Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 28
Jensen's Inequality application: Relation between Average BER of Wireless and Wired system and Principle of Diversity

Hello. Welcome to another module in this massive open online course. So, we are looking at a practical application of Jensen's inequality in the context of a wireless communication scenario. In fact, to look at a comparison correct of a wireless communication system, the performance of a wireless communication system, a comparison of that with that of a conventional digital or a wire line communication system, alright.

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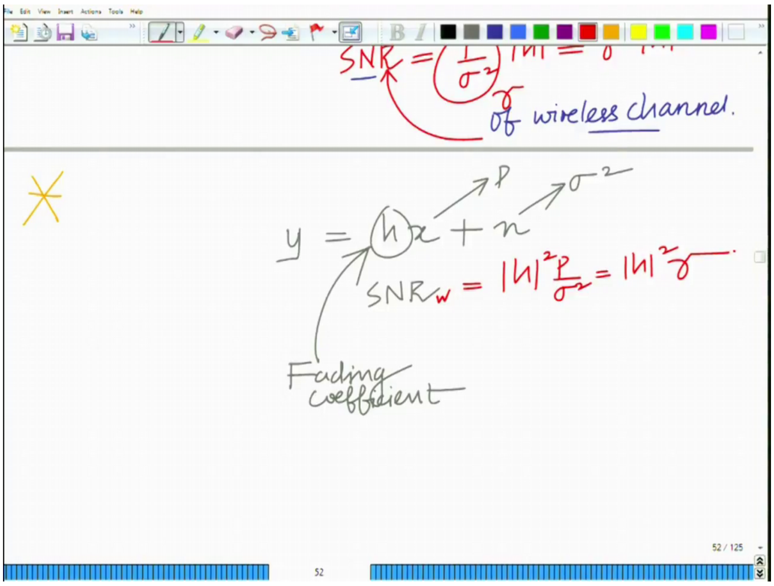

And, what we have seen is that, this relevant channel model for your wireless communication system is h x plus n the signal has power P, noise has power sigma square. Now, because of the fading channel coefficient, this h is your fading channel coefficient or this h is the fading coefficient. What has happened is your SNR of the wireless channel that is, magnitude h square times P over sigma square which is magnitude h square times gamma.

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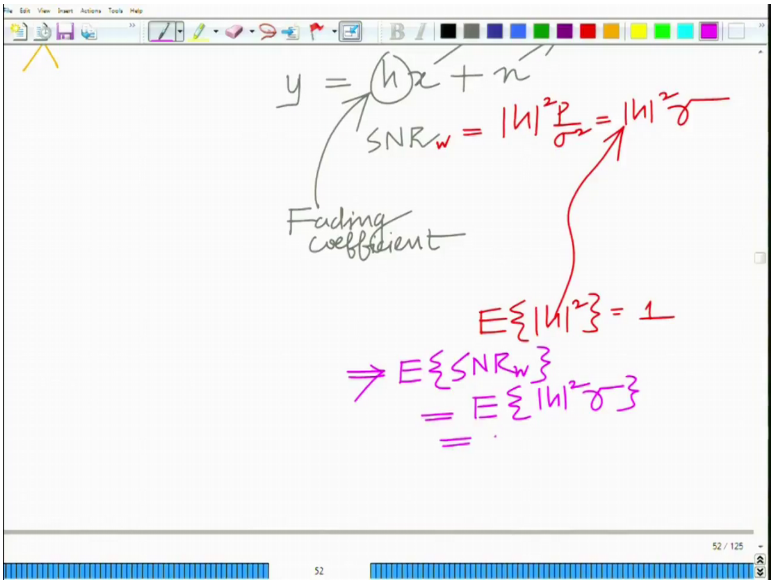

Further, what we will do is that, we will set that expected the average value of this magnitude h square, will set this equal to 1. What happens because of this? That is, if you look at the average SNR of the expected value, that is the mean or the average SNR of the wireless communication system, that will be exact expected value of magnitude h square times gamma which is basically.

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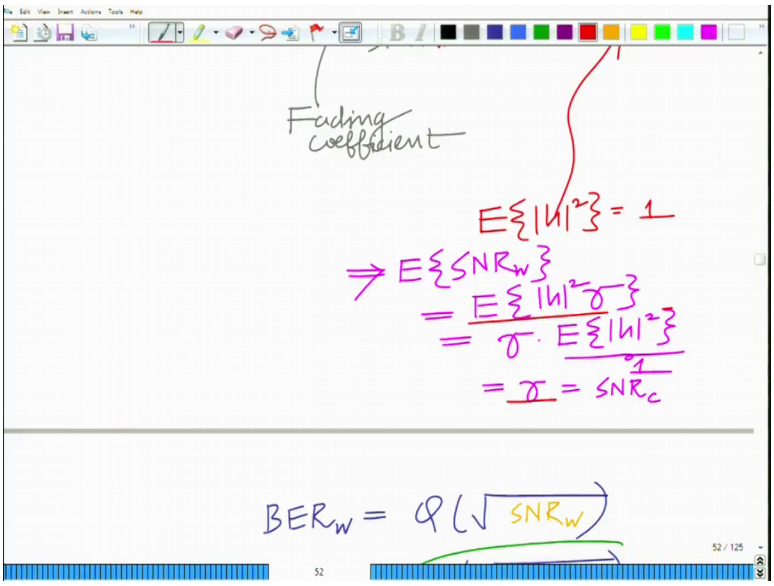

Well gamma times expected value of magnitude h square and we have seen that expected magnitude square equal to 1. So, this is gamma which is nothing but the SNR of your

AWGN channel or let us call it as SNR of the your conventional correct or your wire line communication system. So, this is a fair comparison, alright.

So, we are using the same average SNR for both. It is not the case that a wireless system has a much higher SNR than the than the conventional communication system, alright. So, to perform a fair comparison between these 2 systems, we are choosing, alright; we are setting the SNR or we are choosing about the channel coefficient is basically assumed to follow a random assumed to follow probability density functions such that the average SNR for both the scenarios is the same.

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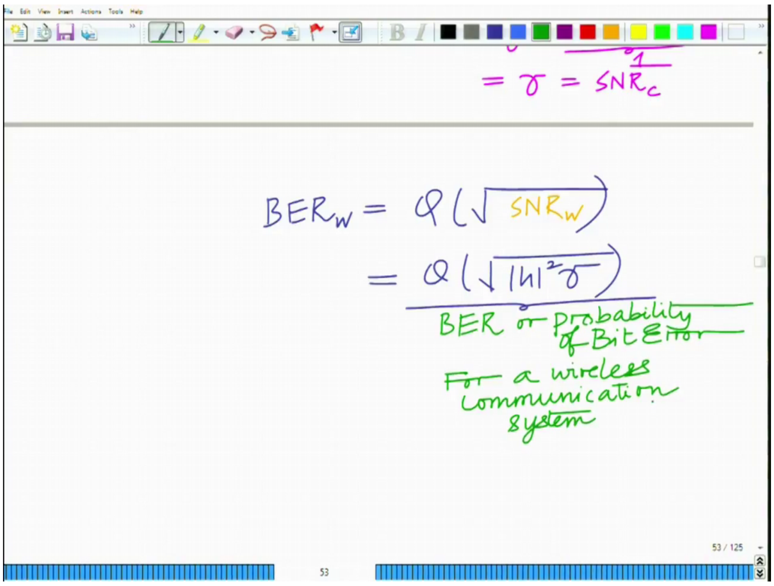

Now, if you look at the probability of bit error for a wireless communication system, so, if you look at the bit error rate for the wireless communication system, this will be Q of square root of the SNR for the wireless communication system. We have already seen that and which is nothing but Q of square root of magnitude h square times gamma ok. So, this is known as the bit error rate. The instantaneous bit error, the bit error rate or this is your basically bit error rate or probability of bit error for a wireless communication system for a wireless communication system.

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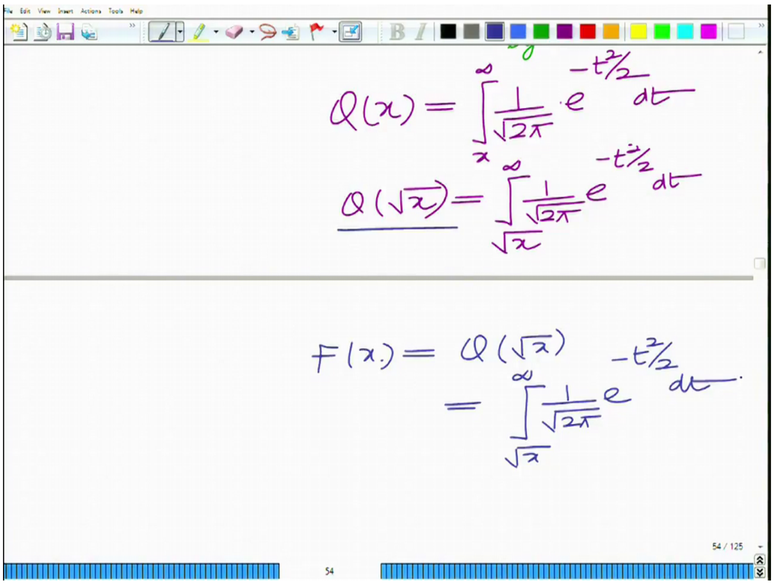

And now, we want to look at this function. Now, we have already seen what is Q of x. Remember Q of x is the CCDF of the standard Gaussian random variable. So, that is given as x to infinity 1 over square root of 2 pi e raised to minus t square by 2 dt and therefore, Q of square root of x.

Now, let us look at this function Q of square root of x. That will be integral square root of x to infinity 1 over square root of 2 pi ok. E raised to minus t square by 2 dt. Simply, replacing x by square root of x correct, that gives me Q of square root of that gives me Q of square root of x. Now, therefore, now what we want to show is, if you denote this Q of square root of x by F of x, so, F of x equals Q of square root of x which is integral square root of x to infinity 1 over 2 pi e raised to minus t square by 2 dt.

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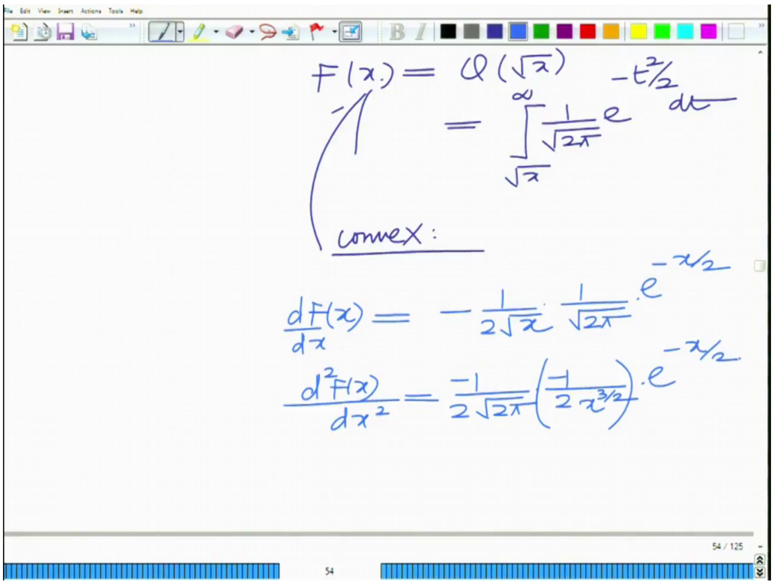

We want to show that this function F of x, this is a convex function. We want to show that and that is easy to show, all right. So, we already know that Q of x is a convex function. We want to show that your square root of x is also a convex function. And that is easy to show if you differentiate this dF by dx. Remember, first derivative of a derivative of the top limit, but top limit is infinity.

If this is the constant derivative is 0 minus derivative of the bottom limit derivative of different, when if you differentiate square root of x you get 1 over 2 square root of x 1 over square root of 2 pi which is a constant times, you have to substitute the lower limit in the integral. That is, e raised to minus x by 2. And now, d square F of x by dx square that is simple to evaluate. First, let us evaluate the derivative of 1 over square root of x.

So, this is 1 over 2 square root of 2 pi times that will be 1 over the derivative of well. So, minus 1 over 2 square root of 2 pi derivative of square 1 over square root of x. That is, half x to the power of 3 by 2 and there is a minus sign times e raised to minus x by 2 minus 1 over again 2 square root of 2 pi 1 over square root of x times derivative of e raised to minus x by 2 which is minus half e raised to minus x by 2.

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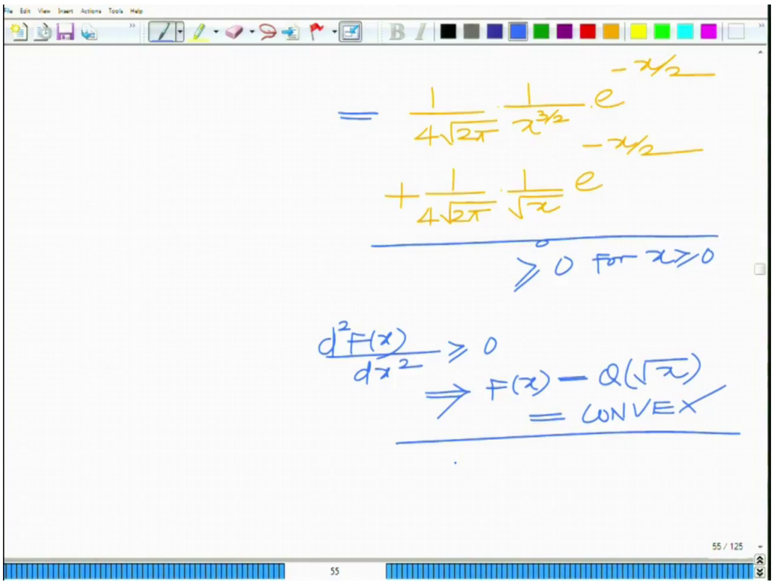

And now, if you simplify this, you can see that this derivative is nothing but 1 over; you can simplify this 1 over 4 square root of 2 pi 1 over x raised to the power of 3 by 2 into e raised to minus x over 2 plus 1 over. Well, this is actually 1 over 1 over again 1 over 4 square root of 2 pi times 1 over 4 square root of 2 pi times square root of x 1 over 4 square root of 2 pi m square root of x raised to minus x by 2. And essentially, the important thing is, here you can see that the second order derivative is greater than or equal to 0; implies that and this is greater than equal to 0 for x greater than or equal to 0.

So, implies that d square F x by dx square is greater than or equal to 0 implies F of x equals Q of square root of x. This is convex, ok. So, that shows that this bit error rate function Q of square root of x is a convex function, ok. So, we start with that, ok. So, F of x Q of x is convex as well as Q of square root of x and now, we are demonstrating this for Q of square root of x because, that is relevant in this context.

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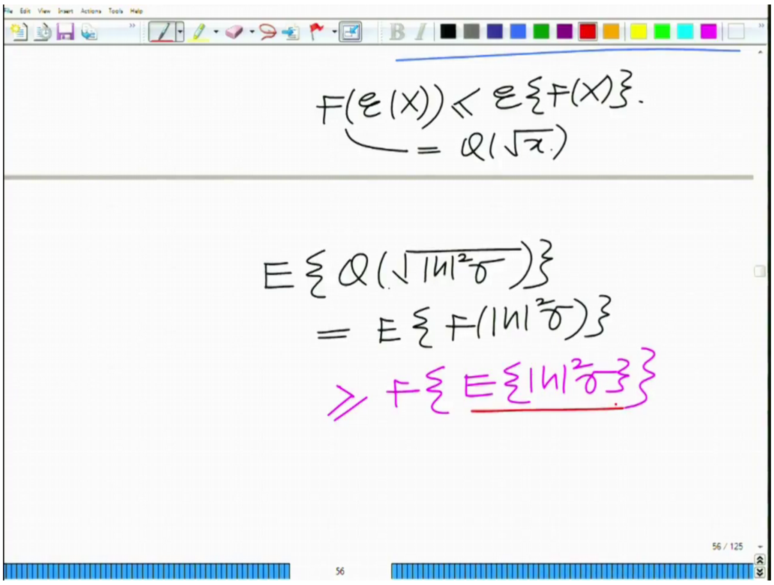

Now,, what is the Jensen's inequality tell us? Let us recollect. The Jensen's inequality that tells us that, F of well F of expected value of X any random variable is less than or equal to expected value of F of X, ok. Now, this implies that if you look at our function F of X equals Q of square root of x, this implies that your expected value of Q of square root of your SNR for the wireless channel which is magnitude h square gamma which is nothing but your expected value of F of X, where F of x is your magnitude h square into gamma.

Expected value of F of X or so, expected value of it is greater than or equal to the is greater than equal to F of expected function of the expected value of F. So, this is greater than equal to the function of the expected value of magnitude h square gamma. And now, you observe something interesting. If you recall or expected value of magnitude h square gamma, that is basically nothing but expected value of magnitude h square gamma. This is equal to gamma.

Because, expected value of magnitude h square equals 1; that is precisely the condition that we set. So, that both these communication systems have the same SNR on the average. And, what you can see is, this is greater than or equal to F of magnitude. It is and this is gamma which is equal to F of.

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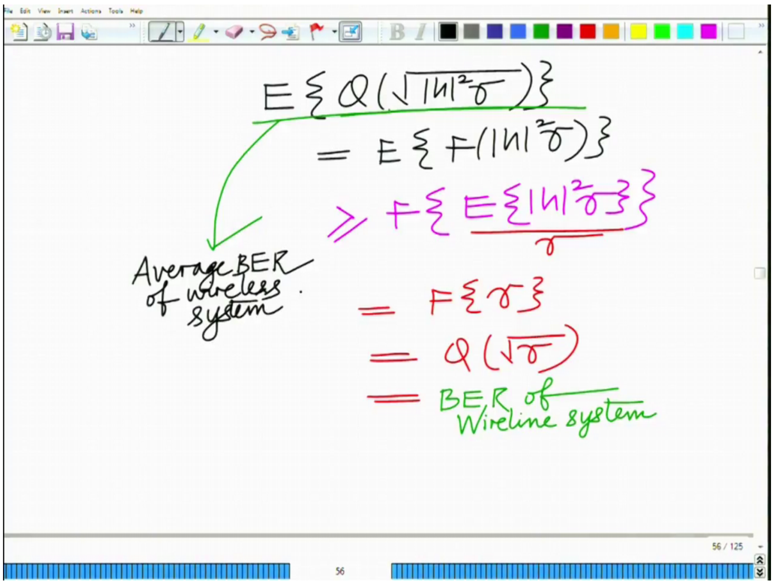

Now, you can see this is greater than or equal to F of gamma which is nothing but Q of square root of gamma. So, this is the now Q of X square root of gamma. This is basically bit error rate of conventional or your wire-line system ok. And what is this? Remember Q of square root of h square magnitude h square gamma? This is the instantaneous beta rate of the wireless communication system which implies that expected value of this. This is the average bit error rate of a wireless communication system.

So, this basically implies that the average bit error rate, correct? This basically shows you know 3 theoretically and rigorously that the average bit error rate of a wireless communication system for the same average. Remember, both these systems of the same average SNR get the average bit error rate of a wireless communication system is significantly is higher. In fact, you will see in practice that, it is significantly higher than that of a conventional or a digital wireless communication system, ok.

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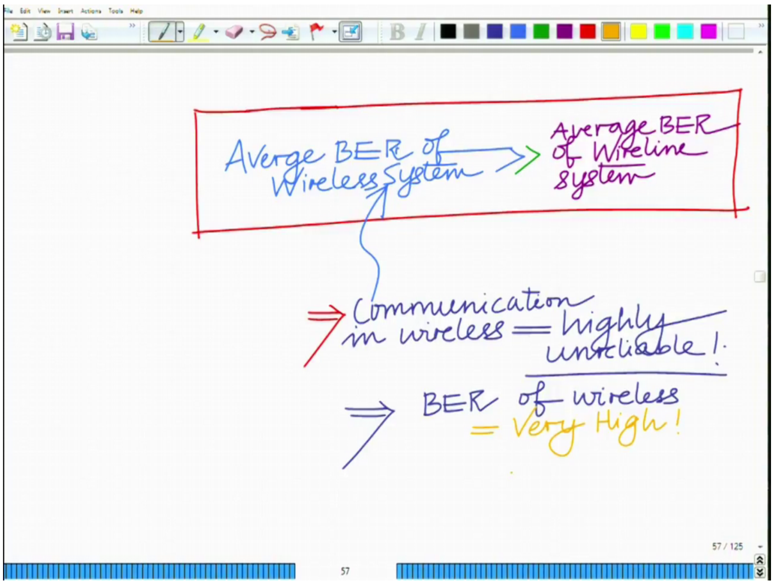

So, this tells us the interesting. In fact, the very interesting one of the fundamental results that underpins the entire study of wireless communication systems is that, average BER of wireless average BER of a wireless communication system is greater than average is greater than average bit error rate of a wire line or you can also say that as a conventional communication system.

The communication system in which there is a wire or a guided media guided propagation medium or a wired medium or the wire medium between the transmitter and receiver. And, in fact, in practice, this often greater than greater that is, it is significantly greater. It is not just greater, but the average bit error rate of a wireless communication. It is a wireless communication system is significantly greater than that of a wire line communication system or a wired communication system which implies that the wireless communication of a wireless communication over communication in a wireless system is highly unreliable.

Because, the probability of bit error is very high so, this quality so, this communication is highly unreliable implies communication is highly unreliable which is basically bit error rate is very high. And why is this arising? That is important. Remember, this is arising. All of this is arising because, what is the fundamental difference between the wire-line and a wireless communication system? The fading nature of the wireless channels the fading channel coefficients. So, all this is arising because of the random nature of the fading channel coefficient ok.

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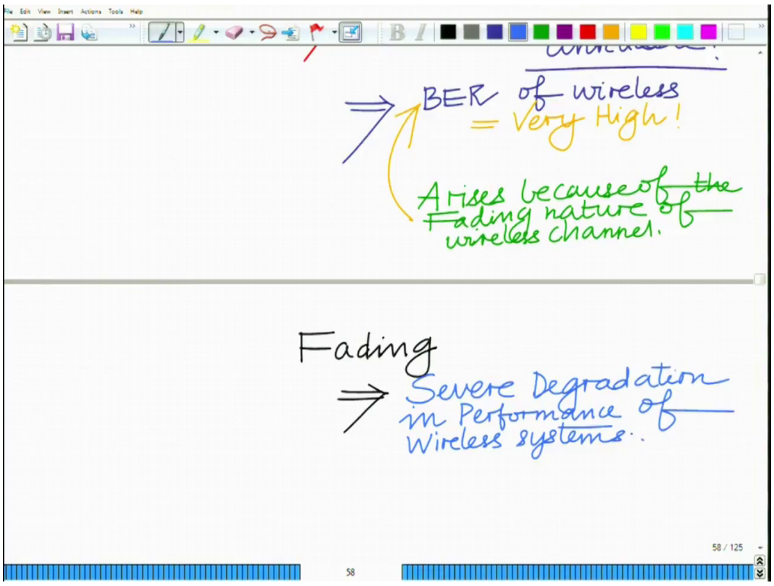

So, this is arising because of the fading nature of the wireless channel coefficient. It is arises because of the fading nature of the violation. And therefore, fading has a significant impact on the nature and in fact, the nature and the performance of communication.

In fact, fading leads to a severe degradation in the performance of a wireless communication system. So, fading leads to a that is the challenge in wireless communication, ok. Fading implies or leads to a severe degradation in performance. So, fading is basically a challenge.

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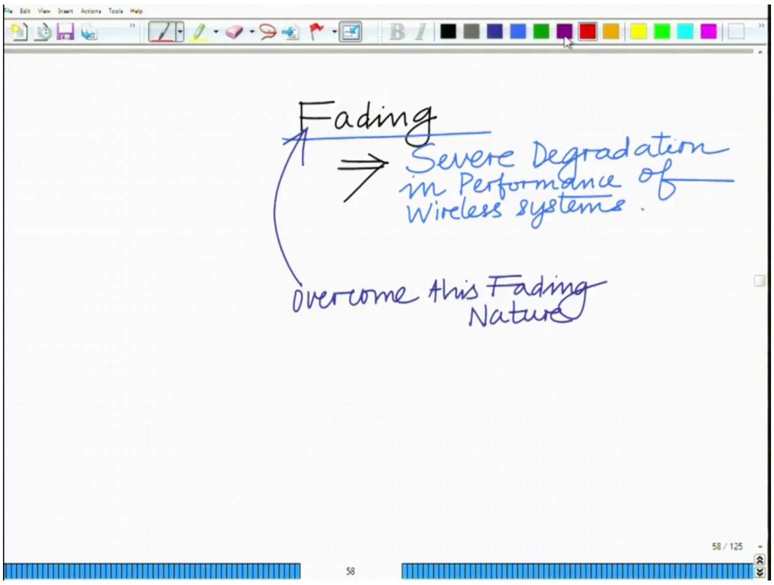

So, this is a challenge and one has to therefore, overcome or surmount this challenge, overcome this fading nature and that is why, we need technologies to overcome this fading nature and one of the most important technologies to overcome the fading nature or this degradation that arises due to fading is termed as diversity.

Diversity is basically, we have multiple received signals at the receiver and you combine these receivers to enhance the signal to noise power ratio as well as the reliability of a wireless communication. This principle is known as diversity. For instance, in a multiple antenna system where you have multiple receive antennas. You receive multiple copies of the signal combine the signal copies to enhance the performance of the wireless communication system, ok.

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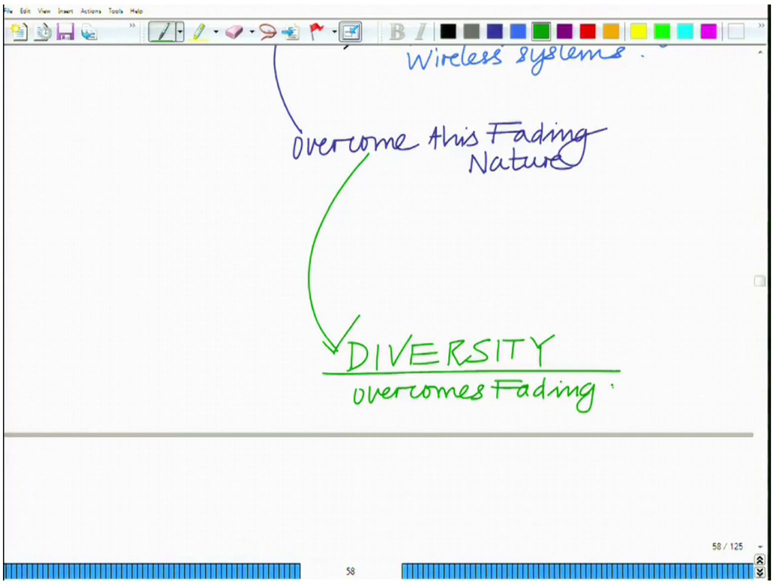

So, the whole this fundamental property of wireless communication, alright leads to technologies that overcome the fading an example of diversity is multiple antennas.

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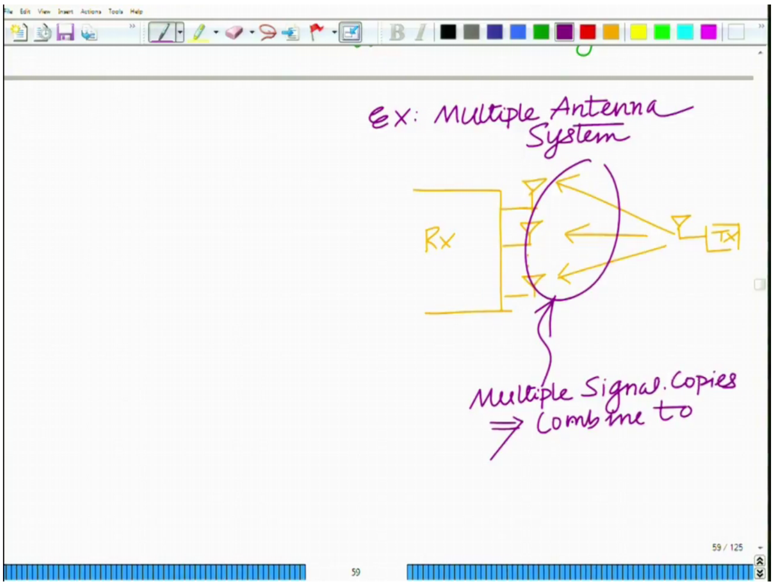

What happens in a multiple antenna system? We have already seen schematically you have the receiver, all right. So, you have from the transmitter you have multiple copies. So, these are your, so, this is your receiver this is your transmitter.

So, you have these multiple copies or basically it is also diversity means, several right. That is the English meaning of the word diversity and these multiple signal copies you combine to enhance the signal (Refer Time: 19:17) power ratio as well as reliability.

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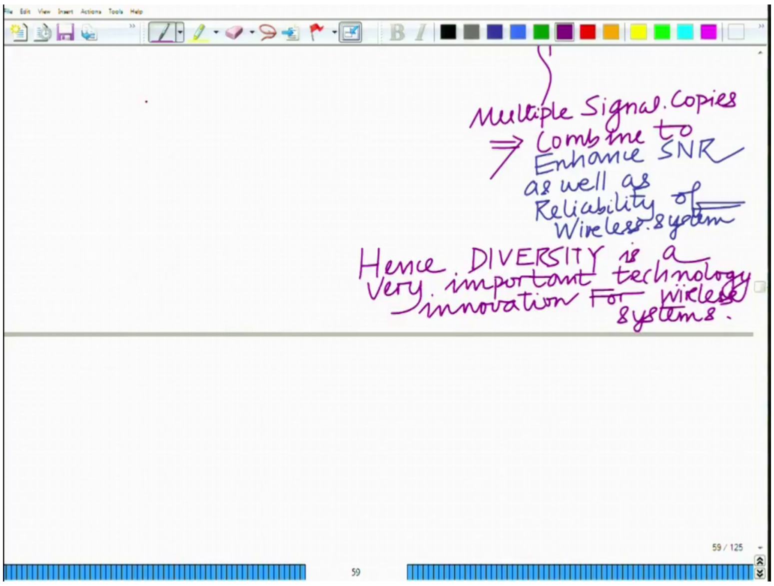

Hence, diversity is a key principle in a wireless communication system and in fact, one of the key technologies. This is a very important or you say you can also say technology innovation diversity is a very important. Technology innovation diversity is a very important technology innovation for wireless communication systems all right. At that and at the root of all this is basically the fading nature of the wireless channel and the performance the poor performance of the wireless communication system the very high beta rate which can in fact, be demonstrated using the Jensen's inequality.

And therefore, Jensen's inequality has a lot of applications. Especially, especially to put fundamentals results like this. It can also be used in as I have already noted, can also be used to derive and prove several results in the context of information theory. So, that basically summarizes the Jensen's inequality and also shows, it is application in a very interesting context. In that, in the context of performance comparison of a wireless versus a conventional wired communication system all right.

So, we will stop here and continue in subsequent modules.

Thank you very much.