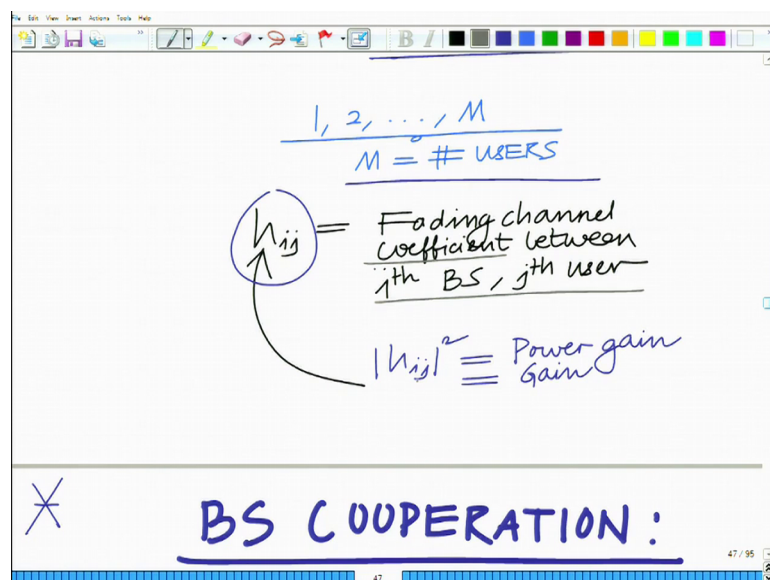


**Applied Optimization for Wireless, Machine Learning, Big Data**  
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**Lecture – 16**  
**Applications: Cooperative Cellular Transmission**

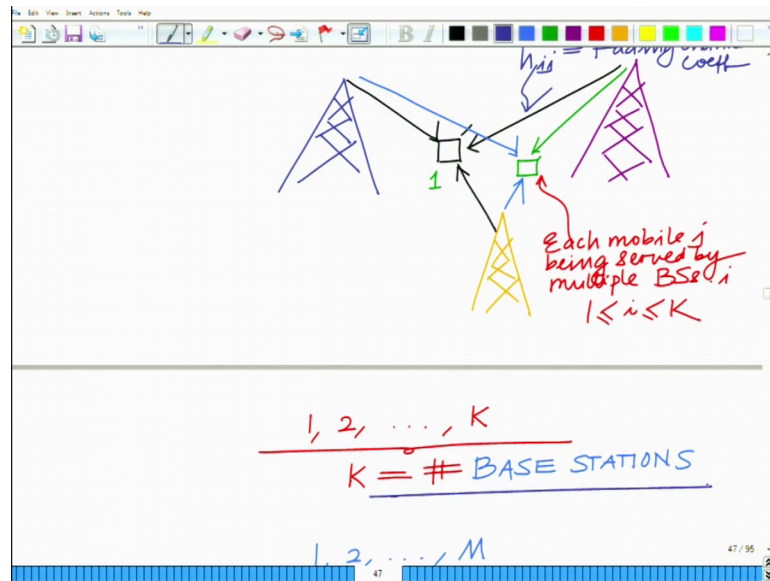
Hello, welcome to another module in this massive of an online course. So, we are looking at a wireless base station cooperation scenario, in which several base stations are cooperating to transmit to a single user or group of users alright.

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So, we are looking at this scenario, which is very practical example in cellular network contrast termed as base station cooperation ok. And what happens in base station cooperation, if you look at it we describe it in the previous module that is we have  $K$  base stations ok.  $K$  is the number of base stations which are cooperating to transmit to  $M$  users ok.

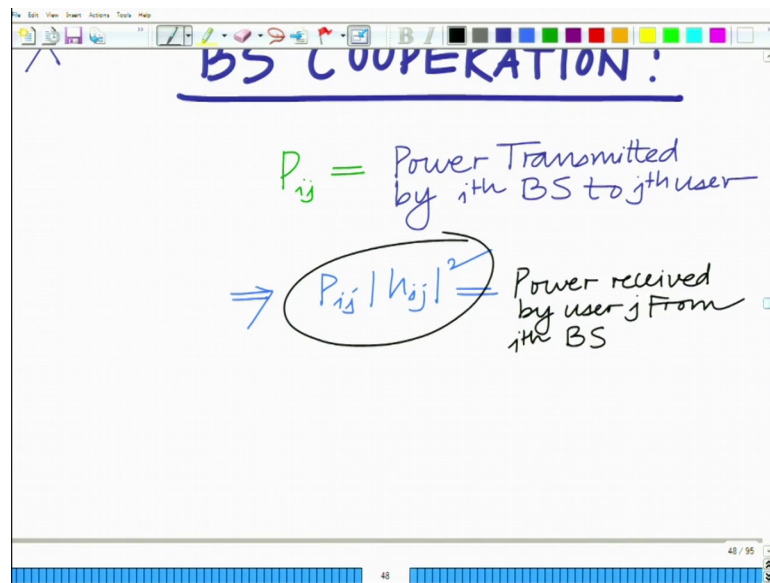
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And these are the base stations and these users are typically located in a region where they can receive the signals from the multiple users such that at the intersection of these various cells. So, we have three cells in the intersection you have some of these user, and these can be served by multiple base stations not just a single base stations. So, the base stations can cooperate with other to enhance the signal to noise modulation at each user ok.

And  $h_{ij}$  this quantity denotes the fading channel coefficient, fading channel coefficient between the  $i$ th base station and the  $j$ th user and therefore, magnitude  $h_{ij}^2$  this is the power gain. And what this means is, if you look at the power received by  $j$  user  $j$ th user from the  $i$ th base station. So, let say so, we already said i think or let us say that now we have another quantity.

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Let say  $P_{ij}$ .  $P_{ij}$  is the power transmitter by  $i$ th base station to  $j$ th user ok. So,  $P_{ij}$  equals power transmitted by  $i$ th base station to the  $j$ th user. So,  $P_{ij}$  is the loads a transmitter power magnitude  $h_{ij}$  square is a power gain, which implies that if I multiply; so, this is the transmit power. So, if I multiply  $P_{ij}$  by magnitude  $h_{ij}$  square, now this quantity this denotes, this quantity this denotes the power received by user  $j$ . So, this is the power, user  $j$  from the power received by user  $j$  from the  $i$ th base station. So, this quantity is given by  $P_{ij}$  magnitude  $h_{ij}$  square alright.

So, now, what we can look at is, let us look at the total power received by user  $j$  any particular user  $j$  from all the base stations. So, to compute the total power at any particular user, we have to sum the power that is received from all base station and that is given as.

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The image shows a whiteboard with handwritten text and a mathematical formula. At the top, it says "Total Power of user j" followed by "= Sum of Power From all BSs." Below this, a summation formula is written: 
$$= \sum_{i=1}^K P_{ij} |h_{ij}|^2$$
 The whiteboard interface includes a toolbar at the top and a status bar at the bottom showing "49 / 95".

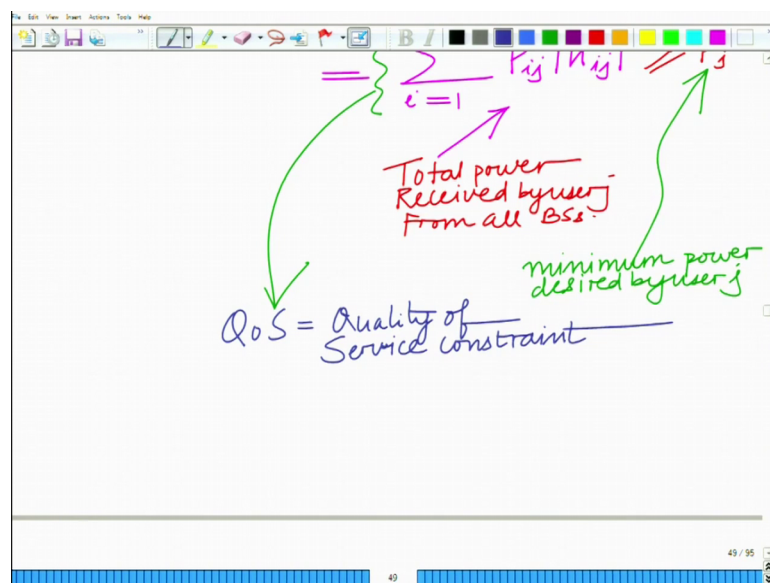
So, what is the total power of user j? The total power of user j I mean total power of signal received by user j that is basically the sum of power from all base stations, from all base stations and that is basically you remember the base station index is i. So, you have to sum over all i. So, i equal to 1 to K, K is the total number of base stations,  $P_{ij}$  magnitude  $h_{ij}$  square. This is the total power received of total power received by user j from all base stations.

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The image shows a whiteboard with handwritten text and a mathematical equation. At the top, it says "From all BSs." Below this, a summation formula is written: 
$$= \sum_{i=1}^K P_{ij} |h_{ij}|^2 \geq \tilde{P}_j$$
 There are two green arrows pointing from the summation part of the equation to the text "Total power Received by user j From all BSs." and from the  $\tilde{P}_j$  part to the text "minimum power desired by user j". The whiteboard interface includes a toolbar at the top and a status bar at the bottom showing "49 / 95".

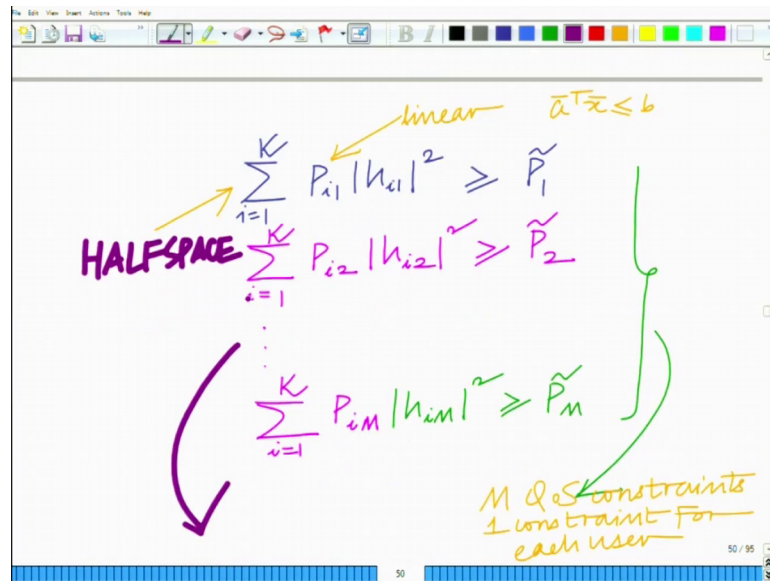
From all base stations and not we will say is this has to be greater than equal to some quantity  $P_j$  tilde. This is the minimum power that is desired by user  $j$ , this is the minimum power that is desired by user  $j$ . So, the total power at user  $j$  received at user  $j$  has to be greater than or equal to some quantity  $P_j$  tilde, which is the minimum power alright the minimum desirable you can say the minimum desirable signal quality at that particular user. This is known as a QoS constraint or a Quality of Service constraint for that particular user ok. So, this constraint is also termed as a QoS or a Quality of Service constraint its quality.

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So, a signal quality has to be such that the received power has to be at least greater than equal to  $P_j$  tilde. So, we will have one quality of service constraint for each user. Remember we have  $M$  such users therefore; we will have  $M$  quality of service constraints if you remember correctly where  $M$ ,  $M$  is a number of users. So, therefore, we will have  $M$  such quality of service constraints; what are those quality of service constrains?

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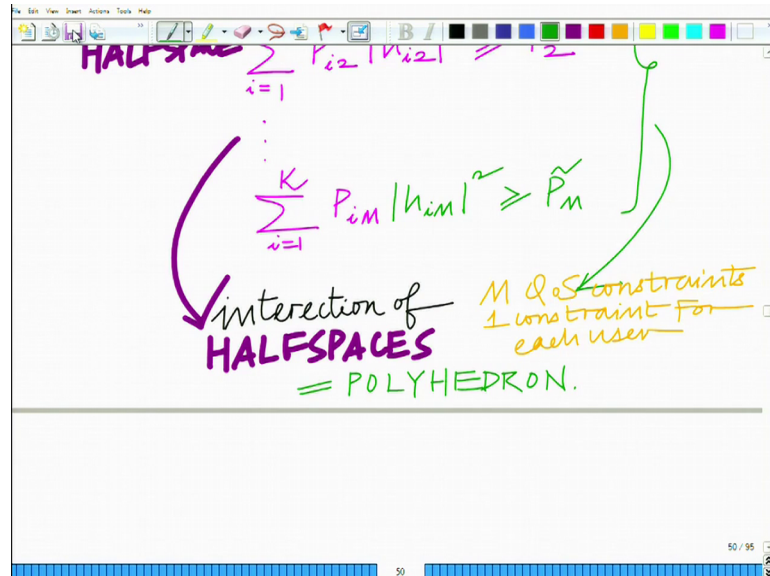
When for user one remember we must have  $P_{i1}$  equal to 1 magnitude  $h_{i1}$  square  $i$  equals 1 to  $K$ . This has to be greater or equal to  $P_{t1}$  tilde right this has to be greater than equal to  $P_1$  tilde summation  $i$  equal to 1 to  $K$  now we can write for user 2,  $P_{i2}$  magnitude  $h_{i2}$  square; that is a  $h_{i2}$  remember is a channel coefficient between base station  $i$  and user 2 this has to be greater than equal to minimum desirable power  $P_2$  tilde of user 2 so on so fourth. You will have summation  $i$  equal to 1 to  $K$ ,  $P_{im}$  for the  $m$ th user magnitude  $h_{im}$  square greater than equal to  $P_m$  tilde these are the  $q$  s constraints these are the  $m$   $q$  s constraints remember these.

What are these are the  $M$   $Q$  s or quality of service constraints there is one constraint for each user. There is one constraint for each other and if you look at each constraint remember in the powers this is linear combination, this is a linear combination of the powers this is of the form  $c^T x \leq b$  less than or equal to less than or equal to this is of the form  $a^T x \leq b$  where this we are looking at  $x$  is basically nothing, but the powers. So, each of this represents the hyper plane, each constraints represents you can see each constraint represents a hyper plane.

So, this implies correct this implies this implies that this is an intersection. I am sorry each constraints represents a half space not hyper plane I apologize each constraint

represent a half space, because this is a which means that this is an intersection of half spaces this is an intersection of half spaces.

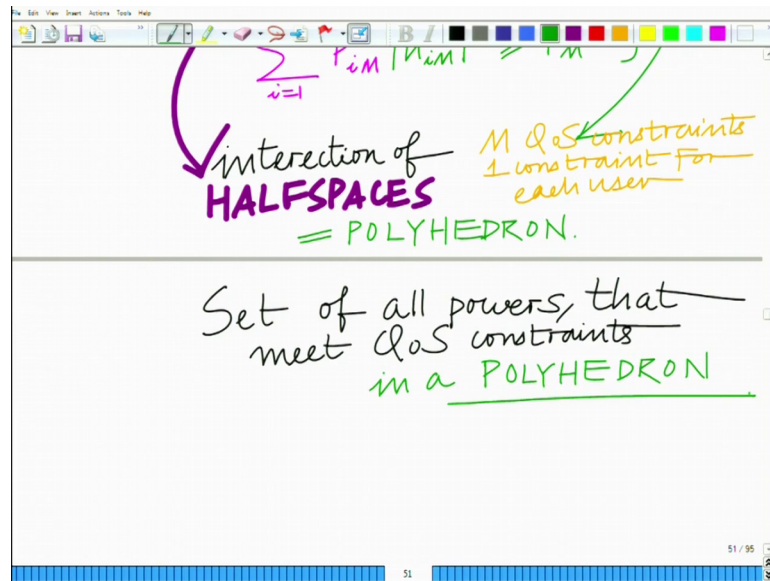
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And implies this which implies that this is equal to a which implies this is equal to a polyhedron. Remember that what you said an intersection of half spaces and hyperplanes is nothing a polyhedron and this is interesting primary interesting practical application.

So, the set of all possible powers, which meet the quality of service constraints of these different users in this base station cooperation setup right this cooperative multiple setup, the set of all possible powers lie in a polyhedron, which is obtained as the intersection of the half spaces given above. So, the set of all possible powers; so, this is very important.

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So, in the practical optimization problem set of all powers that meet the QoS constraints, these are in a polyhedron these lie in a polyhedron or a polyhedral regions. So, to optimize this powers that are transmitted to the different users by the base station correct what has to consider? The set of all possible powers that; lie inside a polyhedron for his optimization problem.

Now, similarly remember each base station also has a possible power constraint total power constraint. So, you can consider that also right. So, now, each base station; now looking at for perspective of base station each base station. So, let us look at the base station power constraint.



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meet QoS constraints  
in a POLYHEDRON

BS POWER CONSTRAINT

Each BS has  
max Transmit Power  
 $P_i = \text{max Transmit Power of BS } i$

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Now, if you look at the base station power constrain, now each base station has a certain maximum power. Let us call this maximum transmit power as  $\bar{P}_i$   $\bar{P}_i$  equals max transmit power of base station  $i$ . This is the maximum transmit power of base station  $i$ , now what does it mean? That is the power that has to be transmit the power that is transmitted to all the users all the  $m$  users by each base station  $i$  has to be less than or equal to this quantity  $\bar{P}_i$ . Because this is the maximum possible transmit power correct?

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$\Rightarrow \sum_{j=1}^M P_{ij} \leq \bar{P}_i$

Total Tx Power  
of BS  $i$  to all  $M$  users

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So, this implies that if I look at any possible any particular base station and sum the power over all users. So, I must have  $P_{ij}$  transmit the summation of all transmit powers has to be less than or equal to  $\bar{P}_i$  and what is this? This is the sum of transmit total transmit power total TX power of base station  $i$  to all users to all  $M$  users. In fact, you can say this is total transmit power to all the  $M$  user. So, this has to be less than or equal to  $\bar{P}_i$ . So, we can write one constraint for each base station.

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Total TX Power  
of BS  $i$  to all  $M$  users -

$$\sum_{j=1}^M P_{1j} \leq \bar{P}_1$$

$$\sum_{j=1}^M P_{2j} \leq \bar{P}_2$$

$$\vdots$$

$$\sum_{j=1}^M P_{Kj} \leq \bar{P}_K$$

HALF SPACE

= K Power constraints

So, what does that mean? You will have  $j$  equal to 01 to  $M$   $P_{1j}$  total transmit power of base station one to all users that is less than or equal to  $\bar{P}_1$ ,  $j$  equal to 1 to  $M$ , summation  $P_{2j}$  total transmit power base station 2 to all users less than or equal to  $\bar{P}_2$  bar.

So on and so forth you can write  $k$  constraints, one constraints for each base station. So,  $j$  equal to 1 to  $M$   $P_{Kj}$  less than or equal to  $\bar{P}_K$  these are what are these? Well these are  $K$  power constraints. Now, you can see again each is a half space each is a each is a half and again you are this  $K$  power constraints each is a half space. So, you have the intersection of the  $K$  half spaces a finite number of half spaces. So, there is also represents a poly heat ok.

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HALF SPACE

$$\sum_{j=1}^M P_{Kj} \leq \overline{P}_k$$

Intersection of K Power constraints, one for each user  
= POLYHEDRON.

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There is an intersection of a K power constraints, one for each user, which is equal to polyhedron. So, either you look at the P o S constraints one for each user total of intersection of M constraints M half spaces, that is the polyhedron and now if you look at the total power constraints of the base station of each base station. For each base station i we have total power constraints. So, this is the i in the intersection of K in the half spaces also polyhedron the set of all possible powers so, as that you meet the total transmit power the transmit power constraint at each base station this satisfies all this satisfies this set of constraints is also a polyhedron.

So, therefore, this polyhedron which basically is a region that is formed by the intersection of either hyper planes or half spaces, which is the convex has a significant utility and arises frequently in various optimization problems especially in the context of signal processing and communication, and this illustrates one such simple application scenario alright. So, we will stop here and continue in the subsequent modulus.

Thank you very much.