Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 14
Ellipsoid and its Practical Applications: Uncertainty Modeling for Channel State Information

Hello, welcome to another module in this massive open online course. So, we are looking at convex sets, let us continue our discussion by looking at another very important convex set this is the ellipse or the ellipsoid, alright.

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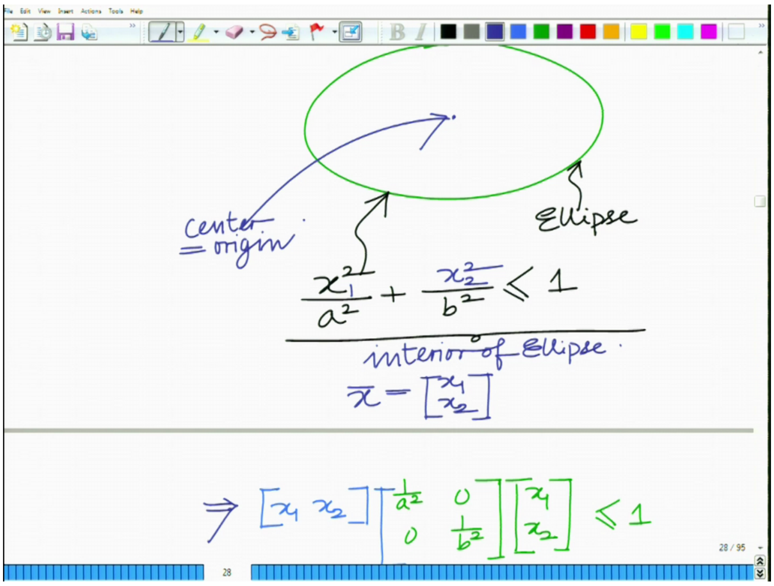

So, we want to look at the ellipse or also in n dimension also on ellipsoid or an ellipsoidal region, alright. And, an ellipse as you know from knowledge of a high school is look something like this and it is typically described by the equation. We are going to come to the general model in a little bit, but first look at a very simple equation for an ellipse described by the equation x square by a square plus y square by b square equals 1.

So, this is an ellipse. Well, this is the equation of the ellipse and the interior of the ellipse including the boundary is described by this inequality that is x square by a square plus y square by b square equals 1, this describes the interior of the ellipse.

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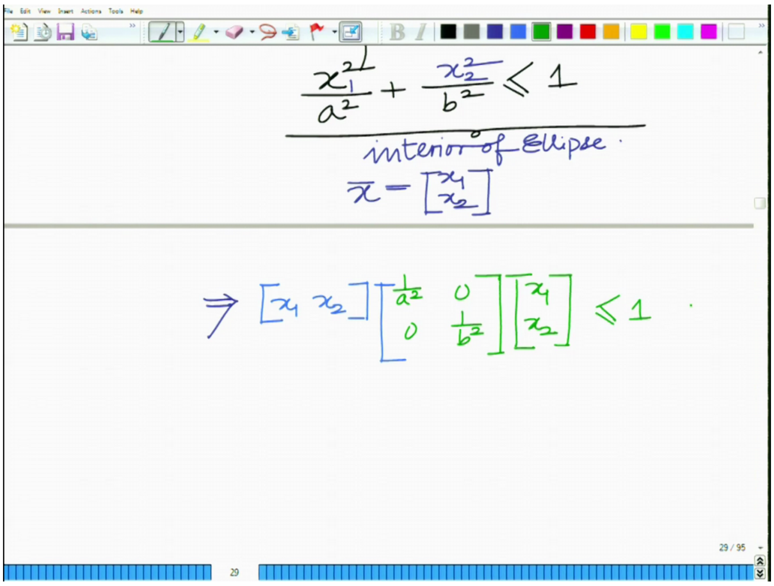

Now, this can be simplified as follows to get the general expression for an ellipse or an ellipsoidal region. I can write this as well, let us write this also or let us write this instead of y square by b square let us write this as a x 1 square by a square plus x 2 square by b square is less than or equal to 1, ok, where x 1 and x 2.

So, x 1 is denoting your conventional x coordinate and x 2 is denoting your conventional y coordinate, and now I can write this as. So, I can denote this by vector x bar equals x 1 and x 2, x 1, x 2, two components. So, this will help me generalize it to n dimensions. So, this will be x 1, x 2 times 1 over a, 0, 1 over b. In fact, let me just write one more step I can write this as 1 over a square, 1 over b square times x 1, x 2 less than or equal to 1, writing this in vector and matrix notation.

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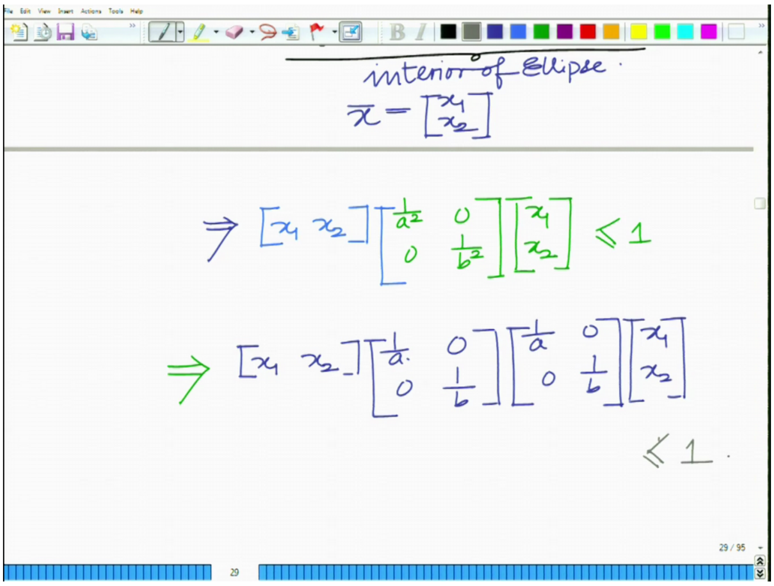

So, this implies we have x 1, x 2 times 1 over a, 0, 0, 1 over b into the matrix itself because it is a diagonal matrix it to itself will give me 1 over a square or 1 over a square and 1 over b square, 1 over a, 0, 0, 1 over b into x 1, x 2; well this is less than or equal to this is less than or equal to 1, ok. And, this you can see now this is basically nothing, but transpose of the vector x bar.

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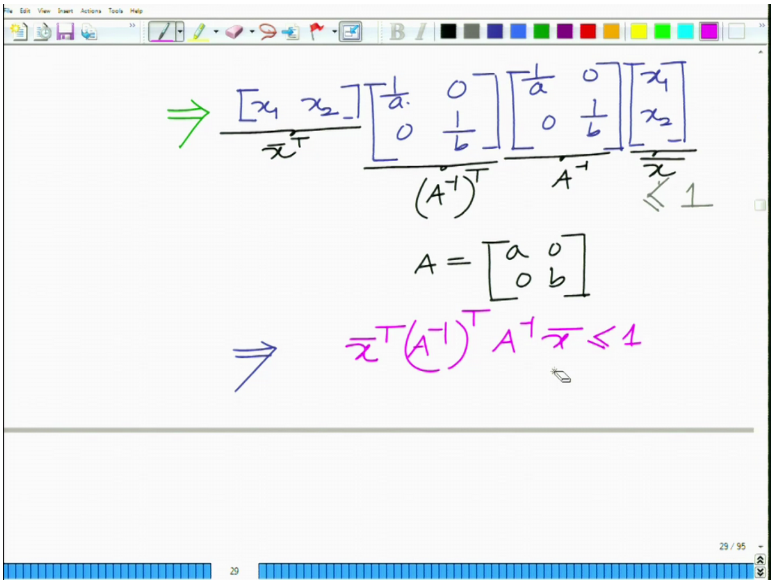

So, this is x bar transpose, this is the vector x bar and you can see if I call this matrix as A inverse, remember I can define A as the matrix diagonal matrix A small a and b on the diagonal. So, this is A inverse and I can write this as a inverse transpose because this is a diagonal matrix, the matrix is equal to its transpose. So, A inverse and A inverse transpose. So, I can simplify this now interestingly as x bar transpose A inverse transpose into A inverse into x bar less than or equal to A inverse into x bar less than or equal to 1. Remember this is our matrix A, that is a diagonal matrix with a and b on the diagonal.

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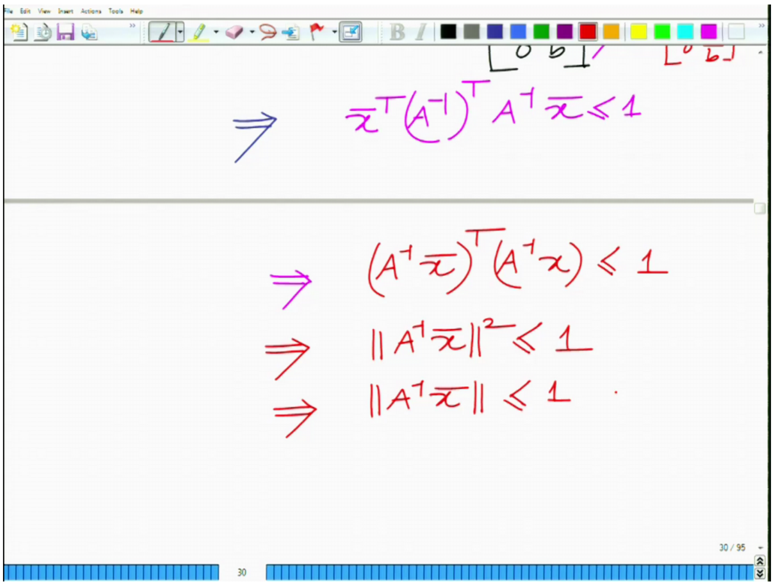

And, this implies and of course, you can see that this implies A inverse equals simply 1 over a, 0, 0, 1 over b, ok. Now, the above inequality implies now, I can write this as follows: I can write this as A inverse x bar transpose into A inverse x bar less than or equal to 1 and now, you can clearly see the vector transpose itself is nothing, but the norm of the vectors vector space, that is, if u bar is a vector we have already seen that u bar transpose u bar is basically norm u bar square.

So, I can write this now very interestingly as norm A inverse x bar square less than or equal to 1 that implies A inverse x bar is norm of A inverse x bar is less than or equal to less than or equal to 1 and this is the equation of ellipse equation of ellipse above.

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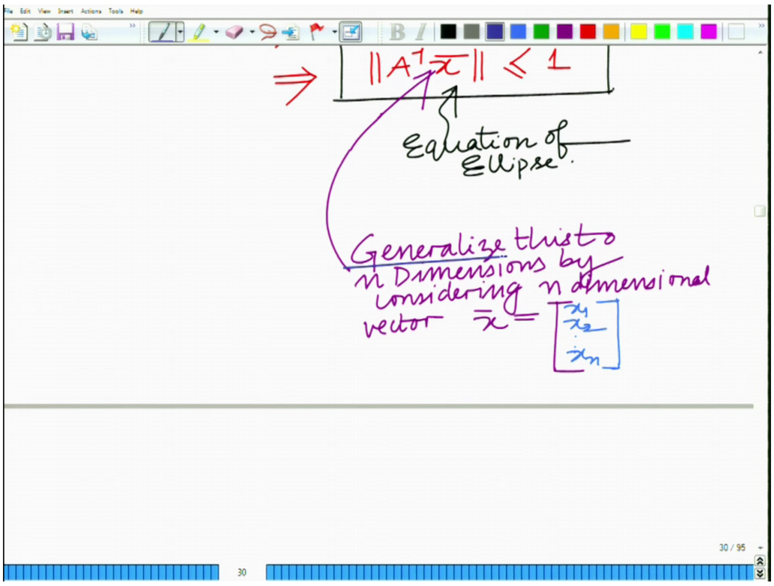

And, now you can generalize as you know ellipsoid by considering in n dimensional vector so, alright. So, you can generalize this n dimensions by considering by considering n n dimensional vector x bar. So, as you consider instead of x 1, x 2 if you consider a n dimensional vector x 1, x 2 up to x n norm A inverse x bar less than or equal to norm A inverse x bar less than or equal to 1, this becomes an ellipsoid, an n dimensional ellipsoid, ok. Generalize this to n dimensions, so, that becomes an ellipsoid, in n dimensions.

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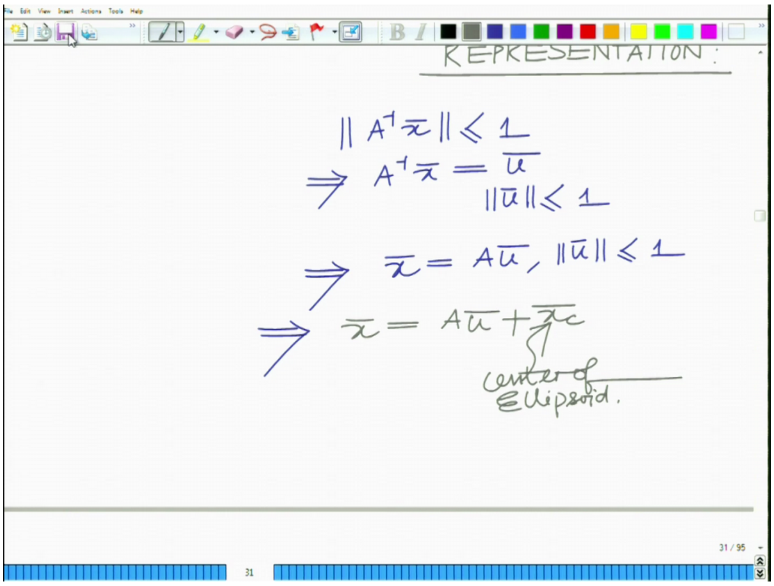

And, now the alternative representation of an ellipse or an ellipsoid now the alternative representation similar to that of a norm ball can be derived as follows. Well, we have norm A inverse x bar is less than or equal to 1 implies I can set A inverse x bar as a vector u bar with norm u bar less than or equal to 1 that implies x bar equals A times u bar with norm u bar less than or equal to 1. Now, this is for centre as origin ellipse with remember this equation here we have started with this is a centre has centre is origin.

Now, if centre is not the origin then I can simply modify this to include the appropriate centre as x bar equals a times u bar plus x bar say. So, this is the centre of the ellipse or the ellipsoid this is the centre of the ellipsoidal region, ok.

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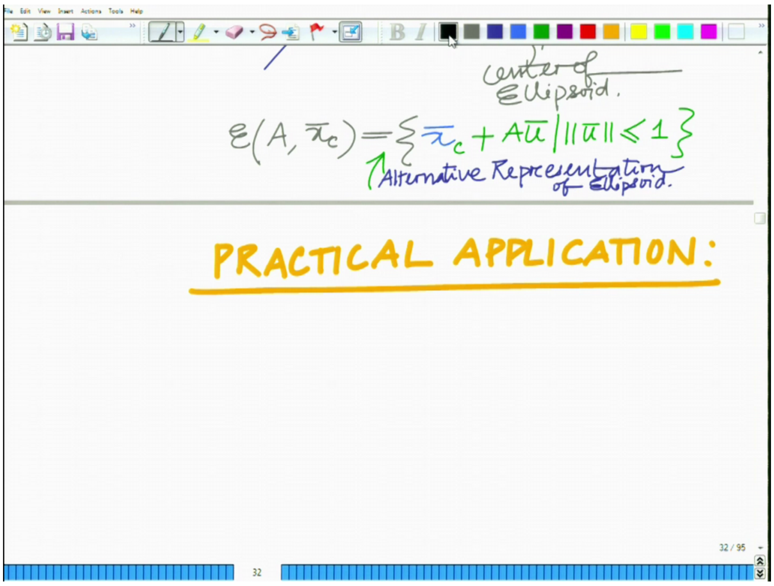

And therefore, the ellipsoid can now be represented as the ellipsoid with the corresponding to matrix A and centre x c bar is the set of all vectors x bar x c bar plus a u bar such that norm of u bar is less than or equal to 1, this is the alternative representation of the ellipsoidal region. This is alternative representation of the ellipsoidal region, alright ellipsoidal region corresponding to a matrix A and the centre x bar c, ok.

Similar to the previous cases let us look at a practical application of this. So, another interesting one of the aspects of this course is also look at is to also look at a practical applications of these concepts, is to also look at a practical application.

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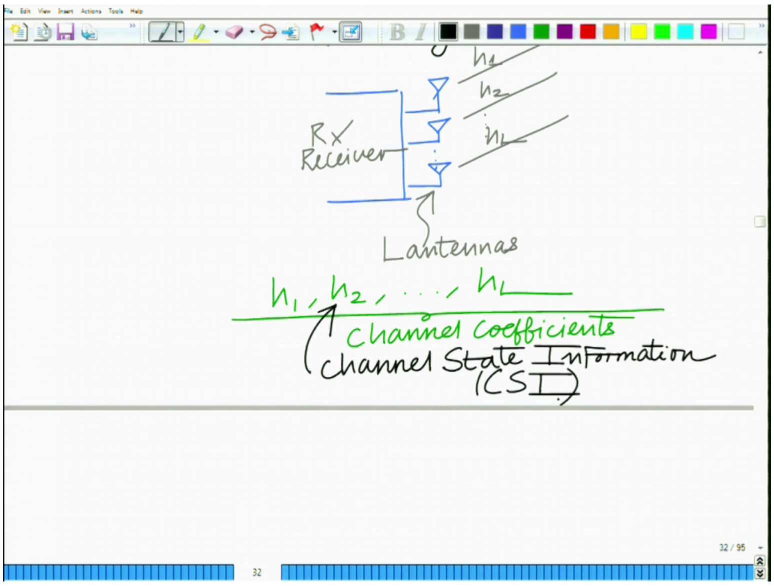

Again, we will look at a multi antenna wireless system. Let us consider a multi antenna wireless system again similar to what we have seen before. Remember, multi antenna wireless system basically has multiple antennas to over improved performance of such system. So, I have multiple antennas and corresponding to this multiple antennas I have multiple channel coefficients h 1, h 2 to h L. So, these are the L antennas. So, these are let us say L antennas. So, this is your receiver, in the wireless communication system we have the L antennas. So, h 1, h 2, h L are the channel coefficients.

Now, these channel coefficients also in wireless communication systems the knowledge of these channel coefficient, this is also termed as channel state information alright. So, the channel coefficient characterize the channel state and knowledge of this channel that is knowing this channel coefficients, having the values of these channel coefficients is also termed as channel state information in the wireless communication system. So, the knowledge of these channel coefficients this is also termed as this is a frequent term, this is termed as channel state information, ok. Knowledge of these channel coefficients is termed as channel state information now this knowledge is important.

Now, to develop enhanced signal processing scheme we need knowledge of this channel coefficients or we need the channel state information at the receiver to develop improved or to basically develop schemes that yield improved performance after signal processing at the receiver, ok.

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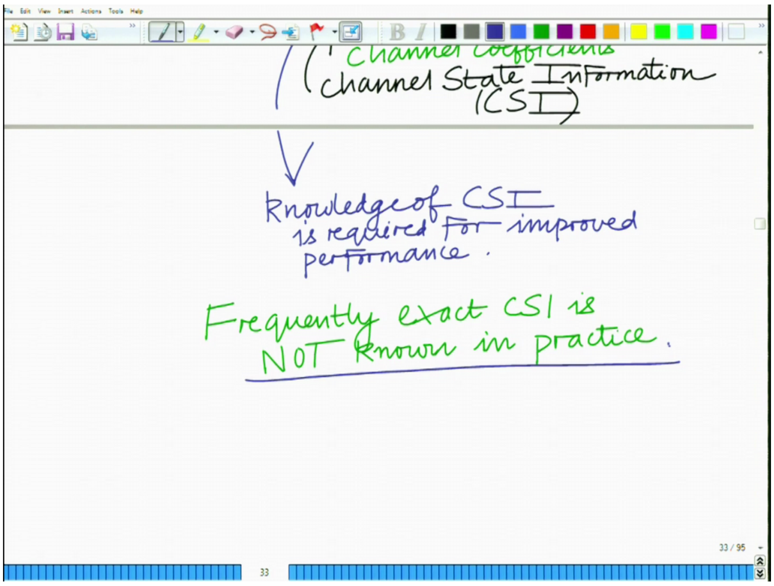

So, this knowledge of CSI knowledge of CSI is required for accurate performance improved performance; however, frequently the exact. So, frequently the exact channel state information that is frequently the exact channel state information is not known in, this is not known in practice. Now, what is known because remember these channel state coefficients have to be estimated and whenever you estimate them there is going to be an estimation error.

So, only approximate channel values of the channel state channel state information or approximate values of these channel coefficients are known, that is, the corresponding to the approximate values or the estimates of these channel coefficients are frequently known in practice, ok.

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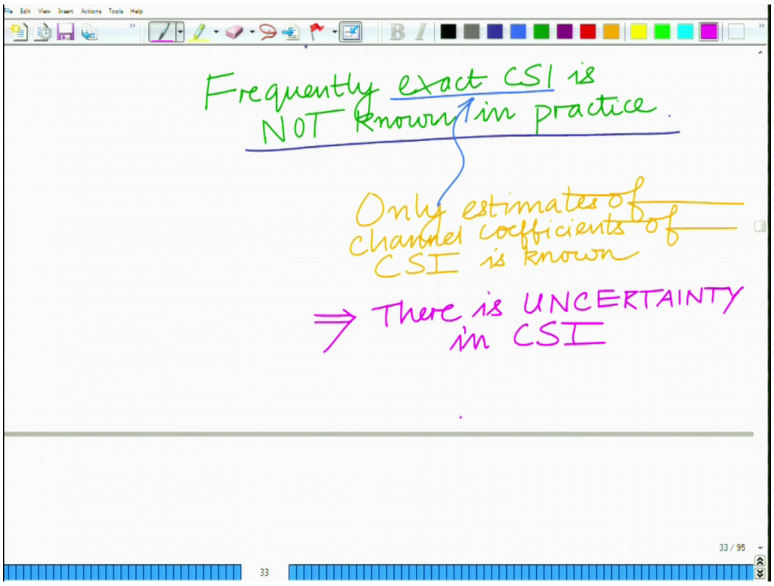

So, only the estimates exact CSI not known only estimates only the estimates or basically, you can think of these also approximate values only estimates of channel coefficients or CSI or CSI is known. This implies that there is uncertainty in the CSI, implies this is termed as uncertainty CSI uncertainty this is uncertainty in the CSI arising from the estimation errors. There is uncertainty in the channel state information.

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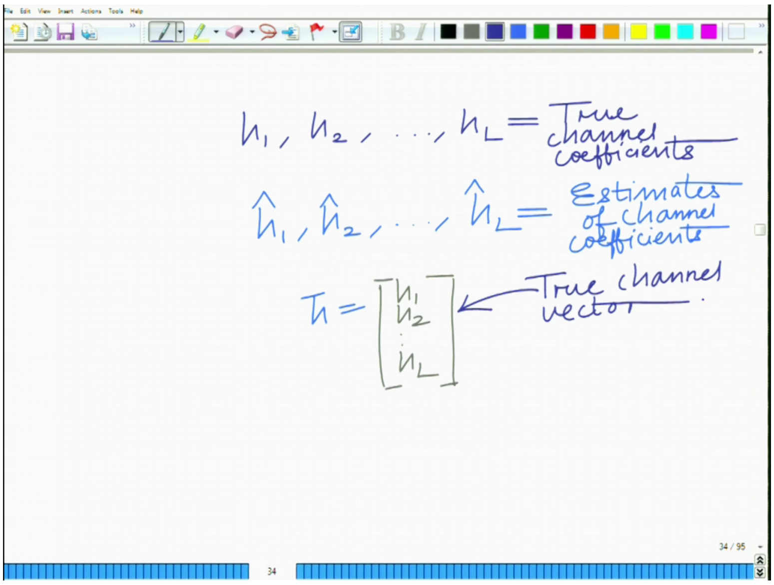

So, we have this estimate. So, we have these true channel coefficients. The true underlying channel coefficients, these are not known and what are known are there estimates that are denoted by this hats h 1 hat, h 2 hat, up to h L hat. These are the these are the estimates these are the estimates of the channel coefficients, and therefore, we have our true channel vector h bar this is h 1 hat h 2 hat up to h L, and this is your true channel vector true channel vector meaning the actual channel vector in the wireless system.

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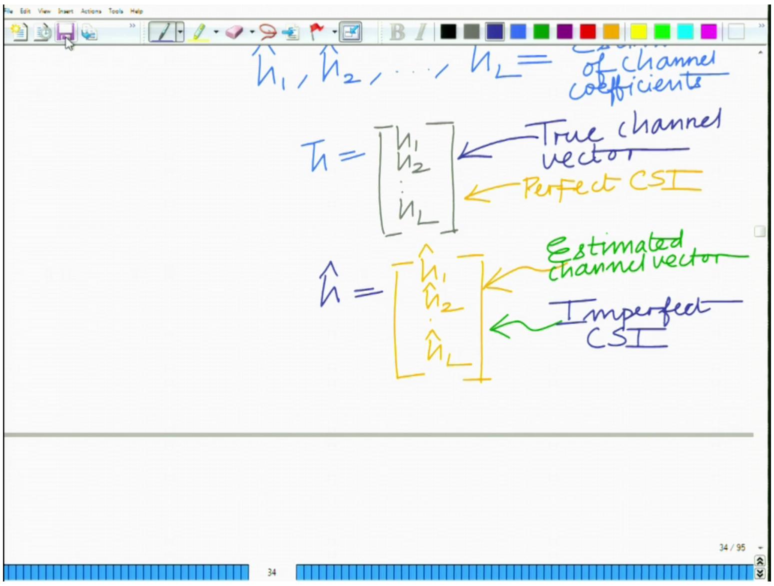

And, you have the estimated channel vector h hat which is equal to comprises of the estimates and this is the true CSI or what is also known as perfect CSI. This is the estimated channel coefficient vector which is also known as this is the estimated channel vector. This is also termed as the imperfect CSI, ok. This is all the, this is known as imperfect CSI. Now, we know that this imperfect CSI is close to the actual CSI that is h hat the estimate is close to h bar, but it is not exactly equal to h bar, ok.

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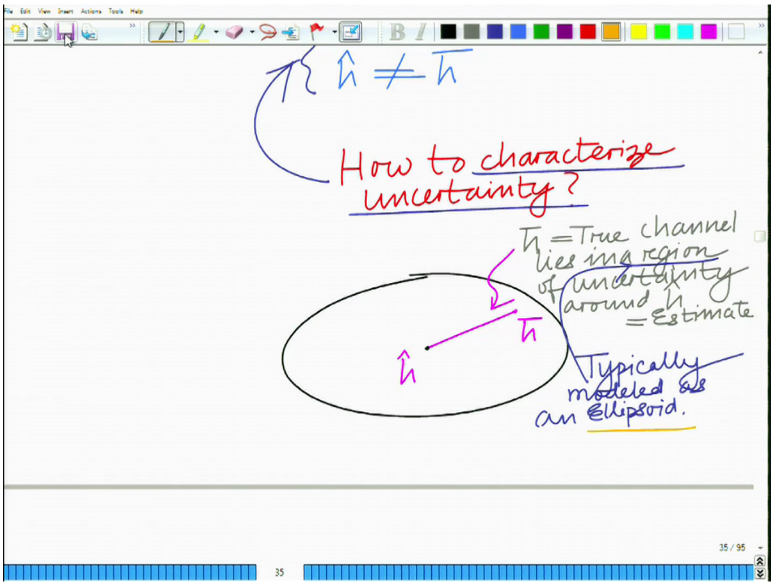

So, h hat we know is approximately equal to h bar, but h hat is not exactly equal to h bar. And, this is an important this is an important consideration in practice because in practice the perfect channel state information is very difficult I mean estimating the underlying channel state coefficients without that is with 100 percent accuracy without any estimation error is impossible, alright. So, in all practical scenarios the channel state information or the channel coefficients are only approximately known, alright.

So, one has to characterize this phenomena this phenomenon of uncertainty in this in the CSI has to be suitably characterize to design signal processing schemes that take into account this uncertainty into CSI and yield improved performance, ok. So, we have to have. So, h hat h hat h h hat is close to h bar, but h hat is not equal to h bar. So, how to characterize this uncertainty? How do we characterize the important question now is how to characterize now how to characterize this uncertainty?

And, therefore, what one can say is that h hat this estimate lies close to h bar or h bar the true channel lies close to the estimate h hat, we can say that h bar h bar lies in a region of uncertainty around h hat and this region is frequently modeled as an ellipsoid, ok. So, what we have and this where the application of the ellipsoid comes in we say that if you consider an ellipsoidal region with the known estimate as the centre then h bar the true channel lies in a region uncertainty regions.

So, h bar equals the true channel it lies in a region of uncertainty around h hat which is the estimate and this uncertainty region typically modeled. So, this is an uncertainty region, this is typically modeled as an ellipsoid in n dimensions this is typically modeled as an. This uncertainty region is typically modeled as an ellipsoid. So, we say that a true channel vector true channel lies in an ellipsoid lies somewhere in an ellipsoid around h hat.

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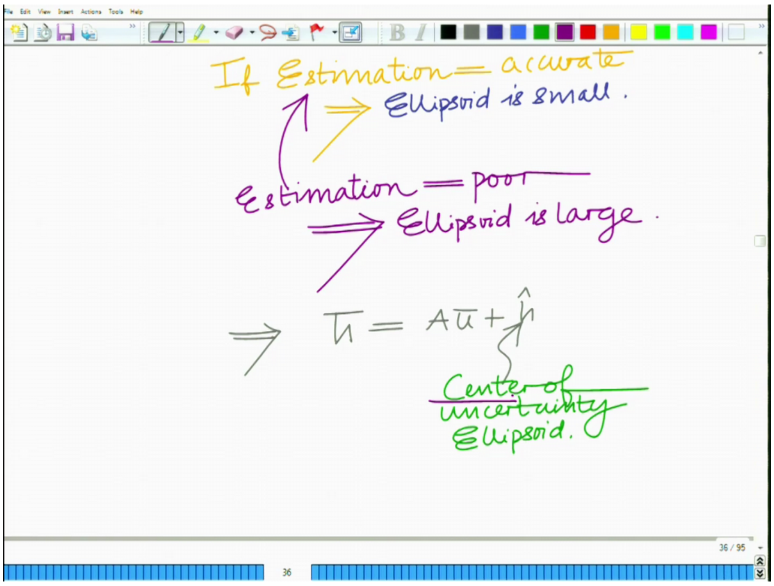

Now, obviously, if the ellipsoid is large; that means, the uncertainty region is large, which means the estimation error is high, alright. Now, if the estimation error is low that is a estimation process is very good then the ellipsoid will suitably small; that is you can localize h bar to a much smaller region around h hat. So, if estimation is accurate that imply that implies ellipsoid that is the size of ellipse is small, ok.

On other hand, inaccurate estimation or poor estimation, the estimation poor implies ellipsoid is large. So, one can characterize the ellipsoid based on the estimation process also, because estimation process what results in the estimation errors. If the estimation errors is large then the uncertainty will be large so, the size of the ellipsoid will be large, that is, there is a lot of uncertainty in where h bar can lie. If the estimation is of good quality then naturally h bar will be close to h hat. So, the size of the ellipsoid will be much smaller.

And, asymptotically you can see when the estimation error become 0, h bar the true channel coincides with h hat, that is, for a large number of pilot symbols that is when the SNR; SNR of estimation tends towards infinity alright. And therefore, now you have an interesting model to characterize the true channel vector I can represent h bar as A times u bar plus remember this ellipsoid has centre e which is nothing, but it h hat. So, this forms your centre of the ellipsoid; h hat is centre of the uncertainty ellipsoid. So, h form. So, h hat is nothing, but the centre of the uncertainty centre of the uncertainty ellipsoid, ok.

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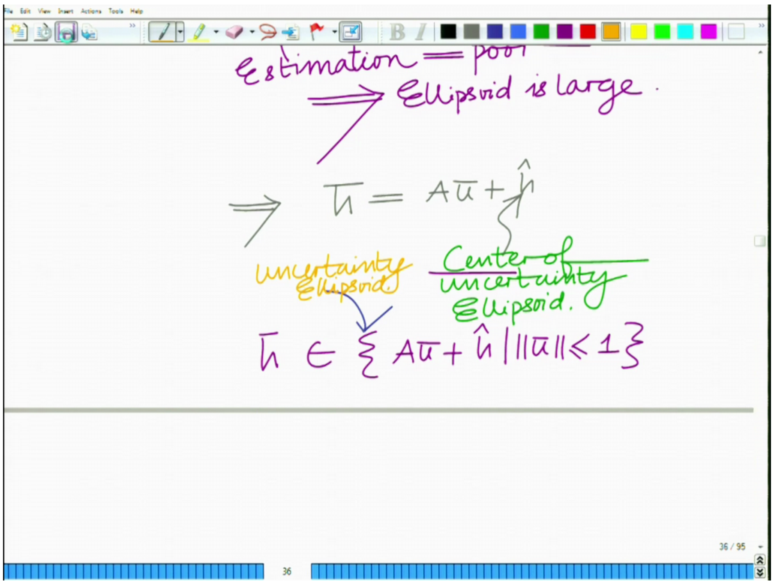

And, therefore, h bar belongs to this uncertainty ellipsoid which is given as A u bar plus h hat such that norm u bar is less than or equal to 1, and this is termed as I already said this is termed as on the uncertainty ellipsoid. This is termed as for this practical scenario this is termed as the uncertainty ellipsoid.

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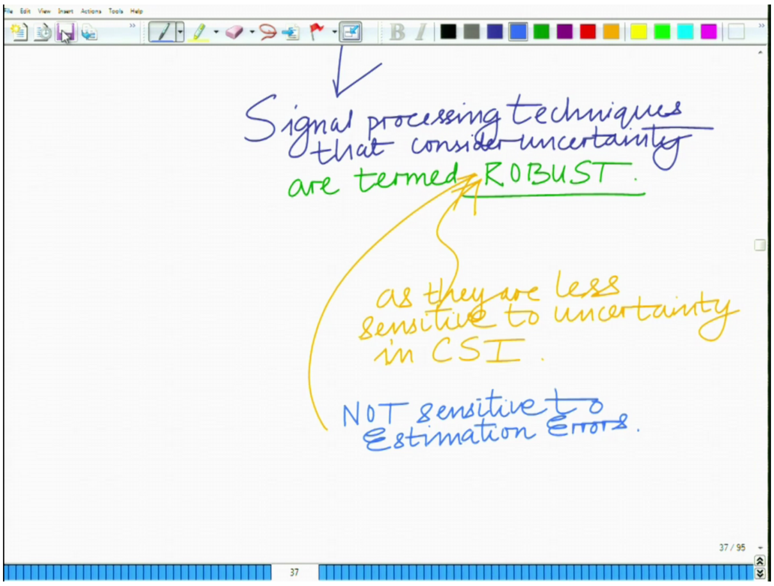

This is termed as uncertainty ellipsoid and now, signal processing techniques that concerned this that consider this uncertainty. Signal processing ellipsoid that consider uncertainty are termed as robust. These are termed as a robust since they are not sensitive to the uncertainty in the channel state information as they are not sensitive or less sensitive, as they are less sensitive to uncertainty in the channel state information, or they are not sensitive to errors; not sensitive to estimation errors as they are not sensitive to estimation errors, alright.

So, an interesting application of this ellipsoid or ellipsoidal region in wireless communication or for that matter signal processing and various other applications is the following. Several quintiles have to be estimated such as, the channel coefficients or even a signal processing alright and underlying filter has to be estimated, alright. So, the true coefficients we do not know where that what the true coefficients are, but we know that they lie close to the estimated values. So, these can be considered to lie in an ellipsoid is region around their corresponding around the respective estimated values alright and that ellipsoidal region is known as the uncertainty ellipsoid in the context of wireless communication this arises because there is uncertainty in the CSI channel state information or the channel coefficients, alright.

So, let us stop here and consider other aspects in the subsequent modules.

Thank you very much.