Principles of Signals and Systems
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Lecture – 71
IIR Filter Structures: Direct Form – I, Direct Form – II

Hello, welcome to another module in this Massive Open Online Course all right. So, in this module let us start looking at the implementation of IIR filters using various IIR Filter Structures all right. So, we are going to outline the structure of these various IIR filters ok.

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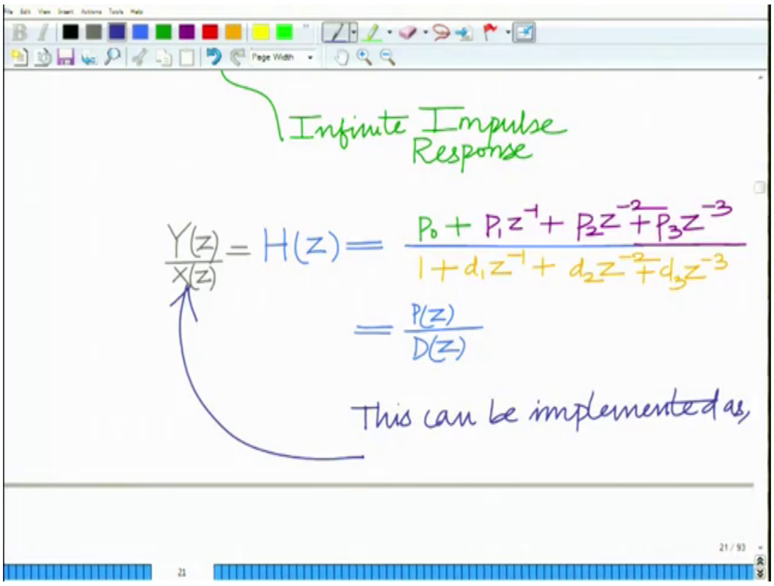

And so, the title is IIR Filter Structures which plays an important role in the implementation of digital filters and IIR as you know this stands for Infinite Impulse Response. To illustrate this consider the third order filter, given as follows. So, we have consider the third order filter given by the transfer function H z equals P naught plus P1 z inverse plus P 2 z minus 2 plus P 3 z minus 3 divided by 1 plus d1 z inverse plus d 2 z minus 2 plus d 3 z raised to minus 3, which I can write as P z over D z and remember this is the output; z transform Y z divided by X of z and this can be implemented as, this can be implemented as follows.

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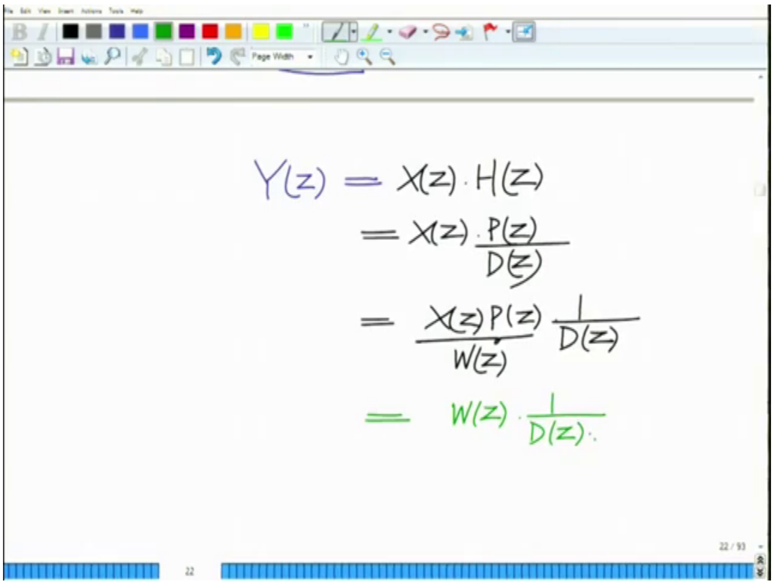

I can write this Y z equals X z into X z which is X z into P z divided by D z and this is X z into P z into 1 divided by D z. So, I can call this implement this as a cascade of 2 systems; I can call this X z into P z this can be defined as W z.

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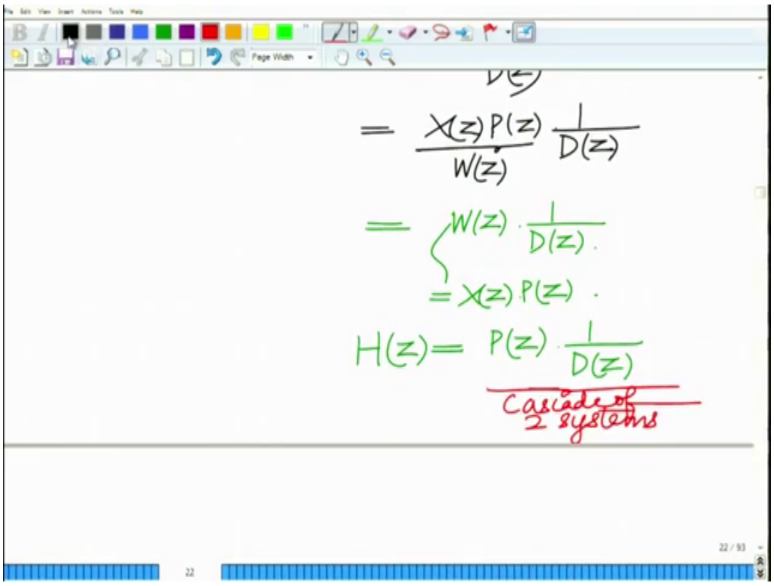

So, I can write this as W z into 1 over D z Y z equal to W z into 1 over D z where, W z equals X z into P z. So, this is a cascade of 2 systems that is P z into 1 over D z. So, H z is the cascade of these 2 systems ok.

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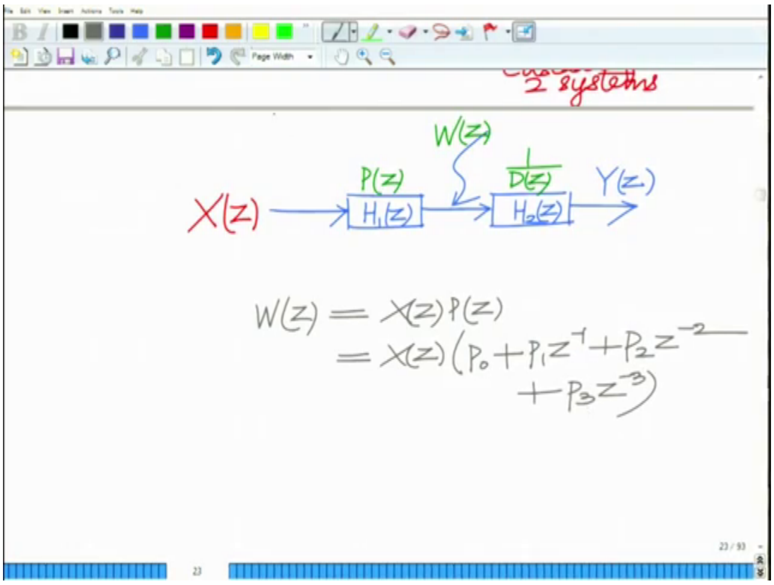

Now, let us look at, so I have Xz given as input to the first system; H1 z, the output Wn is given as input to the second system H 2 z and this is Wz and the intermediate signal is given by the z transform Wz. So, H1 z is basically your Pz and this is your 1 over Dz ok.

And now let us look at Wz you can see Wz equals Xz into Pz which is Xz into P naught plus P1 z inverse plus P2 z minus 2 plus P 3 z minus 3.

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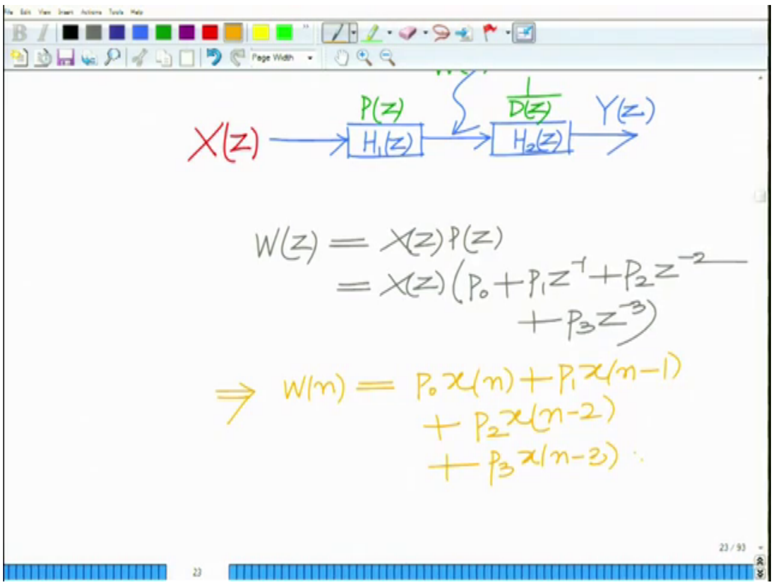

And therefore, now if you look at the corresponding, if you look at the corresponding time domain expression; already equivalent time domain difference equation for this, you will get taking the inverse z transforms that will be Wn equals P naught times Xz corresponding to P naught times xn plus P1 z inverse Xz that is xn minus 1 plus P2 z raised to minus 2 Xz that is xn minus 2 plus P 3 X of n minus 3. And the structure for this it can be given as follows.

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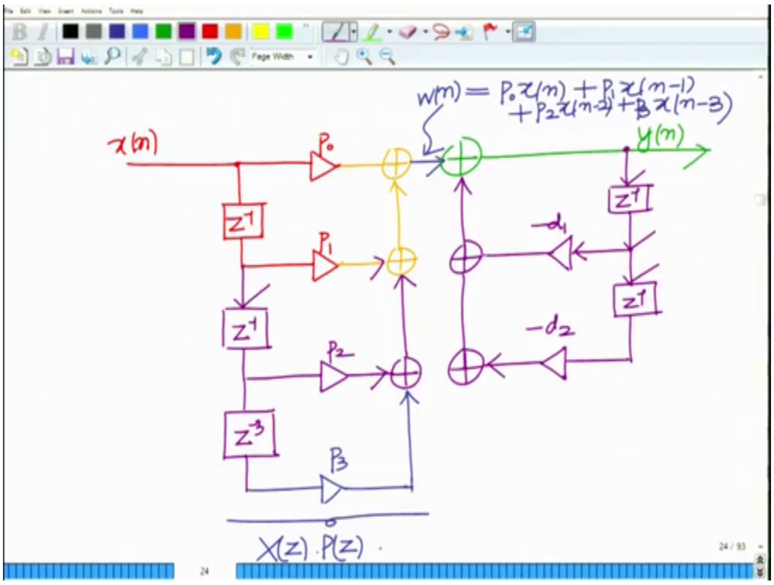

So, you have the input x of n. So, let me draw it over here. So, I have this is your x of n. So, this is P naught, this is z inverse and the element z inverse correct and you are multiplying that by P1, that will give you xn minus 1. So, this is performing the addition of P1 xn minus 1 plus P naught xn and again going through another element z inverse that will then multiplied by this xn minus 2; that is multiplied by P2 and then what you have over here is you take this and you add to this.

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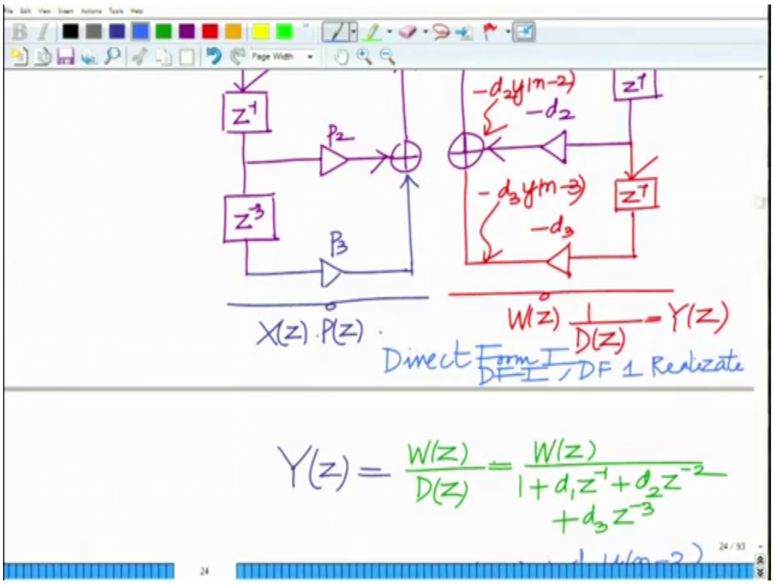

And finally, what you have over here is another element z minus 3; that you will multiply by the gain P3 and add to the existing. And you can see what you have over here is basically your wn. So, that is P naught x n plus P1 x n minus 1 plus P 2 x n minus 2 plus P 3 x n minus 3. So, this is wn equals your P naught x n plus P1 x n minus 1 plus P2 xn minus 3, P2 x n minus 2 plus P 3 xn minus 3 ok. Now we come to the second part. So, this basically implements your Xz into Pz. Now we come to the next part that is Yz equals Wz divided by over Dz ok.

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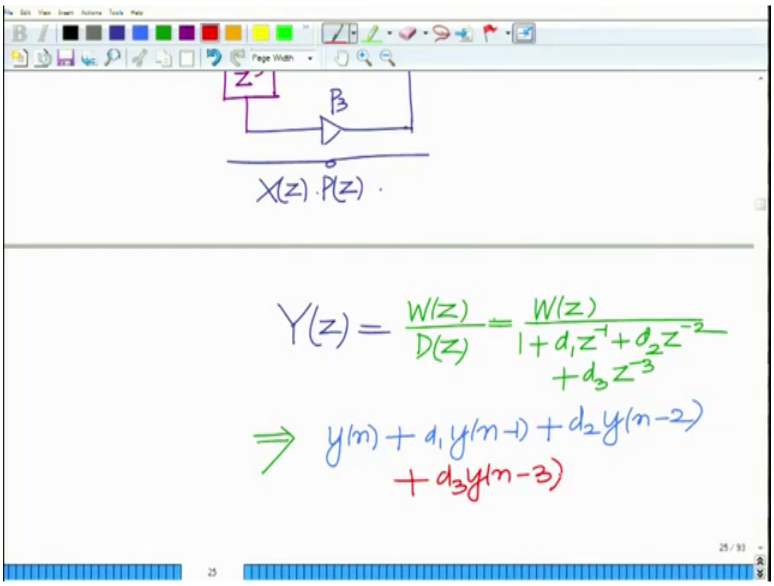

So, we have Yz equals Wz over Dz; which is basically you are Wz over 1 plus d1 z inverse plus d 2 z minus 2 plus d 3 z minus 3. Now once again taking the inverse z transform; this implies that you have yn plus d1 yn minus 1 plus d 2 yn minus 2 plus d 3 yn minus 3 equals your wn.

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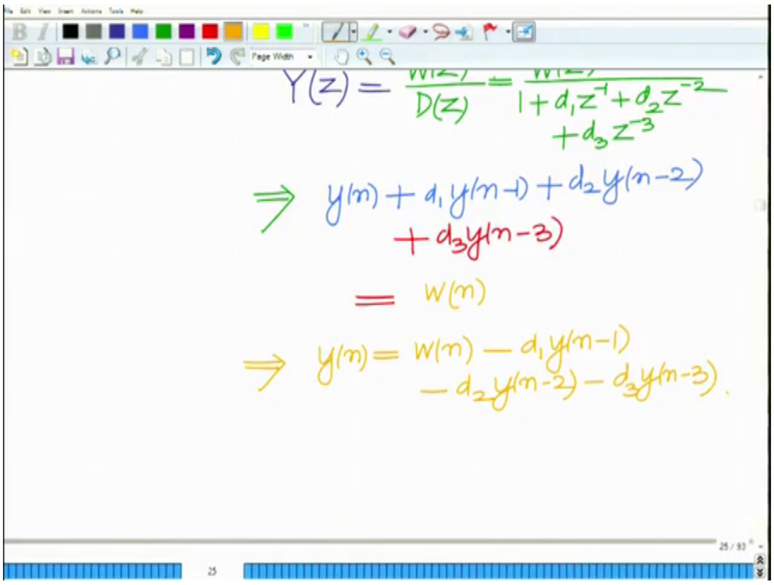

Now, this can also be written as yn equals wn minus d1 y n minus 1 minus d 2 y n minus 2 minus d 3 y n minus 3. And now, therefore I can write this thing over here as follows; I can incorporate the IIR structure corresponding to this as follows. So, wn minus now let us look at this wn minus so, I have over here. Let us say your outcome that is yn ok.

Now, I am going to here perform so, you have minus dn. So, this is w1 minus dn I am sorry there has to be a delay over here. So, that will give you yn minus 1 correct. So, minus dn yn minus 1 and this will be minus d 2 and pass it through a delay z inverse. So, this will be minus d 2 yn minus 2 and finally, you have another gain here minus d3 and this will be your third delay z inverse. So, that will be minus d 3 yn minus 3. So, wn minus d1 yn minus; so, so at this point you have minus d1 yn minus 1 at this point you have minus d2 yn minus 2 and at this point of course, you have minus d3 yn minus 3 and wn and this when added to wn gives you your yn.

So, basically now this part represents your Wz into 1 over Dz equals Yz ok. So, you have implemented it as a cascade of 2 systems all right, so into Pz into 1 over Dz. So, this is basically the cascade of 2 systems all right and this is known as the direct form 1 and this kind of realization all right, this cascade of 2 systems all right; this is known as the direct form 1 realization ok. So, this is realization is known as your direct form 1; that is your DF one, also termed as DF 1 or DF 1 Realization ok. So, that gives you your DF 1 realization.

Now, what you can do is first you can interchange these 2 blocks. Remember when you have a cascade of 2 LTI systems you can always interchange; that is your H1 z H2 z. You can interchange them to have H2 z followed by H1 z all right because convolution is committed to the resulting remains resulting system remains unchanged. So, basically I can interchange these 2 systems ok.

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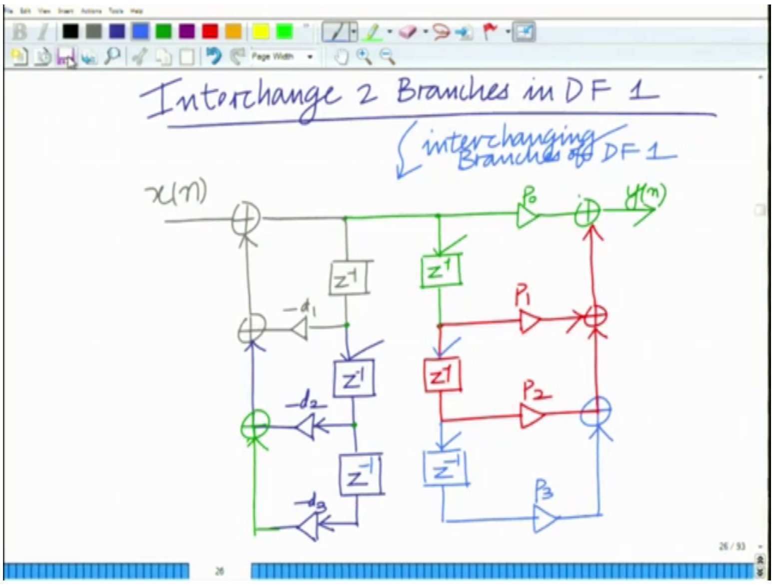

So, now, what I am going to do is I am going to interchange the 2 systems or interchange the 2 branches in DF 1. And what that gives me something very interesting; when I interchange the 2 branches I will have well I will have xn. Let me just draw this gain and I have an adder and that will give me z inverse ok.

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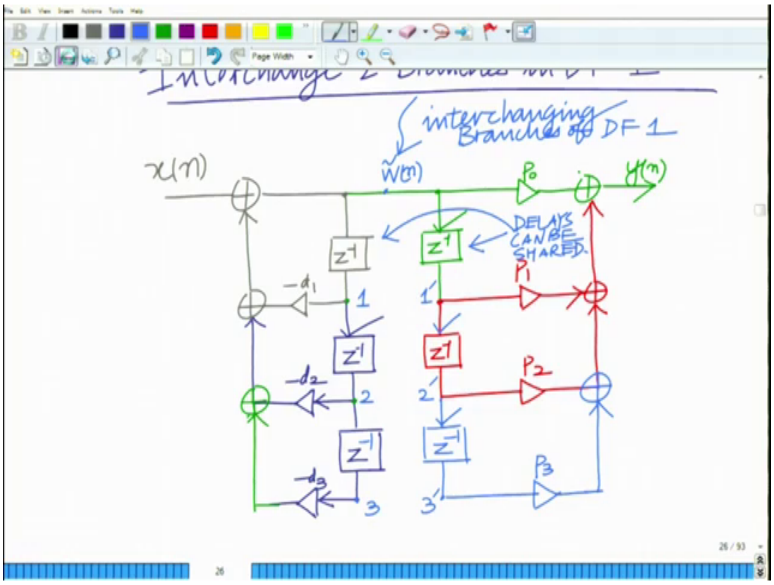

This will be multiplied by minus d1 added to the previous one; z inverse ok, multiplied by minus d 2 and added and finally, what we will have is z minus 3 multiplied by minus d3 and then you have here you have a adder minus d3 minus d1 minus d2 ok.

And now you can here you have z minus 1 all right. Basically I am interchanging the 2 systems ok. So, I have z minus 1 and this will be z minus 1 and z inverse that and first you have here gain of P naught. So, this is your yn or let us put it this way. So, P naught and this will be P1 and further you have your second delay z inverse multiplied by gain P2 output of this multiplied by gain P2 and you have another adder and then you have another element z, I am sorry z inverse, this has to be z inverse multiplied by let me just check the delay elements; this has to be I am sorry gain z inverse ok.

And therefore, now you multiply this by P3 ok. So, they will multiply this by P3 and what you have is basically you have the interchanged all right. So, this is basically interchanging of after interchanging, after interchanging branches of DF 1 this is what you have ok. And what you can realize at this point is that if you look at this pairs of signal points 1 and 1 prime and if you call this 2 and 2 prime and finally, if you call the signals at these points 3 and 3 prime; these signals at these points 1 and one prime, 2 and 2 prime, 3 and 3 prime. These are the same; that is whatever the signal is at this point all right let us call this w tilde n. So, this will be w tilde n minus 1, w tilde n minus 2, w tilde n minus 3 all right.

So, basically what that means, is I can fuse these 2 signal points and therefore, the resulting what that does is basically the advantage of that is basically that eliminates the duplication of these delays. So, I can fuse I can merge the delays ok. So, basically I can fuse so the signal points 1-1 prime.

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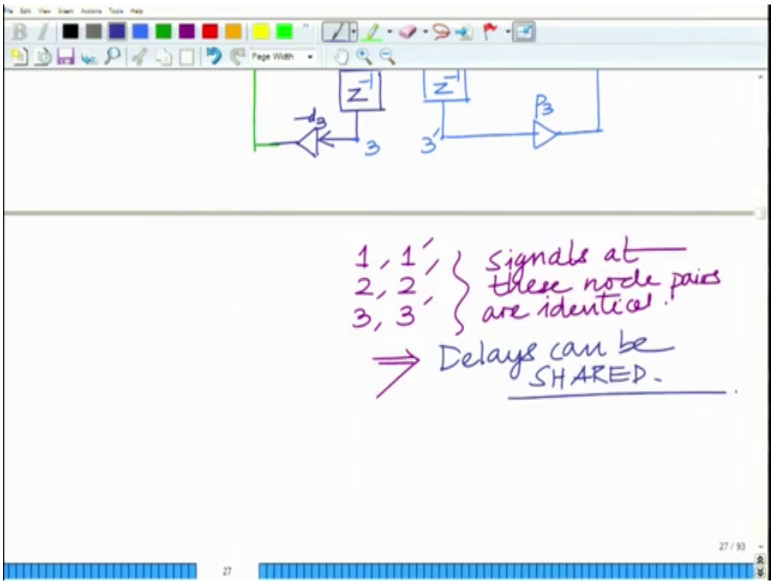

Now, these pairs are basically signals. This implies the signal that these node pairs are identical. This implies that you can delays can be shared; implies basically those signal points can be merged. So, delays these delays can be shared in a common.

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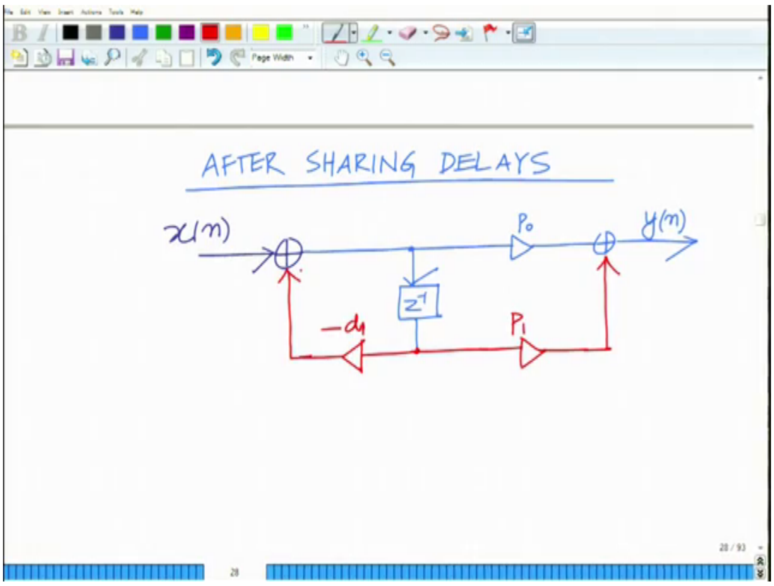

So, for instance the delays can be shared and once you merge these and share the delays the net, after sharing of delays on the common and basically eliminating the duplicate delays what you have; is you have your xn that is basically so, you have your xn that is basically adding. And you have your point, you have z inverse and this is multiplied by gain P naught. Again there is an adder here and you have yn and now here from this itself this node itself you can have 2 branches or 2 out. So, this is minus d1 and this part here is basically your P1 and this is basically what is giving you this.

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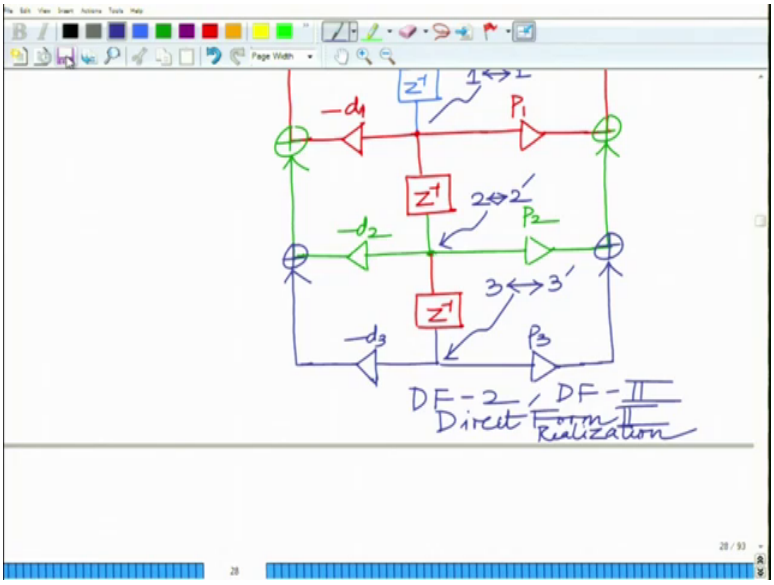

And now you can have you have another delay which is basically because you have merged the nodes 2 and or 2 and 2 prime and this gives you P2 and you have an adder here and similarly you have an adder here and you have a gain of minus d2 and then you have this point. And finally, you have this point here z inverse P3 and you have minus d3; another adder and then you have this thing ok.

So basically, now you have what you have? So, this was previously for instance I think let us look at this point. This was previously 1 and 1 prime. So, this was merging 1 and 1 prime. So, basically I am representing it by this symbol 1 and 1 prime, this is after merging 2 and 2 prime and this is after merging 3 and 3 prime and what you have is; so, the 3 delays, 6 delays have become 3 delays and basically this is basically this is termed as the Direct Form 2 realization; DF 2 DF 2 or the system does the direct form 2 realization all right.

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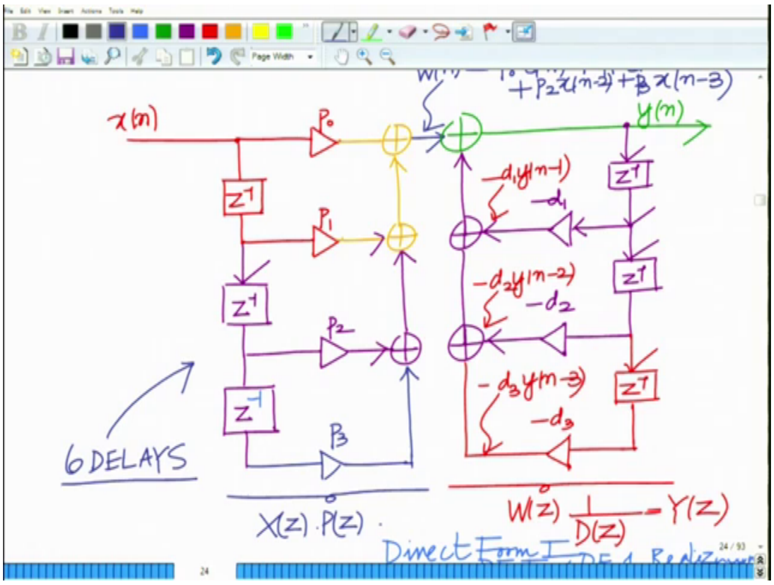

So, previously what you see is we had here we are DF 1 notice we had 6 delays ok. So, notice here we had 6 delay elements all right. So, there are 6 delay elements here and from this 6 delays what we have here is we have only we have managed to reduce this to basically 3 delays.

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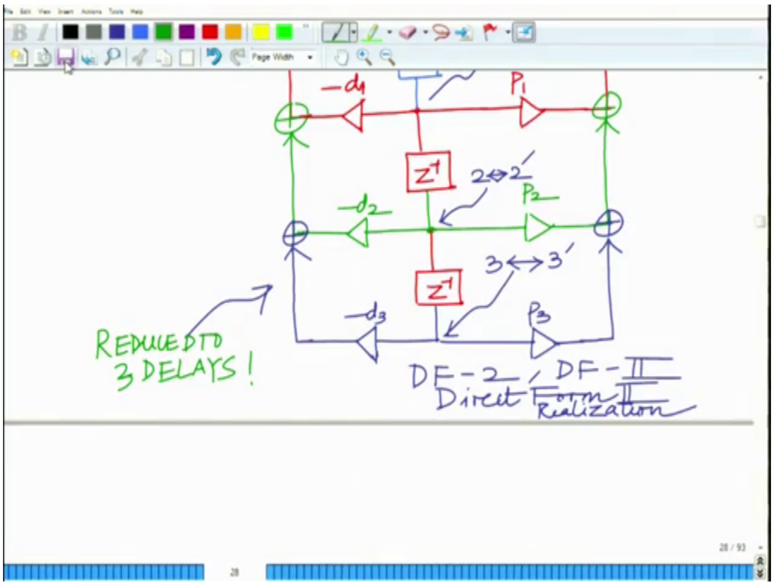

By fusing them suitably so what we have managed to do is we have reduced it to, so reduce the delays by a factor of 6. We have come down to 3 delays already. That is the advantage of the direct form direct form 2 realization all right. So, in this module we have started looking at the direct form that is the realization of various IIR filters all right. We have looked at the direct form 1, direct from 2 realization and will continue with other realizations in a subsequent modules.

Thank you very much.