Principles of Signals and Systems
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Lecture - 67
Example Problems: DFT

Hello welcome to another module in this massive open online course. So, we are looking at example problems in the Fourier analysis of discrete time signals. In particular, we are looking at the discrete Fourier transform, all right. So, let us continue our discussions.

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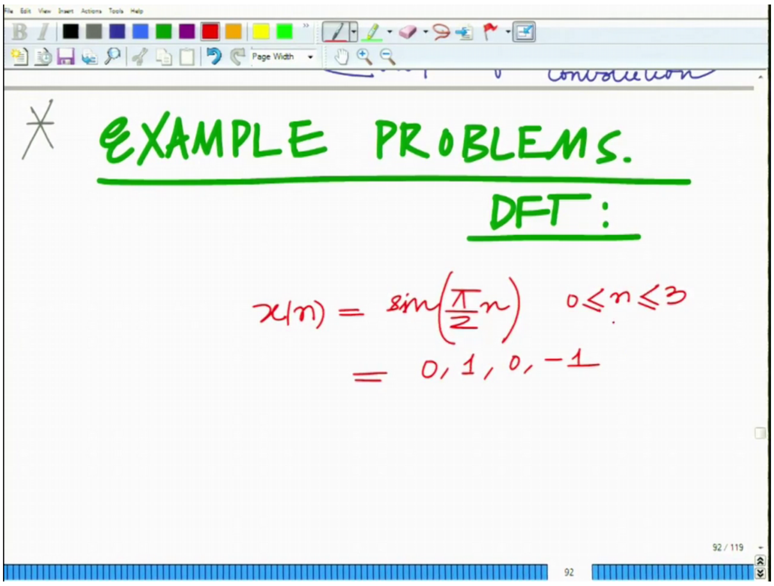

So, we are looking at, example problems in DFT and well, we are let us continue our discussion of the previous problem. So, we have the signal x n equal sin pi by 2 n, 0 less than equal to n less than or equal to 3, which basically turns out to be the sequence 0 n equals 0 this is 0, 1, 0 minus 1, all right.

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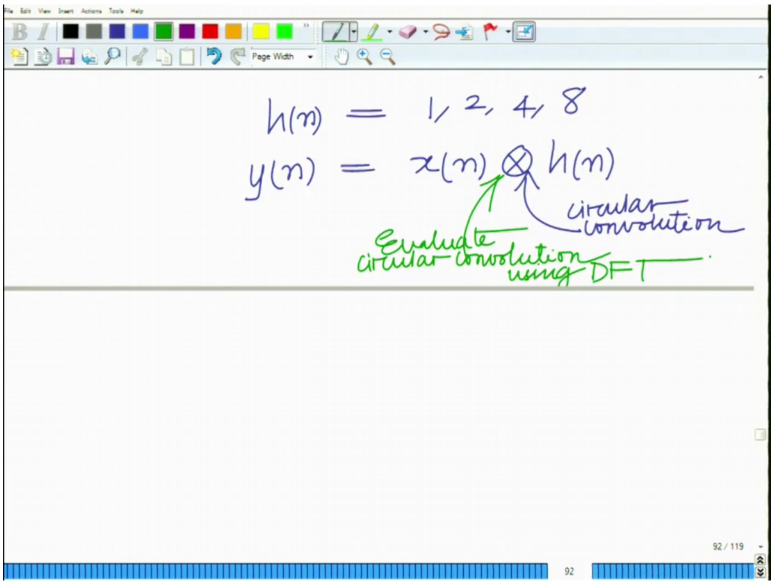

And further, the sequence h n or you can consider the impulse response of the system h n, to be 1, 2, 4 comma 8 and y n, equals x n circularly convolved with h n. So, this is x n circularly convolved with h n correct this is your circular convolution. The h n circularly convolved with h n, we have implemented this in the time domain correct the circular convolution between these two signals in the time domain, now we want to implement it using the or now we want to evaluate it using the DFT ok.

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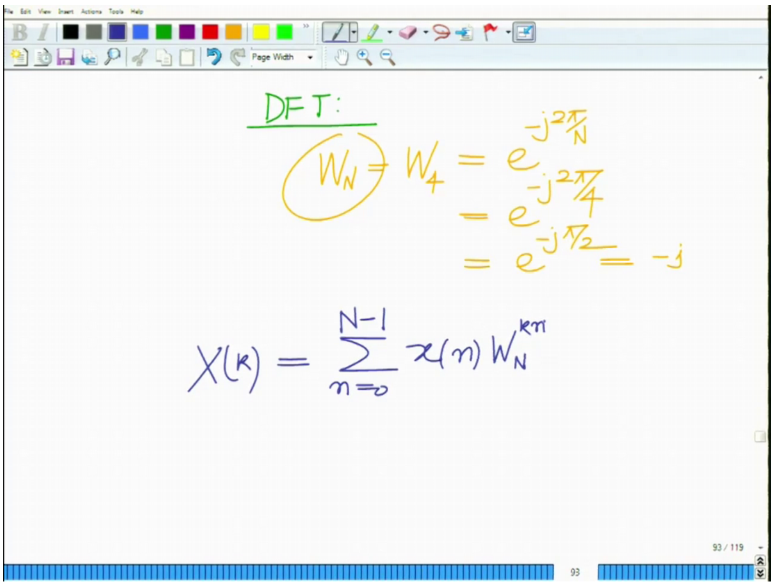

So, Evaluate the circular convolution, using the properties of DFT and what we have is?, now, the DFT can be evaluated as follows, Now, if you want to use the DFT, realize that we need this factor W N which is, in fact, W 4 which is e raised to minus j 2 pi divided by N, which is e raised to minus j 2 pi divided by 4, which is e raised to minus j pi by 2, which is equal to minus j. So, W N is minus j ok. So, this quantity W N which we use in the evaluation of the DFT is minus k.

Now, therefore, our quantity X k, the k th DFT coefficient can be written as, summation n equals 0, n equals 0 to Nminus 1, that is 3, 4 minus 1 is 3. So, N minus 1, x n W N, to the power n, W N raised to the power k n am sorry, W N raised to the power k n.

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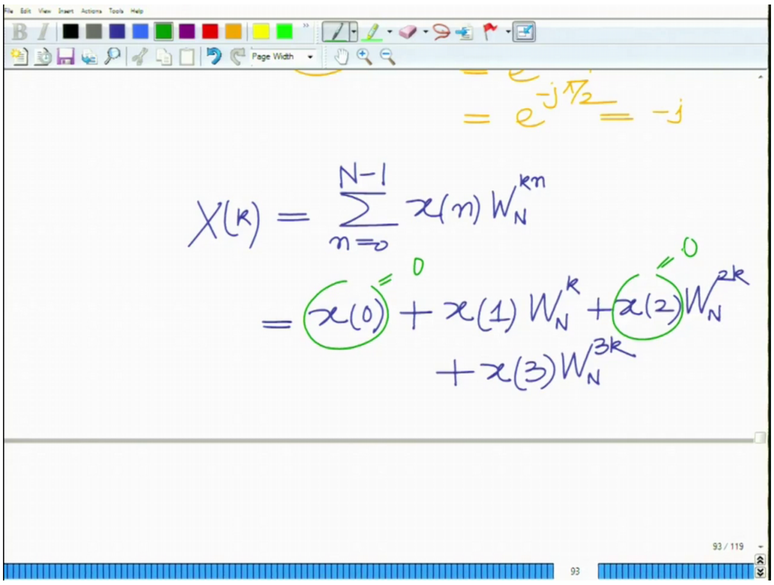

So, this will be n equal to 0. So, x of 0 is 0. So, this will be x of o into W N raised to the power of 0, that is 1 plus x of 1, W N raised to the power of n or sorry, W N raised to the power of k, plus x of 2, W N raised to the power of 2 k, plus x of 3, W N raised to the power of 3 k. All right, this is the expression for this.

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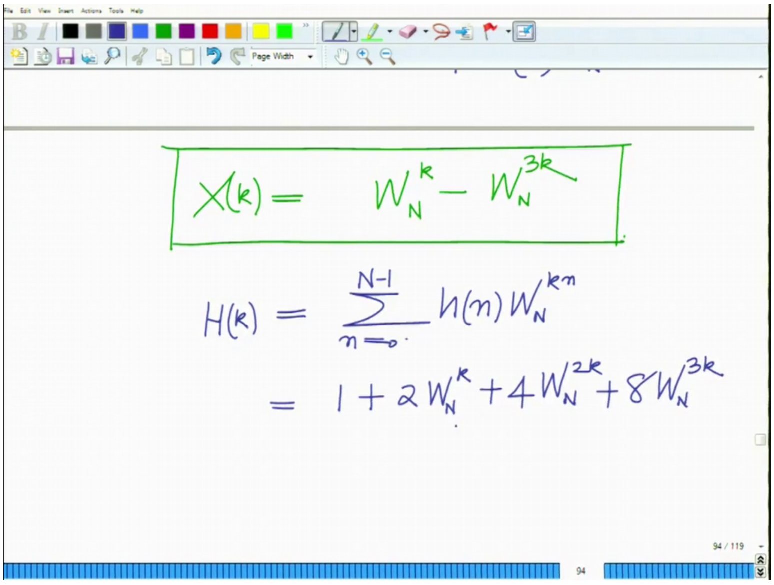

Now, out of this, observe that x of 1, observe that x of 0 is 0, x of 2 is also equal to 0. So, these two terms are 0 and what remains therefore, is the term corresponding to x of 1, which is basically, 1 raise into W N raised to the power k, plus x of 3, that is minus 1, minus 1 into W N raise to 3 k. So, the k th DFT coefficient, of the signal, sin pi by 2 n is W N raise to k, minus W N raised to 3 k, where W N recall is minus j, for this case ok all right?

And, similarly now, Y of k equals summation n equals 0 to n minus 1, that is, I am sorry, this is H of k, this is H of k, is H of n, W N raised to the k n, which is H of 0 is 1. 1 plus H of 1 is to, 2 W N raised to the power k, plus 4, W N raised to the power 2 k, plus 8 W N raised to the, 8 W N raised to the, I am sorry, W N raised to the 3 k. This is the expression for your H of k. Ok?

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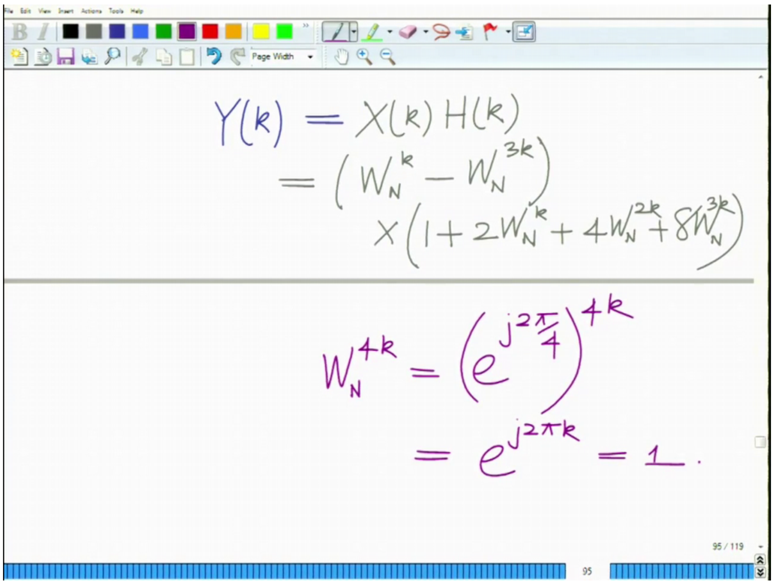

And now, observe that the DFT of the output is the multiplication of the DFT coefficients on the input. So, Y of k, that is, you will have Y of k, because you have a circular convolution in the DFT domain, it becomes multiplication that become X of k, into H of k, which is basically, W N k minus W N 3 k and its product with 1 plus 2 W N k, plus 4, W N 2 k, plus 8, W N 3 k and we will multiply this realizing the property, using the property that, W N here W N is W 4. So, W N to the power of 4 k, is, in this case, e raised to j 2 pi by 4, to the power 4 k, which is equal to e raised to j, 2 pi k which is 1.

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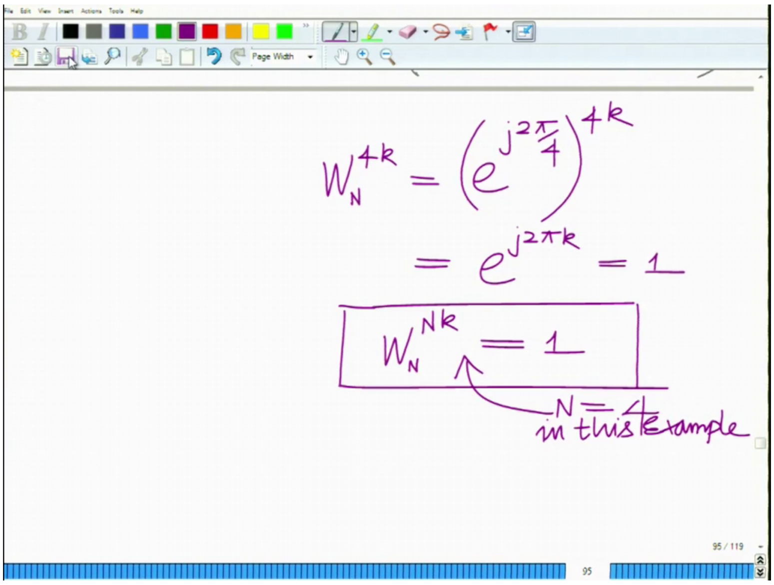

So, W N raised to the. So, in general, WN, raised to the power of N k equals 1. Ok?. In this case, N is equal to 4. So, N is equal to 4, in this example. So, N is equal to 4, in this particular example. So, W N raised to the power of 4 k, is unity form we will use this property in the simplification. Ok?

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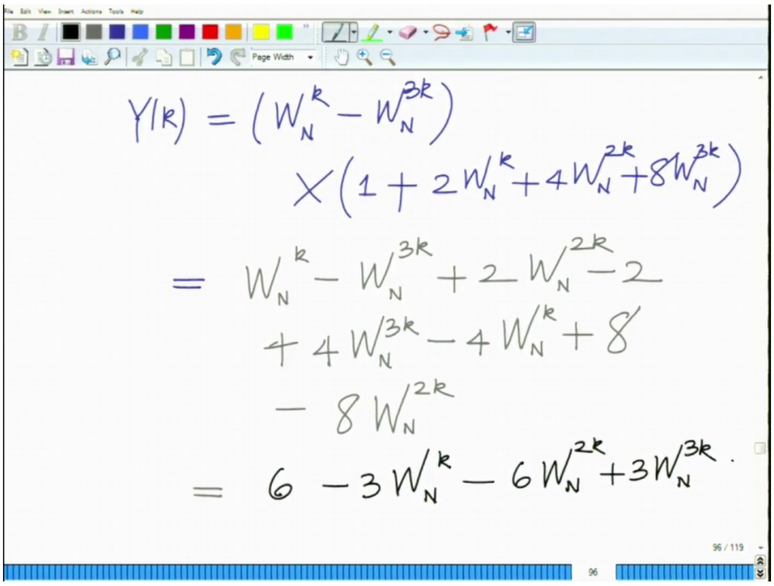

So, we have Y k, you have Y k equals W N k minus W N 3 k, into 1, plus 2 W N k, plus 4, W N 2 k, plus 8 W N 3 k, this is equal to now, you can simplify this as W N k into 1 minus W N 3 k, plus 2 W N k 2 W N k that is W N 2 k, minus 2, W N 3 k into W N k that is W and 4 k which is 1. So, this is simply minus 2 plus 4, W N 3 k, minus 4, W N 3 k into W into that is W N 5 k, but W N 4 k is 1. So, this will be simply be W N k, plus 8 into W N 3, 8 W N 3 k into W N k, is W N 4 k which is 1. So, this is simply a plus 8 minus, 8 W N 3 k into W N 3 k is W N 6 k, which is W N for k in to W N 2 k. So, this is simply, W N 2 k.

So, this is equal to now, if you look, at this you have 8 minus 2 is 6 6. Now collect the coefficients of W N k, that is, 1 minus 4 minus 3, W N k. Now collect the coefficient of W N 2 k, that is, plus 2 W N 2 k, minus 8 W N 2 k, that is, minus 6 W N 2 k and W N 3 k. Now you have minus you have 4 W N 3 k minus W N 3 k so, 3, W N 3 k and, if you compare this, with the DFT, that is, y 0 plus y 1 W N k.

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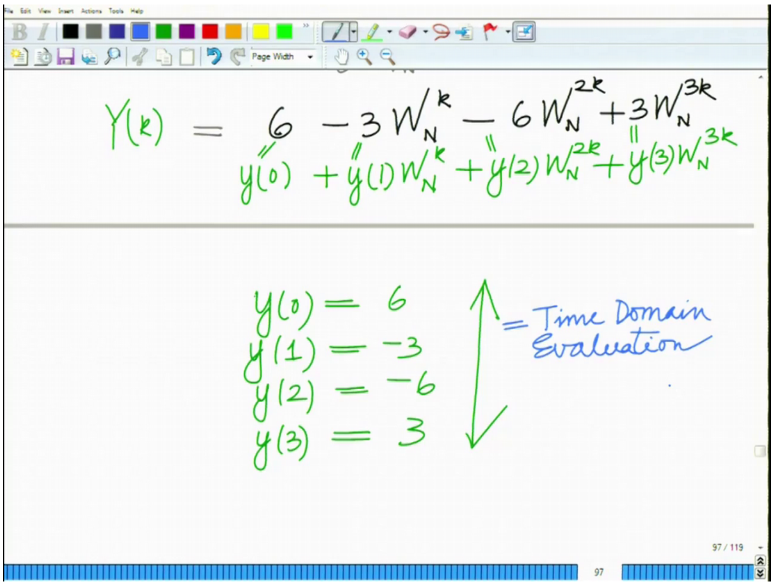

If you do a term by term comparison, y 2 W N 2 k, you can see W y 0 equals 6, y 1 equals minus 3, y 2 equals minus 6 and y 3 equals 3. So, what?. So, by doing a term by term comparison of the DFT, you can reduce the coefficients of the DFT of Y.Tthere is a coefficient Y k. All right? by looking at the coefficients of the expression ok.

And therefore, what you have is y 0 equals 6, y 1 equals minus 3, y 2 equals minus 6 and y 3 equals, y three equals and remember, this is identical to our what we have derived from the time domain evaluation. So, we have evaluated this using the DFT, also evaluated the circular convolution and the time domain and of course, both of them yield the same answer and that is basically, this serves as a check. So, each serves as a check, on the other. Ok? all right?.

. So, let us proceed to the next problem, this was I believe, if I remember correctly this was the 18th, this was the 18th problem. So, let us proceed to the next example, that is example 19.

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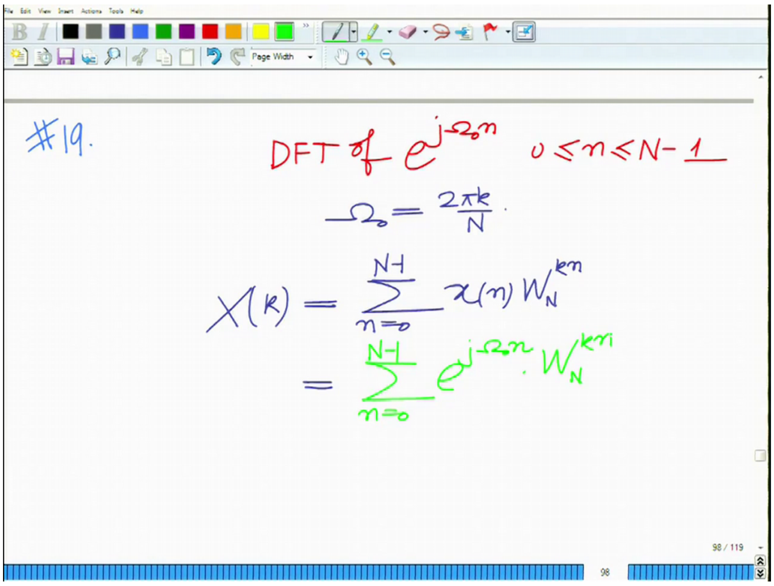

Example number 19, what we want to do is, we want to find the DFT of e power j naught e raise to j omega naught n 0 less than equal to n less than equal to, N minus 1. Ok? And under the condition, omega naught is not equal to 2 pi k over N.

In this case, X k equals, summation n equal to 0, to N minus 1, xn W N kn, which is equal to well, this is equal to summation n equal to 0, to N minus 1 e raise to j omega naught n W N kn.

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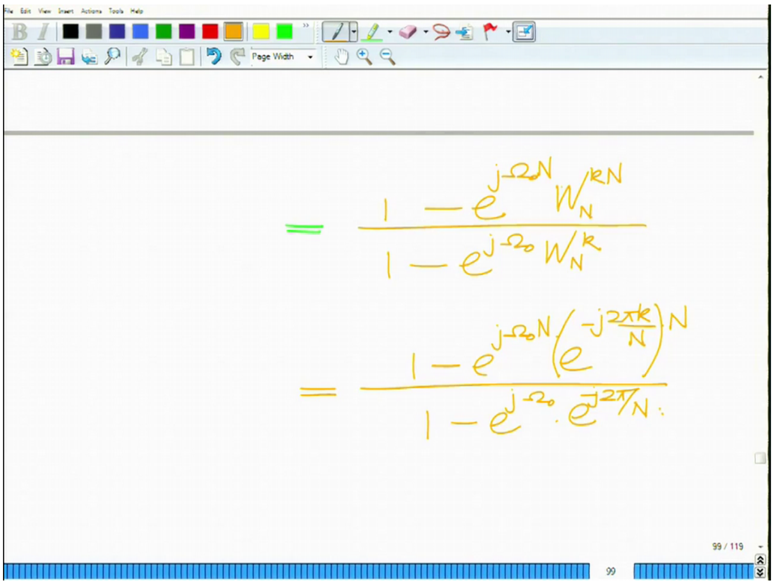

Which is equal to, is a finite summation, so this is summation, n equal to 0 to n minus 1 e raise to j omega naught into W N k raised to the power n. Ok?. Which is equal to, in this case, well, this is 1 minus, e raised to j omega naught N W N kn over 1 minus, e raised to j omega naught, W N raised to k.

Which you can now simplify this as well, this is 1 minus e raised to j omega naught N, e raise to minus j W N k N, is again 1, or you can just simply write it as e raised to minus j 2 pi k, over just write this although this is 1, one can just write this as e raise to minus j 2 pi k, over N raised to the power of N e raise to minus j omega naught, e raise to minus j 2 pi over N or e raise to minus j, 2 pi k over N.

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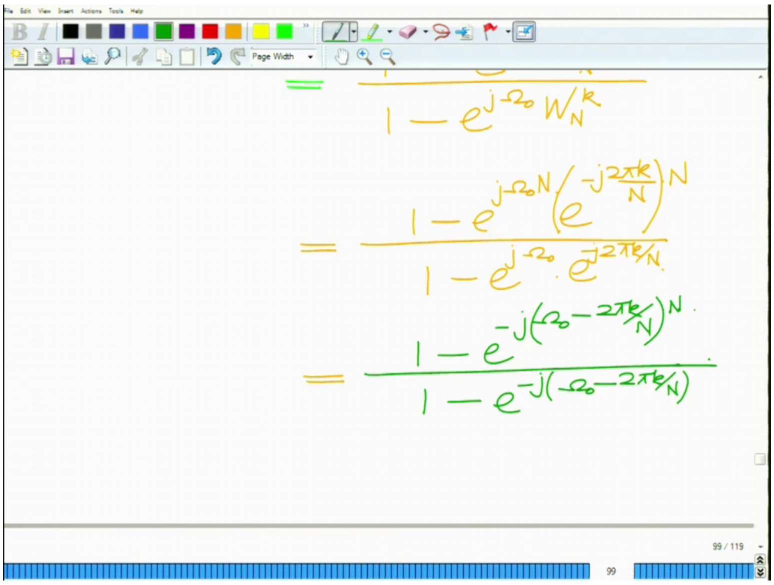

Which is equal to this, will be if you can check this, this will be 1 minus e raise to minus j omega naught, minus 2 pi k over N raised to the power N. 1 minus over 1 minus e raise to minus j omega naught minus 2 pi k over N. Which is equal to, simplify this, e raise to j omega naught, minus 2 pi k over N into N by 2 e raise to j omega naught, minus 2 pi k over N into half.

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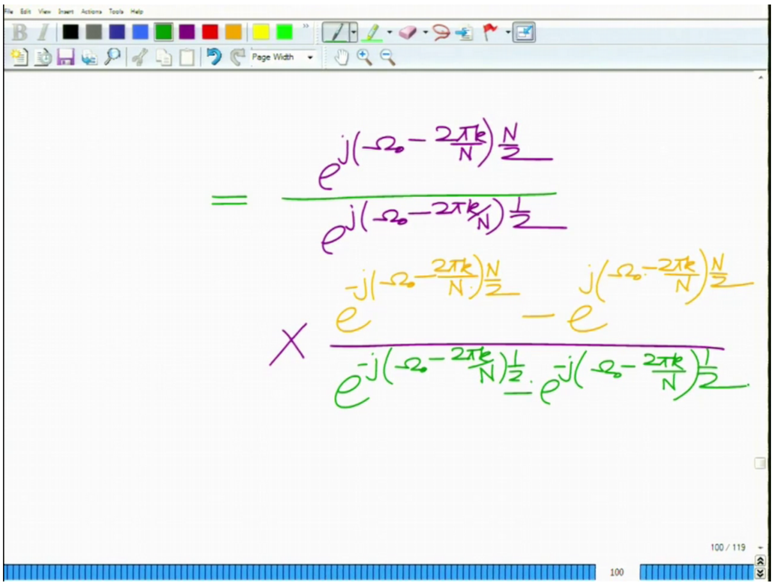

This multiplied by, e raise to minus j, omega naught minus 2 pi k, over N into N by 2, minus e raised to j, minus e raise to j, omega naught minus 2 pi k over N, into N by 2, divided by, divided by, e raised to minus j omega naught minus, 2 pi k over N into half, minus e raised to minus j omega naught minus 2 pi k over N, into half.

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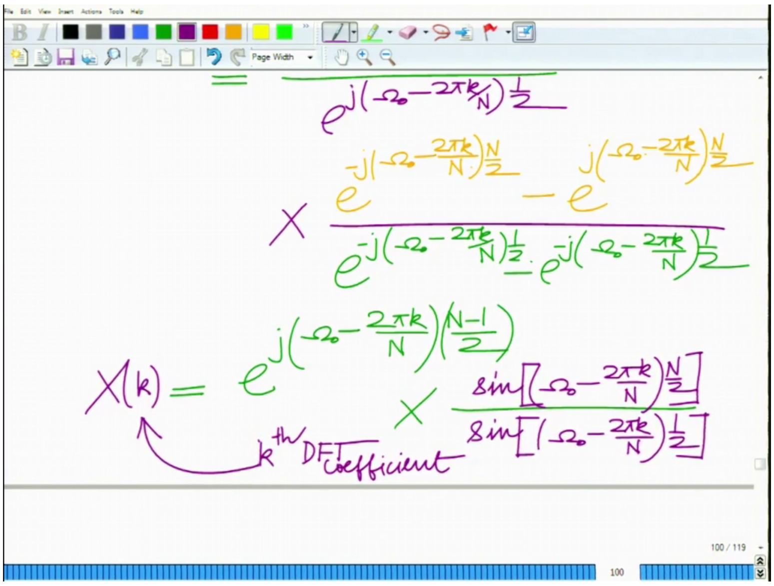

And this, if you simplify this, what you will obtain is, e raise to minus or e raise to j. In fact, e raise to j, omega naught minus 2 pi k, over N, into N minus 1 by 2 into sin omega naught minus 2 pi k over N into N by 2 divided by sin omega naught minus 2 pi k over N into half.

So, this is the expression for your DFT coefficient that is x of k this is the expression for the this is the expression for the kth DFT coefficient that is e raise to j omega naught, minus 2 pi k over N into n minus one over two times sin omega naught, minus 2 pi k over N, in to N by 2 divided by sin, omega naught minus, 2 pi k over N, into half. So, this is the expression for your DFT coefficient that is X of k. This is the expression for the, this is the expression for the kth DFT coefficient. That is e raise to j, omega naught minus 2 pi k over N, in to N minus 1 over 2, times sin omega naught minus 2 pi k over N, times 1 by 2, divided by sin omega naught minus 2 pi k over N in to half all right? So, this is the expression for the kth DFT coefficient,of the given signal all right?

Now, let us do another interesting problem, which is to express the DFT as a linear transform, ok. So, what you want to do is we want to express this DFT as a linear transform or DFT in the form of a matrix.

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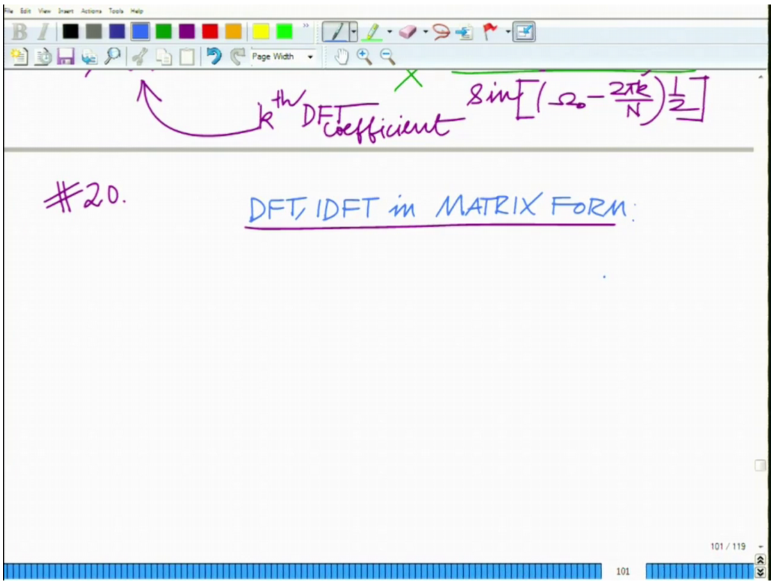

And, this is not mistaken. This is problem number 20. So, the DFT, can be expressed or DFT and in fact, IDFT DFT. So, the DFT and IDFTt can basically, be expressed in matrix form all right? So, what we will do is? Because, we have done a couple of examples, let us stop this here and we will look at this, fundamental aspect in the next module.

Thank you very much.