

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

NPTEL

**NPTEL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

**Course Title
Electromagnetic Waves in Guided and Wireless**

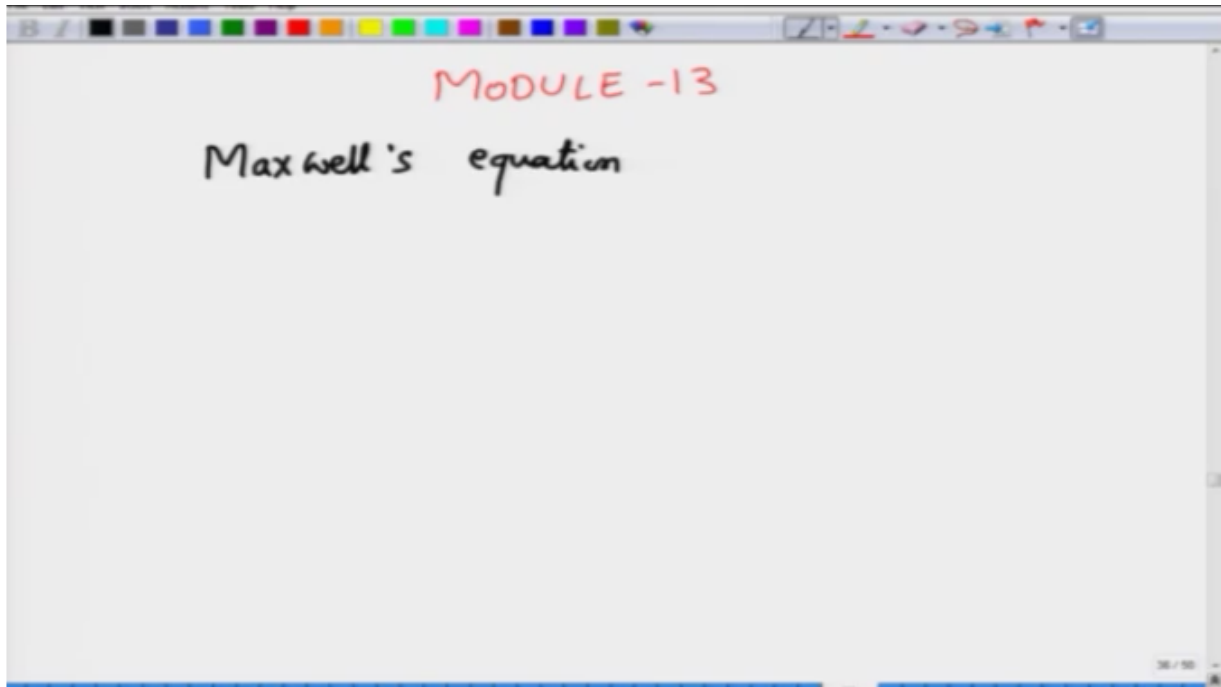
**Lecture - 13
Introduction to Propagation of Electromagnetic Waves**

**By
Dr. K Pradeep Kumar
Department Of Electrical Engineering
IIT Kanpur**

Hello, and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. This is our module 13, we have finished modules that correspond to transmission lines, we will have something to say more about transmission lines in a slightly different context as we go through over the next few modules, okay.

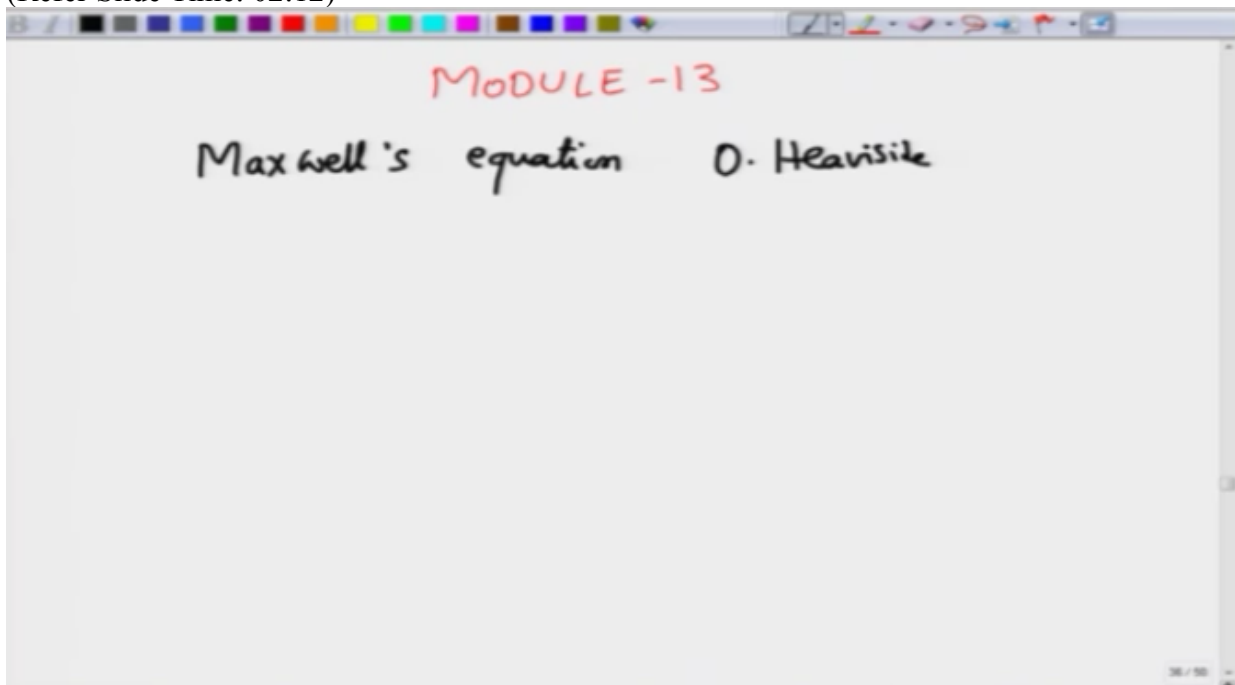
In this module we will begin the study of propagation of electromagnetic waves, okay, so far we have briefly at some points or time mentioned this word electromagnetic definitely we have mentioned it at the beginning of the modules, however we have so far not seen these electromagnetic waves, we have not described them and we have not studied them, so seen of course you can't see electromagnetic waves unless it happens to be light, but for us we want to capture the basic principles of electromagnetic waves using a certain set of equations so that we can talk about it in a much more intelligent manner, okay.

The starting point for any electromagnetic analysis is what is called as Maxwell's equations, okay, Maxwell was a English physicist who wrote down the equations which bear the name, which bear his name not exactly in the form that I'm going to write but in a slightly different form, the form that we are going to use to call Maxwell's equation uses what is called as Vector field quantity, so I'll talk to you about what a Vector field quantity is in a short while, (Refer Slide Time: 01:48)



but I would want to put down this Maxwell's equation to begin with so that we have a sense of what we are looking at, okay.

These equations as I told you were put down by Heaviside who was another English physicist whom we have already seen in the Heaviside condition that we talked about in the transmission lines and the equations,
(Refer Slide Time: 02:12)

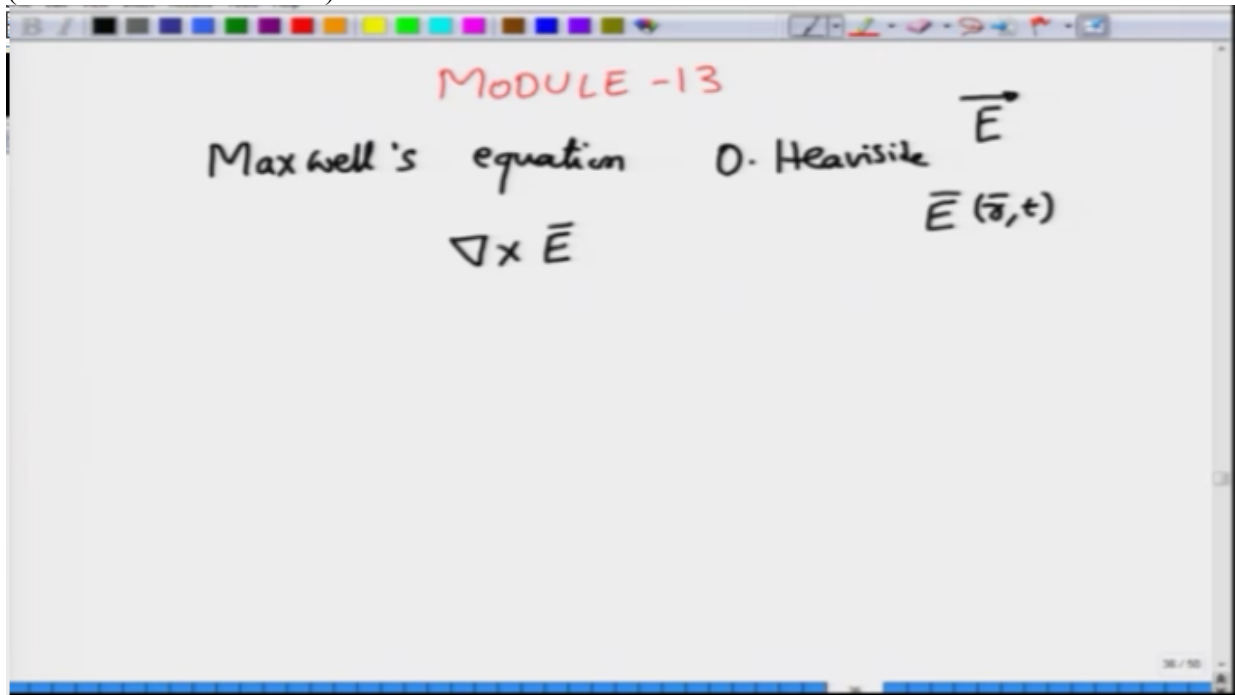


there are four equations basically supplemented with another equation called as you know the material equation, okay, those are mostly phenomenologically done, meaning that they don't

really come from first principals but they are kind of waste mostly on observations, okay, and they actually depend on what type of material medium we are considering this equations in.

So what are those Maxwell's equations? There are 4 equations, okay, and they are all return in terms of certain quantities called as electromagnetic quantities, there are 4 basic electromagnetic quantities that we are interested in and these electromagnetic quantities are all vectors, and only that they are vectors you know they are actually vector fields meaning that there value is different at different points in space as well as that value keeps on changing, by value I mean the electric field you know the magnitude of those, so by value I mean the magnitude of those is different at different points in time, sorry different points in space as well as those magnitudes also keep changing with respect to time, together we use this notation you know where we write E with the bar on the top, this bar indicates a vector, now other ways of indicating a vector is to actually put an arrow here and I differ it not to use an arrow but instead use a bar,

(Refer Slide Time: 03:28)

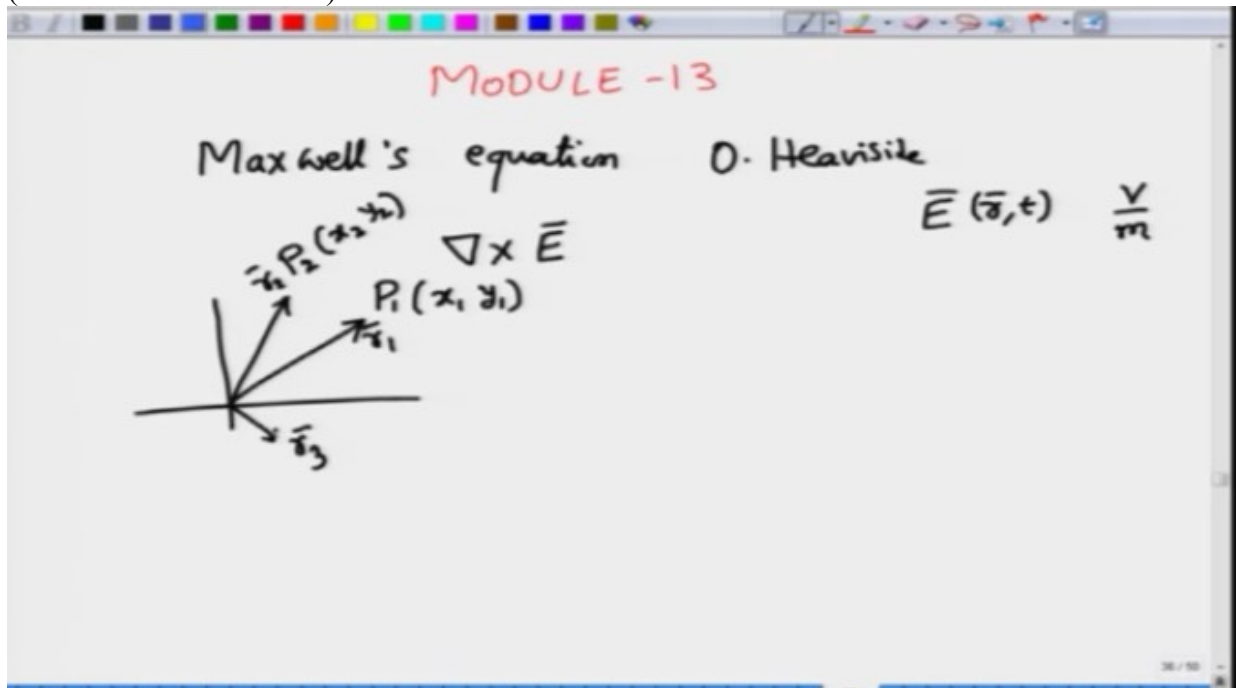


makes my writing easier in practice or in a print you will actually see that this electromagnetic quantities or any quality which is a vector is usually given in the bold form, okay.

So this is a electromagnetic quantity which is called as electric field, sometimes called as electric field intensity, it is measured in units of volts per meter in the standard SI units, and it is a function of space which is represented by the coordinate vector and time T.

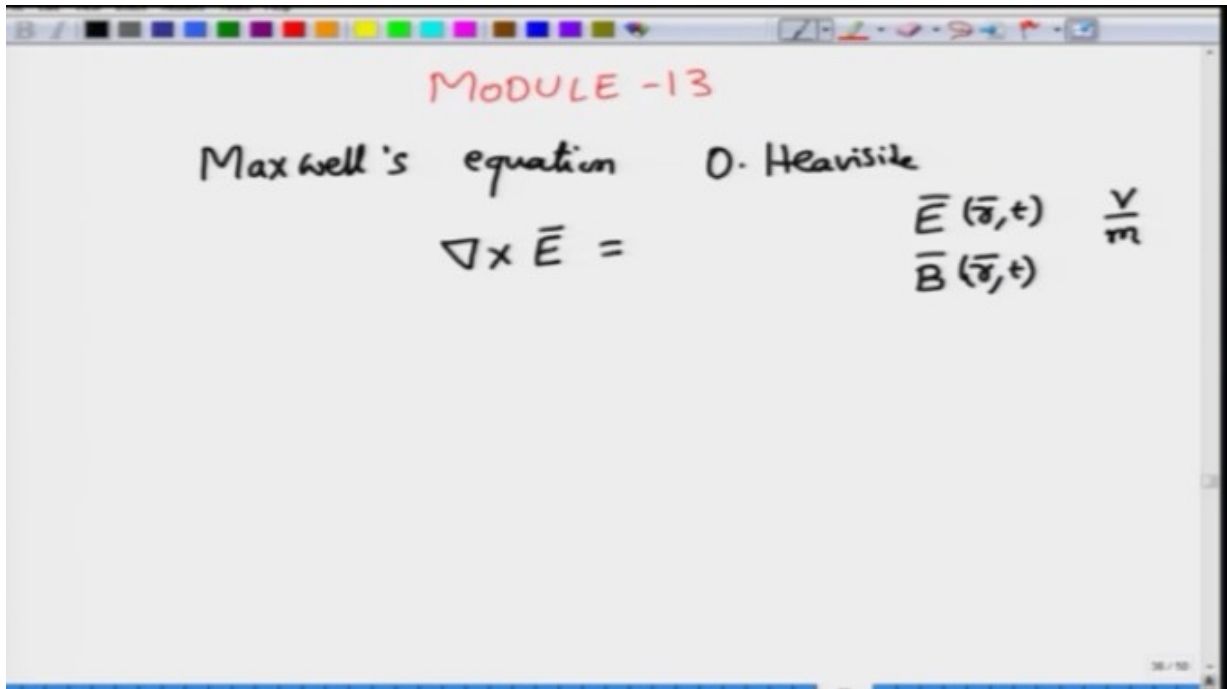
Now I would refer you to other NPTEL courses which will talk about vectors and vector analysis as well as to know little more basics about this electromagnetic quantities, our focus is not to be bogged down by the basics of electric and magnetic field but rather assume that you already know that first, you know you already know that and then we build on that knowledge to understand the electromagnetic waves themselves, okay it's kind of a second course as I have told you that you would normally take after you are finished first course on electromagnetics.

So this electric field quantity you must have seen earlier is a vector field it is function of both position vector \vec{r} , the position vectors are all given in this manner, so this is a two dimensional position vector \vec{r} so this maybe a position of a point P_1 which has a specific X_1 and Y_1 points, then you have another position vector which is say \vec{r}_2 the vectors, right, so these are all the directions of them, so position vector \vec{r}_2 represent the point P_2 which says X_2, Y_2 , this could be another position vector \vec{r}_3 and so on,
 (Refer Slide Time: 05:08)

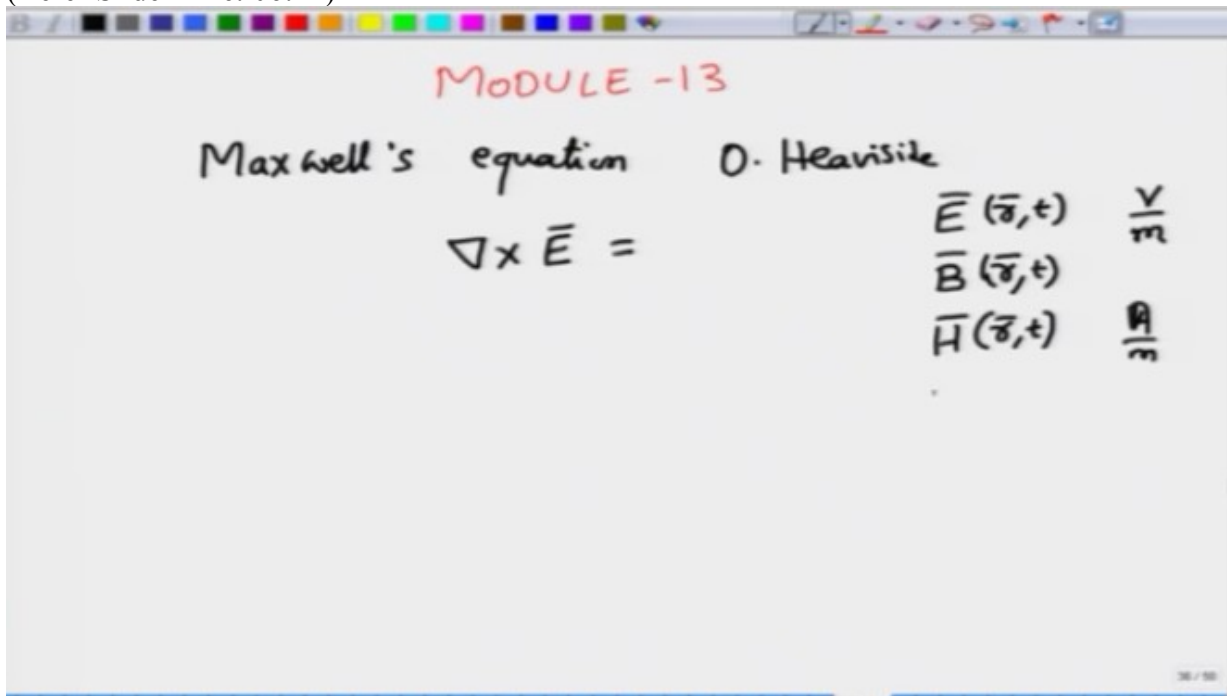


so at each position vector you have an electric field itself right, so electric field magnitude and direction itself can be written which would be the electric field, and not only that these magnitudes can change over time that is what we mean by space and time dependent vectors and that is entire thing is called as a vector field, okay, so this vector field is what we have and we will be dealing with these type of vectors fields, the one that we considered or that we wrote here is called as a electric field vector you know, electric field or sometimes called as electric field intensity and that is measured in volts per meter.

And there are additional electromagnetic field quantities, vector fields, this one is called as a magnetic field, sometimes also called as a magnetic flux density,
 (Refer Slide Time: 05:59)

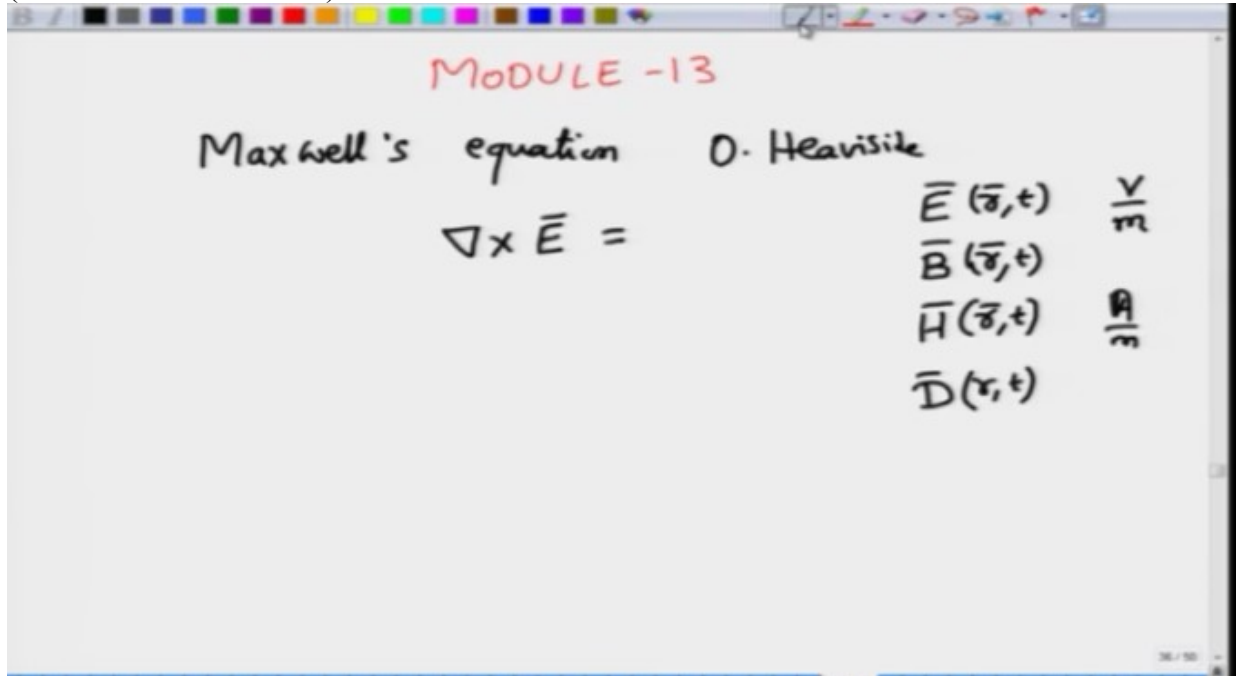


the way you call this as magnetic field or magnetic flux density depends on whether you're looking at literature from engineering point of view or from physics point of view, from physics point of view this is usually called magnetic field, but we call it as a magnetic flux okay, this is measured in some per meter square quantities, okay, we'll not really worry about that B, because we don't usually work with B in these equations, okay, we instead work with H which we call as magnetic fields, okay, most engineers would call, electrical engineers would call H as the magnetic field or sometimes called as magnetic field intensity, okay, and this would be measured in ampere per meter, okay, so this would be measured in ampere per meter H, (Refer Slide Time: 06:44)



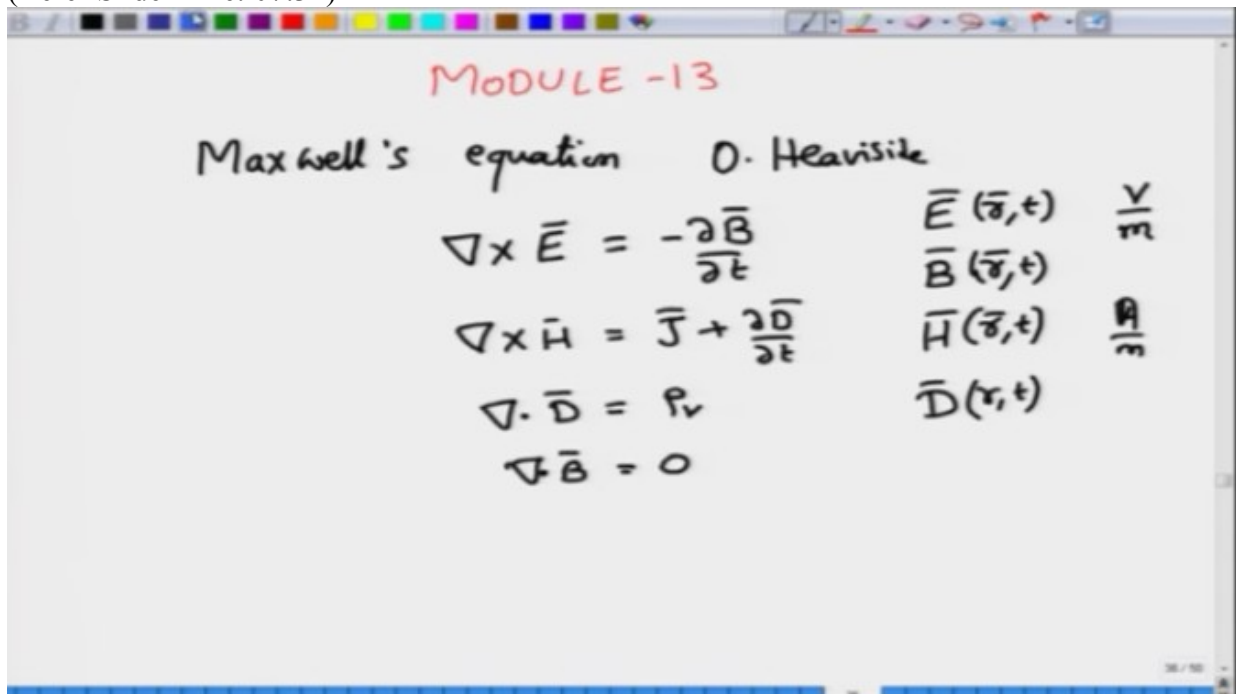
and then finally you have E, B, H and D which is another quantity which is very similar to B in that, we call electrical engineers call this as electric flux density, we measure this one in some coulombs per meter square, but again we will mostly not be dealing with D, but instead only be dealing with electric field E, okay.

(Refer Slide Time: 07:03)



However the equations that we write which are valid for any general matter as well is given by this $\nabla \times E = -\nabla B / \nabla T$, $\nabla \times H = J + \nabla D / \nabla T$, then you have two equations $\nabla \cdot D = \rho v$, and then $\nabla \cdot B = 0$.

(Refer Slide Time: 07:31)



Now many of these equations were known before Maxwell, okay, in fact this equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ is actually Faraday's law, okay, Faraday's law, okay. Faraday did his experiments and then found out that if you take a conductor okay and then you take a magnet and then move this magnet in and out of this conductor then if you measure EMF here, the electromotive force or the potential this movement of the magnetic field in and out of this conducting loop will actually lead to nonzero EMF value which usually we denoted by this different type of a letter \mathcal{V} , okay, a Greek \mathcal{V} kind of a thing, okay.

(Refer Slide Time: 08:16)

The image shows a whiteboard with handwritten notes. At the top, it says "MODULE -13". Below that, it says "Maxwell's equation O. Heaviside". To the left, there is a diagram of a blue loop with a battery and a resistor, labeled "Faraday's laws". An arrow points from this diagram to the equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$. To the right of this equation, there are three more equations: $\vec{E}(\vec{r}, t) \frac{V}{m}$, $\vec{B}(\vec{r}, t) \frac{A}{m}$, and $\vec{D}(\vec{r}, t)$. Below these, there are two more equations: $\nabla \cdot \vec{D} = \rho_v$ and $\nabla \cdot \vec{B} = 0$.

He also found out that it doesn't have to be the case that only the magnet has to move you keep the magnet stationary but you move the, you know the conducting loop okay, even then there will be EMF induced and the nature of this EMF that is induced in the loop will be such that it will oppose the very force that is actually causing the creation of this EMF that is this will actually oppose the mechanism which is generating this EMF, okay, and this minus sign was, and this phenomenon is called as Lenz's observation and this is called as Lenz's law the minus sign will, and the EMF that is generated as a result of this back action is also called as a back action EMF, okay, this part was known this is not something that Maxwell discovered, this was all known before Maxwell wrote down the equation in fact Maxwell followed much of Faraday's you know research and he was very impressed by Faraday's research and therefore he you know, I mean he did not have to invent this, he was already known.


These two laws are called as divergence laws, sometimes also called as Gauss's divergence laws, the first one tells you that if you take a closed surface okay, or rather if you take a close volume and then see that the volume you know as the volume size actually shrinks down and down, then you will eventually see that the density or this divergence which we would call it the vector field operation divergence would actually be equal to the charge density that is sitting here, so this ρ_v is called as the charge density,

(Refer Slide Time: 09:50)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's law \rightarrow



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{D} = \rho_v \\ \nabla \cdot \vec{B} = 0 \end{array} \right.$$

$$\vec{E}(\vec{r}, t) \quad \frac{V}{m}$$

$$\vec{B}(\vec{r}, t)$$

$$\vec{H}(\vec{r}, t) \quad \frac{A}{m}$$

$$\vec{D}(\vec{r}, t)$$

these are the equation that we have written in the differential form, we will write down the corresponding equations in the integral form to better talk about this things.

And then the third or the fourth equation this $\nabla \cdot \vec{B} = 0$ indicates that no matter how small you make this volume, you will not be able to find the divergence of B, okay.

A vector field is set to have a divergence when the electric field or the magnetic field or any kind of a vector field exhibits this kind of a field behavior, okay, so the field lines actually look something like this as though they are emanating from a source here, okay, you can imagine that you know there is some sort of a charge you know enclosed in some close surface, and then the electric field lines from the charge assuming they are positive, they will all be coming out, right, so there is a source actually to generate that electric fields and in a matter we instead of talking about the electric field E we talk about this vector D which usually doesn't really have a name, but we do called as electric flux density or electric displacement vector field.



The source of the electric flux or the electric flux density is actually the charge, okay, and we know that you can always you know isolate a positive charge and a negative charge therefore this phenomenon of D having a nonzero divergence simply follows from the fact that you have charges which are positive or negative and they can be isolated with respect to or whether, okay, however no such luck is there in the magnetic fields, okay, you can take a magnet you can break it up that magnet into smaller and smaller fragments, no matter how small you go, you will not be able to find anything that would be saying say this is north pole and this is a south pole, okay, these charges positive and negative are analogous to positive or they are north pole and south pole and you won't find these two poles isolated, wherever there is a north pole there is always a south pole, okay, so the field lines of this B would always have to loop back, the field lines of B always has to loop back indicating that this lines have no beginning and no end and therefore they have no divergence,

(Refer Slide Time: 12:00)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws \rightarrow

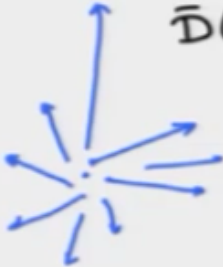



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{D} = \rho_v \\ \nabla \cdot \vec{B} = 0 \end{array} \right.$$

$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
$\vec{B}(\vec{r}, t)$	
$\vec{H}(\vec{r}, t)$	$\frac{A}{m}$
$\vec{D}(\vec{r}, t)$	



okay, it's like you know turn on a tap of water and when the water starts to flow if you just isolate the water, you know tap portion itself it seems that the water is coming out of that tap, but you follow the water you know going back to its sources and other thing and the water eventually reaches back to the tap, then from your close loop that would be the analogous situation for B having no divergences, okay.

So $\text{del. } B = 0$ is sometimes therefore called as no isolated magnetic poles can exist, okay, magnetic poles or sometimes called as monopoles, okay mono means single and pole being like the charge equivalent of a positive charge or negative charge, but in the magnetic case so we say that no magnetic monopoles exist, all magnetic poles or rather magnetic basic, magnetic that you are going to have will have both north and the south pole, so these are called as dipoles, okay, so there is nothing that has only a positive pole or a negative pole, so this north and south does not actually exist, I mean isolated does not exist, okay.

So these divergence loss were also known it was not that it was kind of discovered, although there is some you know controversy or some kind of an observation with respect to this zero. Now there was another English physicist called Dirac who won a noble prize for his work in quantum mechanics,
(Refer Slide Time: 13:31)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws	→	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
		$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

Dirac

he postulated or he actually showed mathematically that for this set of equations to be you know consistent you also need to have a magnetic charges, that is magnetic monopoles, okay, he did it in a very different context and the arguments are quite involved, we don't want to go there but he showed that magnetic monopoles have to exist for the charges to be quantized, remember the charges are always quantized in terms of the basic electric charge, right, the electric charge will be the electric charge of a single electron which is about -1.6×10^{-19} coulomb, right, and this minus comes because you know we consider their charge and electron to be negative and the charge in a proton to be a positive, (Refer Slide Time: 14:16)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws	→	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
		$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

Dirac

$e \sim -1.6 \times 10^{-19} C$

e_m

so this is the fundamental unit of charge, okay.

We can say that the current has a value of something, but you know what we actually mean is that we have so and so many number of electrons passing that particular you know point where you are measuring them, and you can't say oh well I will have a charge which will be like say 1/3rd of E or 0.67 times of E, these are not allowed because the fundamental unit of charge is quantized, and the quantity magnitude will be 1.6 x 10 to the power -19 coulombs, okay, because we decided to use coulombs so this shows that electronic charge is actually quite small, but if you take the fundamental unit of electric charge to be equal to 1, then you can see that there are you know huge number of electrons in 1 coulomb, okay.

So anyway that is the conversion problem that we don't have to worry about, but what he showed was that for the electric fields to be quantized in this, why should electric fields be quantized he says that it is because the magnitude monopoles must exist in nature, okay, however so far no one has figured out this magnetic monopoles experimentally, therefore we do not write a rho M term here, at least not in the physics way, mathematically you can still write rho M and then introduce another quantity here to make this entire equation consistent, but you do know that there are no magnetic monopoles and there are no magnetic currents, at least so far no one has been able to find anything here, okay.

Now comes the second equation which I deliberately omitted in discussion, I went straight to the third and fourth equations, what was known before Maxwell was this, we knew that $\nabla \times \mathbf{H} = \mathbf{J}$, okay, what this is actually known as this amperes law, what it simply means is that a current carrying wire, okay, if there is a current I that is being carried by a wire it will generate magnetic fields, okay, and this magnetic fields are all represented by this H, the corresponding integral equation for this is that if you integrate the magnetic fields around the closed loop that would actually be equal to the total current that is enclosed by that particular loop, okay, (Refer Slide Time: 16:40)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws	→	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
	→	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{Wb}{m^2}$
		$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

Ampere

$$\nabla \times \vec{H} = \vec{J}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

so you have this magnetic fields in space and then you draw a circle around the magnetic fields and then when you integrate the magnetic field along that particular line you will actually see that, that would be given by the, I mean that would actually tell you what is the total current that is enclosed by that loop, okay, so this was known, there was no problem with this one, and in the vector notation this ampere's law is given by $\nabla \times \mathbf{H} = \mathbf{J}$.

However vector field relationships also tell us that when you take a dot product on to $\nabla \times \mathbf{H}$, please remember that $\nabla \times \mathbf{H}$ is what is called as a curl operation, we will say slightly more about the curl operation later on, and this $\nabla \cdot$ is a divergence operation, what we are saying is that divergence of the curl was always of any vector field quantity will always give rise to 0, okay, so this is mathematically done, so you can't do anything about this portion here.

(Refer Slide Time: 17:36)

MODULE -13

Maxwell's equation O. Heaviside

	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
Faraday's laws	$\rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{Wb}{m}$
	$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	
	$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

Ampere $\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$

$\text{div curl} = 0 \quad \oint \vec{H} \cdot d\vec{l} = I_{enc}$

Now what is $\nabla \cdot \mathbf{J}$? It turns out that $\nabla \cdot \mathbf{J}$ actually represents the flow of the current because in the, this one, in the equation you can actually show that $\nabla \cdot \mathbf{J}$ integrated over a certain volume is actually equal to the closed surface integration of the current over that surface area of that you know volume that is bounded by that surface area, and what we have seen is that this integration of $\nabla \cdot \mathbf{J}$ which is what that divergence of \mathbf{J} , there has to be some source for that current, \mathbf{J} is called as a current density and the source for that current is actually the charge that is decaying inside, okay, so as the charges are moving out they will constitute a current, okay, so $\nabla \cdot \mathbf{J}$ is nonzero in general in fact $\nabla \cdot \mathbf{J}$ would actually be equal to $-\frac{\partial \rho}{\partial t}$, okay, this was also known, of course for sustain a current unit charges and these charges have to be moving, so this is nonzero in general, whereas the left hand side is equal to zero all the time, so you have zero on one side, nonzero on one side, so clearly there is some inconsistency in this entire equation, okay.

What Maxwell did was a very interesting thing, although mathematically it may look very simple to us, but what he did was to postulate that there is this extra term, okay, which is called as the conduction current, okay, he showed that this conduction current or conduction current density when you add it to this equation amperes law, (Refer Slide Time: 19:13)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws $\rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\vec{E}(\vec{r}, t)$ $\frac{V}{m}$

$\rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\vec{B}(\vec{r}, t)$ $\frac{Wb}{m^2}$

$\nabla \cdot \vec{D} = \rho_v$ $\vec{H}(\vec{r}, t)$ $\frac{A}{m}$

$\nabla \cdot \vec{B} = 0$ $\vec{D}(\vec{r}, t)$ $\frac{C}{m^2}$

Conduction Current density

then the equations become consistent, why because now when you take del. del x H which would be equal to 0, this would actually be equal to del. J + del/del L interchanging the del operation with respect to the time derivative operation, but I know that del. D is actually to rho V and therefore I can write this equation as del. J = -del rho V/del T, right, and this equation is called as continuity equation which simply tells you that the charges have to move away from a close surface in order to current to actually be nonzero, okay, (Refer Slide Time: 19:53)

MODULE -13

Maxwell's equation O. Heaviside

	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
Faraday's laws	\rightarrow		
	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{W}{m}$
	$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	
	$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	
	$\nabla \cdot \nabla \times \vec{H} = 0 = \nabla \cdot \vec{J} + \frac{\partial (\nabla \cdot \vec{D})}{\partial t} \Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$		
			Continuity Equation

Conduction current density

so this continuity equation is something that comes from apart from Maxwell's equation, this is you know experimentally done so this was known earlier, what Maxwell did was to introduce this continuity equation into this del x H, not exactly in the way that you know we have written, but he did introduce this term del D/del T and called it conduction current density and that actually fixes all of this problems with electromagnetic field, so he made the equations consistent.

And in one stroke of genius he actually combined electric field and magnetic fields into a single entity, although the idea was there before you know from Ampere, Faraday, Biot Savart, Gauss, there was all these ideas floating around, but Maxwell was the one who kind of unified this electric and magnetic fields and therefore these equations are rightly called as Maxwell's equations.

This term may appear very simple, but it really you know kind of fixed lot of problems with electromagnetic theory itself, for example if you take a simple capacitor, right, you can see that there will be current going in and current coming out, this current and current but if the space between the two plates is nothing, then how can there be current continuity, right, (Refer Slide Time: 21:07)

MODULE -13

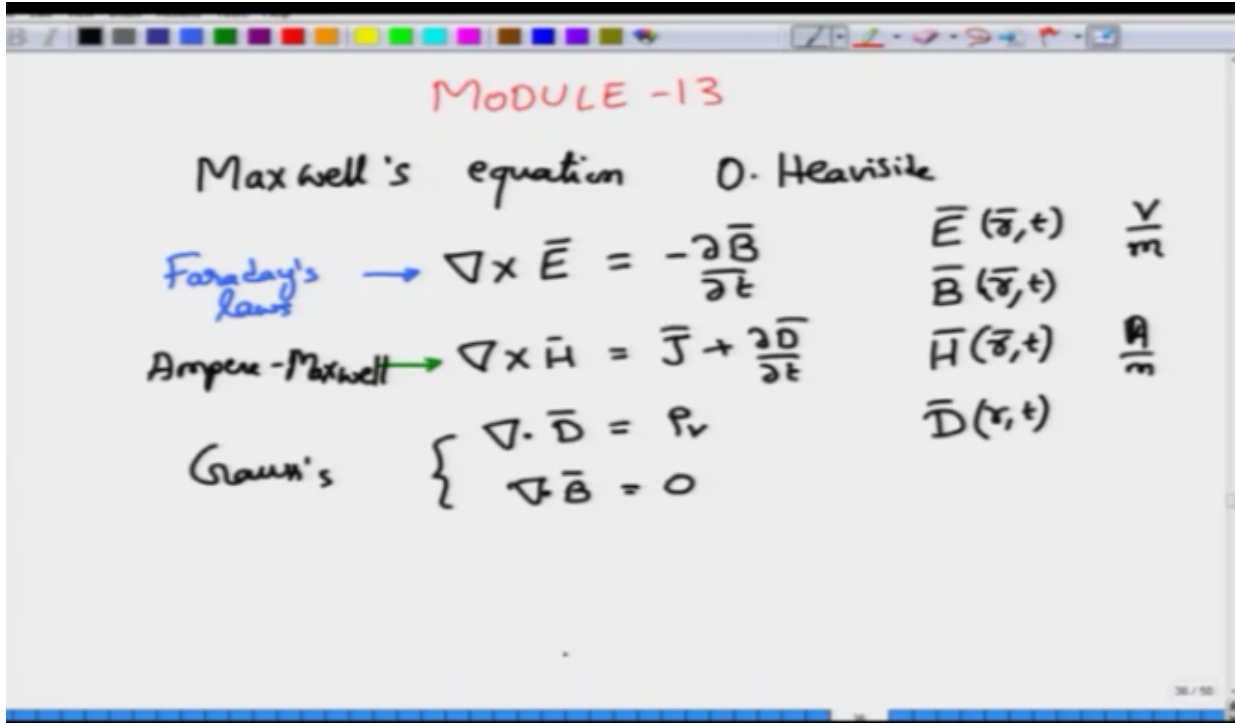
Maxwell's equation O. Heaviside

	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
Faraday's laws	\rightarrow		
	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{A}{m}$
	$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	
	$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

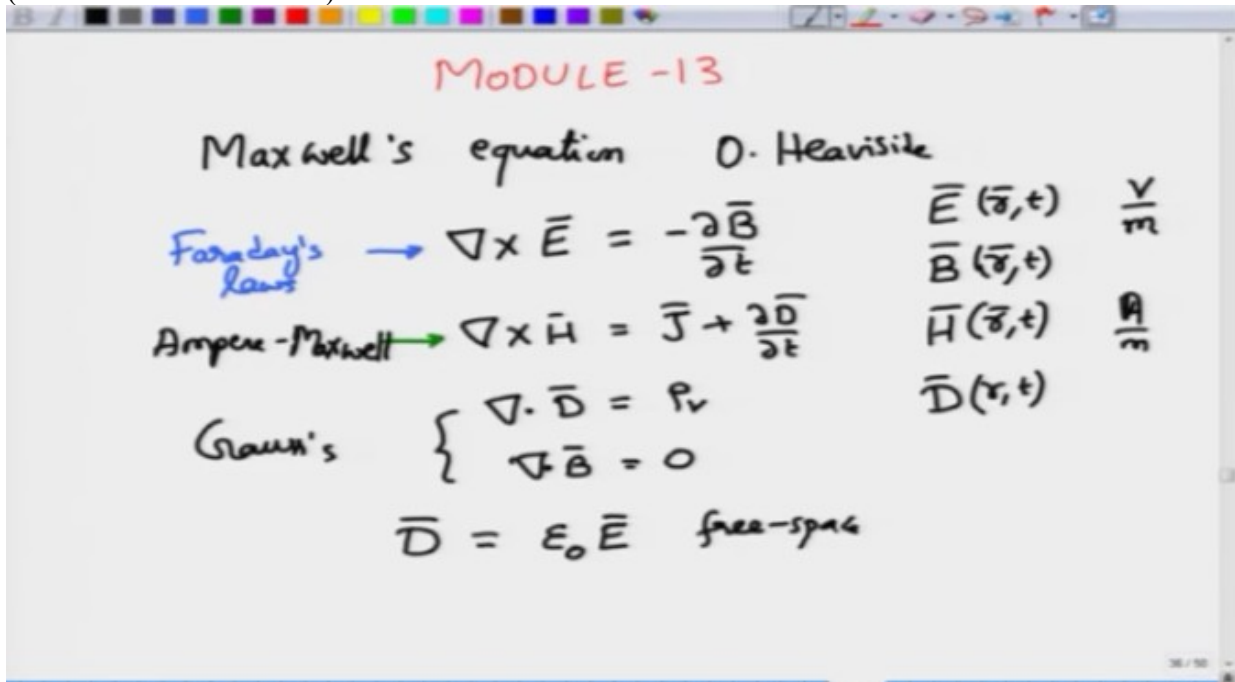
$\nabla \cdot \nabla \times \vec{H} = 0 = \nabla \cdot \vec{J} + \frac{\partial (\nabla \cdot \vec{D})}{\partial t} \Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$
 Continuity Equation

$\vec{J} \rightarrow | - | \rightarrow \vec{I}$

and without continuity who is you know generating this current I on to the right hand I mean someone has to be generating that, right, and what he found out was current continuity can be restored if you postulate the existence of $\nabla \cdot \vec{D} / \partial t$ as the displacement current, okay, so this is what Maxwell did and because of that lot of problems got sorted out in electromagnetic theory and now they form the basis of whatever we want to steady, okay, or whatever we study with respect to electromagnetic, so this two are called as Gauss's laws, okay, so we'll call this as Gauss's laws, divergence laws and this is called as ampere, because he did the initial experiments and Maxwell because he figured out this unifying conduction current, this is called as Ampere Maxwell law or sometimes called as modified Ampere's law, okay.
 (Refer Slide Time: 22:02)



In a matter you have to you know relate D with E for this to actually be I mean applicable in the matter as well, and if it is free space meaning it is vacuum or it is you know just general air most of the times, then D is approximately equal to epsilon naught E in air, and exactly equal to epsilon 0 times E in free space, okay,
(Refer Slide Time: 22:28)



and this epsilon is called as epsilon naught which has a value of about 8.85×10^{-12} farad per meter is called as permittivity of the medium, and in this case this is permittivity of free space when the medium is simple, scalar, homogeneous, isotropic and linear then the D

field will be proportional to E field, and this proportionality constant in addition to epsilon 0 will also include this term epsilon R which is called as relative permittivity, and relative permittivity will always be greater than 1 and this is the relation between D and E.

(Refer Slide Time: 23:07)

MODULE -13

Maxwell's equation O. Heaviside

Faraday's laws	→	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{E}(\vec{r}, t)$	$\frac{V}{m}$
Ampere-Maxwell	→	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{B}(\vec{r}, t)$	$\frac{Wb}{m}$
Gauss's	{	$\nabla \cdot \vec{D} = \rho_v$	$\vec{H}(\vec{r}, t)$	$\frac{A}{m}$
		$\nabla \cdot \vec{B} = 0$	$\vec{D}(\vec{r}, t)$	

$\vec{D} = \epsilon_0 \vec{E}$ free-space $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$
 $= \epsilon_0 \epsilon_r \vec{E}$ Permittivity
→ relative > 1

There is a similar relationship between B and H, B is given by mu naught H in free space and it is given by mu naught mu RH, although the relationships are slightly more complicated in the magnetic field, mu naught with a defined value of 4 pi into 10 to the power minus, I think it is 7 henry per meter, I hopefully I have got the numbers correct, and mu R being the permeability, mu R being quantity that is relative, these are called as permeability, okay, they will tell you how easy in some sense it is to magnetize a certain material, the larger the permeability the easier the material can be magnetized okay.

(Refer Slide Time: 23:41)

$$\int \nabla \cdot \vec{J} dV = \oint \vec{J} \cdot d\vec{s} - \frac{dq}{dt}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$= \mu_0 \mu_r \vec{H}$$

$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
 μ_r permeability

Wood for example will have very low permeability, you can't turn wood into a magnet as easily as you can turn an iron into a magnet, okay, so that is where permeability wood actually come in, okay.

And we now need to just look at two operations, one is called as divergence operation as I have told you or I have shown you this operation is called as divergence operation, and it usually has a field structure which would be emanating from a source or it would be terminating on to a source, okay, mathematically of course to find out this divergence we write del. D mathematically as D field integrated over a close surface, so you can imagine that there is a sphere and this is kind of, or any kind of a shape doesn't matter, we have a closed surface and over that closed surface you integrate the D field, okay and then divide this one by the volume that the surface actually occupies, okay and then you let the volume go to zero so that you have what is called as divergence, okay, so you can imagine that there is a closed surface, and you can actually integrate your D field over this, and then shrink the volume down to a point and you get the divergence.

(Refer Slide Time: 25:02)

$$\int \nabla \cdot \vec{J} dV = \oint \vec{J} \cdot d\vec{S} - \frac{dq}{dt}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{B} = \mu_0 \vec{H} = \mu_0 \mu_r \vec{H}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\mu_r \text{ permeability}$$

$$\nabla \cdot \vec{B} = \lim_{\Delta V \rightarrow 0} \frac{\oint \vec{B} \cdot d\vec{S}}{\Delta V}$$

Curl on the other hand is defined as integral of a quantity over a closed loop, okay, you imagine again a surface which is an open surface okay and that is bounded by some contour okay, and then you take the you know H fields over that or any kind of a vector field integrated over the closed contour and then divide this one by sorry, that went off to something else, divide this one by delta S, where delta S is the area of this open surface, okay, this has to be important, the surface area has to be open, okay, and the sense of direction whether it's positive or negative (Refer Slide Time: 25:41)

$$\int \nabla \cdot \vec{J} dV = \oint \vec{J} \cdot d\vec{S} - \frac{dq}{dt}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{B} = \mu_0 \vec{H} = \mu_0 \mu_r \vec{H}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\mu_r \text{ permeability}$$

$$\nabla \cdot \vec{B} = \lim_{\Delta V \rightarrow 0} \frac{\oint \vec{B} \cdot d\vec{S}}{\Delta V}$$

$$\nabla \times \vec{H} = \frac{\oint \vec{H} \cdot d\vec{I}}{\Delta S}$$

which is usually done by the right hand rule, okay and we will come to that one later on in case we would really like to know, but I am assuming that you already know how to work with this, right.

And as you let ΔS go to 0, you see that what the quantity that you are going to get will be called as the curl operation, okay. Finally for completeness sake let me mention the integral versions of these equations, I'm sure you would have seen them you can turn these differential equations into integral equations by invoking Stokes and divergence theorems, something that you may already have seen, okay, so you have $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$, please remember this is the closed line integral, this is an open surface integral, similarly you have $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$ and then $\oint \vec{D} \cdot d\vec{S} = \int \rho_v dV$, ρ_v being the charge density measured in coulombs per meter cube, okay, so you can write down the remaining Maxwell's equation there divergence equation and complete this set of equations.

(Refer Slide Time: 26:58)

$$\begin{aligned}\oint \vec{E} \cdot d\vec{l} &= -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \\ \oint \vec{H} \cdot d\vec{l} &= \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \\ \oint \vec{D} \cdot d\vec{S} &= \int \rho_v dV\end{aligned}$$

In free space \vec{J} will be 0, ϵ_0 will I mean ρ_v will be 0 and that is the case that we are going to start looking for electromagnetic waves. Thank you very much.

Acknowledgement
Ministry of Human Resource & Development

Prof. Satyaki Roy
Co-coordinator, IIT Kanpur

NPTEL Team

**Sanjay Pal
Bharat Lal
Ashish Singh
Badal Pradhan
Tapobrata Das
K. K. Mishra
Ashutosh Gairola
Puneet Kumar Bajpai
Bhadro Rao
Shikha Gupta
Aradhana Singh Rajawat
Sweta
Nipa Sikdar
Anupam Mishra
Ram Chandra
Manoj Shrivastava
Dilip Tripathi
Padam Shukla
Sharwan K Verma
Sanjay Mishra
Shubham Rawat
Santosh Nayak
Pradyuman Singh Chauhan
Mahendra Singh Rawat
Tushar Srivastava
Uzair Siddiqui
Lalty Dutta
Murali Krishnan
Ganesh Rana
Ajay Kanaujia
Ashwani Srivastava
M.L. Benerjee**

**an IIT Kanpur Production
© Copyright Reserved**