Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 38
Practical Application: Robust Beamforming With Channel Uncertainty for Wireless

Hello, welcome to another module, in this massive open online, course. So, we are looking at various types of Beamforming and in this module let us look at, another very, important and very interesting and in fact, a very practical format of beamforming that is termed as Robust Beamforming. And it is going to take, it is a little involved. So, it is going to take a little time to explain this, but nevertheless let us start this topic of robust.

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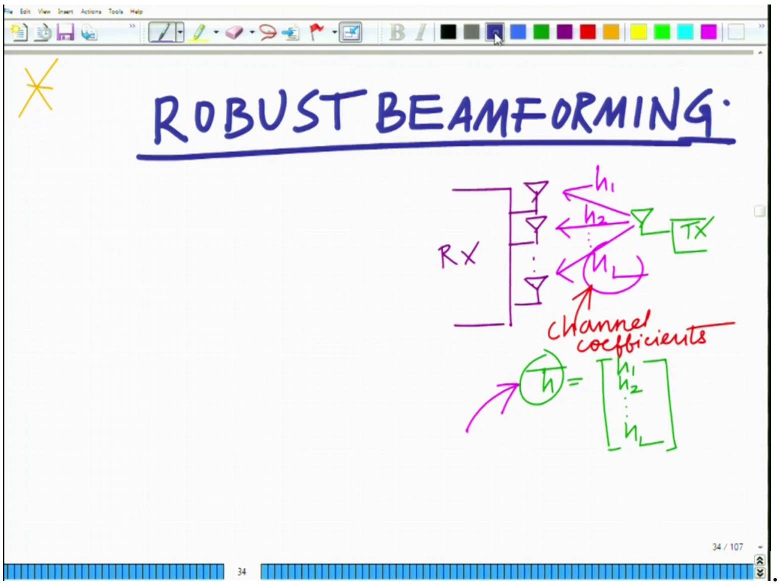

Now, let us go back and look at what we are doing in beamforming. Now, if you go back and look we have this, multiple antenna arrays at the receiver. In fact, this can also be there at the transmitter. So, you have a multiple antenna, array and you have a transmitter and you can have several interference also; that is what we have seen or you can have secondary users and primary users. But essentially what we are doing is the following thing. We have this, channel coefficient, corresponding to the l antennas which were given by h 1, h 2 so on up to h n.

Now, these are the channel, coefficients correct; these are the channel coefficients. And you have your channel vector which is if you put, these, things as a vector, you have the, vector the channel vector and this now, this channel vector; this knowledge of the channel coefficient. This is also termed as the Channel State Information.

If, you look at papers for instance research papers on wireless communication, you will see this is frequently termed as CSI, which is basically your channel.

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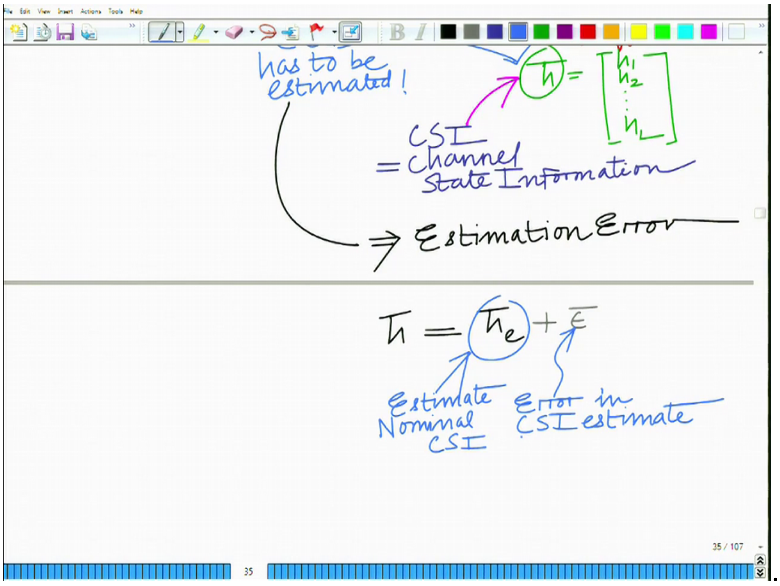

This is also termed as, Channel State Information. Now, the thing about Channel State Information that is this knowledge of this channel vector h, it is not available a priori all right, it is not available in the beginning which means, this has to be, somehow obtained all right because the channel is something that is, varying with time. It depends on several things, it depends on the scattering environment, it depends on the location of the base station location of the user. In general, it depends on the environment and it is changing. So, this knowledge of this channel vector has to be acquired or in other words this channel vector h bar which we have been assuming implicitly to be known has to be estimated initially.

So, that is the important aspects. So, this Channel State Information CSI which is, broadly termed as CSI has to be estimated and what this means is whenever there is an estimation process, there is always going to be, Estimation Error ok. So, no estimation process is hundred percent accurate which means, there is always the there is especially in practical sonorous. There is always a receive dual Estimation Error that depends again on various, various settings you can say.

For instance, how high is the signal to noise power ratio, how fast is this average you can. So, in general there is, Estimation Error all right there is error in the, available knowledge of the channel state information or in general there is error in the available estimate of the channel vector. So, you have your channel vector h bar ok. So, your channel vector is, h bar, but this channel vector is, not known exactly frequently. So, what is known is this estimate h bar and the channel vector is this estimate plus some you can, think of this as error.

So, you have this Error in the CSI estimate that is, Error in the estimate of the channel state information. So, what is known is, this is the estimate or this is also known as the Nominal, C S I something that is available on the face of it. So, this is Nominal channel state information or an estimate of the channel state information. And this is the true underlying channel which is unknown

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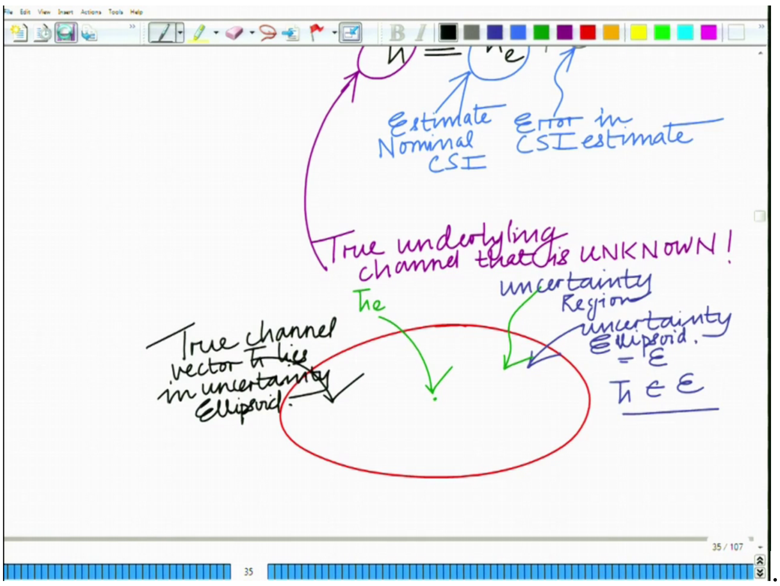

The true channel the True channel vectors are known, but what is known is an estimate and what we know is in general, this underlying channel the True channel vector h bar is close to the estimate that is all we know. Now, how close that has to be characterized at? One of the ways to characterize that is as we have seen again we have seen this before is to basically look at a region; around the estimate.

So, we say this is your estimate h bar p and this is the uncertainty region, typically, modelled as an ellipsoid. So, this is also known as an uncertainty ellipsoid. That is E and what you say is that the true channel vector lies somewhere in this uncertainty ellipsoid all right.

So, this is an ellipsoidal, uncertainty region all right. Around the channel estimate h bar e and the true channel vector h bar lies somewhere in this uncertainty ellipsoid and depending on the nature of the uncertainty. The uncertainty is severe than the ellipsoid is larger the uncertainty is smaller than the ellipsoid shrinks which means, the truth channel vector is actually very close to the available estimate h bar ok. So, the True channel vector h bar lies somewhere in the uncertainty ellipsoid.

True channel vector h bar lies in this uncertainty ellipsoid and so, h bar is not known exactly, but h bar lies in this Uncertainty Ellipsoid. So, how do we model this? So, we model this as follows.

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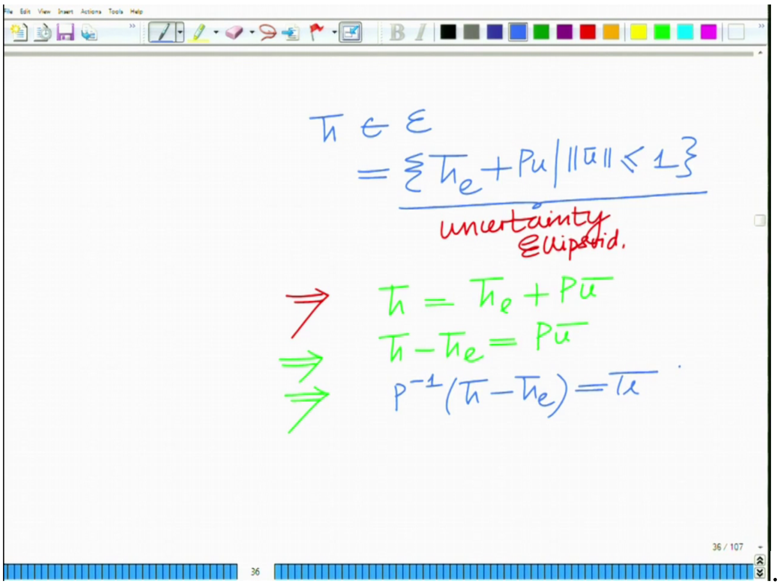

The true channel vector h bar belongs, to this, uncertainty ellipsoid. We know, how to model this Ellipsoid we have already seen this. So, the ellipsoid with centre h, bar e can be modelled as h bar e plus some matrix P times u.

Such that, that is this is a set of all h bar e plus matrix P times u such that norm of u bar is less than or equal to 1. This is the model for your uncertainty ellipsoid. This is the model for the uncertainty ellipsoid and you can see this is clearly an uncertainty. For instance, you can write this as this implies, what does this imply? This implies that your h bar equals h bar e plus P times u bar which implies that, h bar minus h bar e equals P times u bar which implies that, P inverse h bar, minus, h bar e equals u bar. Now, note that norm u bar is less than or equal to 1.

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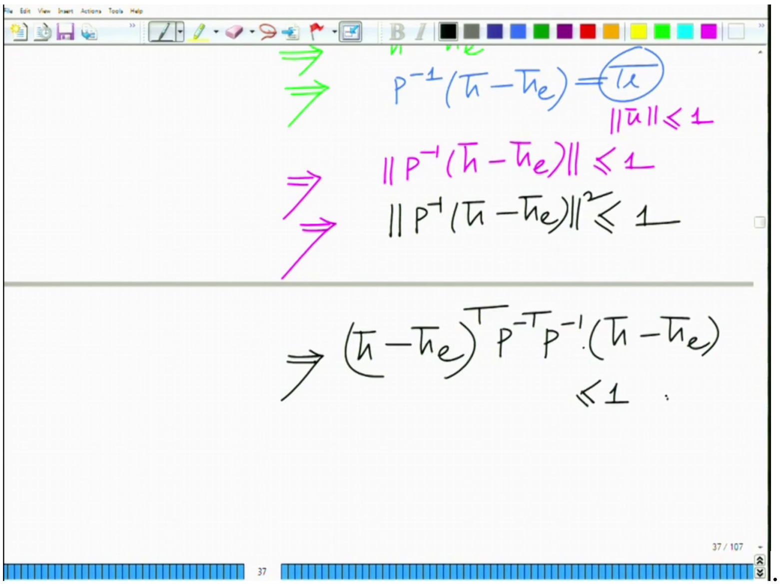

Which implies, norm P inverse, times h bar, minus h bar e, less than or equal to 1. Which implies that, norm square of P inverse h bar minus h bar is square norm square less than equal to norm square l vector is nothing, but the vector transpose times it itself.

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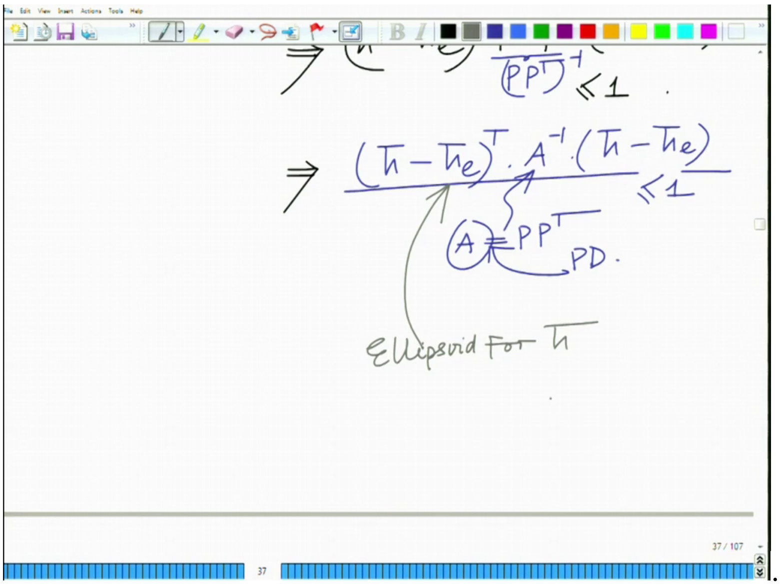

Which implies that, h bar, minus, h bar e transpose P inverse transpose P minus transpose P inverse into h bar minus h bar e less than or equal to 1; which implies that now you can see that now, P minus. So, this you can think of this as P P transpose inverse.

So, I can write this as, h bar minus h bar e transpose some matrix A inverse h bar minus h bar e less than equal to 1where this matrix A equals P P transpose and A is therefore, you can see this is a, P S D metric. In fact, is a P D matrix, positive, definite matrix because you are looking at A inverse.

And, therefore, this you can clearly see therefore, this is the ellipsoid. In fact, this is the ellipsoid ok; this is the ellipsoid, or h bar or rather the uncertainty ellipsoidFor h bar. And, that actual vector, h bar lies somewhere in this Ellipsoid. Now, let us go back let us revisit our original beamforming problem ok.

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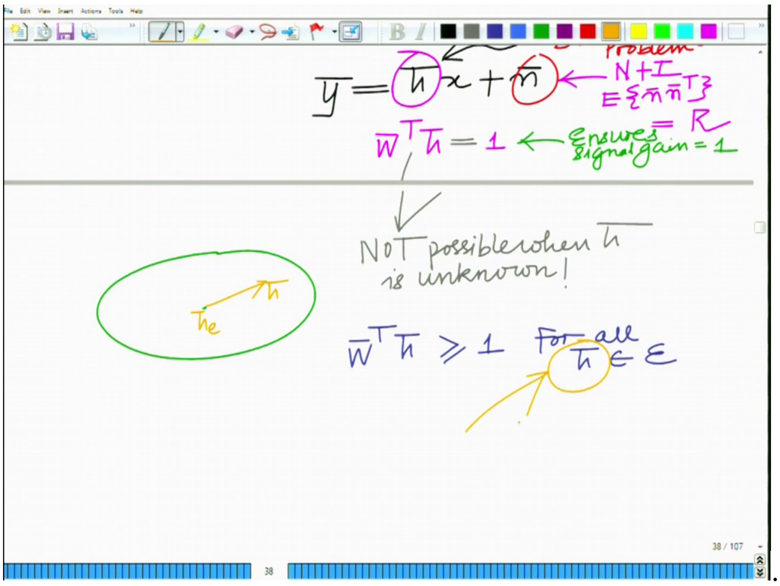

So, we have our original beamforming problem is y bar, equals h bar x, plus n bar, this is your original beamforming problem. Let us say, let us pick it general let us say n bar contains the noise plus interference N plus I with covariance remember we said, if you have noise plus interference instead of a white covariance you can characterize it by a covariance matrix R expected value of n bar n bar, Hermitian or rather you can say expected value of n bar n bar transpose ok, is R, this is the noise plus interference covariance.

Now, what we have doing so far is we have assumed h bar to be known exactly and when we beam forward we assume, we are setting W bar transpose h bar greater than or equal to 1 and this we said basically ensures, signal gain equals 1 and this is a very convenient framework all right. So, when you Beam forming bar W bar, you ensure to ensure that signal gain is constant. You have a constant signal gain while minimizing the noise part because; we said otherwise the solution is the trivial beam formal that is W bar equals 0. Therefore, we said W bar transpose h bar equals to 1.

Now, the problem with this approach is that if h bar is not known then, how are you going to enforce this condition right? So, we do not know h bar. So, it is meaningless to say W bar transpose h bar equals 1 because h bar the actual channel vector the underlying CSI is unknown. So, this is not possible, when h bar is unknown. So, the first thing that you have to realize is this is only possible when the underlying channel vector h bar is not ok.

And, therefore, now what do we do? So, now, there is no way to ensure this condition, but rather, what we do ok, I am sorry this is previous one was W bar transpose h bar equals to ensures unity gain for the 2 channel vectors. But now, what we do is we modify this as follows. Now, we modify this as W bar transpose h bar greater than or equal to 1. For all h bar belongs to the uncertainty ellipsoid.

So, what this says is you take the uncertainty ellipsoid and you look at any h bar belonging to that uncertainty ellipsoid.And for any h bar belonging to the uncertainty ellipsoid you are ensuring a minimum gain of unity all right. So, instead of just fixing the gain to unity for one particular channel vector h bar you are ensuring that for all these, channel vectors that belong to the uncertainty ellipsoid h bar the minimum gain is unity.

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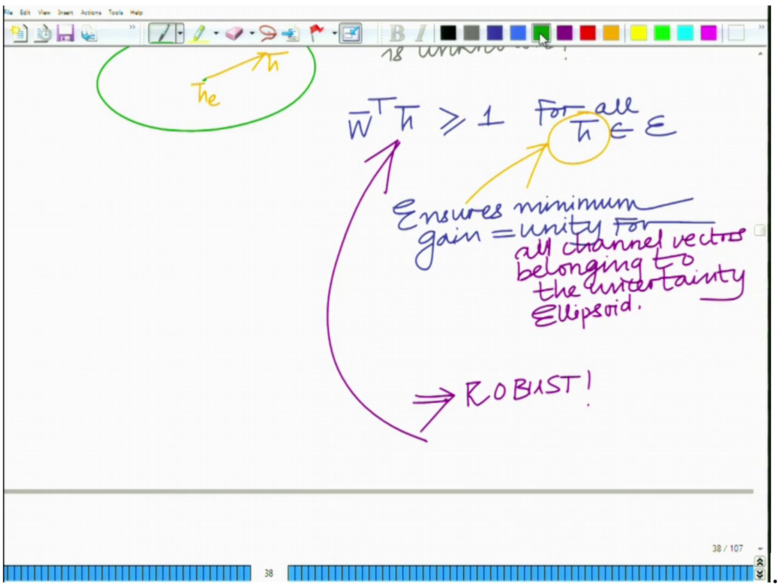

So, thereby this ensures minimum gain equals unity. Ensures minimum gain equal to unity for all channel vectors belonging to the, but all channel vectors belonging to the uncertainty ellipsoid and therefore, in that sense all right in that sense, it is robust!

Now, you can say this is, robust implies that, this is robot why is this robust remember what is the definition of robust? Robust something is robust implies that, something is strong something that cannot be swayed very easily. So, we say when you say a person is robust; that means, the person is resilient all right, even the person is attacked or the person is, for instance say, under an attack or something of that. So, the person has the ability to withstand that all right.

So, in that sense this optimization problem is robust meaning that, even if there is an uncertainty in the channel vector h bar which there is, this formulation is able to withstand it. Because you are ensuring a minimum gain of unity for not just any singular term any single value of the channel vector, but for all the channel vectors that belong to this particular uncertainty ellipsoid.

So, in that sense this robustness criterion makes sure that the designed beamformer is resilient or it can withstand this challenge of this uncertainty or this how do you put it this sort of this kind of scenario all right. This kind of an implement the scenario that is arising because of the estimation error all right.

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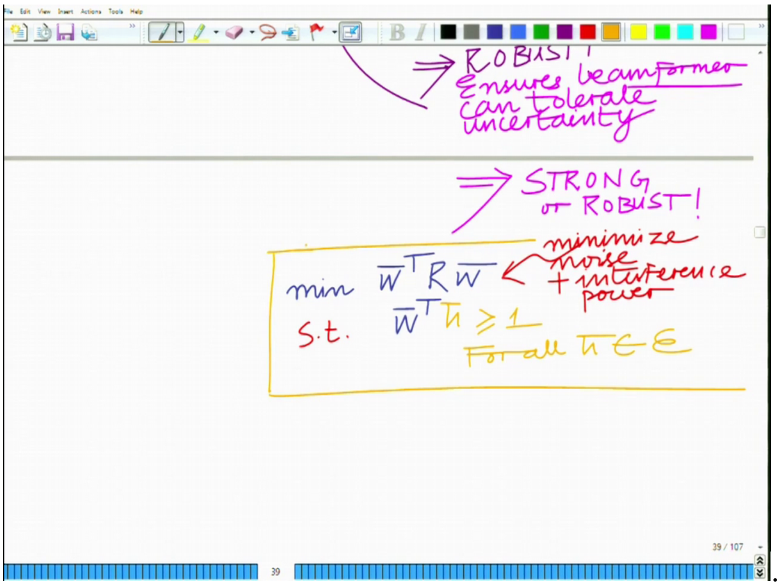

So, the robust framework ensures that the beamformer can tolerate uncertainty implies, it a strong or robust unless, something that was done. Previously if, it does not take the uncertainty into account this thing can withstand uncertainty; therefore, it is robust. And therefore, the robust beamforming problem can be formulated as follows. We similarly minimize the noise plus interference power W bar transpose R W bar

So, again, once again you minimize the noise plus interference power. You minimize the noise plus interference power, but now the constraint instead of W bar transpose h bar equals 1. This becomes W bar transpose, this becomes W bar transpose h bar greater than or equal to 1. For all h bar belongs to the uncertainty ellipsoid.

So, this is basically you are interesting and very interesting and you can say in novel robust beamforming problem. So, this ensures that you are minimizing the noise plus interfering interference power while at the same time ensuring a minimum signal gain for all channel vectors that belong to the Uncertainty set. So, this is your robust beamforming problem.

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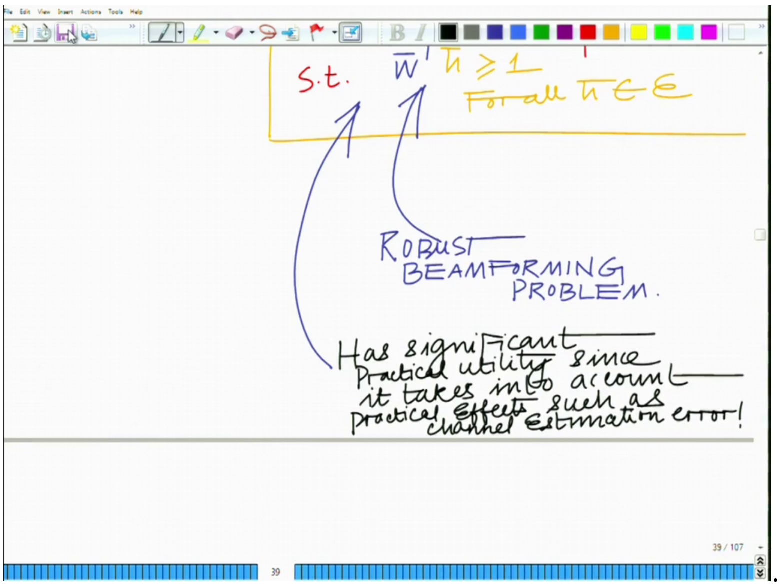

So, very interesting and not just interesting it has a lot of practical applications, of course, all of the beamforming paradigms that we have seen. So, far have immense practical little. But this one especially, has significant practical utility because it takes into account the practical artefacts, the practical effects that arise in systems such as the channel estimation error therefore, this further. So, this further enhances the practical utility.

So, this has a significantly higher practical utility. Since, it takes into account practical effects such as the, channel estimation error and therefore, it is robust and indeed it has significant practical utility and of course, as you can have already it must have noticed the problem. Now, formulating a problem is one thing, but then we also have to solve the problem to derive the optimal beam form also. So, in that sense, the problem has also become, significantly more complicated than that is naturally when you try to build, increased capability in something that the paradigm becomes more complex.

So, in that sense is robust forming problem is more involved than the previous beamforming paradigms that you. So, we are slowly building up the complexity. The first we have seen beamforming. Beamforming with the interference, zero forcing beamforming and now robust beamform it is indeed. In certain sense you can say encompasses, all these paradigms and generalizes to a scenario where this vector h bar is not known precisely and this is significantly more complex we are going to see the solution to this in the subsequent modules.

Thank you very much.