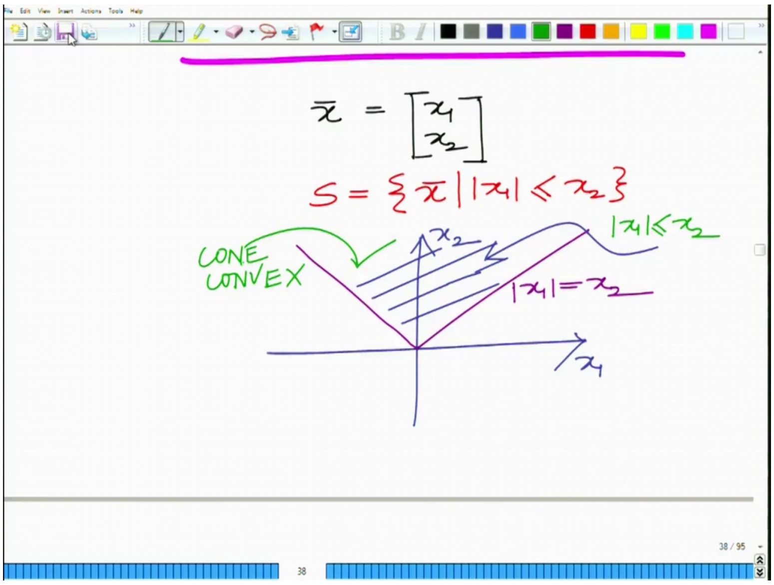
Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 15
Norm Cone, Polyhedron and its Applications: Base Station Cooperation

Hello, welcome to another module in this massive open online course. So, we are looking at various types of convex sets and their relevance to practical applications especially in context of wireless communications and signal processing. Let us continue our discussion by looking at yet another convex set or class of convex sets that is the convex cone, ok.

(Refer Slide Time: 00:38)

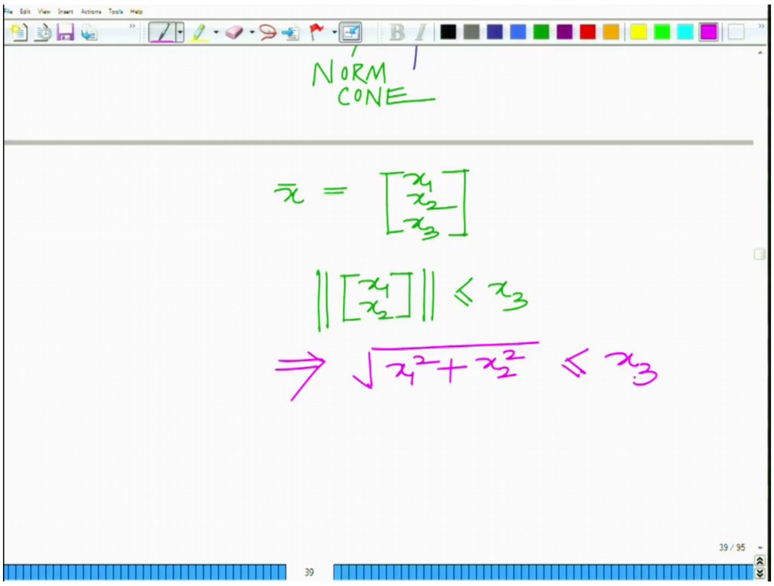


So, what is also start looking at a convex cone or a norm cone this is another prominent class of convex sets and well, what is a norm cone?

Well, let us consider a 2-dimensional scenario let us consider a 2-dimensional vector x 1, x 2 and if you consider the set of all vectors that is the set of all vectors such that magnitude of x 1 is less than or equal to x 2, ok. So, you want to look at the set of all vectors x bar such that magnitude of x 1 is less than or equal to x 2, alright and this can be represented as follows. This is your x 1, this is your x 2 and this line represents magnitude of x 1 equals x 2 or these two lines. So, this is magnitude of x 1 equals x 2, ok.

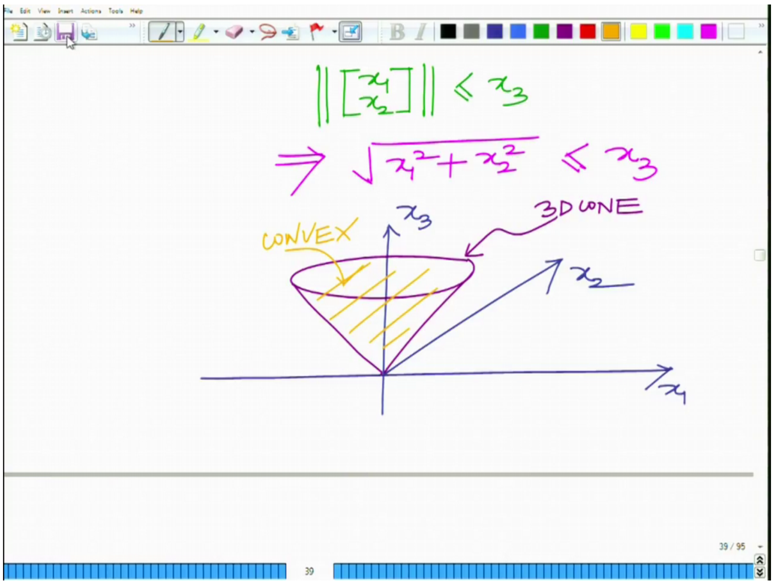
And, if you look at this region this region basically represents the region magnitude of x 1 is less than equal to x 2 and you can see this region is a cone and this is basically also convex. So, this is termed as a simply as a cone or also convex cone that is you are looking at a 2-dimensional vector. So, 2-dimensional plane in which we considering all the vectors such that magnitude of x 1 that is the first quadrant magnitude of x 1 is less than or equal to x 2, and this is a convex region, alright.

(Refer Slide Time: 02:57)



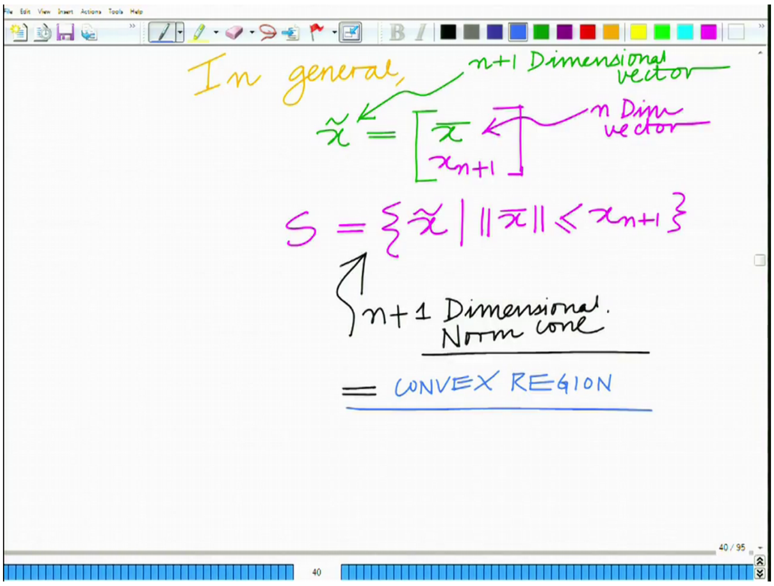
And, now we can similarly form it for and this is also termed as a norm cone,. A convex cone or a norm cone similarly you can look at this for a 3-dimensional scenario that is you have x bar equals x 1, x 2, x 3 and you consider the set of all points such that if we take the first two coordinates that is norm of x 1, x 2 less than equal to x 3, which basically implies that square root of x 1 square plus x 2 square less than equal to x 3, ok.

(Refer Slide Time: 03:53)



And, if you plot that if you plot that in 3-dimensions this is your x 1, x 2 x 3 that looks similar to the cone the shape of a classical cone that we are all familiar with this is the region, and this is the 3D cone, the classical conical shape that we are all very familiar with. And, you can clearly see that this region, if you look at the interior of this cone this is convex and it is also reasonably easy to show that the cone is a convex region.

(Refer Slide Time: 04:42)

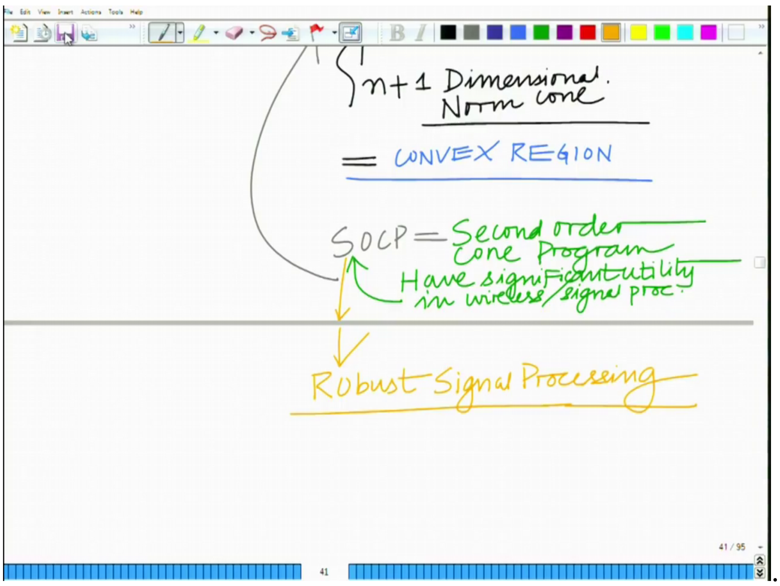


And, in general now you get the idea to generalize this we have x tilde equals let us say we have n dimensional n plus 1 dimensional vector of which we form the first n dimensional. So, this is your n plus 1 dimensional vector. This is your n plus 1 dimensional vector and this is x of n plus 1, this is x of n plus 1, ok. So, this is a n dimensional vector x bar and this is x of n plus 1 this is an additional. So, we have a x n plus 1 dimensional vector x tilde. Now, if you consider the set s of all vectors x tilde such that norm of x bar less than or equal to x x n plus 1.

Well, this represents an n plus 1 dimensional norm cone this represents a n plus 1 dimensional norm cone that is a convex region which is the relevance to us is that this is a convex region, this is a convex region and it is a fairly important class of convex regions, it is very interesting and some more sophisticated convex region and right it is slightly difficult to describe a practical application of the convex cone in the context of resistance wireless communications or signal processing at this point.

But, we will note that or I would like you to note that the convex cone in fact, has a very interesting and very prominent applications which will explore during this course it is just that it is a little difficult to setup the problem right now.

(Refer Slide Time: 07:18)

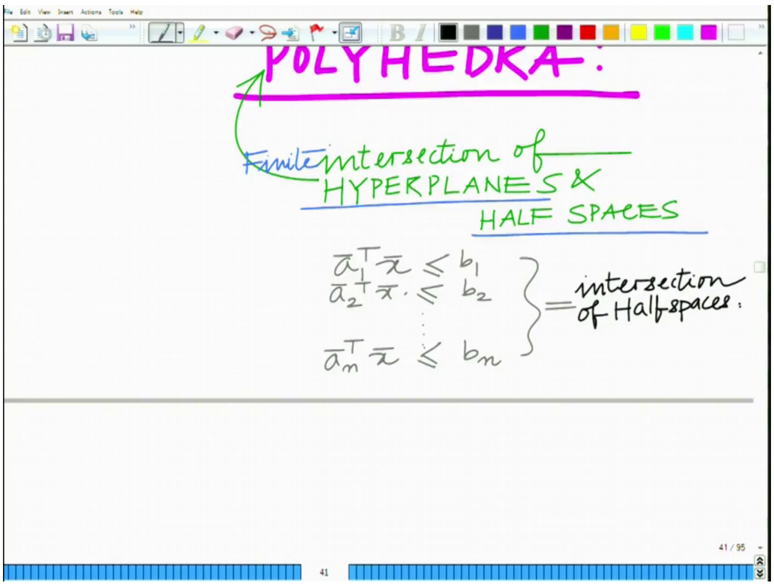


We will explore problems what are known as problems that are known as SOCP or as second order cone programs and these have significant application and utility. So, these are these are SOCP problems second order cone programs that have cone conic constraints and these have significant utility in the context of wireless communication and signal processing.

These have significant utility in wireless communication and signal processing. These are second order cone programs. Especially in the context of robust signal processing robust similar to what we have seen previously in the context of robust for instance estimation or robust signal processing. One of the most prominent applications of this SOCP paradigm is in the context of robust signal processing for instance for instance you can look at applications such as robust beam forming for the same beam forming problem, that we can look at in a multiple antenna wireless communication system.

If you make it robust as we have seen robust is making it resilient to the channel estimation errors it becomes a robust beam forming problem and all such applications can be formulated using the second order cone program SOCP will basically involves conic sets, alright and, we will look at them. In fact, we look at these kind of problems in quite some detail as we proceed through the later modules or in the later stages of this course, ok.

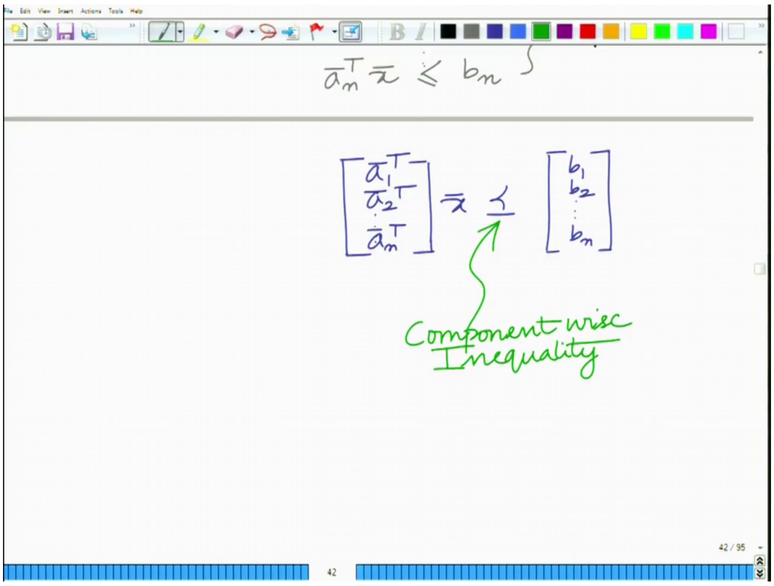
(Refer Slide Time: 09:28)



So, now let us move on to other interesting convex sets which also arrives very frequently and these are termed as polyhedral. Now, polyhedra are basically formed from the intersection of hyperplanes and half spaces. These are formed from the intersection of hyperplanes and half spaces. For instance, in fact, a finite intersection we can note that it is a certain point, but these are finite formed by finite intersection of hyperplanes and half spaces.

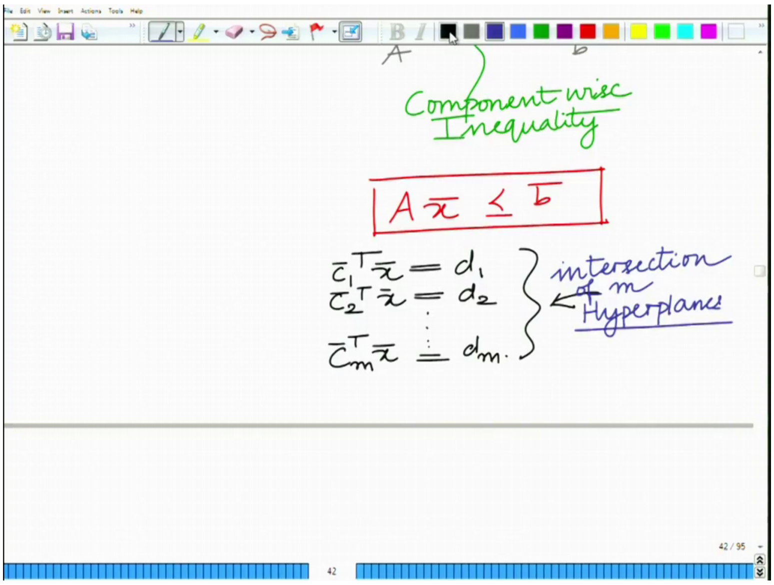
So, for instance we have seen that a half spaces can be represented as follows a 1 bar transpose x bar less than equal to b 1, a 2 bar transpose x bar less than equal to b 2, so on a n bar transpose x bar less than equal to b n. This is a collection of this is an intersection, let us put it this way this is an intersection of half spaces, correct. This is an intersection of half spaces.

(Refer Slide Time: 11:14)



I can write this as follows I can put all these concatenate this in a matrix and I can write this as a 1 bar a 2 bar transpose a n bar transpose times x bar. Let me write the right hand side b 1, b 2 up to b n and what I can use here is what is known as a component wise inequality, which means that a 1 bar transpose x bar less than b 1 that each component on the left is less than each component on the right. So, this is known as a component wise inequality. This is known as a component wise inequality it means each component that is we take two vectors vector a vector let us say u bar less than v bar. which means each component of vector u bar is less than or equal to each component of this vector v bar, ok.

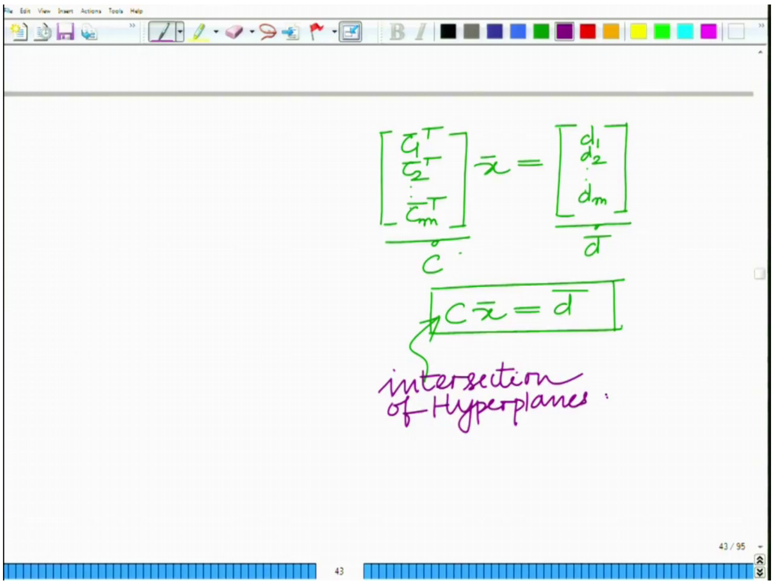
(Refer Slide Time: 12:37)



So, now denoting this by the matrix A and this by the matrix b bar I can represent this intersection of half spaces as A x bar A x bar less than or equal to b bar, ok. So, this basically represent an intersection of your intersection of half spaces.

Now, similarly remember this is an intersection of half spaces now similarly remember I can also formulate the hyperplanes as follows that is your C 1 bar transpose x bar equals d 1, C 2 bar transpose x bar equal d 2, C n bar transpose x bar equals d m, let us put this make this as m. This is a collection of intersection of remember each represents a hyperplane this is an intersection of m hyperplanes, ok. So, this is an intersection of m hyperplanes. Again, I can concatenate this system.

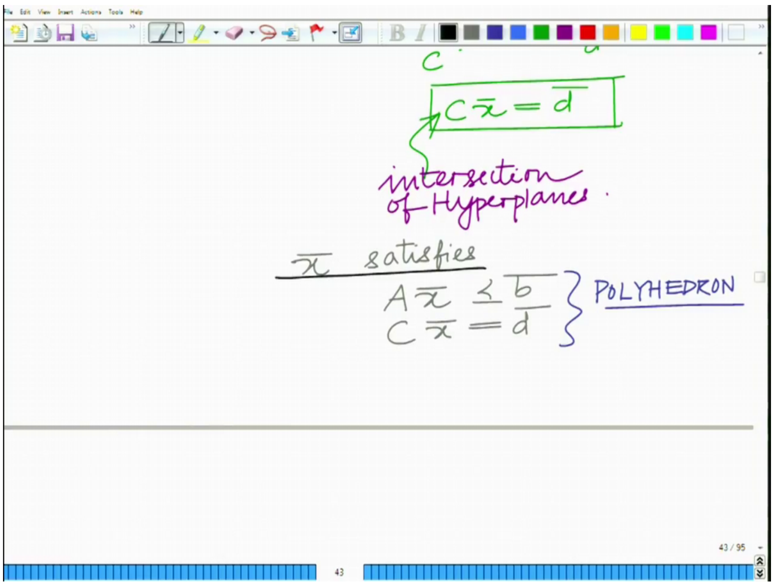
(Refer Slide Time: 14:01)



I can represent this as follows C 1 bar transpose C 2 bar transpose C m bar transpose times x bar equals b 1, b 2, up to b n. So, we have C this is a matrix C this is a matrix let me call this as d yeah, this is d 1, d 2, in fact, as mentioned above this is d 1, d 2, d m. So, we have C into x bar equals d bar this is your intersection of hyperplanes. It is a compact way of representing an intersection of hyperplanes.

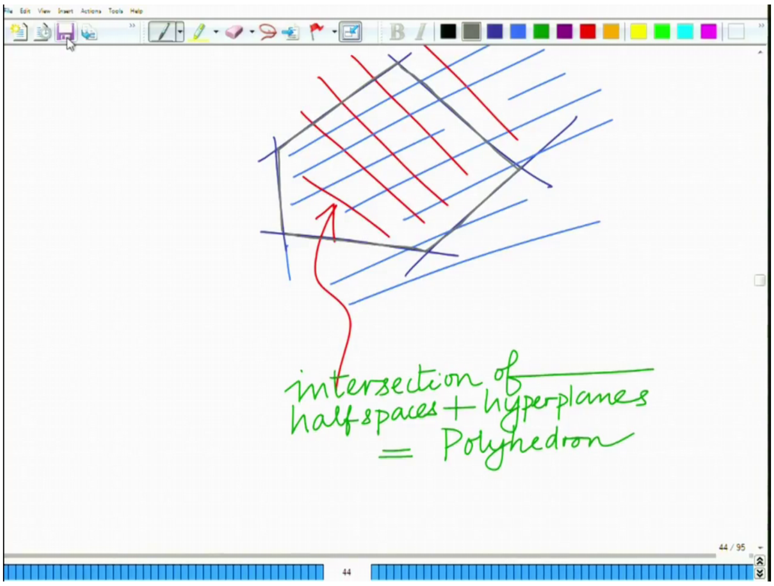
And, now if you put them together you have an intersection of hyperplanes and half spaces.

(Refer Slide Time: 15:21)



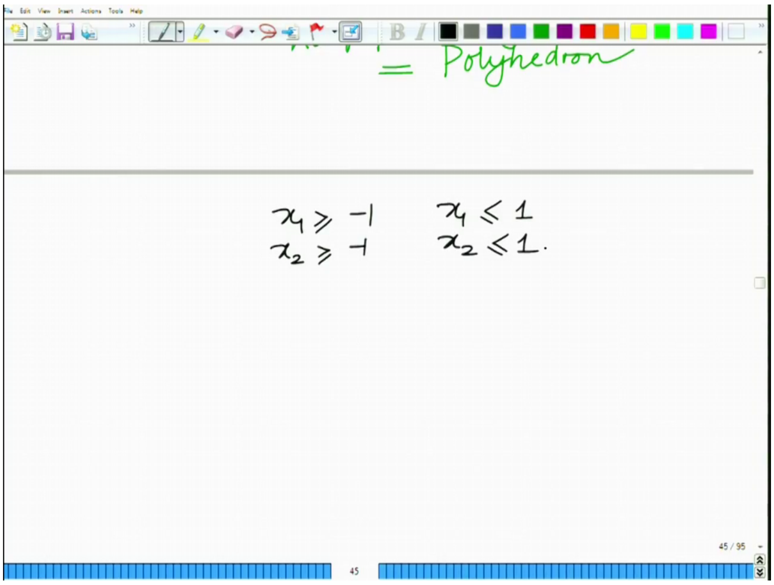
That is vector x bar that satisfies A x bar component wise inequality less than equal to b bar C x bar equals d bar this represents this region represents what you known as a polyhedral. This represents that intersection of a intersection of a finite intersection of a finite number of intersection of a finite number of hyperplanes and half spaces basically represents a polyhedron.

(Refer Slide Time: 16:00)



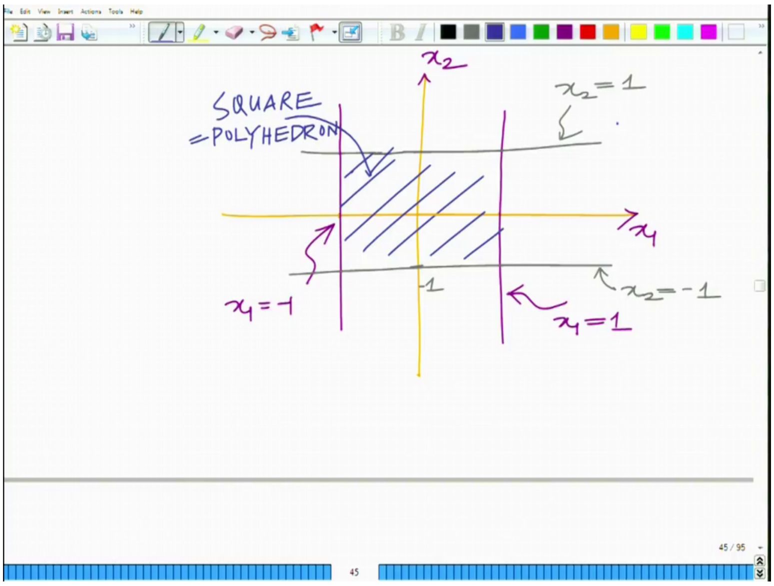
So, it can be just simply shown as follows. So, you can imagine having a large number for instance of a half spaces and for instance you can think of this as one half space corresponding to this and you can think as this as another half space corresponding to this and you can imagine the various half spaces and now, if you look at this region that lies in the intersection of all these half space. This intersection region of half spaces in fact, you can also through hyperplane into that intersection of half spaces plus hyperplanes, this is your polyhedron, ok. So, that is roughly you can see described by this region that is described by this region so, that basically forms your polyhedron, ok.

(Refer Slide Time: 17:53)



And, for instance you can take a simple example again you can look you consider polyhedron form by this four hyperplanes that is x 1 greater than equal to minus 1, x 1 less than equal to 1, again x 2 greater than equal to minus 1, x 2 less than equal to 1.

(Refer Slide Time: 18:11)

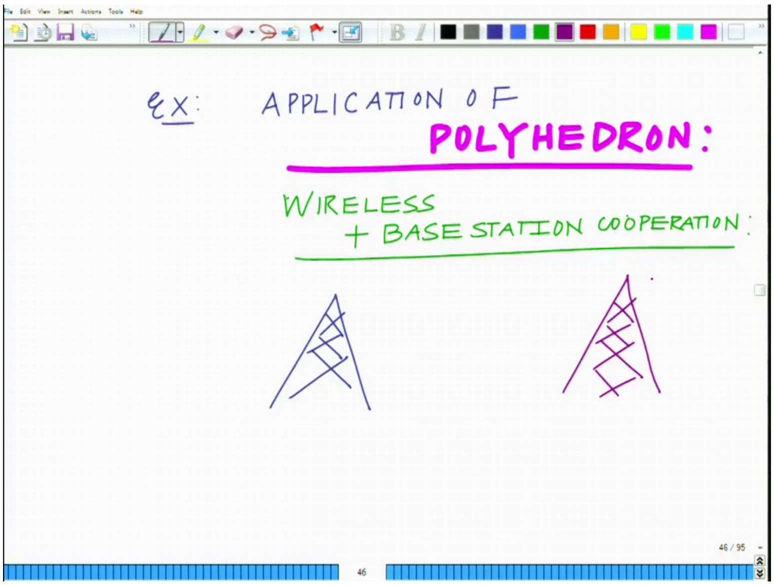


And, if you look at that region that is something that everyone will immediately recognize that is if you take the two points minus 1, 1 or x 1 equal to minus 1, x 2 equal to. So, these are the two hyperplanes these are x 1 equals 1. So, this is your x 1, this is your x 2 and this is x 1 equals minus 1. So, this region is x 1 and this is the region which is basically the strip is the region that is x 1 greater than equal to minus 1 minus 1, 1 less than equal to 1 and if you look at x 2 this is the region pertaining to x 2 equal to minus 1 and so, this is the hyperplane for x 2 equal to 1.

So, this is the hyperplane x 2 equal to 1, this is the hyperplane x 2 equals minus 1 and if you look at this square region, now you can see the square, which is formed by the intersection of these hyperplanes this square is a polyhedron which is convex that is the important thing. So, square is a polyhedron and so, worth noting especially that polyhedron is important because it is convex.

Now, in general intersection of convex sets is convex. The hyperplanes and half spaces are convex sets. So, their intersection is also convex, in particular such a region is known as a polyhedron, and it is very handy and it arrives frequently in several applications,. So, this square is a this square region is basically your polyhedron, which formed by the intersection of a finite number of half spaces and hyperplanes, ok.

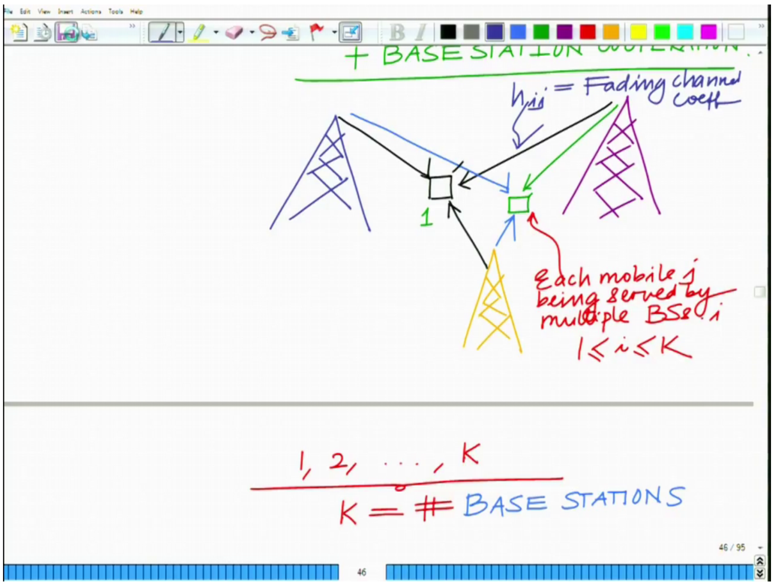
(Refer Slide Time: 20:30)



To understand this better we can start looking at an example which we might continue in the next module. So, let us look at an application of this concept of your polyhedron and this will be in the context of cooperative wireless communication that is, if you look or cooperative base station transmission or base station cooperation. So, again once again looking at a wireless plus base station cooperation also known as quality point cooperative multipoint.

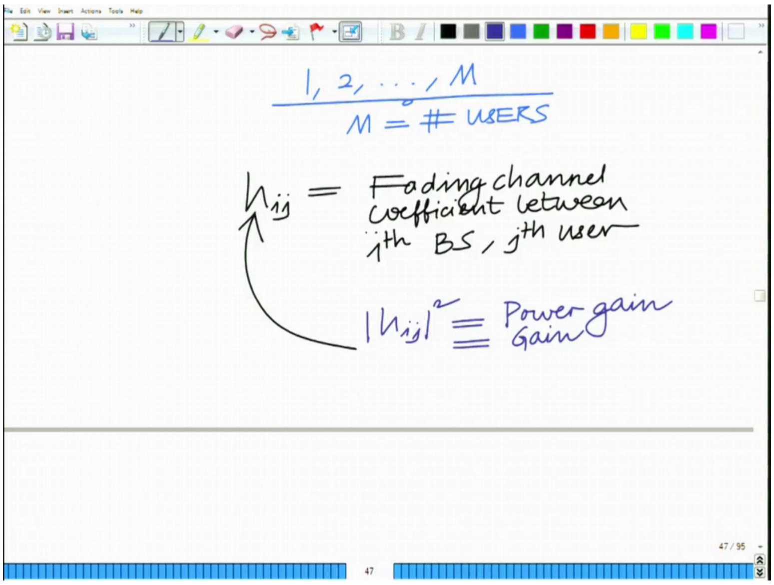
So, base station cooperation, and what happens in this scenario is we have several base stations typically in wireless cellular network what happens is conventionally a single base station transmits to a single mobile, but in some scenarios you can have several base stations cooperating to transmit to a single user or a group of users and this is especially possible. If the users are at the edge of the cell or in the region between multiple cells, where they can be simultaneously served by several base stations, ok.

(Refer Slide Time: 22:12)



So, each of these towers is let us say a base station and you have a mobile that is being simultaneously served. In fact, you have several such mobiles let us say which are simultaneously being served by the. So, each mobile is being served each user j. So, what we are saying is each mobile being served by multiple base stations i, that is, let say we have K base stations 1 less than equal to i less than equal to K, ok. So, we have base stations 1, 2 up to K, these are the base stations. Base station 1, base station 2 so, K equals the number of base stations, ok.

(Refer Slide Time: 23:46)



And, we have M total users, that is 1, 2 up to M these are the number of users these are the number of users.

Now, what we will also have is we will have now remember whenever you have a wireless transmission scenario there is a channel between the transmitter and receiver and this channel is fading channel, because the wireless channel is fading in nature. So, that is characterized by fading channel coefficient. So, you will have a fading channel coefficient h ij equals the fading coefficient fading channel coefficient and h ij is the coefficient between the i-th user. So, h ij channel between the i-th. Let us put this between the i-th this is the fading channel coefficient between the i-th base station and the j-th user, ok.

So, this is the fading channel coefficient between i-th base station comma, this is the fading channel coefficient which means if you look at h ij is the fading channel coefficient if you look at magnitude h ij square this represents the power gain this represents what is conventionally known as simply as the gain, simply this is also sometimes referred to simply as the gain. I mean there can be an amplitude gain which is given by magnitude h ij, this is the power gain magnitude h ij, h ij square which is the power gain which means if magnitude h ij square is strong then this received signal at the user j corresponding to the signal transmitted by base station i is going to be strong, but if this channel is in a deep fade, alright.

Which means, there is a lot of interference in the channel and as a result of this if magnitude h ij square is very low then the power received, alright the received power by user j corresponding to the signal transmitted by base station i is going to be very low that can be attributed to the fading, alright. So, this is the fading nexist arises this varying power level at each user corresponding to the signals transmitted by the base stations it is this varying power level arises due to the fading nature of the wireless of a typical wireless channel, alright.

So, naturally what is the power that has to be transmitted by the various base stations what is so that each user receives the desired amount of power or what is the power that has to be transmitted by the base stations corresponding to the power constraint at each base stations. So, all these are various optimization problems and these can be these are these are things that we going to look at during the course my intention here is to formulate a basic problem related to this and demonstrate the applicability of the polyhedron that is the convex set that we have just seen, alright.

So, we will stop here and continue this discussion this practical application in the context of a cooperative wireless cooperative base station transmitting cooperative base station transmission in the next module.

Thank you very much.