Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture - 10
Matrix Inversion Lemma (Woodbury identity)

Hello welcome to another module in this massive open online course. So, we are looking at the mathematical preliminaries and the examples for the various mathematical preliminaries. Let us continue our discussion and look at another important principle that comes in handy several times this is known as the Matrix Inversion Lemma or the Matrix Inversion Identity.

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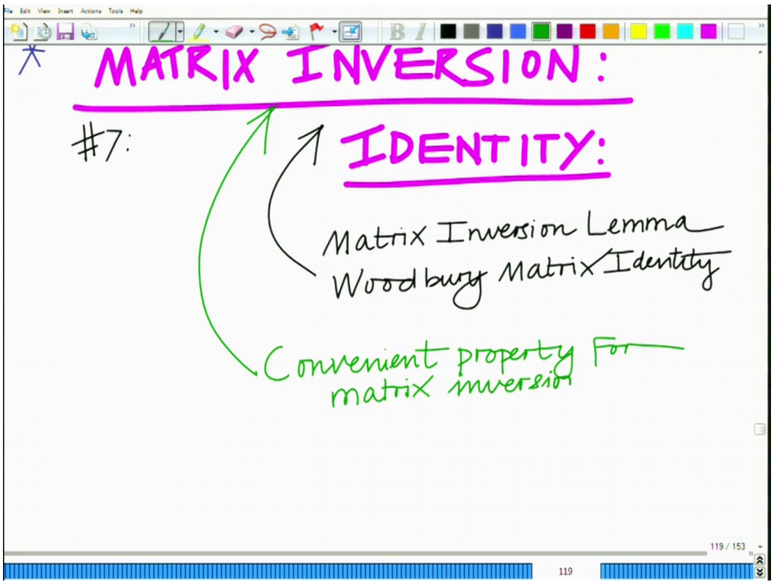

So, what we want to look at is the matrix in this module of linear algebra and matrix preliminaries for optimization. So, we want to look at the matrix inversion identity and this is also often termed as the matrix inversion. So, there are many names for this, this is also termed as the matrix inversion lemma, is also termed as the Woodbury matrix identity; also popularly known sometime as the Woodbury matrix identity. This is our example number 7 and what it is? It is a very convenient principle for the inversions a convenient property for matrix inversions or convenient you can also say trick to compute the inverse of a matrix; convenient property for matrix inversion or to compute the inverse of a matrix. What it states is that if I have a matrix inverse of the form A plus UCV inverse.

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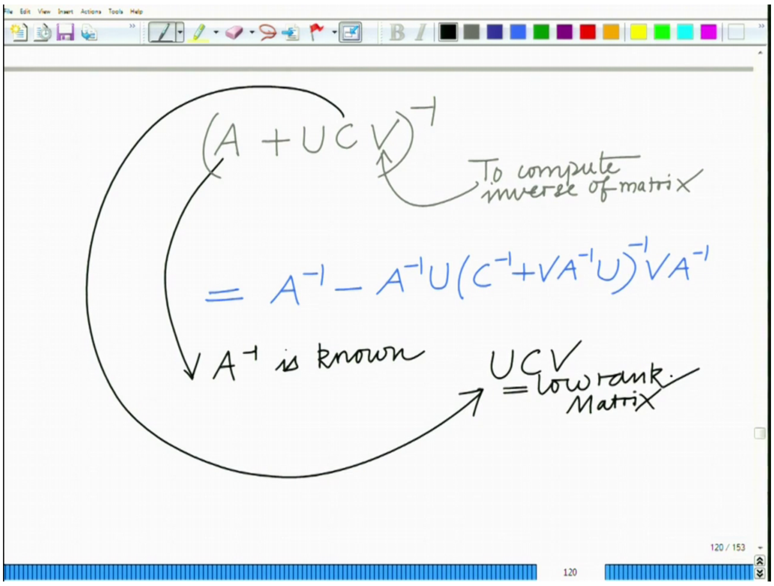

So, I want to compute the inverse of this matrix to compute the inverse of this matrix of the matrix A plus UCV, this inverse is given as follows, this inverse is given as follows that will be equal to A inverse minus A inverse U times C inverse plus VA inverse U inverse into VA inverse and this is the matrix inverse and this is especially convenient if the inverse of A is already known; let us say A is a large matrix, for which the inverse is already known or can be computed rather easily and this quantity UCV is a low rank matrix ok.

So, this is very handy if A inverse is known for instance this requires A inverse you can see this and UCV equals a low rank that is it is not a full rank matrix, although it can be used even when it is a full rank matrix its handy when it is a low rank matrix.

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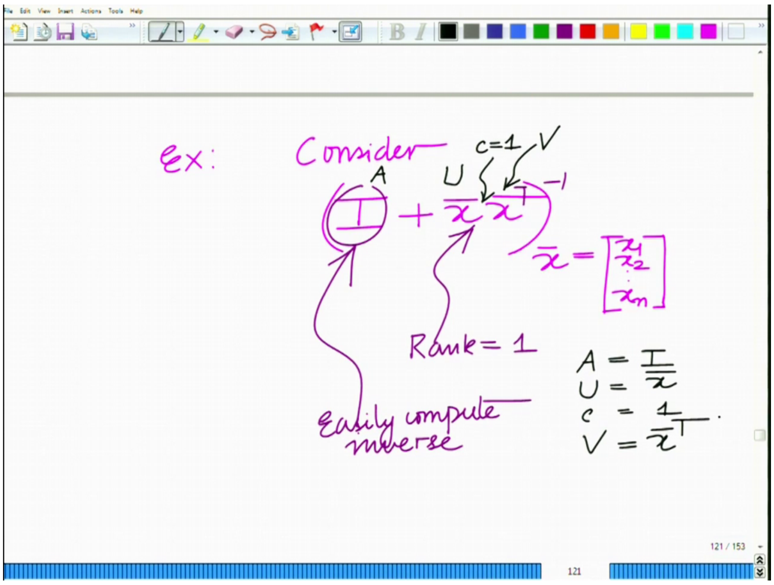

For instance to understand this better let us look at a simple example or let us call this an illustration. Consider, computing the inverse of I plus x bar x bar transpose where and where x bar you can see is a vector; this is the vector x 1 x 2 up to x n and this x bar x bar transpose, I am sorry this x bar x bar x bar x bar transpose inverse. So, we want to compute the inverse of this and you can see this x bar x bar transpose has rank equal to 1; this is a rank 1 matrix that is we compute x bar x bar transpose you will realize that x bar x bar transpose is the rank 1 matrix or basically a rank deficient matrix and if you look at this I, this is a matrix for which you can easily compute the inverse; I inverse is nothing but I.

So, this is what we mean this is its sort of a very illustrative case where this matrix inversion identity can be used and in fact, if you look at or compare or this thing this is our A or x bar is U. Now there is no c, so c is basically the constant its simply 1 and V is this is V equals x bar transpose. So, in our matrix inversion lemma we have A equals I, U equals x bar, c equals 1 and V equals x bar and v equals x bar transpose ok. So, that is what that is a property that we can use and now with this settings we can use this property of the matrix inversion lemma and this can be done as follows.

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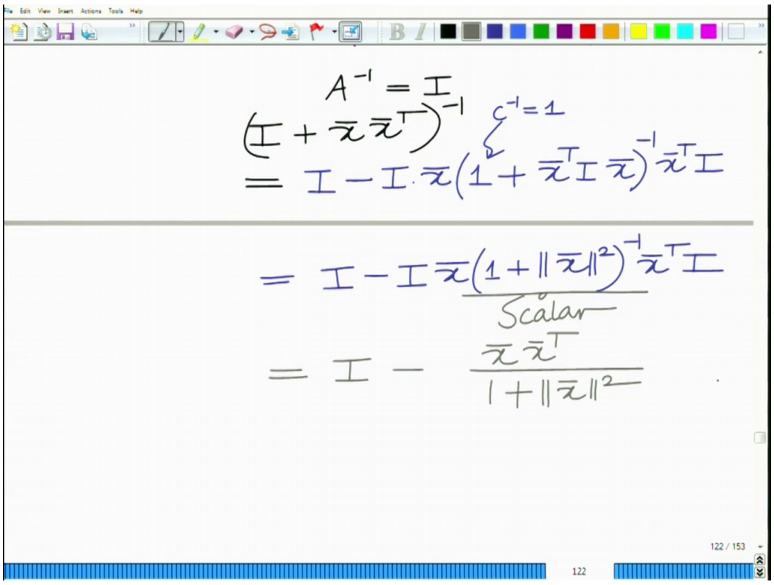

Now, note that A inverse equals identity A is identity, so inverse is identity. So, I plus x bar x bar transpose inverse equals well this equals A inverse, which is I minus, this is A inverse once again I into U that is x bar times c inverse c equals 1. So, c inverse is c inverse equals 1, so c inverse plus V x bar transpose A inverse, which is again identity times U, which is x bar inverse c inverse plus V inverse U inverse into VA inverse; V is again x bar transpose and A inverse is identity, which is equal to now you can see this is I minus I times x bar; this is 1 plus you can see x bar transpose x bar into identity into x bar is simply x bar transpose into x bar this is norm x bar square inverse into x bar transpose into identity.

And what you can see here is that this quantity 1 plus x bar square this is a scalar quantity, this is simply a number because remember norm x bar is a number norm of vector that is length of the vector x bar norm x bar square is also a number, all right.

So, 1 plus norm x bar square is a number. So, inverse of a number is simply the reciprocal that is 1 over that number ok. So, I can simply write this now once you realize that I can simply write this and of course, these are simply identity matrix. So, I can simply write this as x bar x bar transpose divided by 1 plus norm x bar square and this is basically the expression for the inverse of you can readily compute this. So, this is basically your expression for the inverse of I plus x bar x bar transpose.

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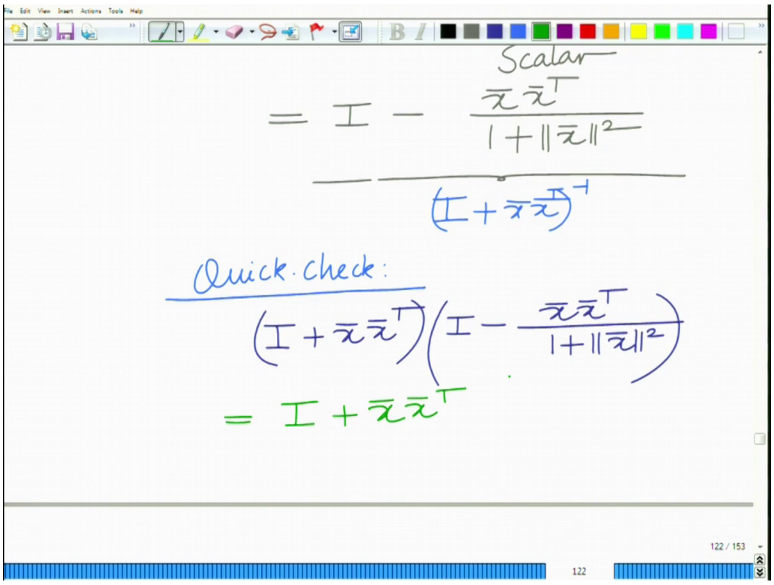

So, the simple trick can be readily used to compute the inverse of such matrices and you can just do a quick check for instance, you can do a quick check to verify that this is indeed the inverse. You can check I plus x bar x bar transpose into its purported or claimed inverse that is x bar x bar transpose divided by 1 plus norm x bar square well this gives I times I plus x bar x bar transpose into identity that is simply x bar x bar transpose minus I into x bar x bar transpose by norm 1 plus by 1 plus norm x bar square, so minus x bar x bar transpose divided by 1 plus norm x bar square minus x bar x bar transpose into x bar into x bar transpose divided by 1 plus norm x bar square.

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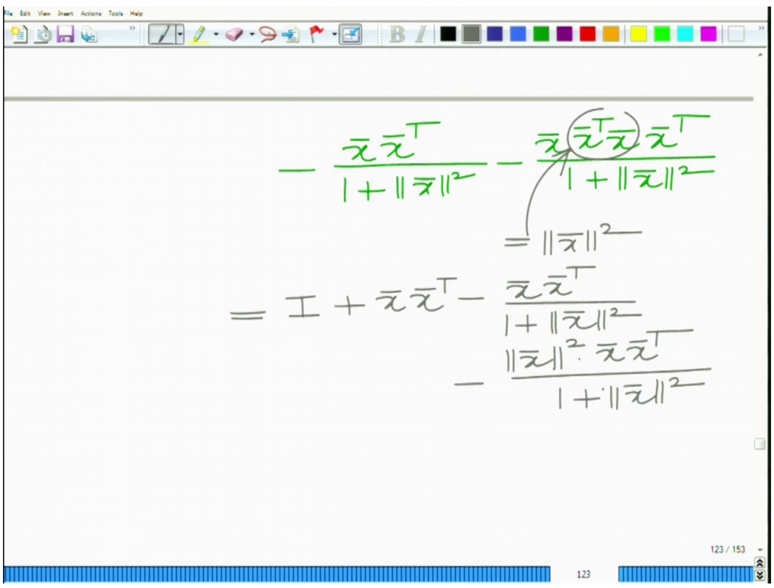

Now, if you look at this quantity x bar transpose x bar that is equal to norm of x bar square. So, this is therefore, equal to I plus x bar x bar transpose minus x bar x bar transpose by 1 plus norm x bar square minus x bar x bar transpose x bar transpose x bar is norm x bar square it is a scalar which comes out norm x bar square times x bar x bar transpose by 1 plus norm x bar square now if you look at these 2 terms you have x bar x bar transpose 1 plus norm x bar square and norm x bar square into x bar x bar transpose by 1 plus norm x bar square.

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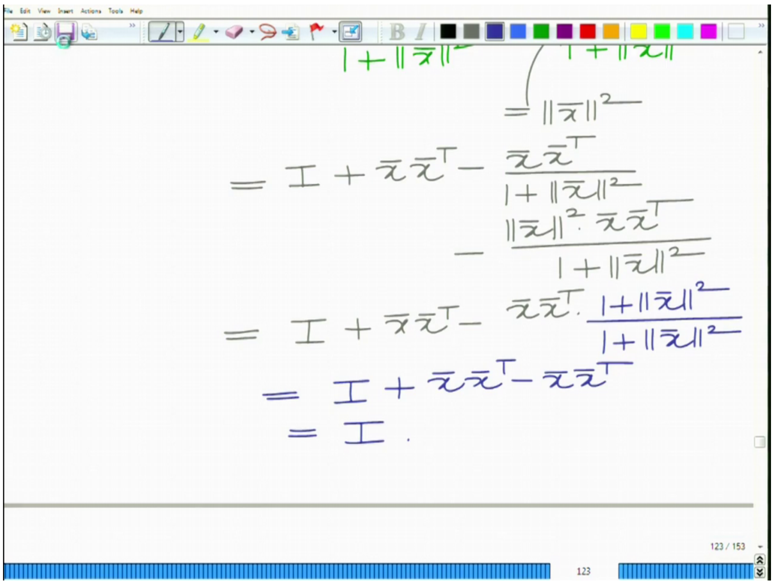

So, this is simply I plus x bar x bar transpose minus x bar x bar transpose times 1 plus norm x bar square correct 1 plus norm x bar square divided by 1 plus norm x bar square, which is equal to I plus x bar x bar transpose minus x bar x bar transpose which is indeed equal to identity and therefore what we have checked is that I plus x bar x bar transpose inverse is indeed I minus x bar x bar transpose by 1 plus norm of x bar square.

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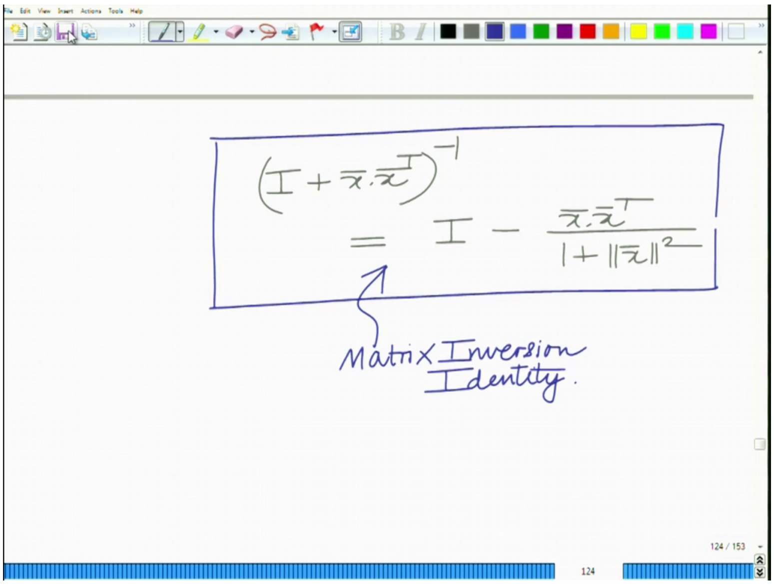

So, we have used this handy property that is the matrix we have demonstrated this using the matrix inversion identity or the Woodberry matrix inversion or the Woodberry matrix inversion lemma all right. So, basically that completes the example.

Thank you very much.