Principles of Signals and Systems
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Lecture - 65
Example Problems: DTFT - Sampling

Hello welcome to another module in this massive open online course . So, we are looking at example problems for the DTFT the discrete time Fourier transforms, let us continue our discussion all right. So, we are looking at example

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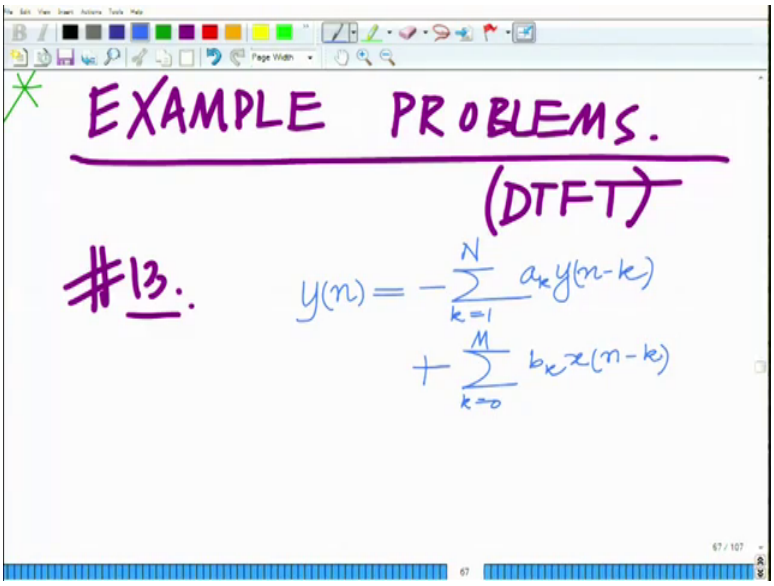

ah. let us start with problem number 13 if I remember correctly and this problem let us look at the following LTI system discrete time LTI system described by the differential equation y n; that is output equals minus k equals 1 to n a k y minus k and plus summation k equal to zero to M b k x n minus k .

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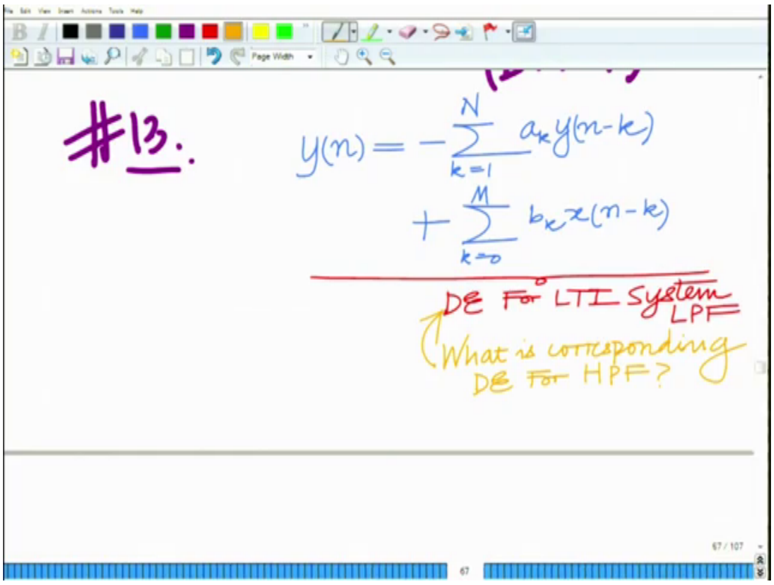

Now, if you look at this what we have over here will be . now this is the difference equation for an LTI system . now if this is LTI system which is a low pass filter ok, this is a low pass filter . now what is the corresponding differential equation or the difference equation for the for a high pass , what is the corresponding difference equation for a high pass filter; that is derived from this low pass filter and we can start as follows. now taking the z transform or taking the DTFT which is very convenient.

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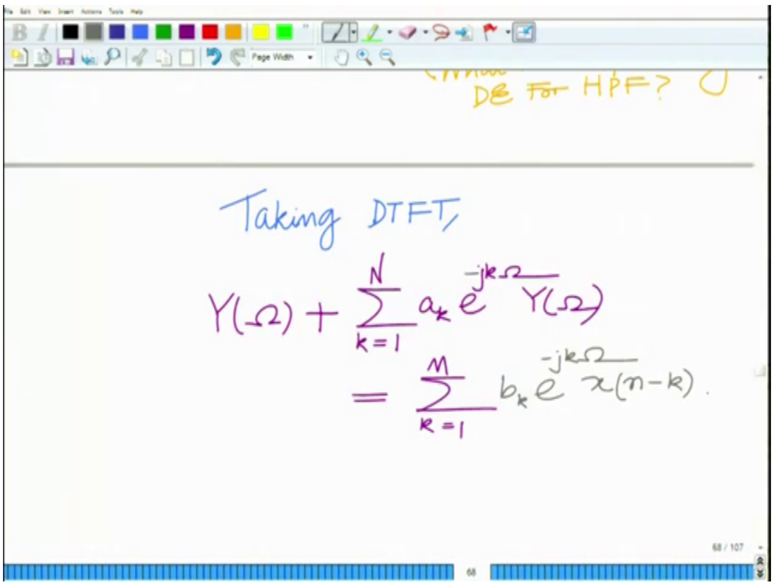

Now as you have seen in all the applications and example problems laptop. So, far the DTFT is a very powerful tool for signal processing discrete time signal processing all right, a discrete signal you can also call it as digital signal processing. So, we have Y omega equals plus k equal to 1 to n ak DT of T of y n minus k is e raised to jk omega into Y omega equals summation k equals 1 to M correct, bk e raised to jk omega ah. I am sorry e raised to minus jk minus jk omega x of n minus k and this would be now, if you find H of omega.

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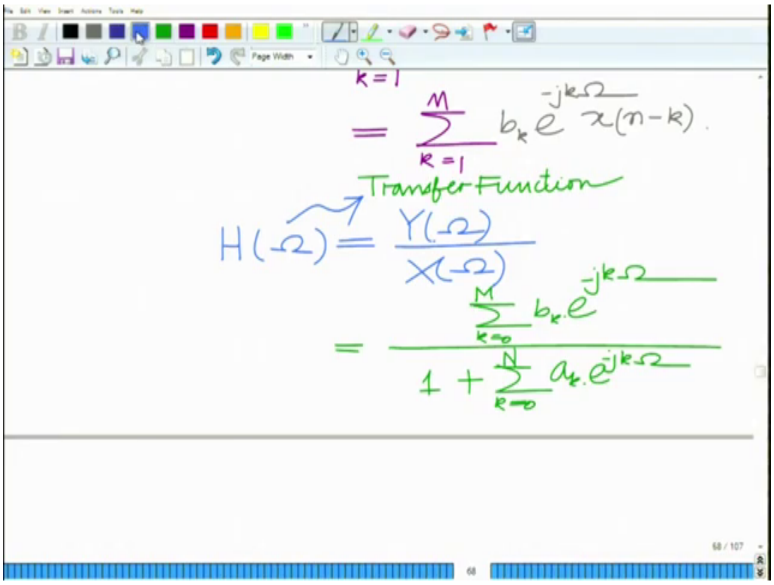

Now, H of omega equals Y of omega . So, this is the transfer function remember H of omega which relates the input and output in the frequency domain. So, H of omega equals ah. well this is summation k equal to zero to M bk e raised to minus jk omega over 1 plus summation k equal to zero to n ak e raised to minus jk omega . And now to get. So, this is the transfer function H of omega

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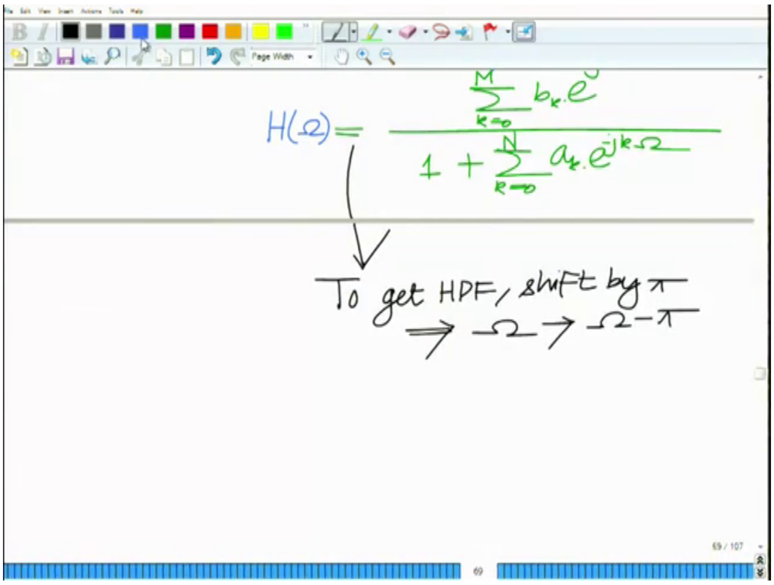

ok. And now to get the corresponding transfer function for the, now this is a low pass filter remember correct. So, get the to get the corresponding transfer function of the equivalent high pass filter I have to shift it to shifted by pi all right. So, omega. So, H of omega becomes H of omega by h of omega minus pi all right. So, we have to replace omega by omega minus pi and that is what we have seen in the previous example to get H PF and this is a neat trick we have to shift by pi implies omega becomes omega minus pi; that is we replace omega by omega minus pi and therefore H tilde of omega of this a high pass filter is summation k equal to zero to M

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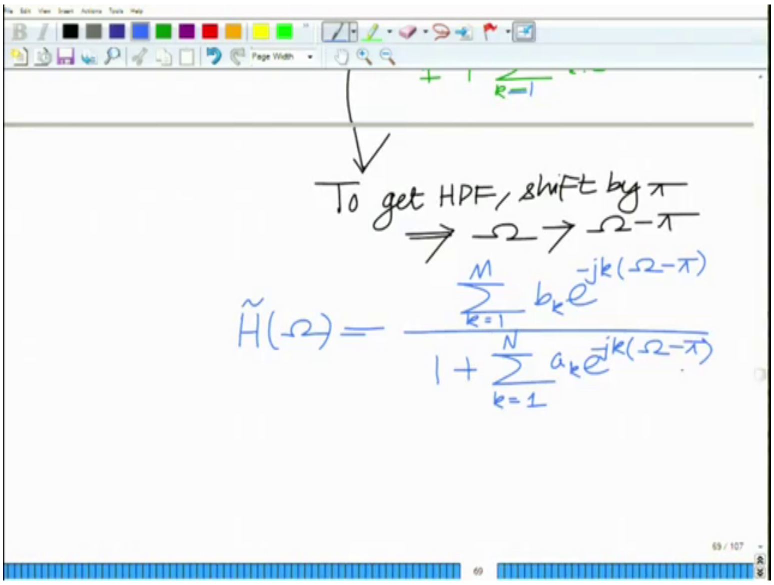

Or in fact, k equal to 1 to M, let me just correct it k equal to this is k equal to, sorry this is boths your both k equal to, these are both k equal to 1 , k equal to 1 correct not k equal to zero or rather k equal to 1 to M bk e raised to j k omega minus pi divided by 1 plus summation k equal to 1 to n a k e raised to j ah, e raised to minus jk omega minus pi

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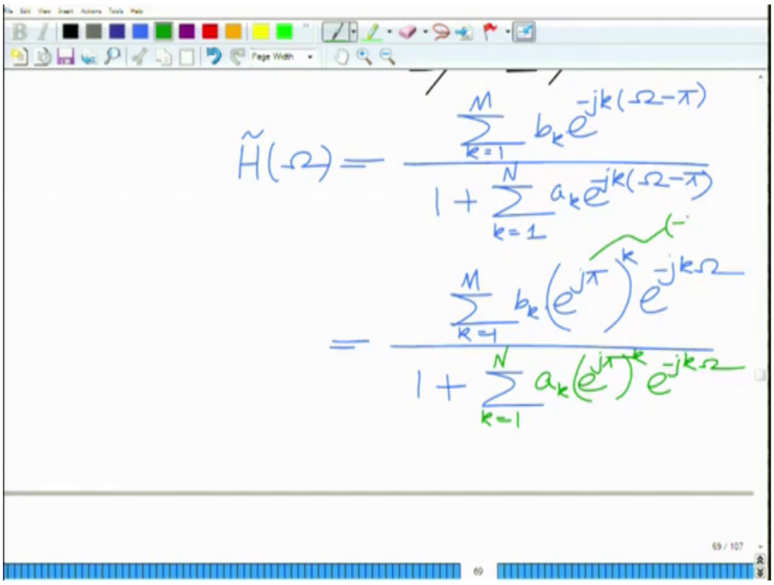

Which now you can simplify as summation k equals 1 to M b k. this will be e raised to j k pi or basically you can write this as e raised to j pi whole raised to k times e raised to minus jk omega. So, that remains as it is over 1 plus summation k equals 1 to n a k e raised to j pi k e raised to minus jk omega.

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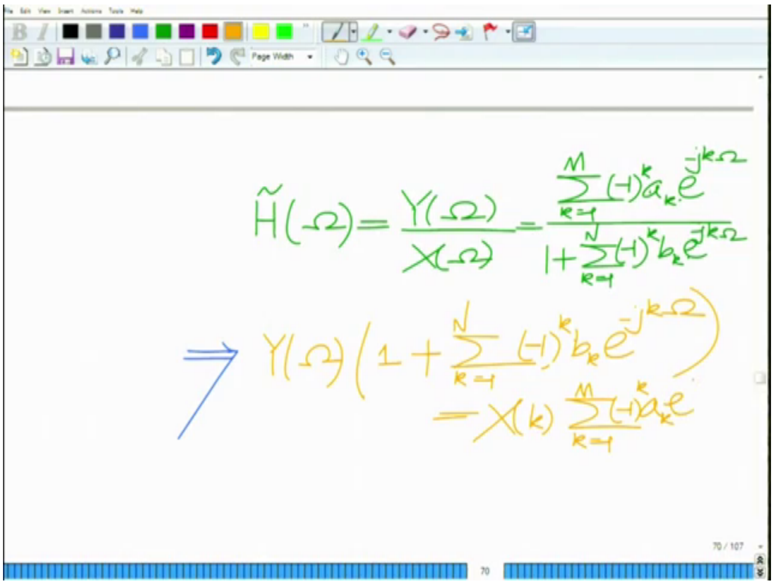

And now you can see e raised to j pi raised to k is nothing, but minus 1 over k. So, H tilde omega, the transfer function, which is the transfer function of the equivalent high pass filter is Y omega by X omega; that is summation k equals 1 to M minus 1 raised to k a k e raised to minus jk omega by 1 plus summation k equals 1 to n minus 1 raised to k b raised to k e raised to minus j omega e raised to minus jk omega.

And therefore, this implies that, if you look at this we have Y of omega into 1 plus into 1 plus summation k equal to 1 to n minus 1 to the power k bk e raised to minus jk omega equals X k summation k equals 1 to M minus 1 to the k minus 1 raised to k ek e raised to minus jk omega . And this implies.

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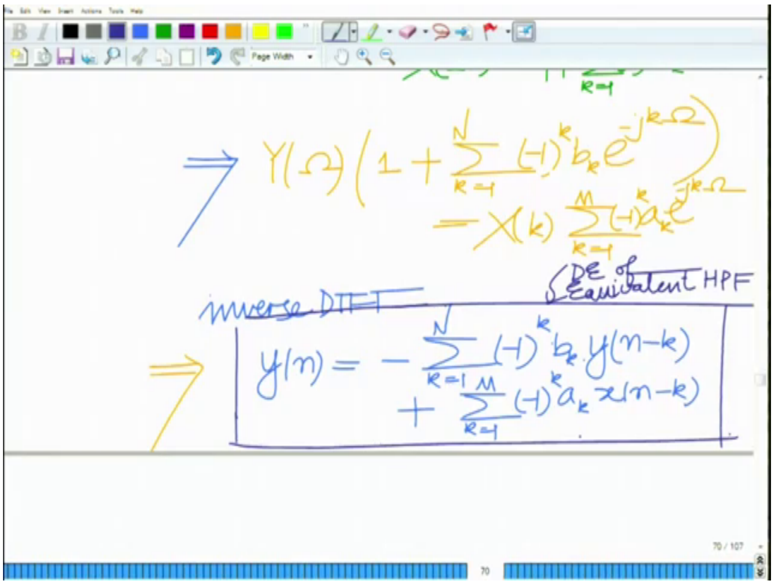

Now, taking the inverse DTFT. So, now, you take the inverse DTFT and this gives rise to the difference equation you can see yn equals minus summation k equals 1 to n minus 1 to the k y n minus k plus summation k equals 1 to M minus 1 k ak x n minus k ok.

So, this is the difference equation of the . this is the difference equation of the equivalent high pass filter; that is developed from the difference equation and the transfer function for that matter of the given low pass filter all right . when we use the trick that to get a high pass filter from a low pass filter you have to shift it by pi or replace omega by omega minus pi in the transfer function all right

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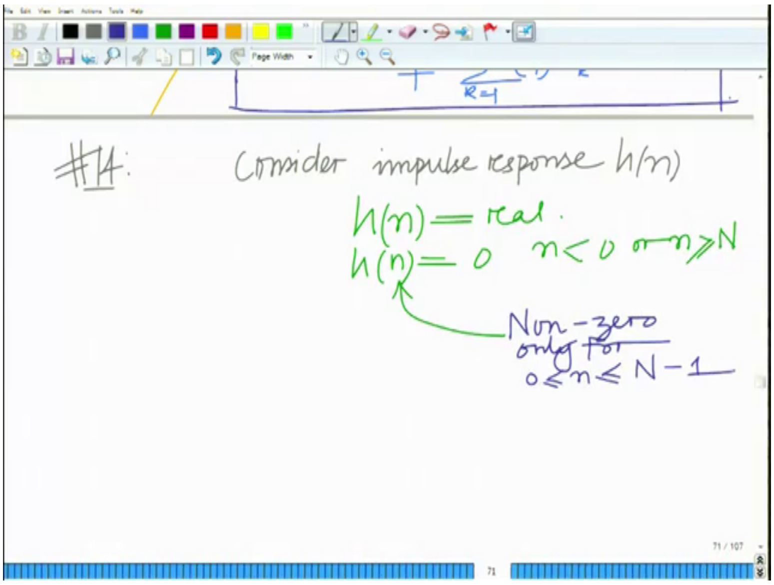

Let us proceed to the next problem, problem number 14 which is the following thing . So, we consider the impulse response h n ; such that h n equals real h n is nonzero only for zero less than equal to n minus 1, which means hn equal to zero for n less than zero or n greater than equal to n. So, this implies this is nonzero only for zero less than equal to n less than or equal to n minus 1.

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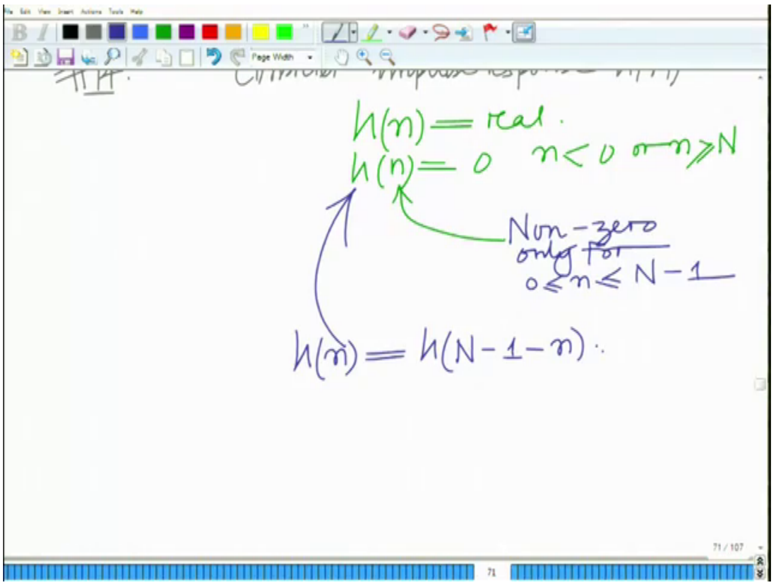

And importantly hn has the property hn equals h of n minus 1 minus n, where n . So, hn equals n minus 1 minus n.

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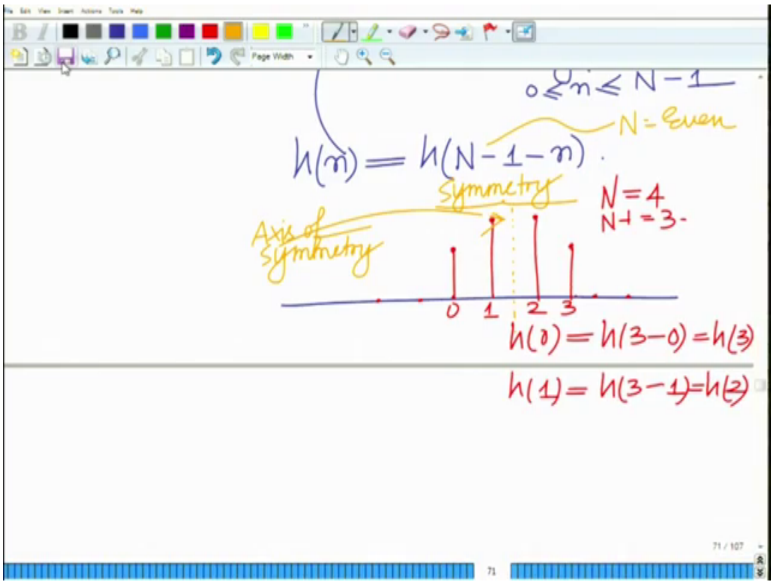

So, this has a symmetry you can see , if you can see it will be something like ok. So, this is your zero 1 2 3, it is zero. So, capital N equals 4 . So, N equals 4 and or and ah. So, N minus 1 equals 3 . So, we have h of zero equals h of 3 minus zero equals h of 3 further h of 1 equals h of 3 minus 1 equals h of 2

So, this has symmetry ok, you can clearly see this as a and here N equal N is even ok. So, this has symmetry about this point. So, this is your, you can think of this as your axis of . So, you can think of this as basically your axis of symmetry ok. So, this is basically you can think of this as a as an axis of ah, you can think of this as a symmetry and axis symmetric axis of symmetry and therefore, this impulse response of this filter is symmetric all right, satisfies the property h small n equals h capital N minus 1 minus small n

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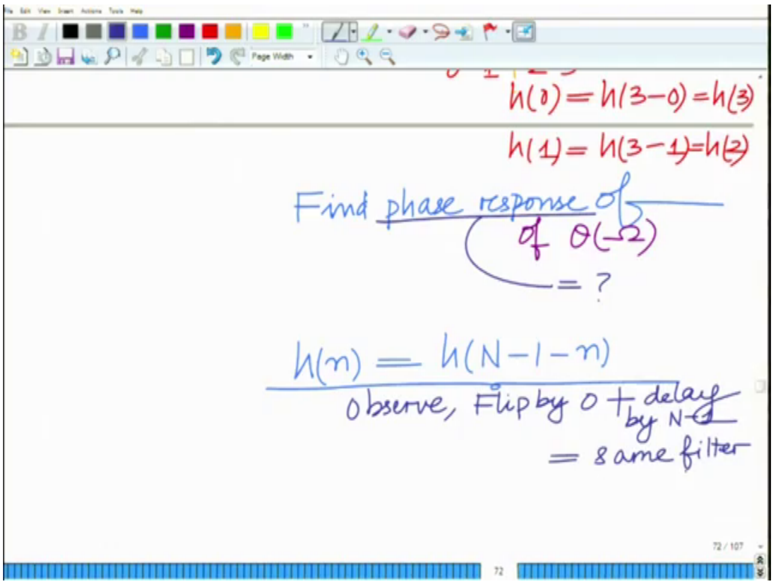

So, now what we want to do is, we want to find the phase response find phase response of this quantity theta omega. now you look at this property now phase response. So, we want to answer the question what is the phase response. now observe that this property h of n equals h of capital N minus 1 minus n from this property what you can see is, if you flip it all right, if you basically flip it about zero this filter about by zero and if you delay it by n minus 1 all right, in the previous case for instance if you consider the mirror image about zero and delayed by n minus 1 then you get again the same filter . So, flip by zero. So, you observe plus delay by n minus 1 equals same filter and this is the interesting property.

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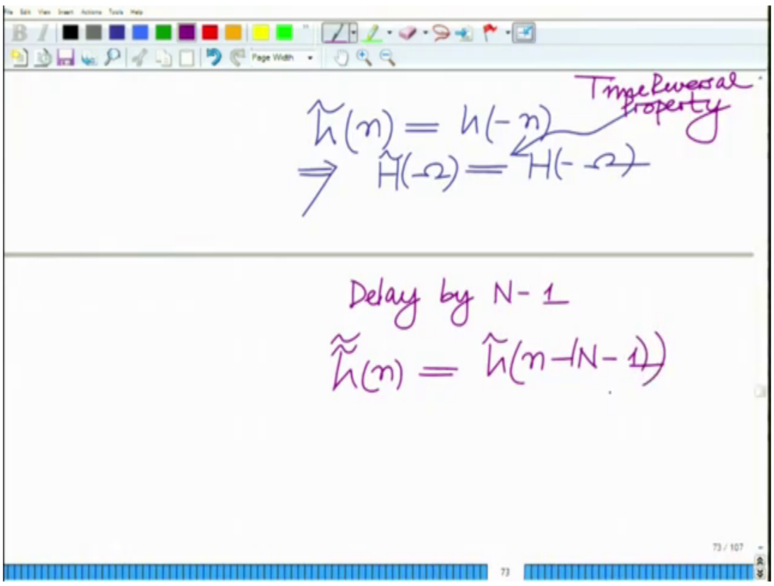

So, if you flip it about the zero axis that is first you take h tilde n equals h of minus n that implies h tilde, taking the DTFT we have a stator remember look at the time reversal. So, we are considering the time reversal, basically time reversal and delay by n minus 1. So, H tilde omega equals H of minus omega, this is from the time reversal property this is from the time universal property H tilde equals H of minus 1

Now, delay by N minus 1. So, we have h double tilde n which is h tilde of n minus N minus 1 which is basically equals

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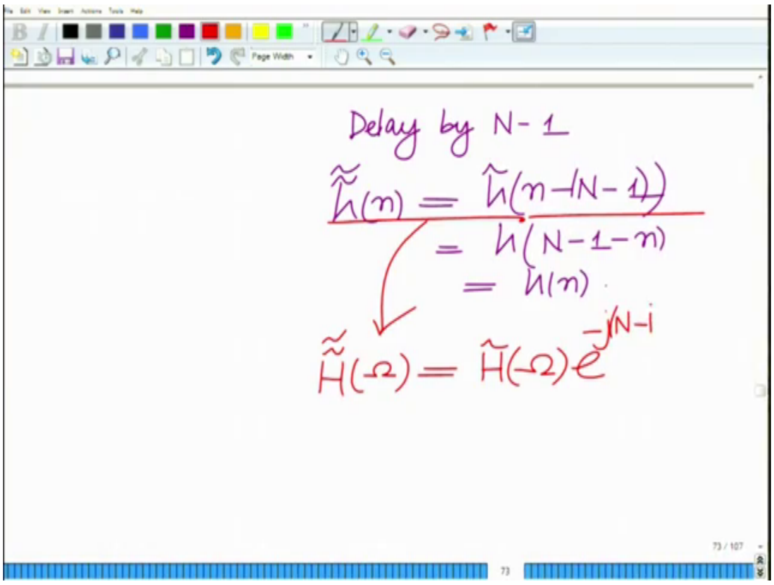

you can see h of N minus 1 minus n , which is equal to now h of n by the property of the filter . and from this you can see you have H double tilde of omega equals H tilde omega, since you are delaying by n minus 1 this becomes H tilde of omega e raised to minus j N minus 1 times omega which is equal to now H tilde of omega you can clearly see this is H of minus omega into e raised to minus j n minus 1 omega this is from the time reversal property

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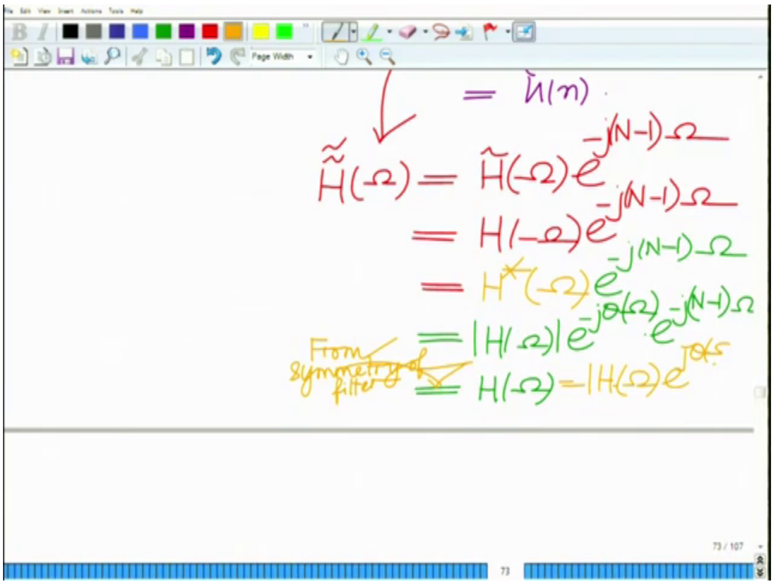

Now, H of minus omega is also equal to H conjugate of omega ok. So, H conjugate of omega times e raised to minus j N minus 1 omega and this follows because H of N is a real filter all right. So, the coefficients of H of N are real and for any real impulse response or for any real signal, we know that H of omega equals H conjugate of that is H of minus omega is basically the same as H conjugate of omega

So, basically I can also write this as mod H magnitude H of omega into the negative of the phase of H omega e raised to minus j theta omega e raised to minus j N minus 1 omega , but this is equal to H of omega from the property of the filter , from property, because H tilde of omega because H of N minus capital N minus n capital N minus 1 minus small n equals H of small n.

So, we have H double tilde omega equals H of omega this is from property of some symmetry of filter which is equal to magnitude H of omega e raised to j theta omega . And therefore, now equating the phase of these two terms, equating the phase, if you call this as term 1 and if you call this as term 2 equating phase of both equating phases

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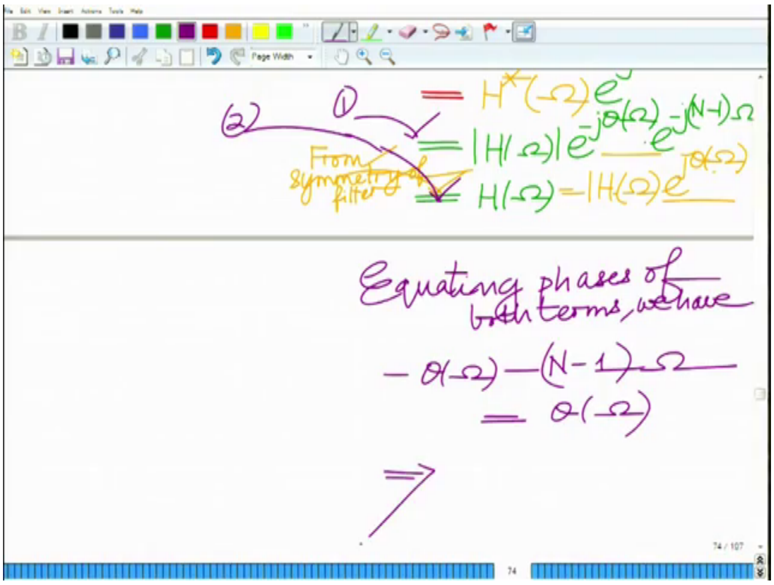

We have minus theta omega minus N minus 1 omega equals theta omega implies twice theta omega equals N minus omega.

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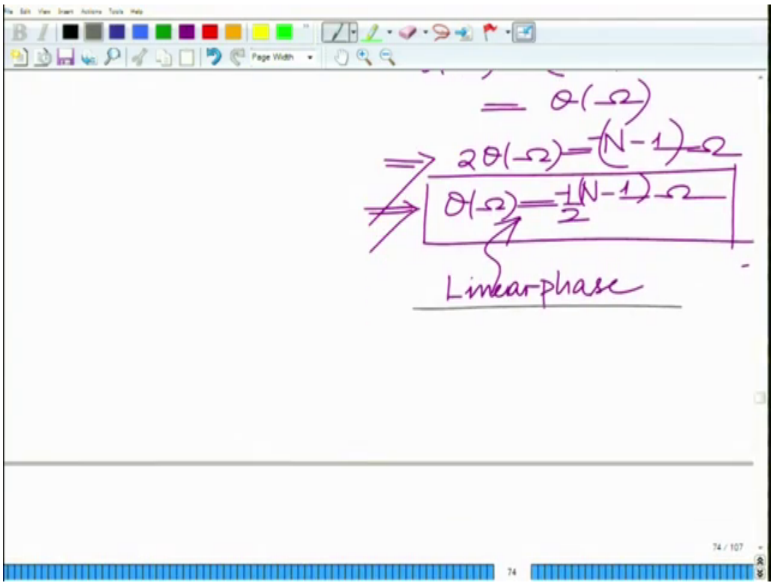

So, this is a neat trick which implies the phase has to satisfy phase of H of omega has to satisfy theta omega equals half or this is basically. In fact, twice the omega is minus n minus 1 omega. So, this is minus half N minus 1 omega ok

This basically satisfies. So, this is a linear phase constraint ok. So, this is a linear phase , phase is linear with omega. So, this has the interesting property that by this symmetry all right. So, the property the symmetric property of this real filter lizard results in a linear phase. In fact, that is something that is very interesting. So, the symmetry property indorse it with a linear phase all right.

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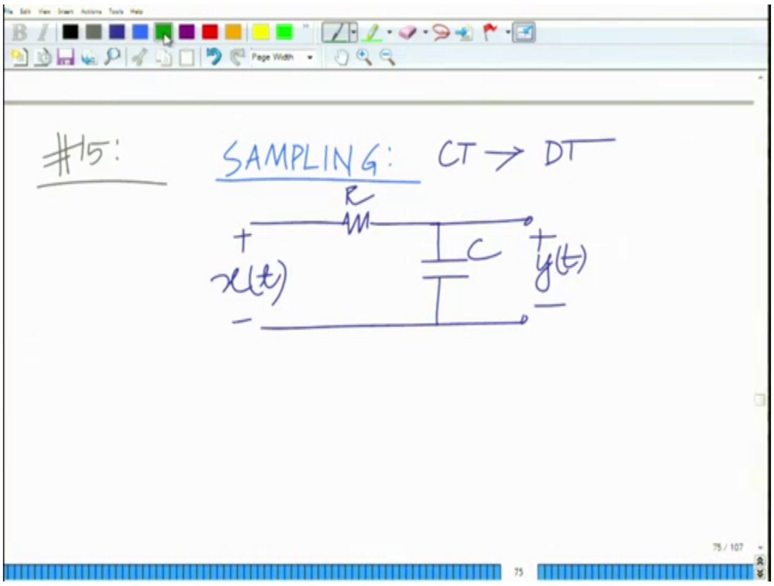

Let us proceed to the next problem which is also again very interesting and it talks about the sampling of it talks about this is the problem number 15 I think , this talks about the sampling of discrete times or a continuous time system . So, sampling from continuous time system, we have the discrete time system .

So, we have an RC circuit resistance R capacitance C, output across the capacitor C input across the RC serial combination. So, we have the input voltage xt output voltage yt and we have seen this several times before.

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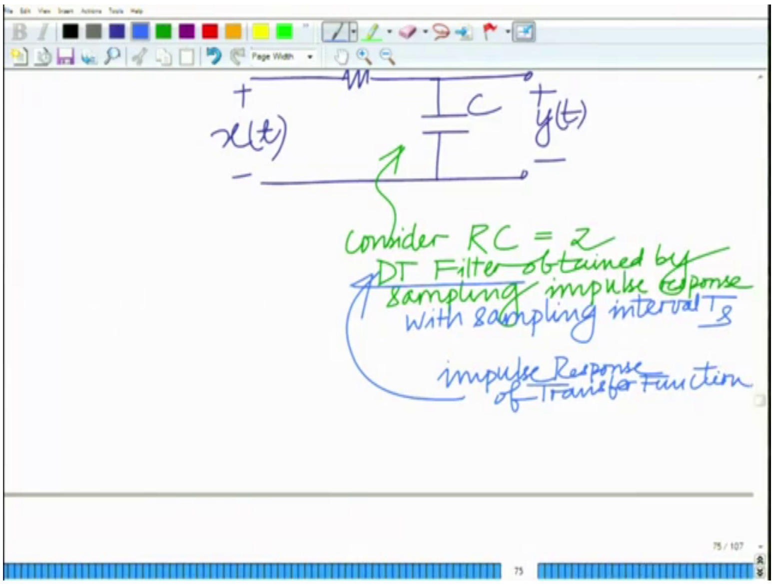

And in RC circuit above consider RC equals 2, what is the discrete time filter obtained by sampling , obtained by sampling impulse response , the above continuous time impulse response with sampling duration , with sampling interval

So, what. So, this is an interesting problem; first you can try to spend some time appreciating it. So, we have this continuous time system all right its a serial RC filter all right if, it has an impulse response. Now I want to sample this continuous time impulse response and derive a discrete time impulse response. So, so that gives us a discrete time LTI system with a discrete time impulse response. We want to find what is either the difference equation or the transfer function that describes the equivalent discrete time LTI system

So, basically what is the discrete time filter or you can say in terms of either the impulse response or the transfer function and this can be found as follows. Now we know the difference equation is given as.

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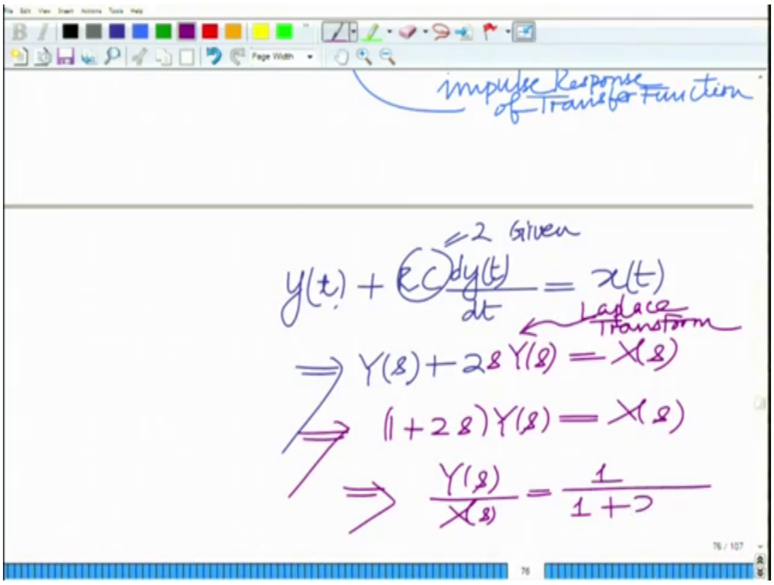

and we have seen this several times before yt is the voltage across the capacitor plus from ohms law R into I that is C dv by dt or C dy t by dt; that is the potential drop across the resistance and this has to be equal to the input voltage that is xt , which implies that y of which implies that now taking the s transform Y of s plus RC and RC we know equals 2 . So, Y of s plus 2, so given RC equal to 2. So, given this RC equal to 2 that is given ok

So, 2 into dy by dt has laplace transform s Ys equals Xs. So, we are basically taking laplace transform. So, this implies 1 plus 2 s into Ys equals Xs this implies Ys divided by X s equals 1 over 1 plus 2 s now this is 1 over 1 plus 2 s .

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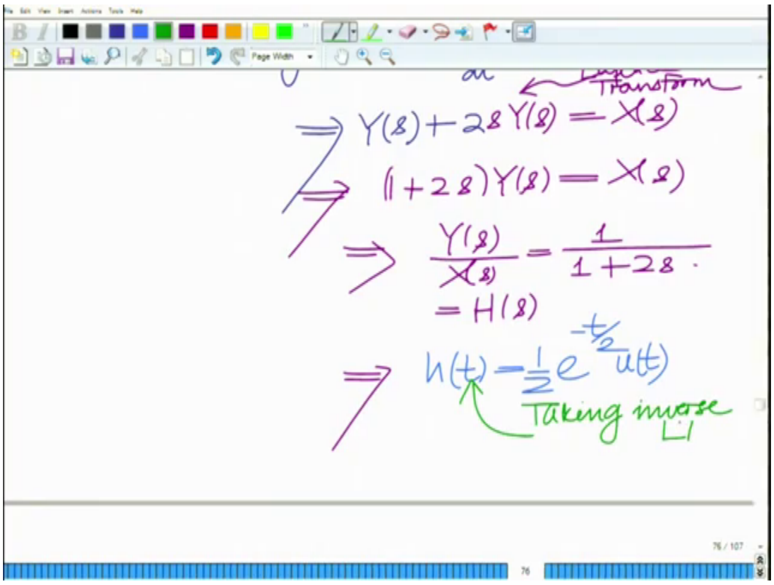

This is your H of s that implies taking inverse laplace transform, we have ht equals half e raised to minus t by 2 ut ok. So, this is basically taking the inverse laplace transform taking the inverse laplace transform.

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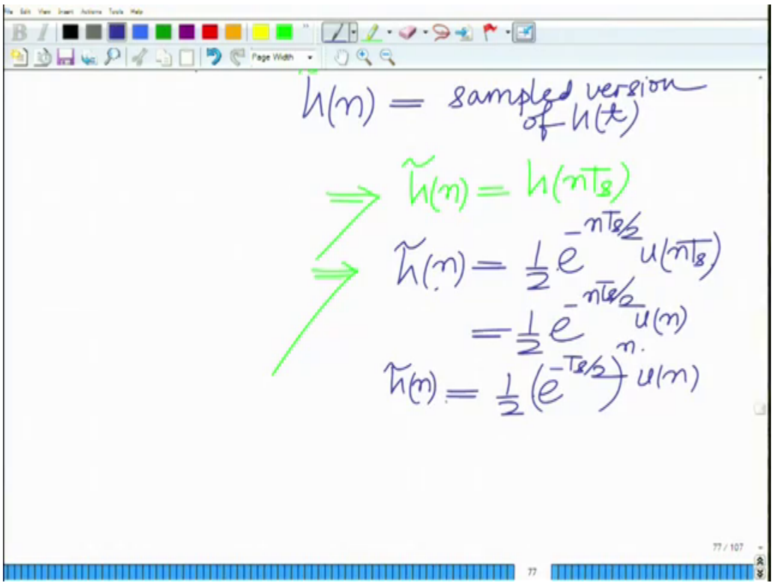

And remember hn is obtained by sampling this, hn equal sampled with sampling duration ts when you sample it, you have you obtain the discrete time impulse response h n. So, h n, now if you look at this its very simple hn equals h of the continuous time . So, let us do a . So, lets call this h tilde n h tilde n equals h of to distinguish them this is h of n ts, where h is the continuous time

Which implies now h tilde n equals half e raised to minus n ts by 2 u n. In fact, that is what u of n ts, but u of n ts. Remember this is e raised to half a raised to minus t by 2 u t. So, this will become u of nts, but u of nt s is the same as u of n. So, this will be half e raised to minus nt s over 2 u of n because t s is greater than zero. this is equal to half e raised to minus t s over 2 raised to the n un , h tilde

So, this is the impulse response of the equivalent discrete time system. this is the impulse response of the equivalent dt system ok.

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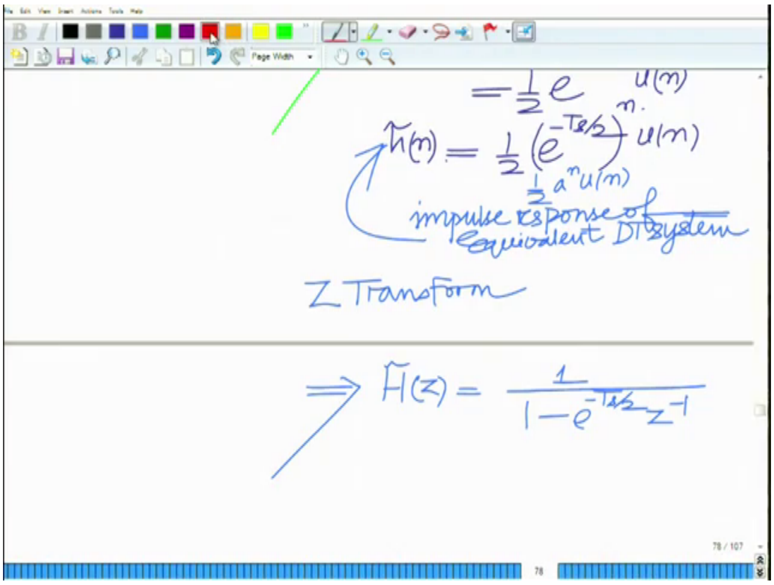

Now taking the z transform yields H of z; that is a transfer function h tilde n is half, now this is you can treat this as half a raised to the n u n where a is e raised to minus t s by 2. So, H tilde z will be half 1 minus e raised to 1 minus az inverse. So, 1 minus e raised to minus t s over 2 z inverse, and this is equal to.

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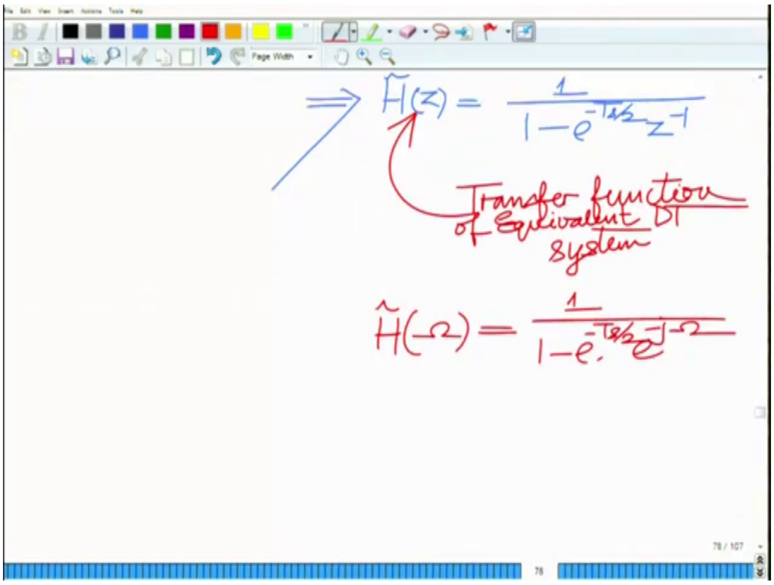

And this is basically the transfer function of the equivalent dt system, discrete time system . this is a transfer function of the equivalent discrete time system .

Now, one can derive the discrete time Fourier transform the DTFT by replacing z by e raised to j omega we know that. So, H tilde of omega. now 1 over 1 minus e raised to minus t s over 2 e raised to minus j omega or 1 over 1 minus a e raised to minus j omega, where we have already seen a is e raised to minus ts over 2 and now we can also find the difference equation as follows.

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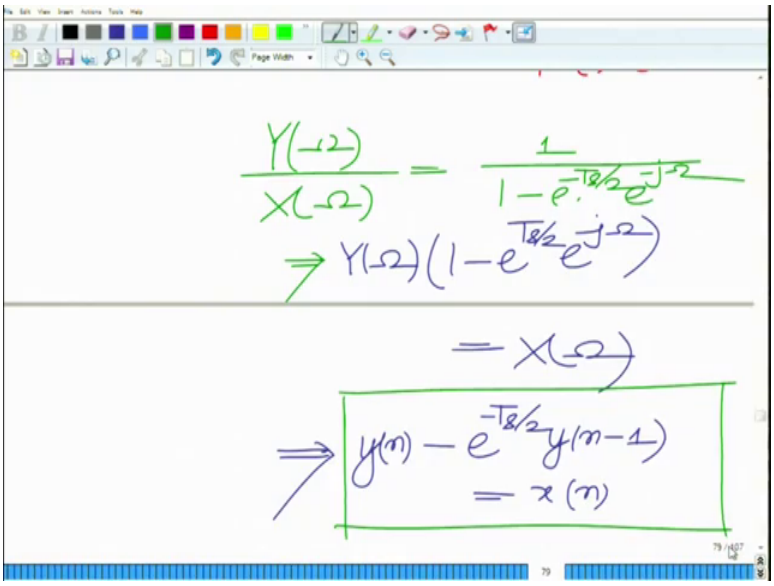

So, we have Y omega over X omega equals 1 over 1 minus e raised to ts over 2 e raised to minus j omega n , which means Y omega into 1 minus e raised to minus t s over 2 e raised to minus j omega is equal to X of omega which implies that difference equation is now given as y of n minus e raised to ts over 2 yn minus 1 equals x of n , this is the equivalent discrete time system or.

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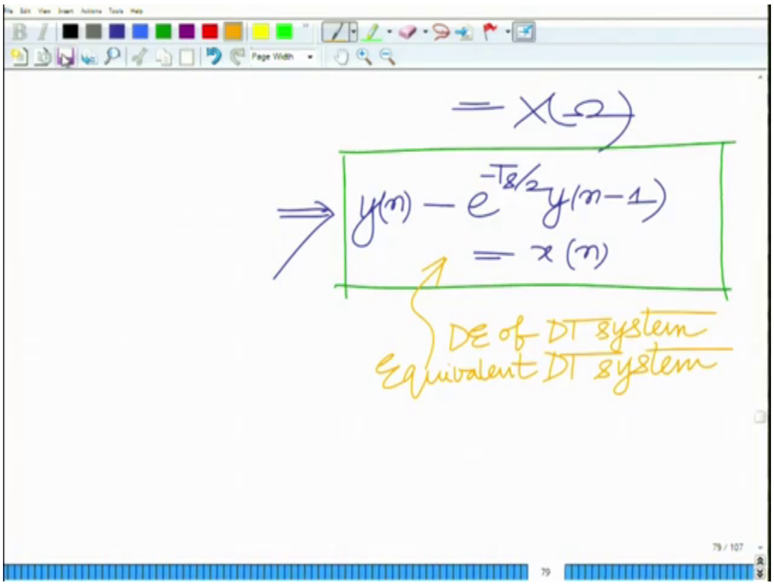

In fact, de of the equivalent discreet are, there is diff difference equation of the equivalent discrete time system all right. So, we have found the impulse response transfer function as well as the difference equation that describes the equivalent discrete time system obtained by sampling the impulse response of the continuous time LTI system all right. So, let us stop here and we will continue with other problems in the future modules

Thank you very much.