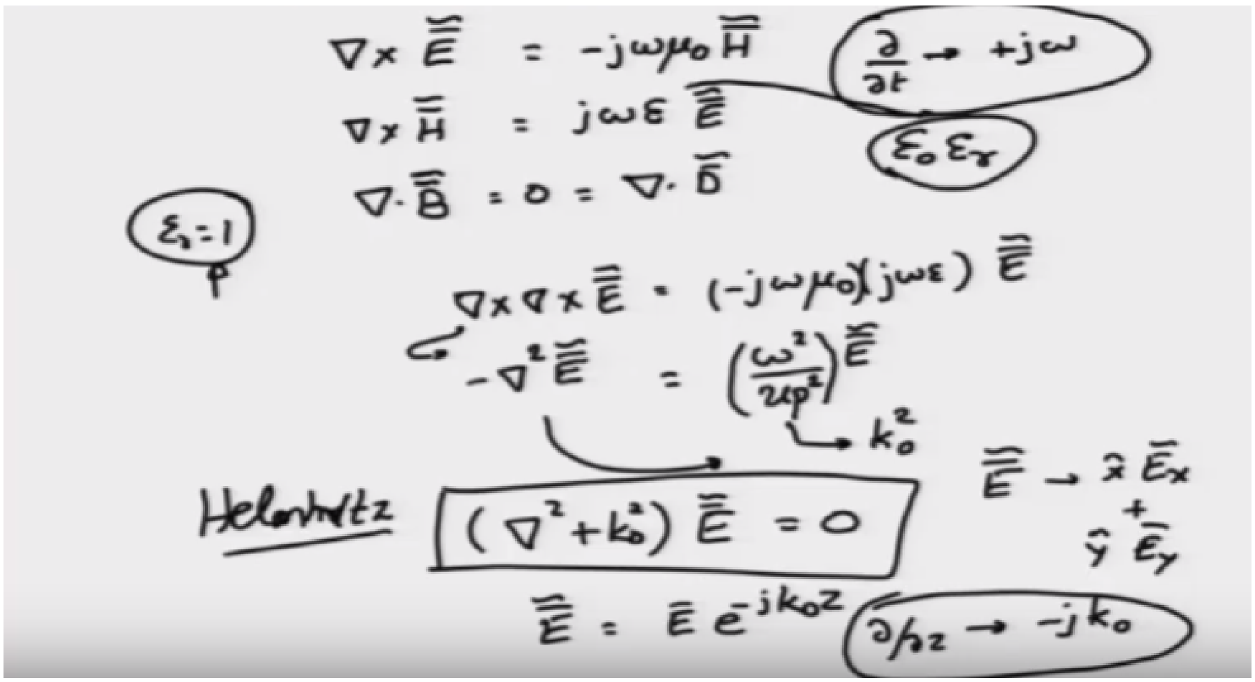

So, when you do that, you can rewrite all these in terms of phasor so I, now have to be forced to use two notations here, the bar representing the vector and this tilde representing the phasor, I am sorry, for this notation but I don't have the bold face in this, you know board, so therefore, I am forced to use a vector as well as the phasor notation. Okay? So this would be equal to now we know that in phasor domain Del by Del T, would correspond to multiplication by plus J Omega. Right? So we have already seen that earlier as well, so in that case this equation which is max sorry! Faraday's equation in terms of phasor can be written as, Del cross E equals minus J Omega mu 0 H okay? And the other equation you had del cross H was equal to or can be equal to J Omega epsilon. I will simply write it as, epsilon because otherwise I have to keep writing epsilon0 and epsilon r, writing to epsilon Two epsilon is little tiring, so I'm going to keep this as J Omega epsilon but you substitute epsilon 2 epsilon 0 epsilon are mentally Everywhere in these equations ok? So, I have this electric field phasor and the other two equations are phasor equal to zero as is equal to Del dot d. I have assumed free space lossless you know all those media things that; we have taken in the same previous class or previous module as well ok? So these equations are now reduced and in fact you can show that when you take the curl of curl off the phaser you are going to get minus J Omega mu naught and then, you are going to get curl of H and in the phasor domain curl of H is, J Omega epsilon. So you simply multiply this one by J Omega epsilon, ok? And then you write down this as e phaser. now minus J times plus J is one, Omega times Omega is omega square and then you have mu naught times epsilon, but epsilon is epsilon 0 epsilon R, therefore, you can write this as Omega square by u P square in the phasor domain and following the same logic, as we did in the last module, you can show that this is going to be del cross E phaser, this would be a minus del cross E phaser and then you can call this Omega square by u P Square, as K0 square, that is a constant, okay? For the given value of Omega this is going to be a constant. I'm going to call this as k0 square, okay? Because I'm going to assume that epsilon R is equal to 1, unless I specifically mention epsilon R to be naught 1, okay? So for this case when you specialize this one - epsilon R equal to 1, you can simply use this k0 square, otherwise, you have to keep using this k0 square, epsilon R, which I find it to be slightly You know somewhere some – right, so I'm going to not write that one simply write it as k0 square, okay?

There is a minus sign here, therefore you pull this one onto the right hand side and therefore, you can obtain what is called as "Helmholtz" equation" which is valid for both electric field phasor as well as for the magnetic field phasor. Okay? So this is the equation, which is in many cases the starting point for our understanding even the wave guide equations are you know reduced down to this equation and then dealt with it. Please also remember r that this phaser actually is a vector meaning, that it actually has two components X, Ex phaser, plus y had Ey phaser in our case, right? Because we have said, he z equal to zero. Okay? So this is the phasor equation and this equation is the Helmholtz equation and one of the solutions of Helmholtz equation, which we have already kind of assumed is that this phasor, okay? Is equal to some vector, okay? Times E power minus J k0z okay? And consequently any Del by Del Z would also be just pulling out minus JK zero and leading the other term as it is so, you need to see the analogous result for time as well as for the space here okay? We can even simplify, as we have said we will talk only about EX and HY components.

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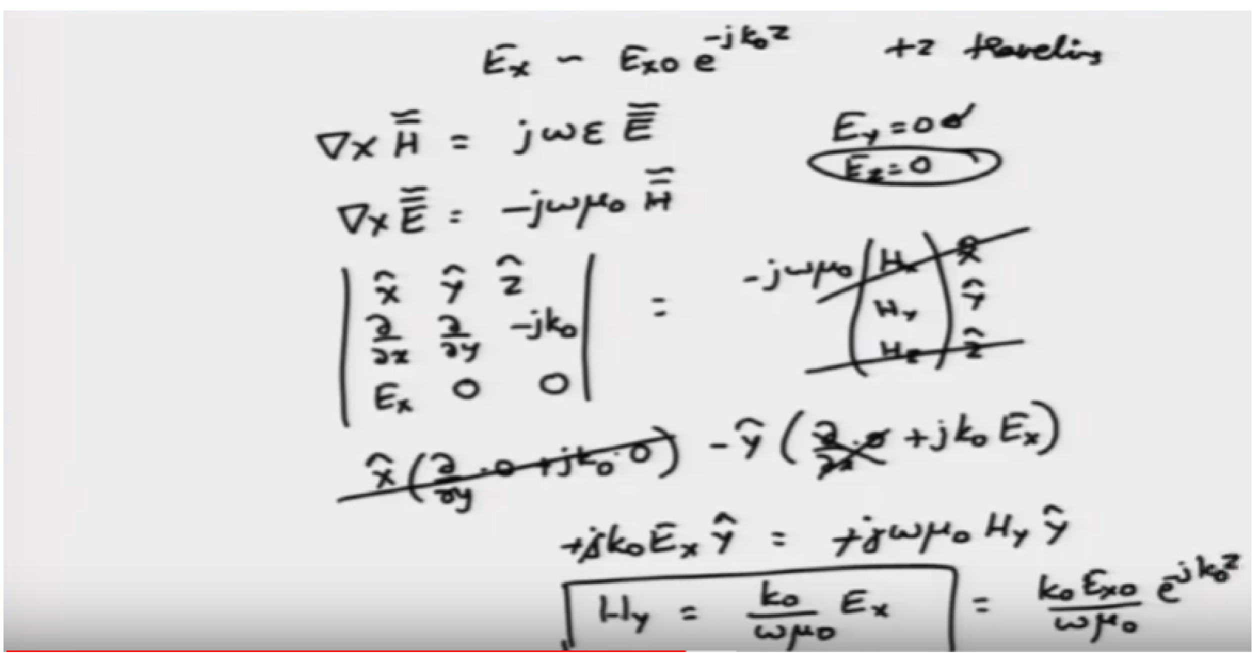

So in terms of only those components your Ex, could be Ex0, e power minus, JK0 Z for a positive z traveling-wave okay? The corresponding phaser for the minus Z traveling wave will be e power plus, JK0

Z. okay? Now we will make that connection between EX and H Y, much more you know I mean show show that it actually comes out mathematically. Okay? What we mean by that is I know? I'm going to consider del cross say H is equal to phaser is equal to J Omega, epsilon times, e phaser, right now let's put down this del cross H equation and on the right hand side we'll also write this one, now I will assume that, EY is 0 Ez is 0, the last one anyway we have assumed which is all right, this Ey equal to 0, I cannot I mean, assume it to be 0 because I can consider purely X polarized wave. I will talk about what polarization means shortly, but I can take this X polarized wave, which has only Ex component and which goes in terms of Z as, e power minus, JK 0 Z, it is not a function of x and y and in our case, we have also removed the time and then considered a specific frequency component. Therefore, Del by Del T has become J Omega. Ok?

So, let us write down this curl expression, so you had or rather we should actually write down the other equation. So we start from Del cross E, is equal to minus J Omega mu naught H phaser and then write down, what is Del cross e. So, I have X hat, I have Y hat and it's red hat, but del by del x is0 del by Del Y is 0 or rather, we will write del by Del X, del by del Y, del by Del Z, can be replaced with minus JK 0 and then I know Ex is the only nonzero component, so I have Ex and then, I have EY, which is 0, Ez which is 0, right and this determinant should be equal to minus J Omega mu Naught, HX, Hy and Hz right. So, this is the equation that you have or you know specifically writing it separately in terms of the X component. Now, you can show that when you take the X hat component here that would actually be zero, Y hat component will exist, Z hat component will be zero, right? You can expand this determinant and then show that only for the Y hat component what you will actually get. so, let's do it everything and then you can see that you know this will be, 0 minus of minus JK 0, makes it JK 0 times, 0, minus y hat, then, you have del by Del X, of 0 and then, you have minus of minus becomes plus J K 0, times e X right and then you have the Z component, which I will not write it so, this anyway was 0, this term is also 0. So, what you get for the Y component is minus JK 0, Ex, Y hat that should be equal to, since the right hand side is actually a, you know a vector, with x and y and z component and on the left hand side you have a vector, which is only the y component, clearly on the right hand side also that means the Z components and the X component should not exist and what you get is minus J Omega mu naught, Hy, right and then Y hat, so taking out minus and minus and J and J, you can clearly show that Hy, right is equal to K0 by Omega mu naught, e X. Okay? This is the phaser relationship. Ok?

And Ex of course is given by Ex0, e power minus, JK 0, so you can even complete that expression, if you would like, so you will have k0, Ex, 0 which is the amplitude, which is assumed to be real divided by Omega mu 0, times e power minus, JK 0 Z**.** So this would be the Hy component.

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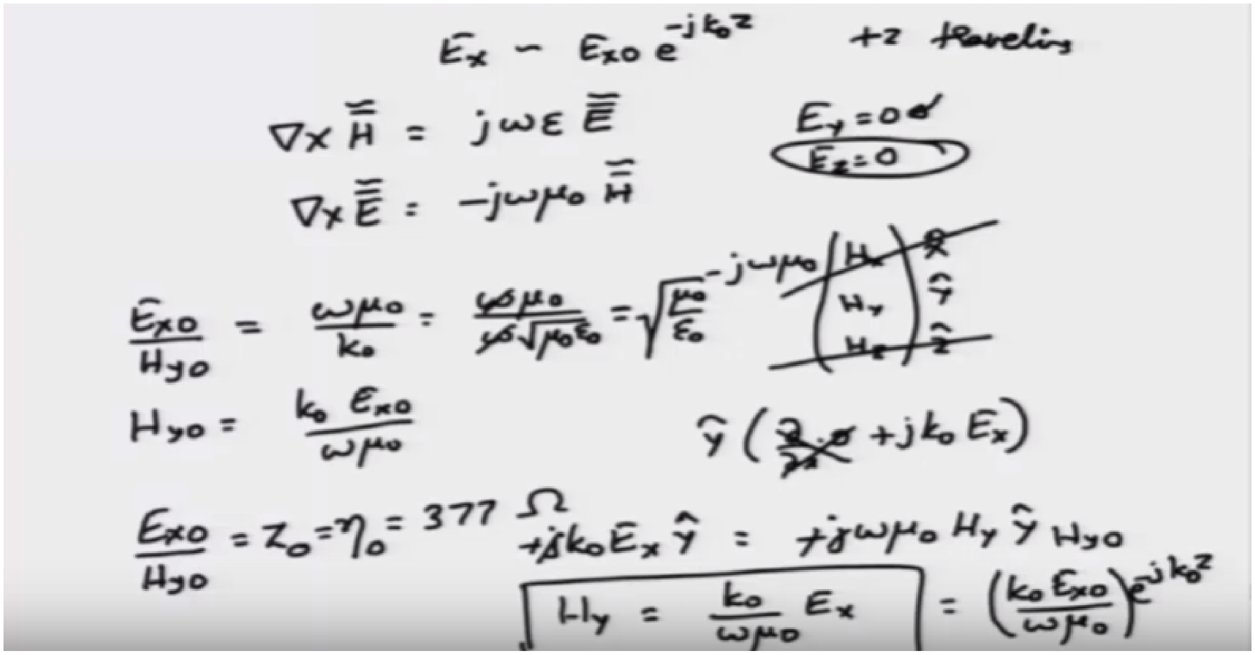

now take the ratio of Ex to HX that is you take the ratio of the amplitudes Ex to HX and call this term which is K0, EX0, by Omega mu 0, as the Hy 0 component, that is amplitude Hy 0. So now you look at Ex 0 by Hy0, that would actually be equal to, so you will have Hy0, equals k0, Ex naught by Omega mu naught, So taking this Ex0 by HY0, will give you Omega mu naught by K0, but I already know what is K0, right? I have Omega mu naught K0, is basically Omega into, square root of mu naught, epsilon naught, because I am considering only free space here, otherwise that epsilon R would be present, so this can be removed Omega can be removed and mu naught by square root mu naught epsilon naught can be rewritten as square root of mu naught by epsilon naught, okay? and when you plug in the values of mu naught and epsilon naught, which we gave you in the previous two other or maybe in one of the previous modules, you will find that the ratio of the electric field amplitude to the magnetic field amplitude, is actually given by or is it is denoted by a special symbol called Z naught or sometimes denoted by ETA naught okay? And that is given by square root of mu naught by epsilon naught, which is roughly 377. so meaning and you can also show that because Ex is measured in volts per meter and Hy is measured in ampere per meter, their ratio would actually be an resistance kind of a dimension. So you will actually see that the amplitudes are given by ohms, which is377 ohms okay? which also, means that the magnetic field amplitude in most of these cases will actually be quite small, it will be much less than the electric field amplitude. Okay? So, this relationship eater naught which I'll also have called by Z naught is suspiciously same as square root of L by C for an ideal lossless transmission line alright so there we call that lossless transmission line the ratio of the plus 2i plus, which is like the forward going voltage to the forward going current, as the characteristic impedance of the line similarly, here we can think of this as a characteristic impedance of the space itself right, where the ratio of the electric field component Ex naught, the amplitude of the electric field component to the amplitude of the magnetic field component of the corresponding Quantity, is actually given by 377 ohms, which is like Z naught, the characteristic of the material itself, square root of mu naught by epsilon naught. What happens when you consider a medium, which is not the same, as free space?

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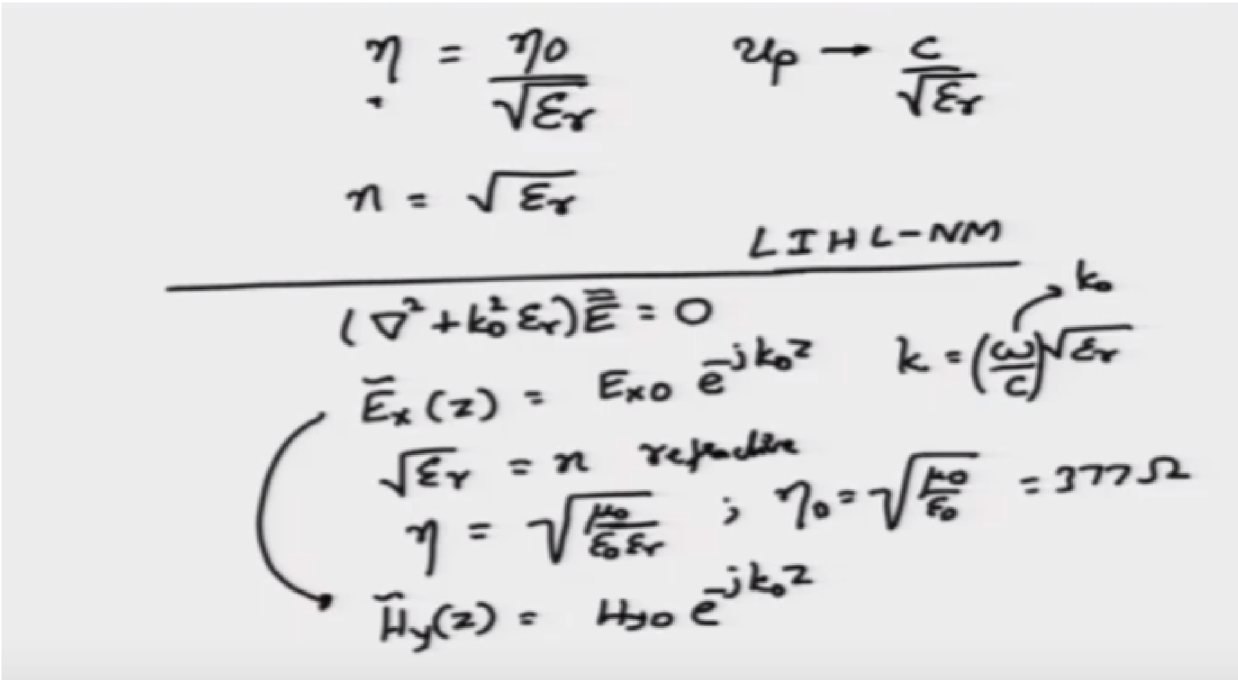

In that case you have to consider the impedance to be ETA and this ETA impedance will be ETA naught, divided by, square root of epsilon R. Okay? Similarly, the phase velocity Up in free space is denoted by the special symbol C and in any material will be C by square root of epsilon R. This is also a good point to mention that refractive index n which suspiciously looks like ETA but it is not ETA. Okay? This refractive index is used mostly by the optical scientists and engineers and this refractive index for a lossless medium can be related as square root of epsilon R, okay? of course this is for the lossless medium in a lessee medium these relationships slightly change, okay? But we will not consider a lossless medium until quite some time now. Okay?

So we are still exploring, what about this one it sells the free space medium lossless medium itself, so, to summarize right? We have Helmholtz equation applicable to any general medium with the condition that it is linear, isotropic, homogeneous, lossless and non-magnetic media, not any general media this is the specific media that we are considering with epsilon R given would actually So, in every electric field phaser and the magnetic field phaser will satisfy the Helmholtz equation specifically, we can show that Ex as a function of Z, in the phaser domain, is given by Ex naught, e power minus, JK0 said where K 0 is equal to Omega by Up, but in the free space or in other medium it would be, Omega by C square root of epsilon R. Okay? So I should actually not call this as K0. I should call this as K right. so I can call this K0 square epsilon R as K square and in that case I have K which is equal to Omega by C times square root epsilon R and call this Omega by C as K0. okay?

we also call, square root of epsilon R as the refractive index n, so this is the diffractive index for this medium the ETA, the medium impedance ETA, which can also be called as Z but we will call it as ETA is given by square root of mu naught by epsilon naught, epsilon R, where square root of mu naught by epsilon naught itself is called as the "intrinsic impedance" of the space or "characteristic impedance" of the space, it is given by 377 ohms, for the free space. Okay? This is another equation, we also know that

Ex corresponds to Hy, that there exists Hy phaser, which will have its amplitude Hy zero and propagating in the same direction, with the same face, that is very important these two actually have the same face and then, they would be propagating in this particular manner. Okay?

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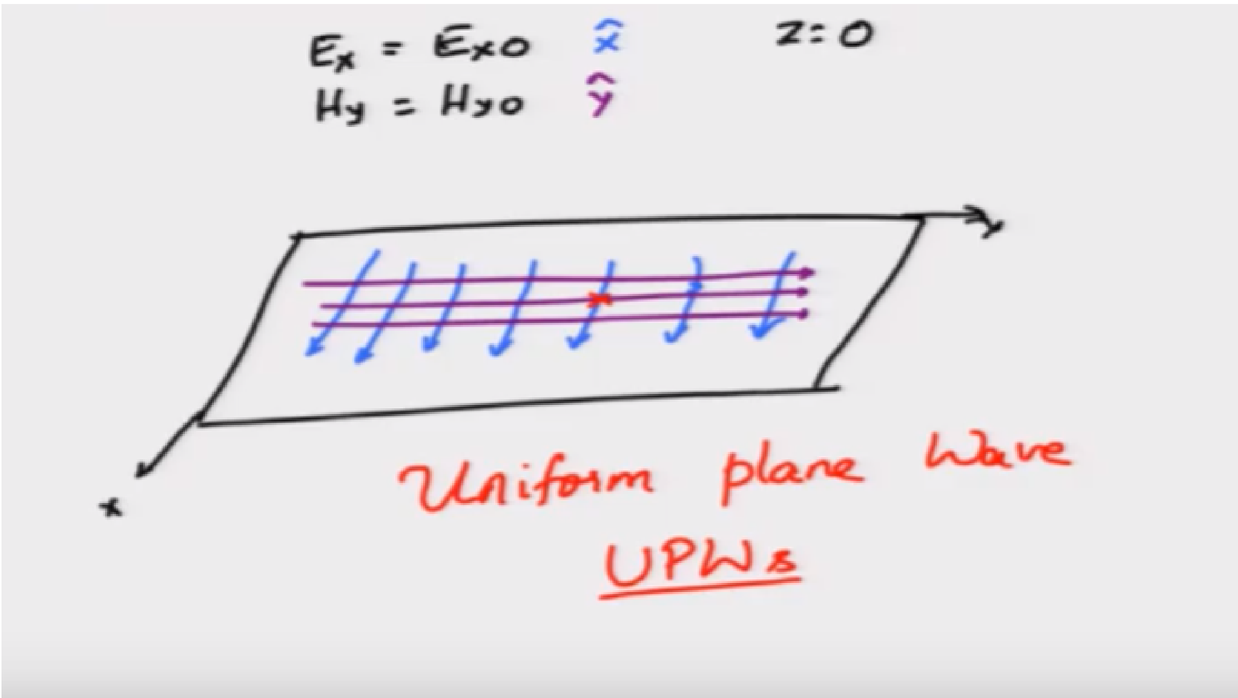

This is all what we have studied in the last two modules and if you actually think of a sketch for EX and HY, you can see that the Ex phasor if you fix Z is equal to constant. Okay? so one of the planes that I can fix that equal to constant would be the Z equal to 0 plane, which of course corresponds to the XY plane right. So, this is your X Direction, this is your Y direction and then Ex will actually be equal to e X naught on this plane correct and what would happen to Hy? Hy would also be equal to a constant Hy0, right however, Ex is a component which is X hat directed, Hy is a component, which is y hat directed. So, if I now write down the Ex field directions, it would be completely independent of the, you know x and y components it would be constant here, okay? And what about the Y component the Y component would be crossing the X component in this manner and it would actually be directed along this way, so it's like a mesh which is directed, one mesh directed along the X direction and the other mesh or rather the lines of this measure directed along the Y direction, Okay? And this mesh is actually the one that is changing at different planes, I considered Z equal to 0. So, which that they're you know amplitudes for ex0and Hy0 but if you consider a different plane and then go back to the real you know way these fields are actually written, instead of writing them in the phaser form what it actually means is that this face is this mesh itself will change in amplitude. so it will shrink and it will change and eventually at some particular plane it may even go to 0 and then again pick up you know the amplitude.

So, if you sit on a particular point and then calculate what would be this, you know the trajectory of that point, over the different planes, you will actually see that over the different plane, you will see a wavy like behavior, for both EX as well as for HY. so, this point, if you consider that would be like a wave which is propagating along the plus Z direction for both Hy and EX and this wave front, which is basically the constant plane the set of all vectors which actually have the same phase right same phase is obtained by fixing up Kz as constant and to collect all the vectors which have the same phase You will see that this phase is actually in the form of a plane, in the form of a wired mesh kind of a thing right, whose amplitude is independent of the X direction or other X direction as well as the Y direction, that is only dependent on the Z direction the amplitude of the wire-mesh changes only along the Z direction along x and y, it would be completely independent and because this mesh looks like a plane right, so this is you know you see this is like a plane to me, right and you have to imagine that plane extending all the way to infinity on every side.

So, that constant phase vectors EX and HY because they form a plane rather than a curve, right you, if this meshes were to be interrelated in the form of a curve, which is perfectly fine we don't really worry about that but then this would not be a plane, right? It would be a plane only when they intersect in this particular manner and then you actually have a plane like intersection of electric and magnetic field lines, right? and because of this nature and this plane wave being this is called as a plane wave and this plane wave is independent of x and y and its amplitude depends only along the Z direction on, xy-plane their values are independent of x and y and therefore this is called uniform plane wave. Okay? This is very important we are going to talk about uniform plane waves in the sequel. Now there is one very specific thing that you may actually object we said because there are no free charges and there is no conduction current density, those terms Rho and J, were actually equal to 0, but from your earlier electromagnetic courses, you may have studied that it’s the charges which are the sources of electric field and it is the current which is the source of the magnetic field, naturally right? I mean You take two charges put them apart you will have an electric field and possibly you Can do the same thing with current, right? it will take a current-carrying wire and there will be a magnetic field around it, granted those fields are steady fields or no static fields, but they have some sort of a source behind them, I mean there is some source, which is generating those fields, but in this plane wave behavior, we said that, J equal to zero, that is medium has no conduction current density Rho V is also equal to zero no charges, so then how exactly are we sustaining non zero values of Ex And Hy? Mathematics can tell us that you can satisfy this homogeneous equation and the solutions will be present but who will decide how Ex0 and Hy zero are to be present at all? I mean, what is the source for this wave behavior right, so where is the source that actually is generating these waves? It turns out that this plane wave is kind of an approximation that we are doing and you do need either, J as well as Rho or at least one of them to be present and to be nonzero, for this wave to be generated. However, we will assume that the source that is these currents and charges are located at far away to the infinity, So, you had Z equal to minus infinity and we are considering a region which is very very far away from it and therefore in this region there is no J, there is no Rho, but implicitly they are both present at infinity, in fact We can show that for these type of waves or for any kind of a wave, electromagnetic wave to exist you have to have charges and, you have to have currents and these charges are not should not be stationary. Right? So, they should actually be oscillating or they should be accelerating, only accelerated charges and currents, can lead to this electric electromagnetic waves. However, for quite some modules, we are not going to consider the generation or the radiation part of it. So, we will assume that, someone somewhere, way passed in the infinity and at infinity has actually produced these waves and these waves are now propagating and we are only concerned, with this part of the wave, where there are no free charges or anything else to interact with it. So the sources are there, but the sources are far away from us, okay? at least they are like say 1billion wavelengths away from us and the wavelengths that we are considering, you can make that approximation, I mean, I’m that billion number was just pulled out of you know this one, my hat it's not really the concrete number, except that mathematically we will simply associate all the sources to be at infinity, right? The second point is that we have seen on the transmission line V and I carry energy right or they deliver power. Will it be the same for this uniform plane waves. Can the plane wave carry energy or power? We will see that one in the next module that they will actually do.

Thank you! Very much!