Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 43
Practical Application: Multi antenna channel estimation

Hello. Welcome to another module in this Massive Open Online Course. So, we are looking at least square. So, we are looked at several aspects including the intuition behind this least square. Let us look at some Practical Applications of this least squares.

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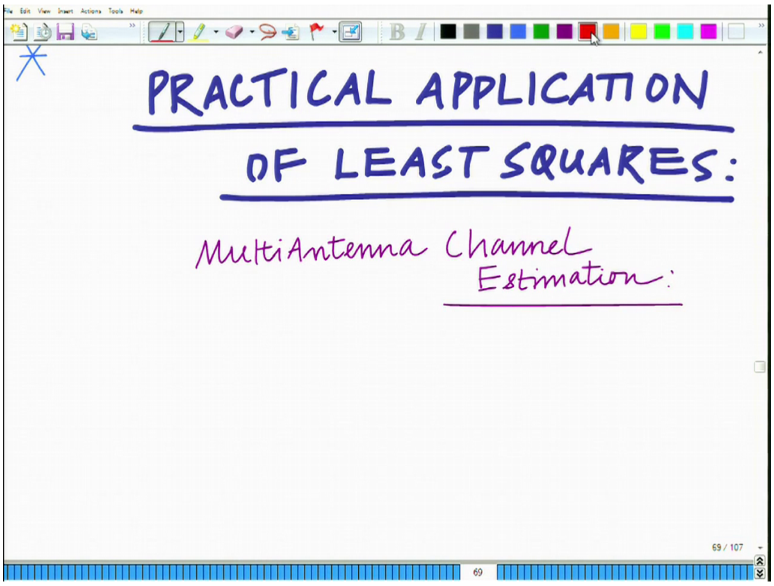

In fact, what we have said is this something that arises very frequently in signal processing and communication. So, we have the practical application of least squares and what we want to do in this is; well, let us consider the first problem is related to wireless communication. Of course, one of the interesting very interesting problems is that in a wireless communication system is that of channel estimation in particular let us look at the problem of multi antenna channel. Let us look at the problem of multi antenna channel estimation.

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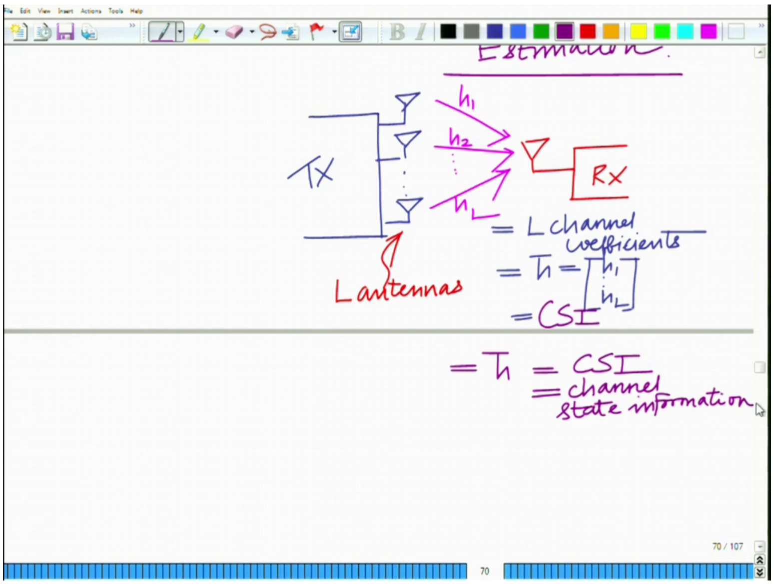

Now, if you look at this typically what we said is now let us consider now let us consider a system with multiple transmit antennas. So, you have a transmitter and this has multiple antennas. So, have a transmitter let us say you have L antennas at the transmitter and for simplicity let us say you have a single receive antenna. Now, you have the channel coefficient since there L antennas there are L channel coefficient. So, you have h 1, h 2 up to h L these are the L channel coefficients, alright. This is a channel and this channel coefficients are unknown alright comprises the alright and we also said this is the channel state information or CSI which is unknown and has to be estimated.

So this channel vector if you call this as h bar this is this channel vector which basically constitutes the CSI, the channel state information and that has to be estimated. So, this is basically your h bar equals CSI.

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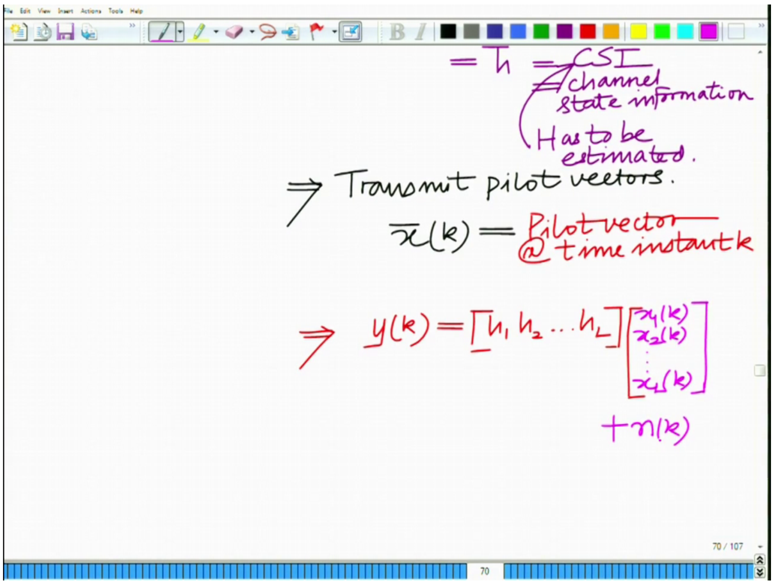

This is the channel state information and the CSI has to be estimated the CSI has to be estimated. Now, to estimate the channel state information we transmit pilot symbols, alright. Now, this pilot symbols are symbols that are known at the receiver. So, implies. So, to estimate the CSI implies we have to transmit pilot symbols. So, in this case pilot vectors you have to transmit pilot vectors.

Now, let us say x k is the k-th pilot vector or pilot vector at time instant k; pilot vector at time instant k. So, this implies that you are transmitting pilot vector time instant k. So, what we have is I can write the receive symbol y of k at time instant k equals the channel coefficients h 1, h 2, h l times x 1 k, x 2 k, up to x l k plus well the noise sample plus n of k.

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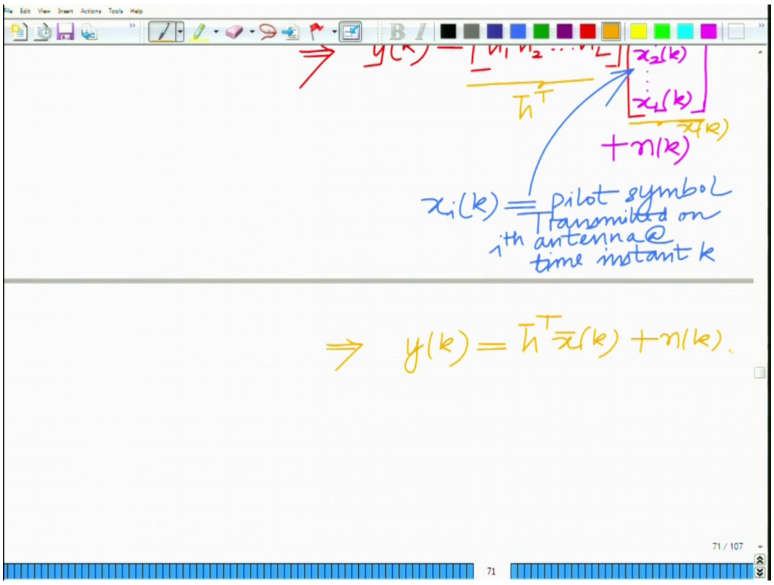

What is x i of k? Now, remember x i of k you can think of this x i of k, equals pilot symbol on the i-th antenna i-th transmit antenna pilot symbol transmitted on i-th antenna, pilot symbol transmitted on i-th antenna at time instant k. So, therefore, I can get this as this implies y k equals, well I can write this as the vector. So, this is your h bar transpose channel vector is h bar times x bar k. So, h bar transpose x bar k plus the noise sample n of k.

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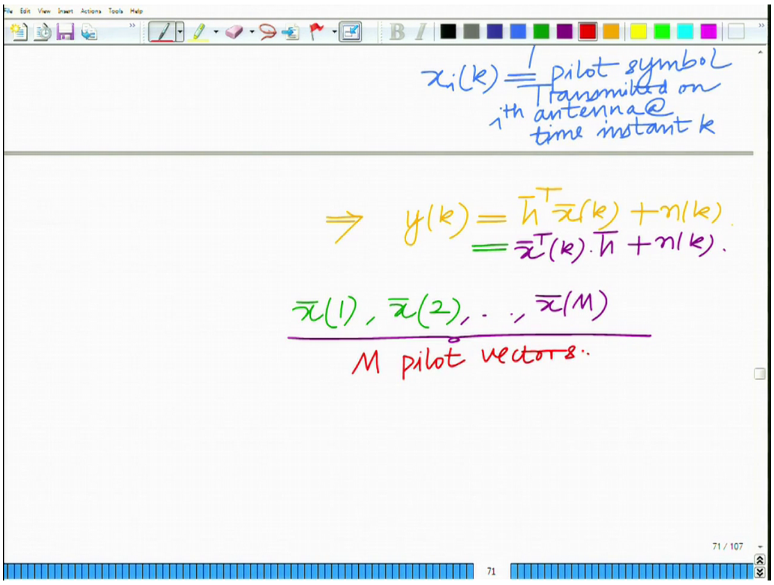

Therefore, now if you transmit let us say M pilot vectors, which you can also try and now remember considering real vector this can also be written as x bar transpose times h bar plus n k, I can write it as h bar transpose x bar k or x bar k transpose times h bar. Now, considering the transmission of M pilot vectors so, let us say you transmit M pilot vectors.

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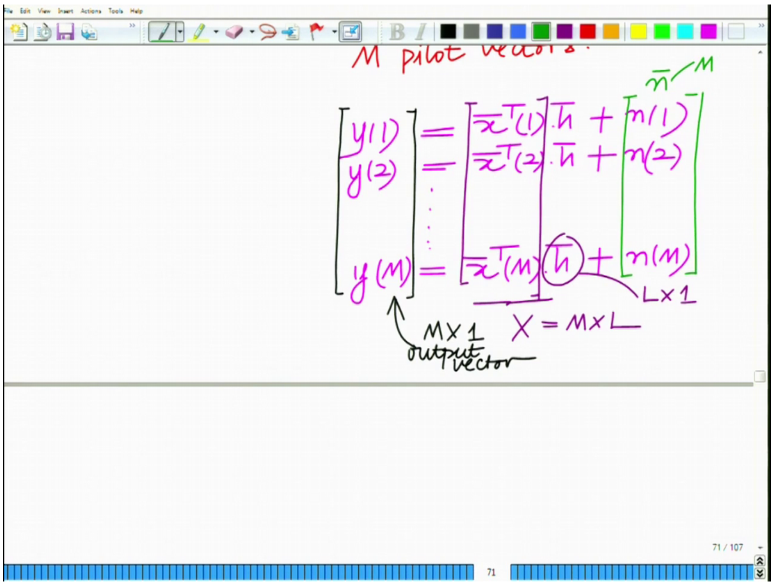

I can write the equivalent system as y 1 equals x bar transpose 1 into h bar plus n 1 similarly, y 2 equals x bar transpose 2 into h bar plus n 2 so on y M equals x bar transpose M into h bar plus n of m. And, now what I can do is I can make a matrix out of this. So, this will become a vector this will become your M cross n, you can call this as your M cross n output vector this will call you can call this as your pilot matrix. In fact, this has M rows each of size L.

So, this will be pilot matrix x will be of size M cross L h bar is a channel vector which is of size L cross 1 and now, once you concatenate this noise elements, you will have this will be n bar of size also again M cross 1.

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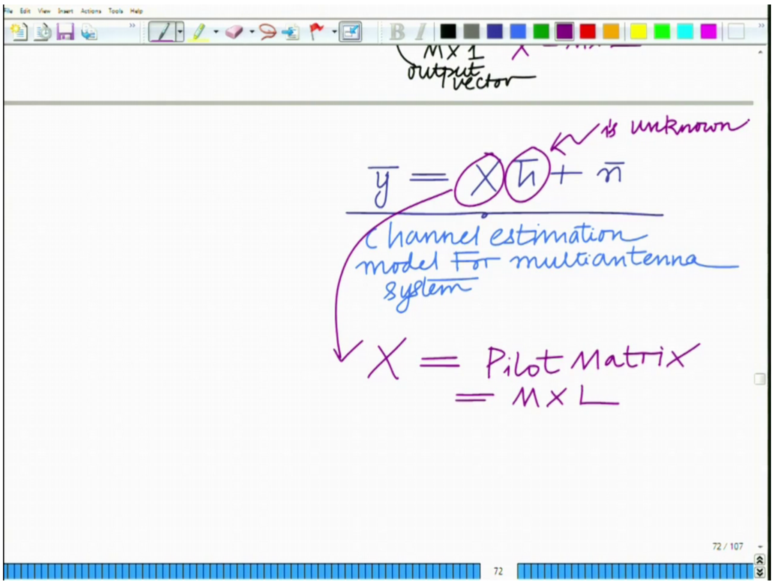

So, next what you have is, this is your model for channel estimation. So, what you will have is you will have y bar equals X times h bar plus n bar. So, this is the model for channel estimation. In fact, channel estimation model for a; this is the channel estimation model for the multi antenna system and in fact, this X is the pilot matrix you can denote it by X or X p the same thing. So, this X equals the pilot matrix and this is of size M cross L.

And, this vector h bar remember this is the CSI, this is unknown. This vector h bar which represent the CSI this is unknown and now to estimate this h bar remember we do not know there are of course, there is noise. This is the noisy observation model. So, you have to look at the best vector h bar which explains the observation observed vector y bar corresponding to the transmitted pilot symbols in the matrix X.

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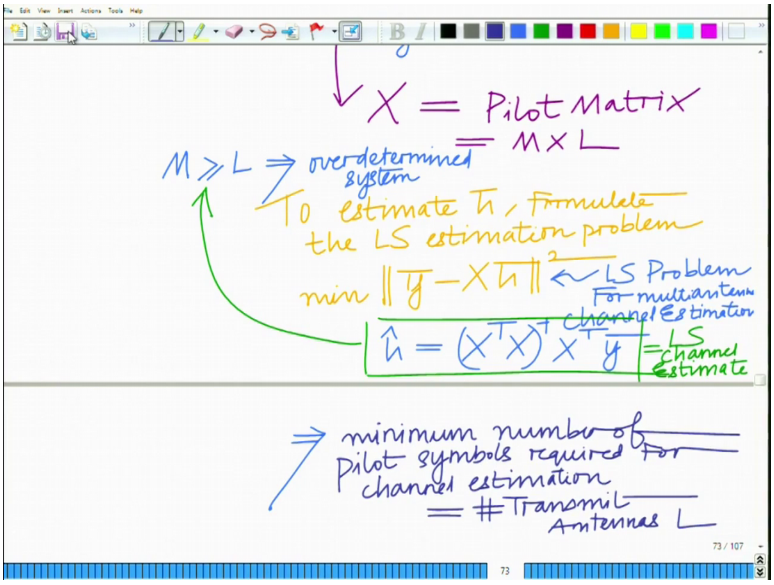

So, therefore now one can formulate for estimation of h. So, to estimate h formulate the least square estimation problem, let us minimize y bar minus minimize y bar minus x h bar norms. So, this is the least square form and we know the solution. So, this is your LS problem for channel estimation, in fact, for multi antenna channel estimation.

And, the solution h hat is given by the least square solution X transpose X inverse X transpose y bar this is also known as the least squares channel estimate. This is very interesting, this is equal to your LS, the least squares channel estimate. So, this is known as the least square channel estimate X is your pilot matrix. So, we have the observation vector y bar in the corresponding pilot vectors. The other thing that we are assuming is that there is an over determined system which is the number of pilot symbols M is greater than or equal to the number of transfer. Remember, this works only with the system is over determined.

So, when we are doing this in simply we are assuming that M number of pilot symbols is greater than or equal to typically M is much larger than. So, this implies this is a over determined system. So, you can also say that for channel estimation using the least square technique the number of pilot minimum number of pilot symbols are required is at least that the number of transmit antenna. So, this is an another interesting result.

So, this implies minimum number of pilot symbols required; another interesting result that is you need at least transmit pilot number of pilot symbols L that is M as to be greater than equal to L for estimation of this multi antenna channel. So, this is an interesting application. In fact, one of the most how; what are the most I would say one of the most popular application, in fact, one of the most practical viable practically prevalent applications of the least square solution in the especially in the context of signal processing for a practical wireless communication system, right. So, we stop here and look at other application is the subsequent modules.

Thank you very much.