Applied Optimization for Wireless, Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 39
Practical Application: Robust Beamformer Design for Wireless Systems

Hello. Welcome to another module in this massive open online course. So, we are looking at Robust Beam forming as an application of convex optimization or the optimization framework that we have seen so far. So, let us continue our discussion.

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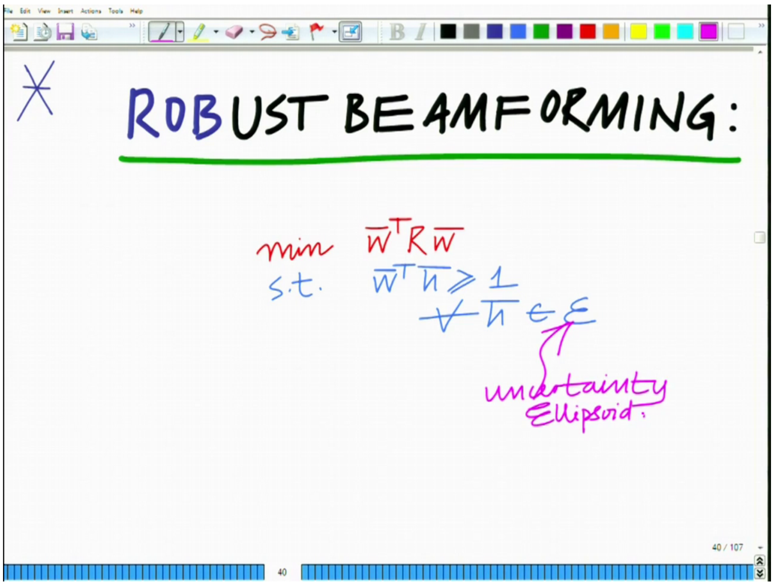

We are looking at robust beam forming or multiple antenna system, remember what beam forming does is to focus the wireless signal in a particular direction formal beam in a particular direction. And robust beam forming is the paradigm where the knowledge of the channel is not known precisely. So, there is uncertainty in the channel knowledge and how to design a beam former that is robust to that uncertainty, we said the robust beam former can be designed as the solution to the following optimization problem w minimize w bar transpose R w bar w bar is the beam former, R is the noise plus interference covariance matrix.

Subject to the constraint that w bar transpose h bar is greater than or equal to 1 for all h bar belongs to this ellipse correct, this ellipsoid this is also termed as the uncertainty ellipsoid, this is also termed as uncertainty ellipsoid.

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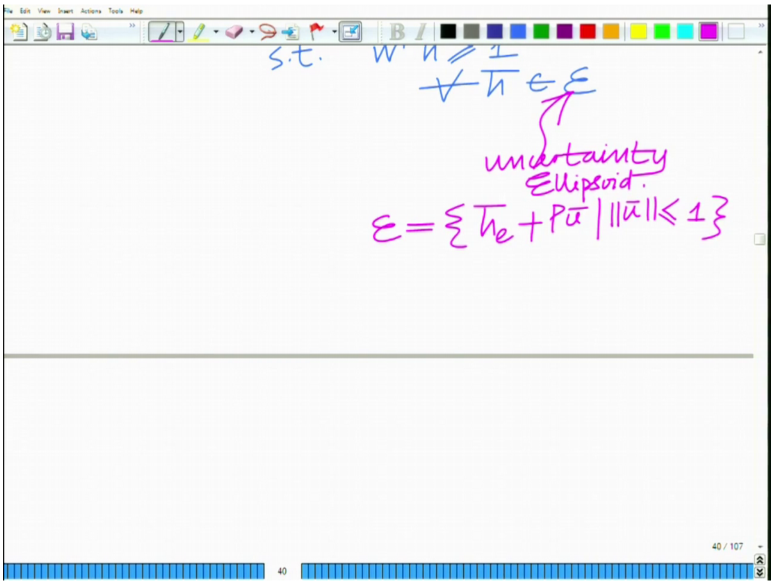

And this ellipsoid is described as follows; this ellipsoid is the ellipse which has the centre h bar e that is the nominal channel or the estimated channel. So, h bar e plus P u bar such that norm u bar is less than or equal to 1 ok. And well now we want to solve this optimization problem to basically determine the optimal beam form the optimal robust be for more if you were ok.

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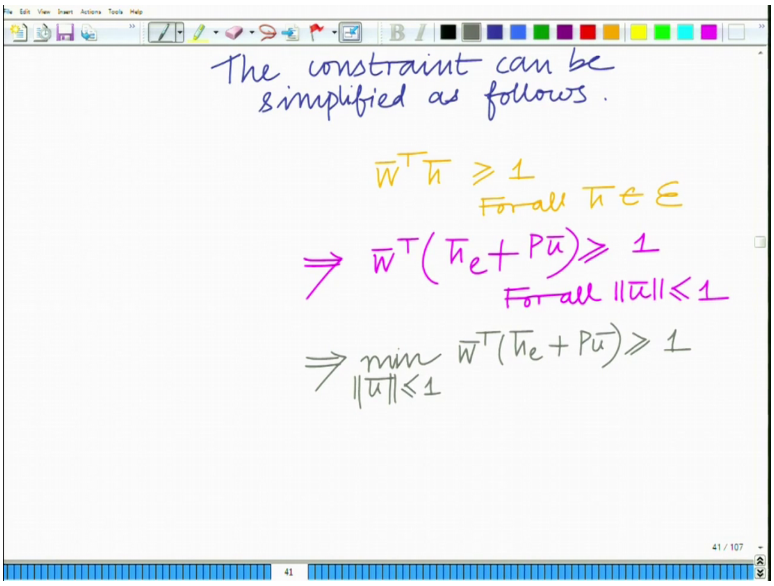

And that solution, first of all lets simplify this optimization problem and this is where it can be done in a very interesting fashion as described below. So, let us look at the constraint and constraint can be simplified as follows ok. Remember the constraint ensures a minimum gain of unity for all vectors h bar belonging to the uncertainty ellipsoid.

So, we have what is the constraint? The constraint is w bar transpose h bar is greater than or equal to v for all h bar belonging to the uncertainty ellipsoid which means now you substitute for h bar w bar transpose h bar is well we have seen that is simply h bar e the estimated channel plus P u bar greater than equal to 1 for all.

Now, for all h bar belong to E. Now becomes because, the equivalent condition is h bar e plus P u bar for all vectors u bar such that norm u bar is less than or equal to 1 for all norm u bar less than or equal to 1 ok. Now this is the interesting part, now this has to be true for all vectors u bar such that norm u bar is less than equal to 1.

This has to be greater than equal to 1 all right, which implies basically this also has to hold at that value of u bar where this is the minimum all right. So, is the minimum of this overall u bar is greater than or equal to 1 that automatically implies that it is going to be greater than or equal to 1 for all u bar such that norm u bar is less than or equal to 1.

So, this can be written equal until and you can convince yourself, this implies that for the minimum of u bar that is you take the minimum over u bar, says in norm u bar less or equal to 1. This w bar transpose h bar e plus P u bar greater than or equal to 1ok.

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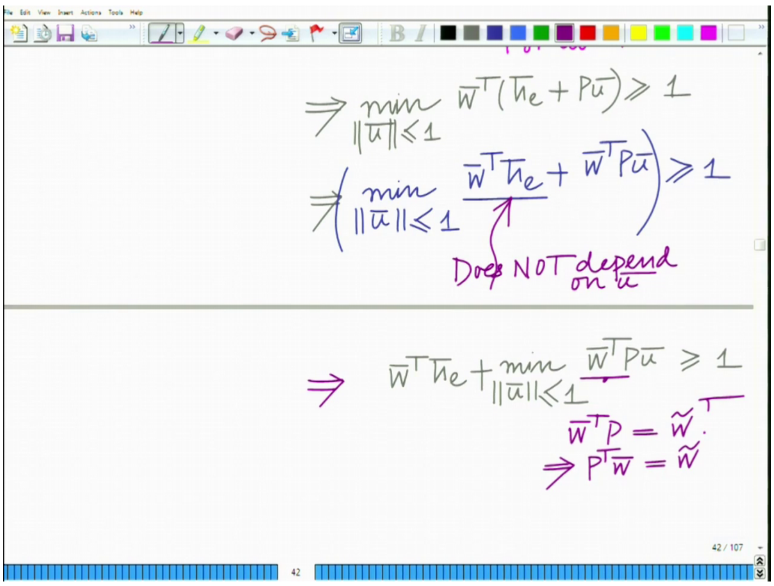

If this definitely implies that about the minimum this has to be greater than equal to 1 and this implies; now I can simplify this further you take the minimum of norm or norm u bar all u bars is that norm u bar is less than equal to 1. Now I can simplify this as w bar transpose h e bar plus w bar transpose P u bar, the minimum has to be over this minimum this has to be greater than or equal to 1.

Now, this is a constant w bar transpose h e bar, this does not depend on u bar ok. So, this will come out of the minimization so, this implies if you look at this w bar transpose h bar e plus minimum of norm of u bar less than or equal to 1 w bar transpose P u bar greater than or equal to 1. Now, what we are going to do? We are going to set this let us set this w bar transpose P as w tilde which implies P transpose w bar I am sorry w tilde transpose so, P transpose w bar will be w tilde.

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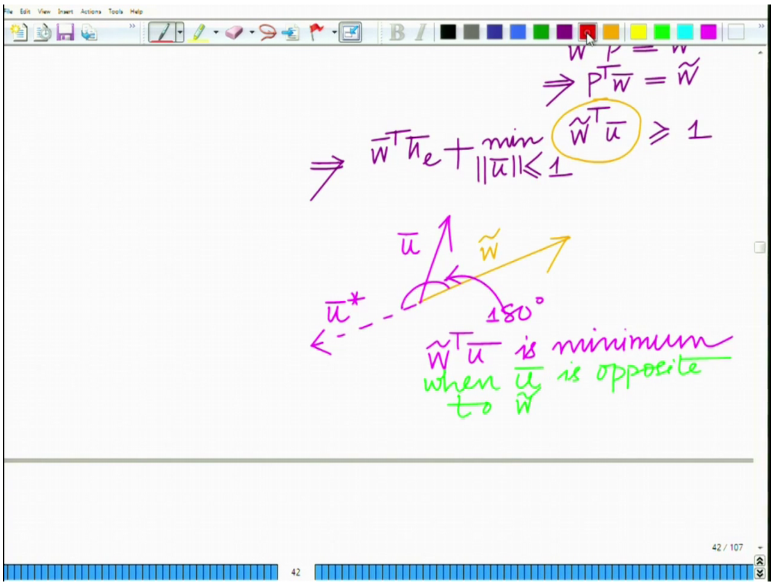

So, I will write this as w bar transpose h bar e plus the minimum over norm for all u bar, such that norm u bar less than equal to 1 minimum or u bars that w tilde transpose u bar greater than equal to 1.

And now this is very interesting, now if you observe now you see what is this is the nothing, but the dot product w tilde transpose u bar. So, we have w tilde and we have this vector u bar correct and we have the dot product. Now when is the dot product between w tilde and you are minimum remember the dot products maximum when you bar is perfectly aligned with w tilde.

And the dot product is minimum when the vector is 180 degree that is it is completely in opposite direction and in a direction opposite the data of the material. So, the dot product is minimum, when u bar forms a 180 degree angle with w tilde. So, this is where so, the this is let us say, this is star. So, this is a 180 degree angle. So, we say w tilde u bar is minimum when u bar is opposite that is forms a 180 degree angle with w tilde.

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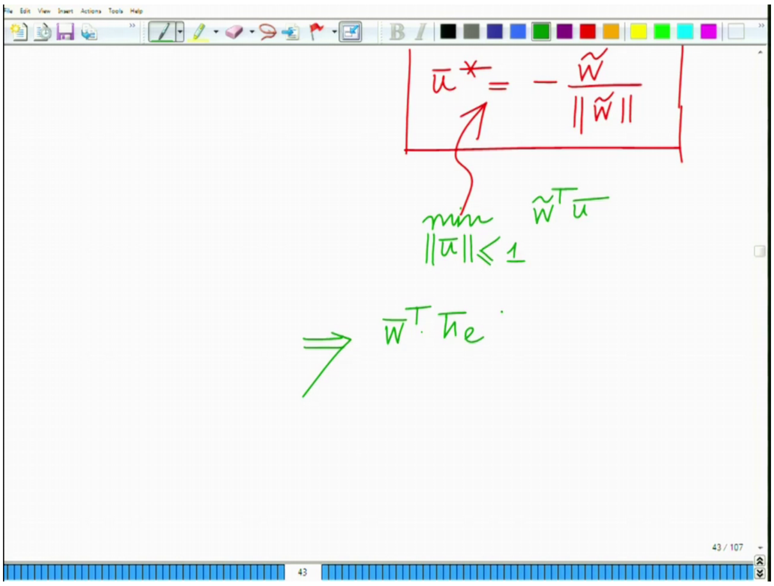

And therefore what we say is u bar star this will be equal to minus w tilde because, the vector that is exactly opposite to w tilde is minus w tilde. However we need u bar to be normally bar to be less than or equal to 1; therefore, we normalize this with norm of the w tilde that is it.

So, u bar is the unit norm vector, that is opposite to w tilde and this is for which this is precisely the u bar for which you have minimum such that norm u bar less than or equal to 1 w tilde transpose u bar ok.

This is where the minimum occurs, that is when u bar is a unit norm vector it is exactly opposite in direction to w tilde. Therefore, the inner product is basically negative number; let us this cosine 180 is minus 1 ok.

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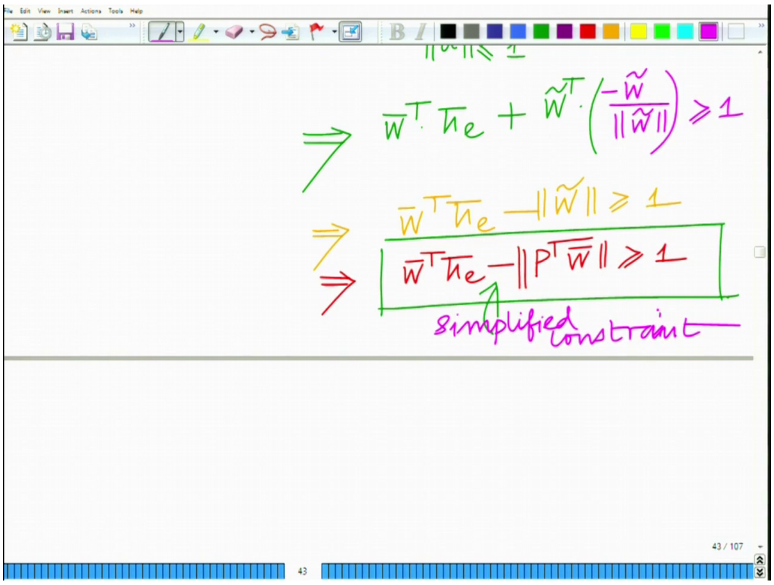

So, this implies that now the minimum will be w bar transpose h bar e plus the minimum over norm u bar u bar such that norm u bar less or equal to 1 occurs when u bar equals minus w when u bar equals minus w tilde divided by norm w tilde. Therefore, and that will be w tilde transpose into and I am now substituting for that value of u bar, which is minus w tilde divided by norm w tilde.

And this has to be greater than or equal to 1, if this is greater than or equal to 1, then it is going to be greater than equal to 1 for all by implication by a. So, by following this argument it is going to be greater than or equal to 1 for all vectors h bar belonging to that else material. And now you see this is mine w tilde transpose into w tilde is nothing, but norm w tilde square divided by w tilde; so, that is norm w tilde. So, this implies w bar transpose h bar b minus norm w tilde greater than or equal to 1.

And now, we substitute for now w tilde, w tilde is nothing, but we have seen earlier w tilde is basically you are a P transpose w bar. So, this implies w bar transpose h bar e minus norm of P bar transpose w bar this has to be greater than or equal to 1. This is the equivalent constraint or this you can say is the simplified constraint, simplified constraint.

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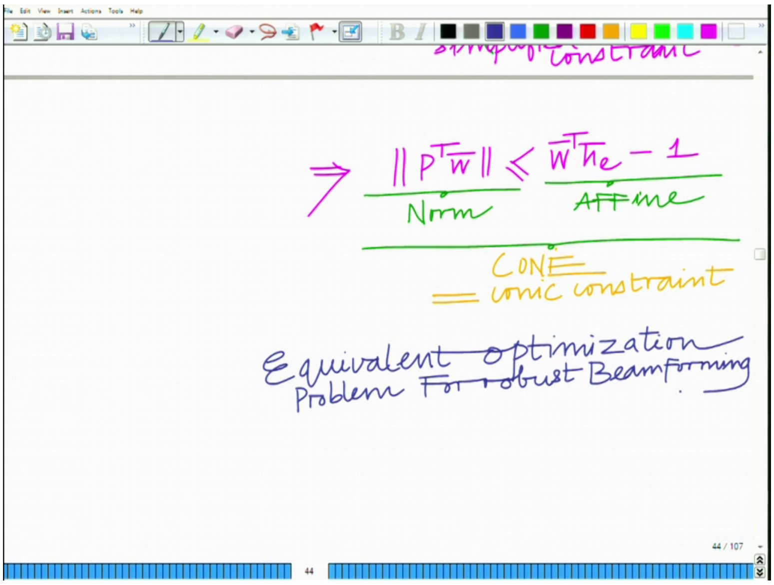

And you can also write this as now w a transpose h bar e minus norm of P transpose w bar greater than or equal to 1, this can also be simplified as follows norm of P transpose w bar less than or equal to w bar transpose h bar e minus 1 ok. And now if you look at this is very interesting, you can recall that this is a norm and this is your a fine, this is the affine portion.

So, we have norm less than equal to something that is affine. So, this is basically you can recall and you can look the notes this is a conic constraint. In fact, this pair, this constraint represents a conic region or this is basically a cone or this is known as a this is let us say this is a cone also known as a conic constraint.

It is a very interesting constraint; it reduces to a cone or a conic constraint. And therefore, now the equivalent optimization problem to find the robust beam former ok, that can be formulated as equivalent optimization problem.

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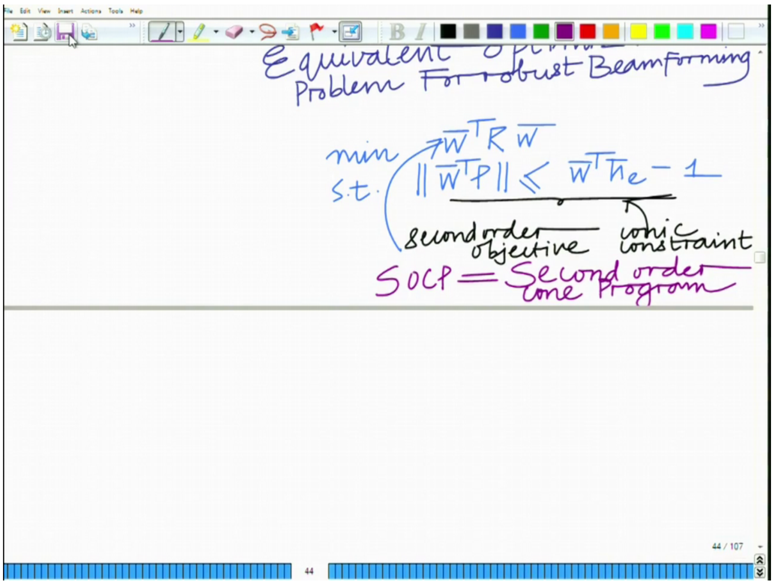

Equivalent optimization problem for robust beam forming that will be minimum of minimum or w bar transpose R w bar, such that w bar transpose P norm is less than or equal to w bar transpose h bar e minus ok.

This is a coning constraint; this is a second order quadratic optimization so, this is a second order objective. This is a conic constraint so; this is known as the second order cone problem. So, this is basically this is known as an SOCP equals second order cone program so, this is a very interesting aspect.

So, the robust beam forming problem reduces to a very interesting optimization for more either a very interesting or belongs to a very interesting class of optimization problems turned as second order cone programs with the objective function is second order. All right our objective function and the constraint is a conic constraint all right. So, this is known as an SOCP problem and it is a very interesting problem.

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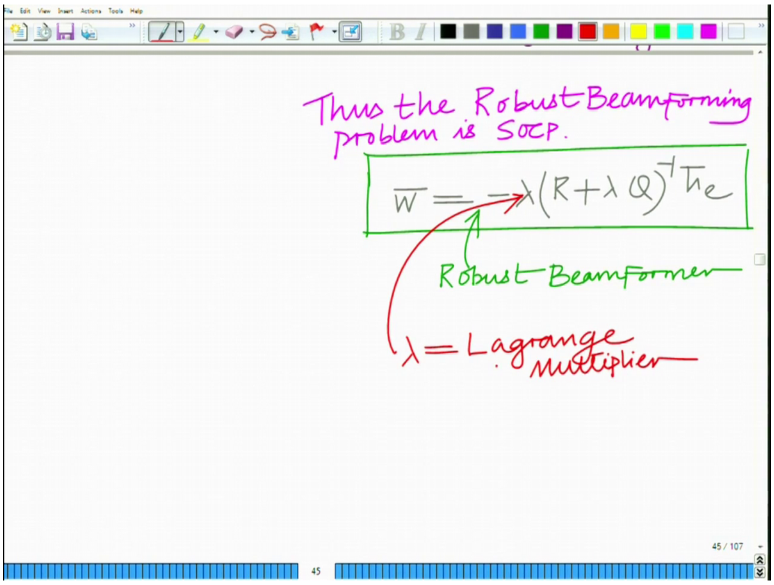

And it can be solved and I will demonstrate it separately because it is a little involved. So, robust beam forming problem is an SOCP let us note that. Thus the robust beam forming problem, thus the robust to inform a problem is in SOCP and the robust beam former w bar; it can be shown that robust beam former is minus of lambda.

In fact, I will show this in a subsequent module plus R Q inverse of h bar e; this is the robust this is the robust beam former. This is the robust beam former and lambda this is the Lagrange multiplier, lambda equals Lagrange multiplier and this has to be determined suitably.

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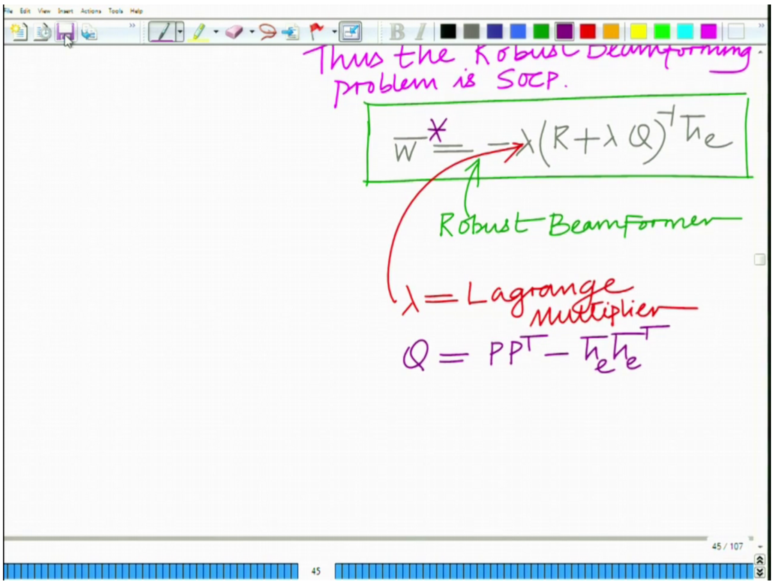

And the matrix Q depends on R and P basically Q equals I am sorry depends on P and h bar e. So, P transpose minus h bar e h bar. Of course you see notice P, P is the matrix corresponding to the uncertainty ellipsoid and h bar e is the estimate the nominal estimate of the channel. And this is the solution to the robust beam forming problem, that is w bar equals you can say this is w bar star w bar star equals minus lambda R plus lambda Q inverse into h bar e all right.

So, I will conclude this module with this all right so, there is a very interesting problem the robust beam forming problem, which can be shown to be an SOCP a Second Order Code Problem and this is the solution. It is slightly involved which I will in illustrate in a separate point.

Thank you very much.