**Principles of Signals and Systems**
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**Lecture - 63**
**Examples Problems: DTFT of Cosine, Unit Step Signals**

Hello, welcome to another module in this massive open online course. So, we are looking at example problems in the Fourier analysis of discrete time signals in particular the Fourier, the DTFT. There is a Fourier analysis of discrete time a periodic signals. So, let us continue our discussion. So, you want to look at example problem on the DTFT correct. So, we are looking at example problems on the DTFT discrete time Fourier transform

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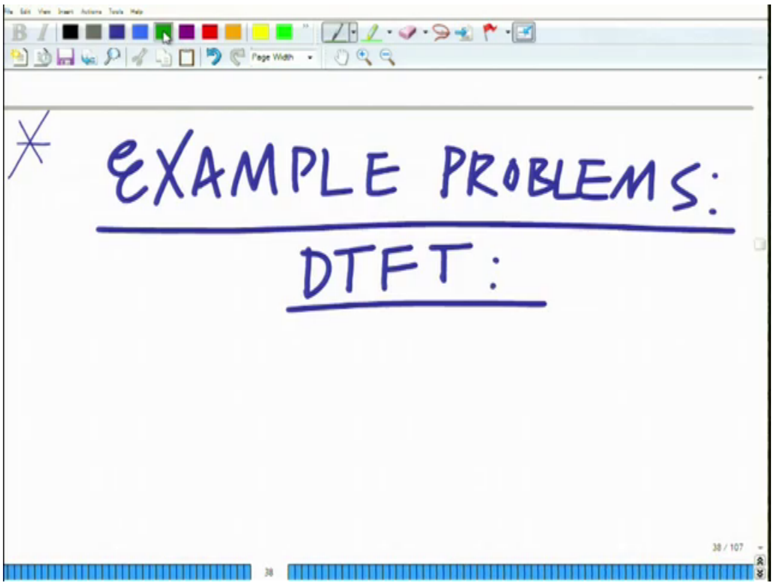


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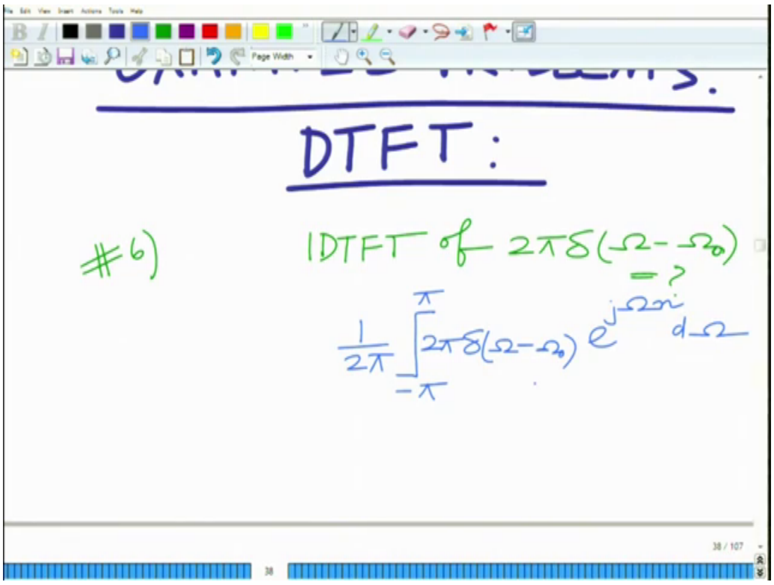


Let us look at example number 6, is fairly simple that is the inverse DTFT, the IDTFT, so we want to find the IDTFT of this function this signal that is 2 pi delta omega naught omega minus omega naught and what is this.

And the IDFT can be found as follows. This is 1 over 2 pi integral over any interval minus pi to pi delta omega minus omega naught e raised to j omega n e raise to j omega n d omega. In fact, this is 2 pi delta omega minus omega naught. So, let me just correct that this is 2 pi delta omega minus omega naught and this will be.

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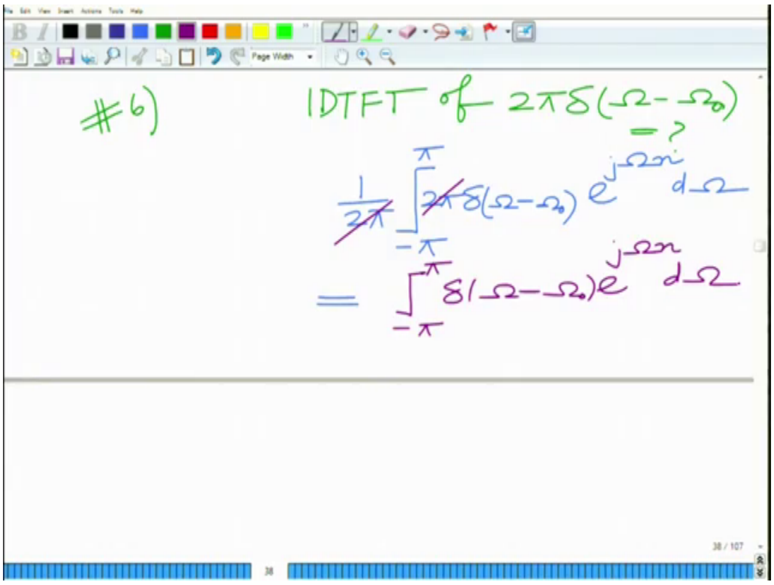

Now the 2 pi is cancel, this will be integral minus pi 2 pi delta omega minus omega naught e raise to j omega and d omega which is basically e raise to j omega n, you substitute here omega equals omega naught and this will be e raise to j omega naught n.

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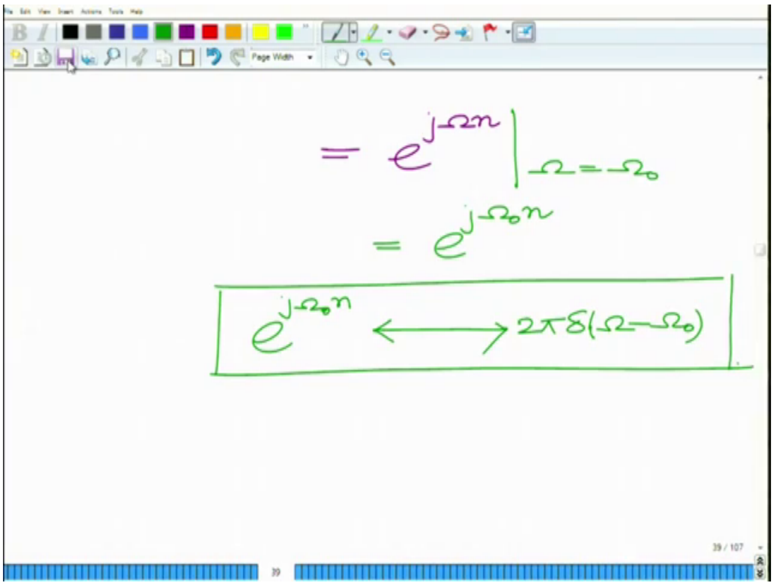

So, that is the inverse. So, we have e raise to j omega naught n basically, that has your discrete time Fourier transform 2 pi delta omega minus omega naught or basically the 2 pi delta minus omega naught has the inverse discrete Fourier transform; that is erase to j omega naught. So, this illustrates how to compute the inverse discrete Fourier transform all right

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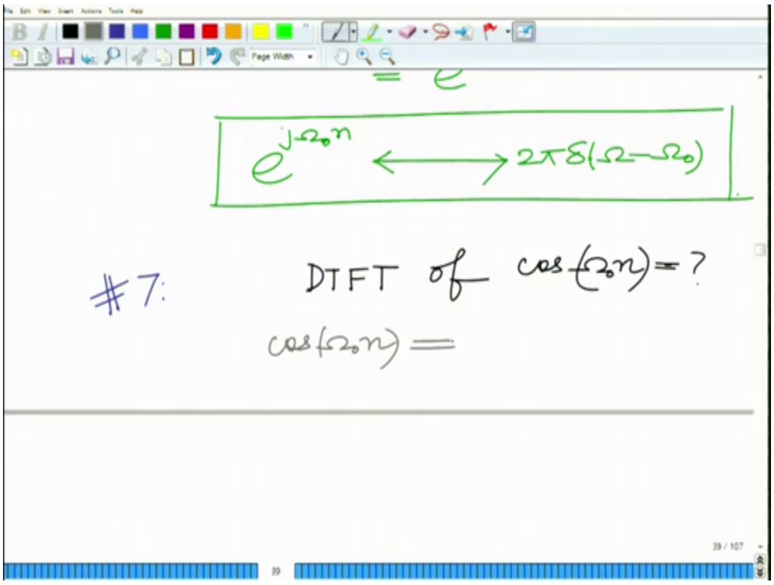


Now let us apply this result to compute the discrete Fourier transform DTF, discrete time Fourier transforms, sorry inverse DTFT of cosine omega naught n and the DTFT of cosine omega naught n. What is the DTFT of cosine omega naught n, and remember cosine omega naught n can be written as, cosine omega naught n equals half e raise to j omega naught n plus half e raise to minus j omega naught n e raise to j omega naught n has DTFT 2 pi omega minus omega naught.

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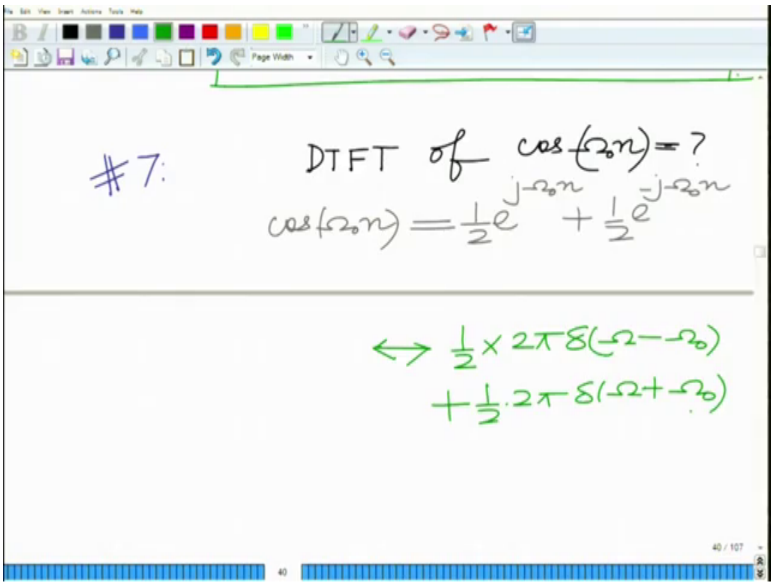

We have as we have seen before in the previous examples or half e raise to j omega naught and has DTFT pi delta omega minus omega naught. Similarly half e raise to minus j omega naught and will have the DTFT pi delta omega plus omega naught. So, this will have half times 2 pi delta omega minus omega naught plus half into 2 pi delta minus minus omega naught; that is delta plus omega plus omega naught

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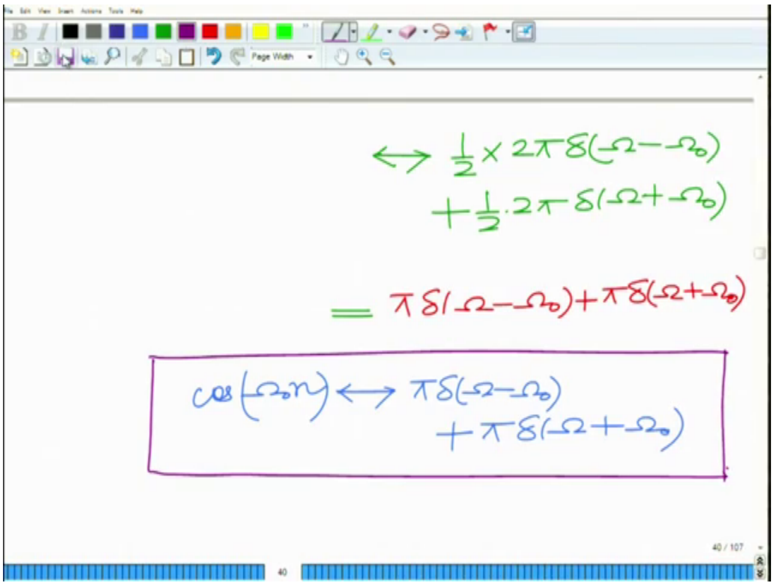


So, this will be equal to basically pi delta omega minus omega naught plus pi delta omega plus omega naught. So, cosine omega naught n has the DTFT pi delta omega minus omega naught plus pi delta of omega plus omega plus omega naught.

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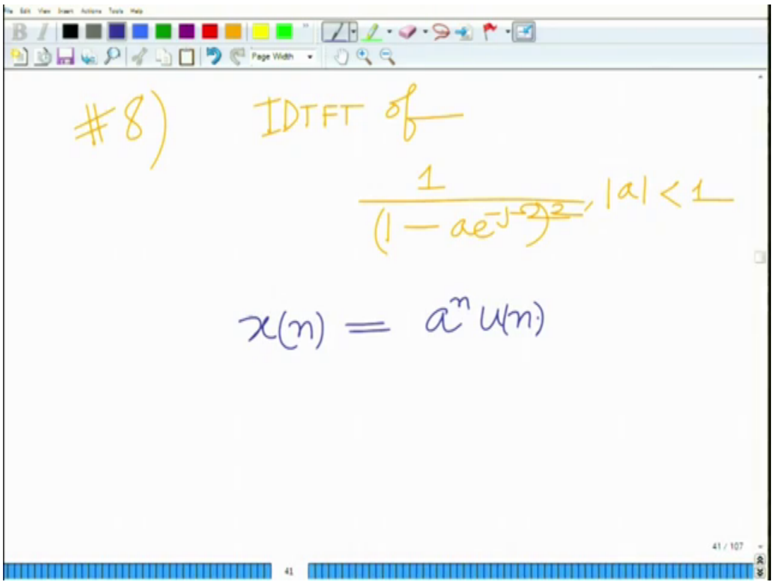


Let us continue to another example. We want to find the inverse DTFT. The inverse, let us find the inverse DTFT of the given signal of, let us call this Y of omega omega equals or let us just call this inverse DTFT of the given DTFT; that is 1 minus ae power minus j omega whole square and also given the condition that magnitude a is strictly less than 1. So, we are required to find the inverse DTFT of this signal.

Now let us start by considering the signal xn is 2 equals a raised to the power of n un all right, where a is magnitude a is less than 1 all right and we will manipulate the DTFT of this signal to derive the inverse DTFT of the given DTFT. So, consider, we start by considering, because this is related to the DTFT of a n; the exponential signal that is e raised to the or the decaying signal a raised to the n u n.

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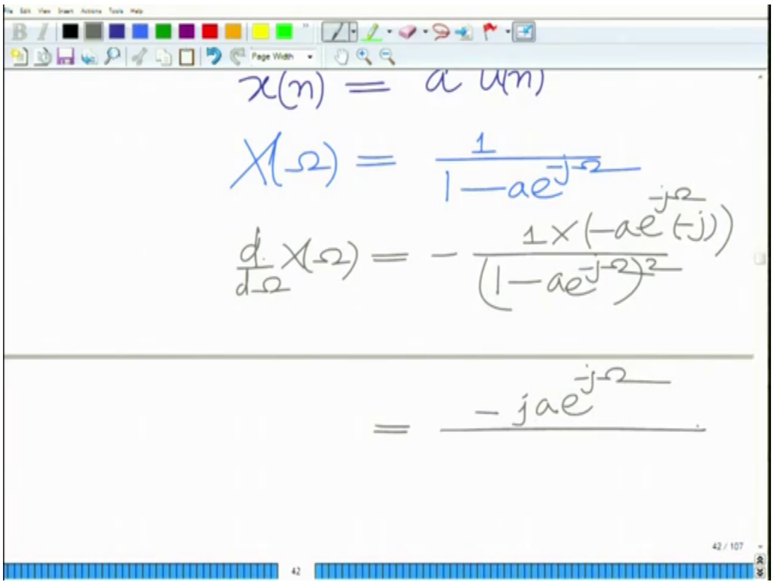

If you look at the DTFT of this you have X of omega equals 1 minus a e raise to minus j omega ok. Now if you differentiate this, if you differentiate this with respect to omega you will have minus or you will get.

First you will get minus 1 over 1 minus ae power minus j omega whole square correct, multiplied by now derivative of minus e raise to minus g omega that is minus a, e raise to minus g omega derivative is minus j omega times minus j ok. And this will basically be.

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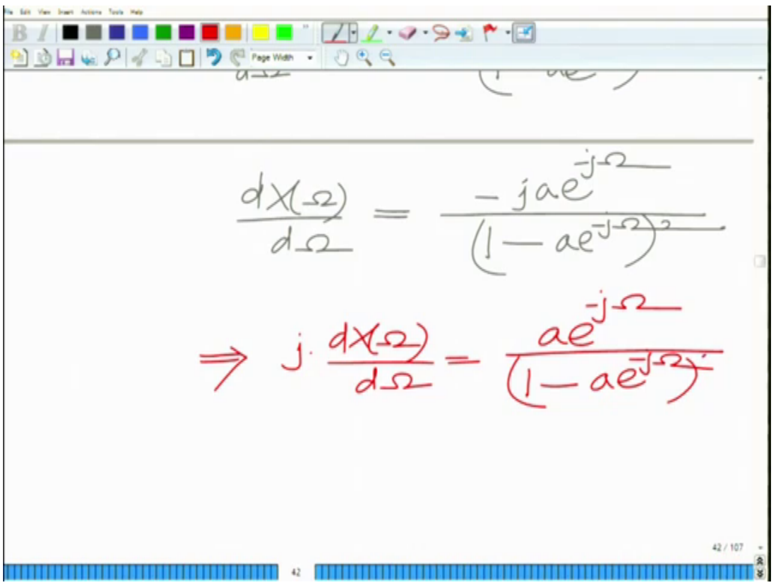

Now if you simplify this, this is minus j a e raise to minus j omega divided by 1 minus a raised to the minus j omega whole square; that is your dx omega by d omega; that is the derivative of X of omega, which implies that j dx omega by d omega will be equal to, multiplying both sides by j you will have minus j into j is 1. So, this will simply be on the right hand side ae raise to minus j omega divided by 1 minus ae raise to minus j omega whole square.

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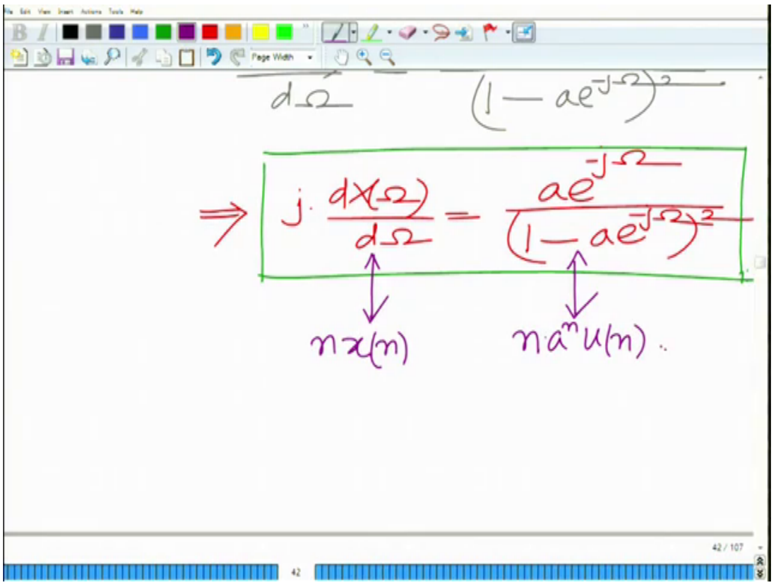

So, that is basically you are corresponds to your j times the derivative of x omega with respect to or not, but realize that jdx omega by d omega, that corresponds to, if x omega is the DTFT of x of n, j dx omega over d omega that corresponds to the DTFT of n times xn. So, which means this corresponds to this is the DTFT of the signal n. So, this j dx omega or d omega, this is the DTFT of n xn.

So, this is basically the, because we know we know we know X of omega corresponds to an un, this is n times x n that is n times an un ok. So, that is your a e power minus j omega 1 by 1 minus ae raise to minus j omega whole square ok. Now let us simplify this quantity ae raise to minus j omega divided by 1 minus ae raise to minus j omega whole square.

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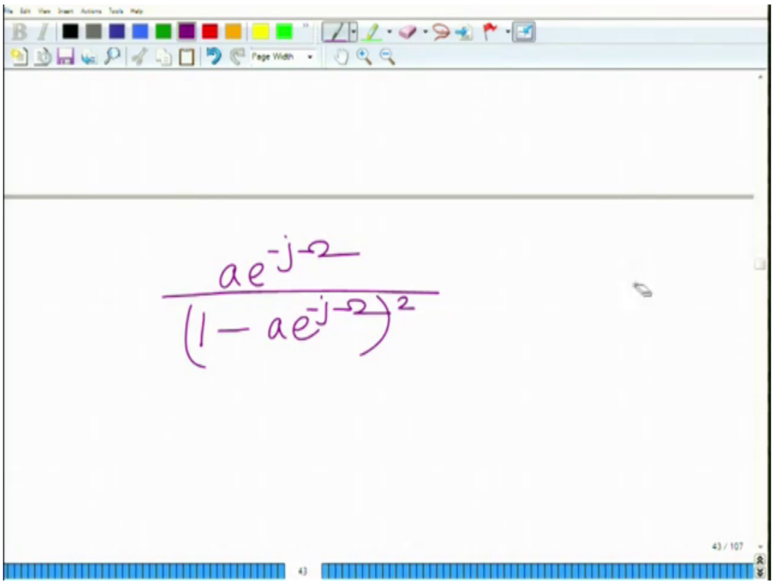


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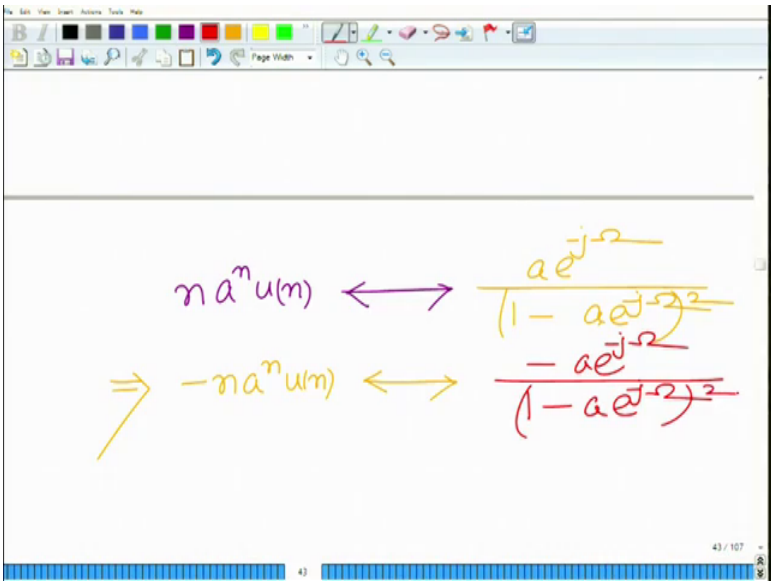


So, this corresponds to or let me just write it this way n a raise to n un, this has DTFT that is your ae raise to minus j omega divided by 1 minus ae raise to minus j omega whole square, which implies if you take the negative of this minus n a raise to n un that has the DTFT which is given by minus a raise to minus j omega by 1 minus ae raise to minus j omega whole square.

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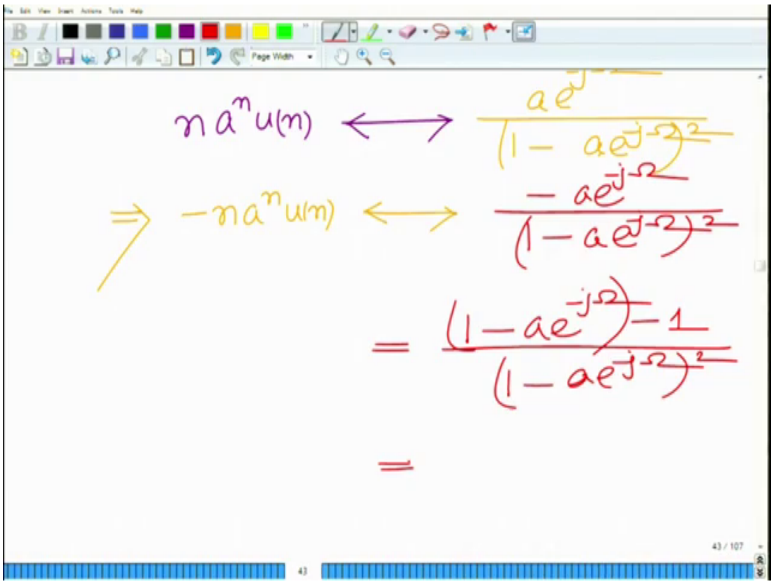


Now you can add and subtract 1, so that will be basically equal to 1 minus ae raise to minus j omega minus 1 over 1 minus ae raise to minus j omega whole square. Now, this you can simplify as 1 minus ae raise to minus j omega by 1 minus ae raise to minus j omega whole square.

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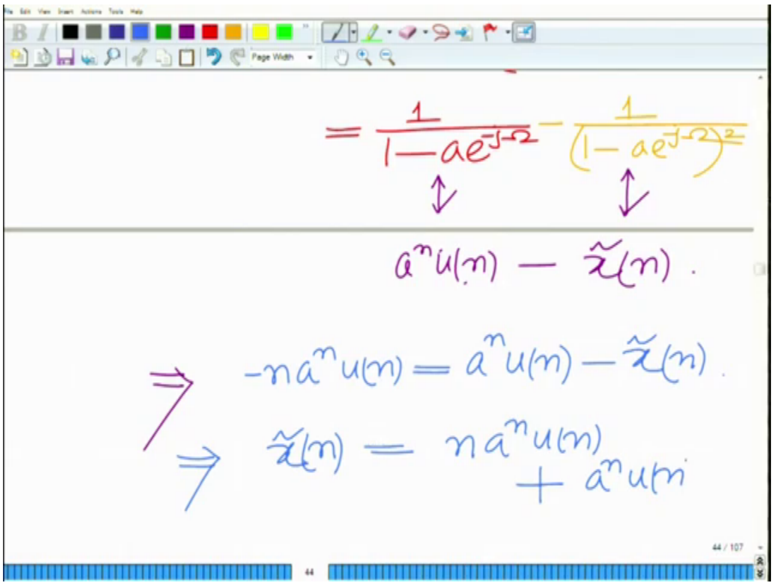


So, that is 1 over 1 minus ae raise to minus j omega minus 1 over 1 minus ae raise to minus j omega whole square. And now you know that this quantity 1 over 1 minus ae raise to minus j omega that has the inverse discrete Fourier transform; that is x of n.

Remember this is simply X of omega. So, that is inverse discrete Fourier transform that is x of n which is an un.

The rest, the remaining term is corresponds to basically, is basically the quantity for which we want to find the inverse discrete Fourier transform. So, if you take the inverse discrete Fourier transform of this you will get a xn; that is a to the power of n un and taking the inverse discrete Fourier transform of this we get, let us call this x tilde n, when x tilde n is the inverse discrete Fourier transform of the quantity 1 over 1 minus ae raise to minus j omega whole square which is the given DTFT.

So, this implies basically what you have is n a raise to n un is a to the power of n or in fact, minus n a raise to nu n is a raise to the un a raise to then un minus x tilde n, which implies that x tilde n is n a raise to n un plus a raise to n un, which is basically equal to n plus 1 into a raised to the n to n.

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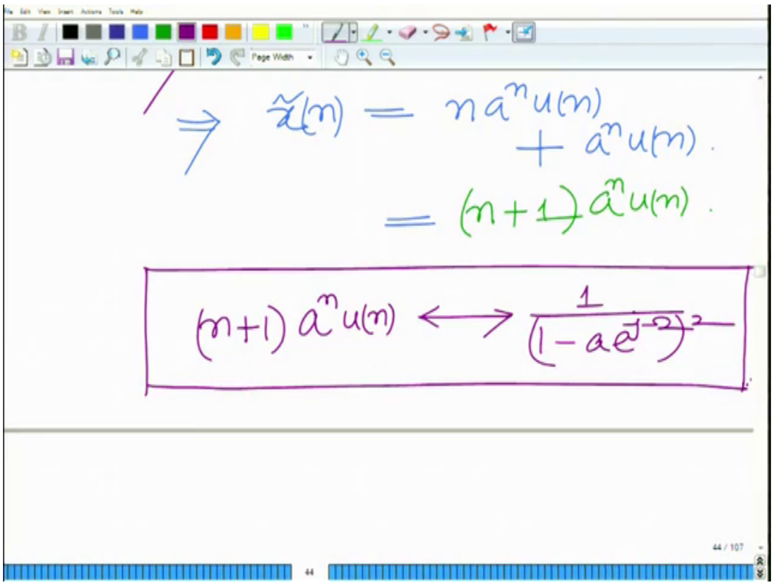

Therefore the inverse discrete Fourier transform of the given quantity is n plus 1 e raise to the power of n un ok, or you can say n plus 1 e raise to the power of n un has the discrete time Fourier transform 1 over 1 minus a e raise to minus j omega whole square, where magnitude of a is less than, magnitude of a is less than 1. So, this is the, basically this is the inverse DTFT of given quantity

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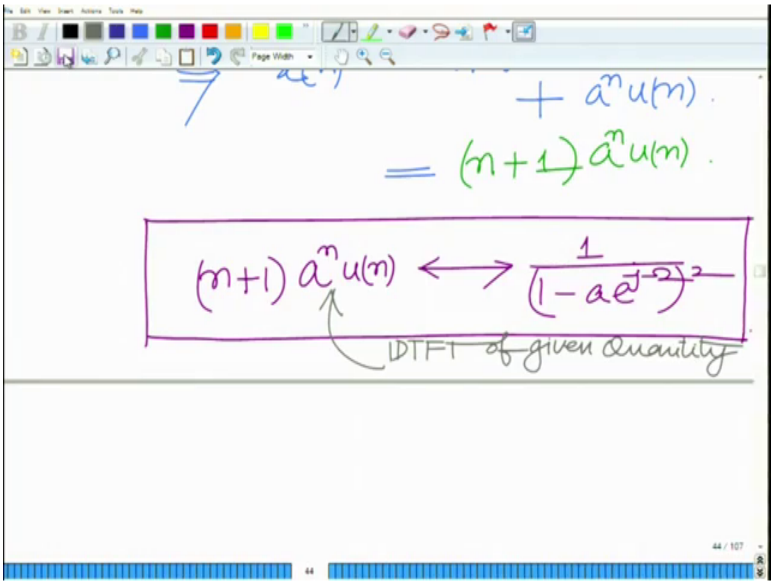


IDTFT, this is the inverse DTFT of the given quantity. Now another way to do, it will be as follows. Another way to do it will be as follows. So, you can see basically that.

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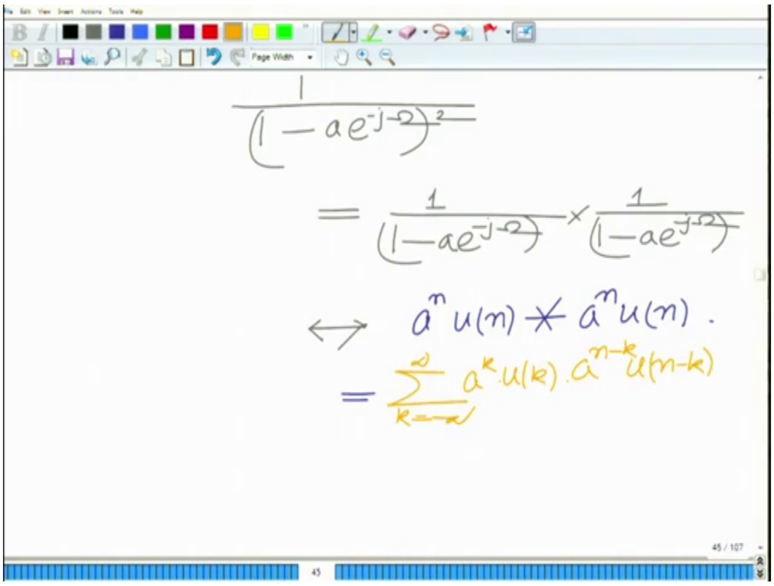


Look at this I have the given DTFT is 1 over 1 minus ae power minus j omega whole square; that is equal to 1 over 1 minus ae power minus j omega times 1 over 1 minus ae power minus j omega.

Now if you look at it, this is basically the multiplication in the, is basically multiplication in the Fourier domain the DTFT domain, which means it must be the convolution of the corresponding inverse DTFT. This is corresponding time domain signals, the corresponding time domain signal for 1 over 1 minus ae raised to minus j omega is a raised to n un.

So, this is basically if you take the inverse DTFT, this corresponds to basically, the inverse DTFT corresponds to. This is the multiplication in the Fourier domain, so in the time domain. In fact, discrete time domain this is the convolution of the corresponding inverse DTFTs, which is a raise to then un.

And that convolution is, basically I can write it as summation k equals minus infinity to infinity a of k uk into a of n minus k un minus k which is basically equal to a of k into a of n minus k is a of n, which is does not depend on k hence comes out of the summation.

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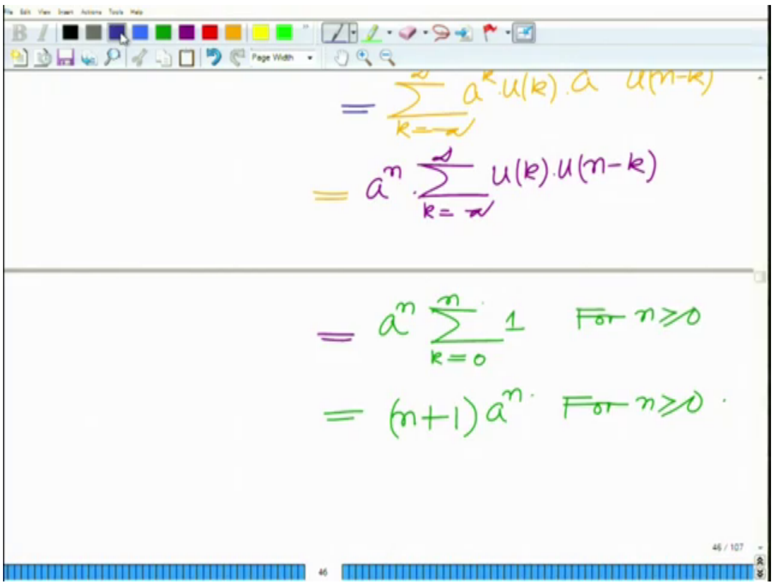


Now if you look at summation k equal to minus infinity to infinity u of k into u of n minus k. This non0 only for this summation you can see, is non0 only for k greater than equal to 0 and k less than equal to n all right.

So, this summation for n greater than or equal to 0 will be, I can write it as, for n less than 0 it is 0, for n greater than or equal to 0 this is simply a raise to n k equal to 0 to n times 1 for n greater than equal to 0. So, this is basically n plus 1 a raised to the n for n greater than equal to 0 and you can write this as n plus 1 into a raised to the n un ok.

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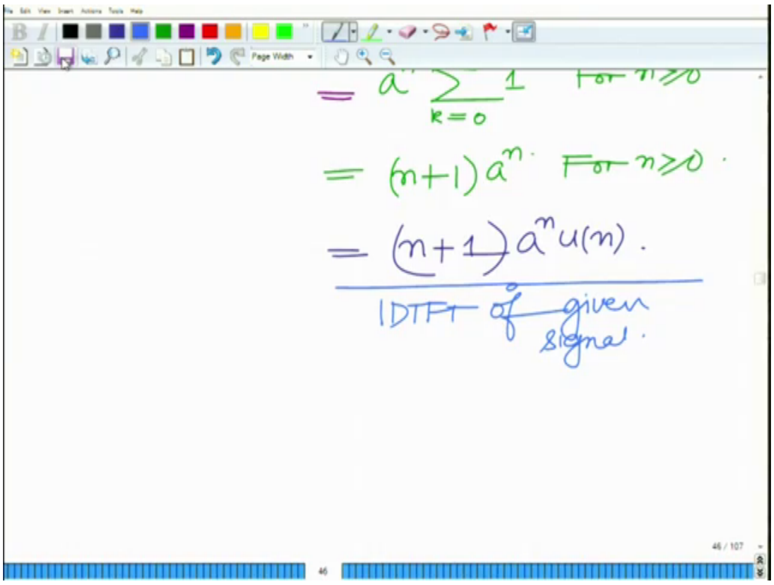


So, this is another way. Again this is the ID at FT of the given signal, which seems to be a slightly simpler way n to, seems to be a slightly simpler way to derive the same result all right. Both approaches yield the same answer, these are two different approaches; one is my differentiation and the other is by simply realizing, that its a convolution of two known signals in the time domain all right. So, let us do another example problem. So, this was problem number, I believe problem number 8. So, we will have problem number 9.

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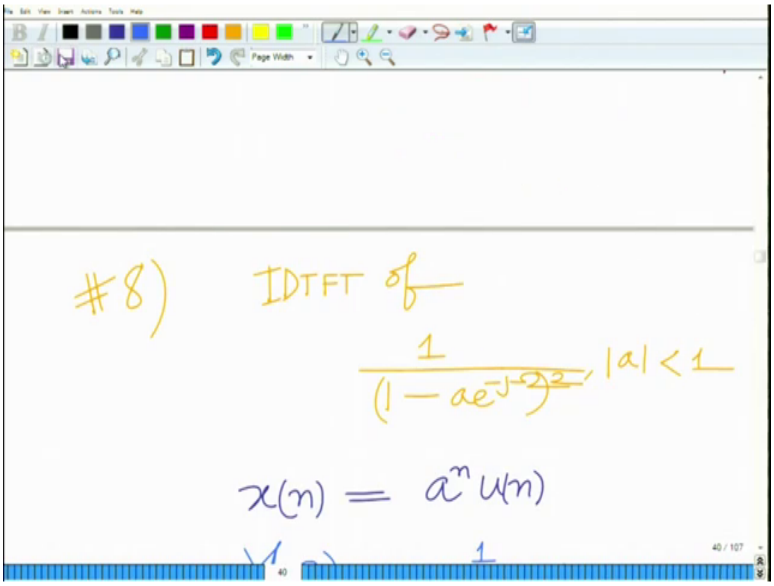

So, problem number 9 is as follows.

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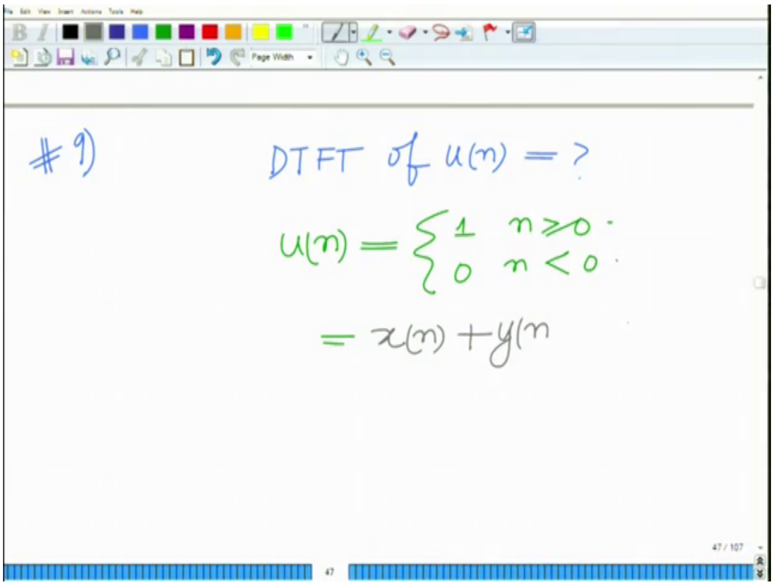


This again another standard problem to find the inverse, to find the DTFT of u of n; the unit step function; what is the DTFT. Remember u of n equals 1 for n greater than equal to 0, 0 for n less than 0. So, this is basically your x of n. I can write u of n as x of n plus y of n, where x of n equals half.

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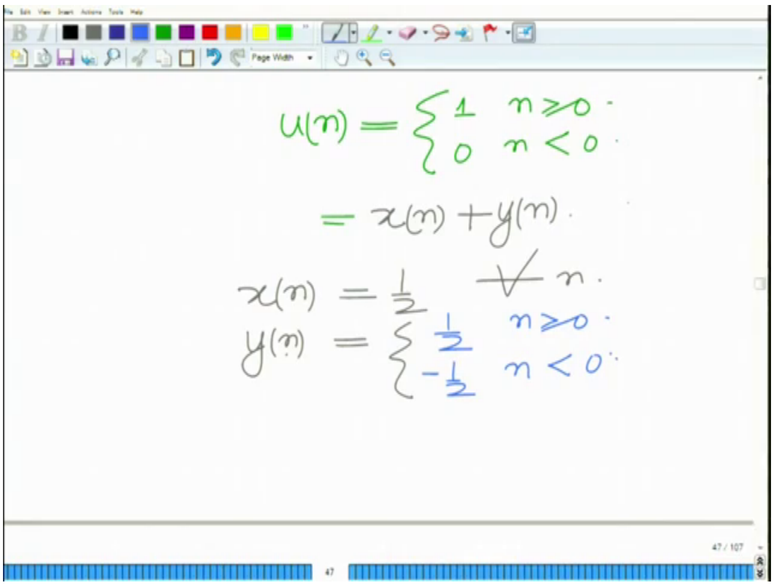

For all n and y of n equals half for n greater than equal to 0 and minus half for n less than 0 ok. So, I can write it as x of n plus y of n, where x of n is half for all time samples n and y of n is half for n greater than equal to 0 and minus half for n less than 0 ok.

Now if you look at the DTFT of x of n, x of n equals half for all n, implies this is basically X of omega equals half.

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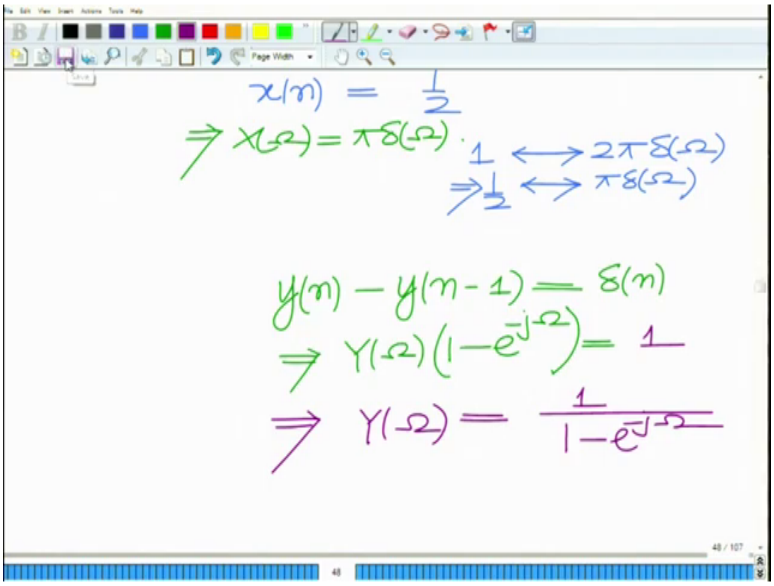


So, if you look at 1, remember one has the DTFT 2 pi delta omega. So, this implies half has the DTFT pi delta omega. So, x of n equals half implies X of omega equals pi delta omega. So, that is the first point.

Now, if you look at yn which is half and greater than equal to 0 and as minus half n less than 0 if you look at y n minus yn minus 1; that is equal to delta n ok, yn minus yn minus 1 equals delta n.

This implies y omega into, that is by the time difference property 1 minus e raised to minus g omega equals taking the DTFT of delta n; that is 1 which implies y omega equals to 1 over 1 minus e raised to minus j omega. And finally, we have the given sequences xn minus plus yn un.

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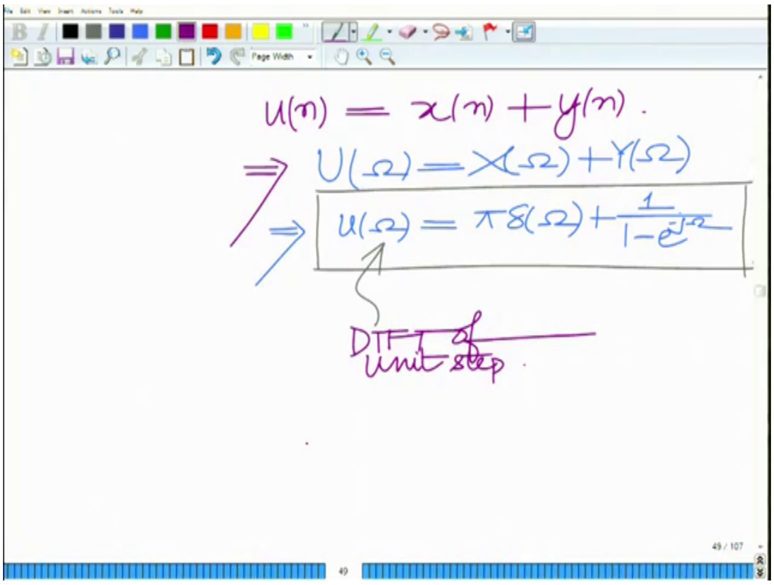


We have written as xn plus yn. By linearity we have U of omega n equals x omega plus y omega implies U of omega equals, this equals X of omega which is basically pi delta omega plus Y of omega; that is 1 over 1 minus e raise to minus j; that is the DTFT of the standard unit step function, unit step signal. So, this is basically your DTFT of unit step. This is the DTFT of the unit step signal all right

Now let us look at the accumulation property that is the next property which is the accumulation property. So, continue in the same example, what is the accumulation property

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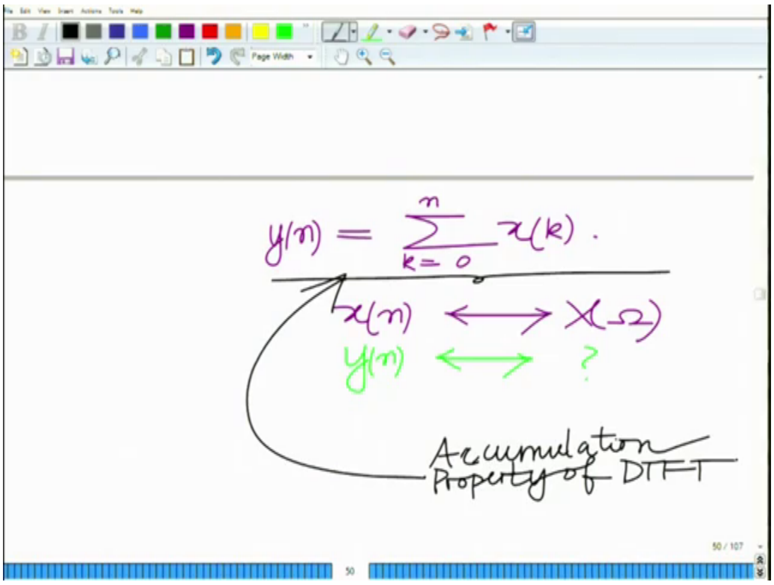


Consider y of n equals summation k equals 0 to n x of k. So, let us say xn has the DTFT X of omega. What is the DTFT of y of n? This is the accumulation property remember. This is termed as a accumulation property of the. Now this system is termed as a accumulator, accumulation property all right.

This is the accumulation property, is similar to integration in the continuous time correct, this is similar to integrate that is you are integrating signal input signal x tau from minus infinity to, from minus infinity to t or 0 to t this known as, its known as the integrator all right. So, similarly this is the accumulator ok. So, this is similar.

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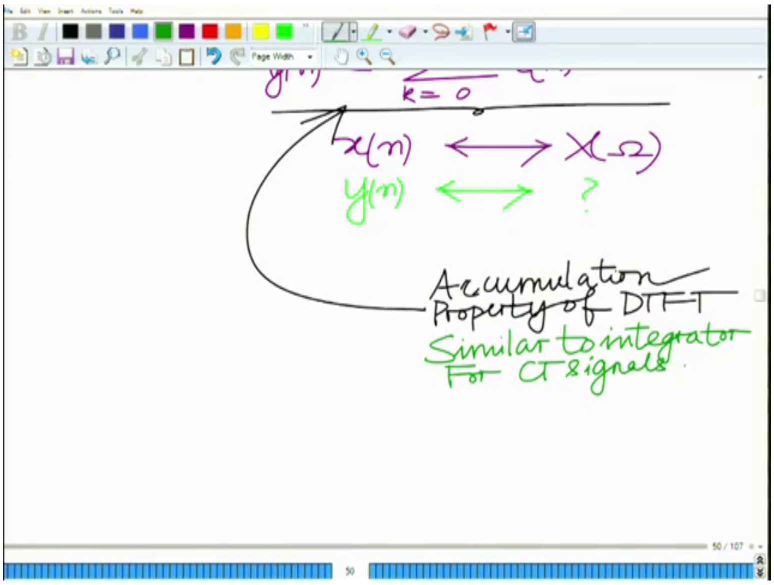


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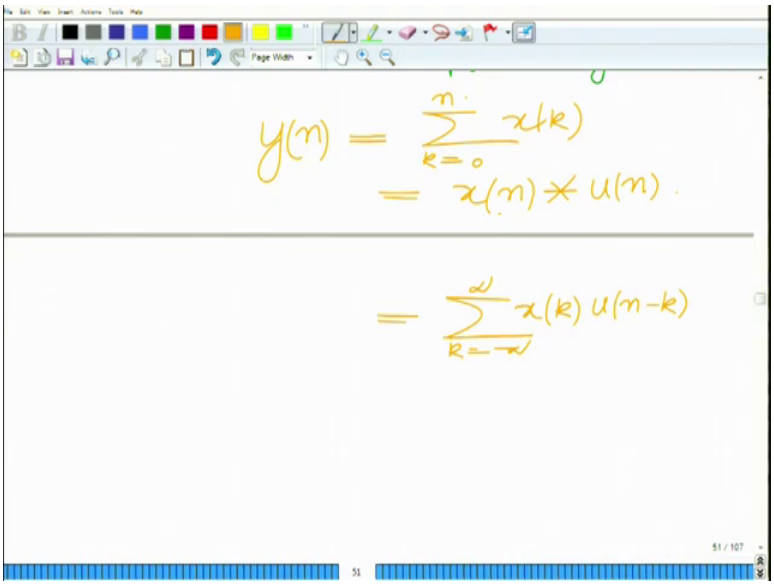

Similar to the integrator for continuous time signals and you can see, now realize very simply that yn is nothing, but basically summation k equals 0 to n xk, which we can see is nothing, but the convolution of xn with un, because convolution of xn with un is similar, simply xk equals minus infinity to infinity xk un minus this simply convolution from.

Sorry this has to be I think, this is convolution from minus infinity to n. So, this is basically, you can see this is simply k equal to minus infinity to n all right. And u n minus k, this is non0 only for k less than equal to n. So, this is basically summation k equals minus infinity to n x of summation k equal to minus infinity to n x of k correct.

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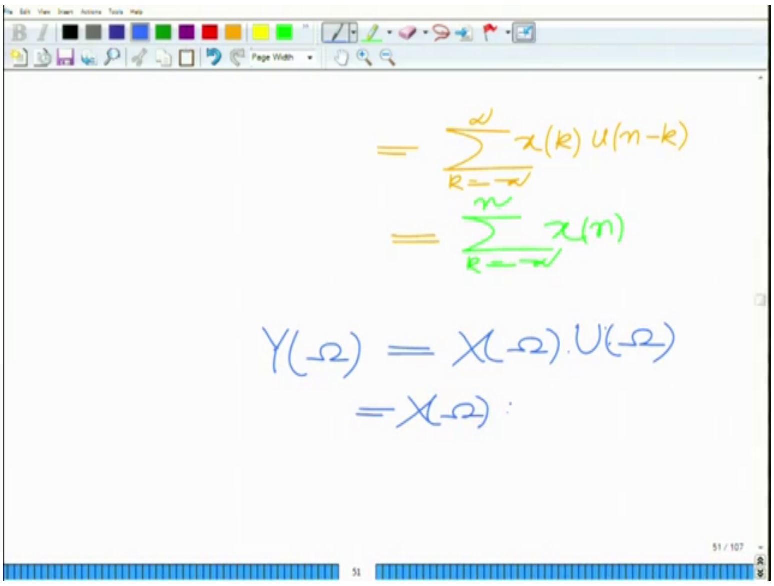

So, basically what you have is that implies, now convolution in time in frequency domain it, because multiplication. So, implies Y of omega which is the DTFT of the accumulator is basically X of omega times U of omega. This is simply X of omega and we have derived the DTFT of the unit step function U of omega; that is given as pi delta omega plus 1 over 1 minus e raised to minus j omega ok,

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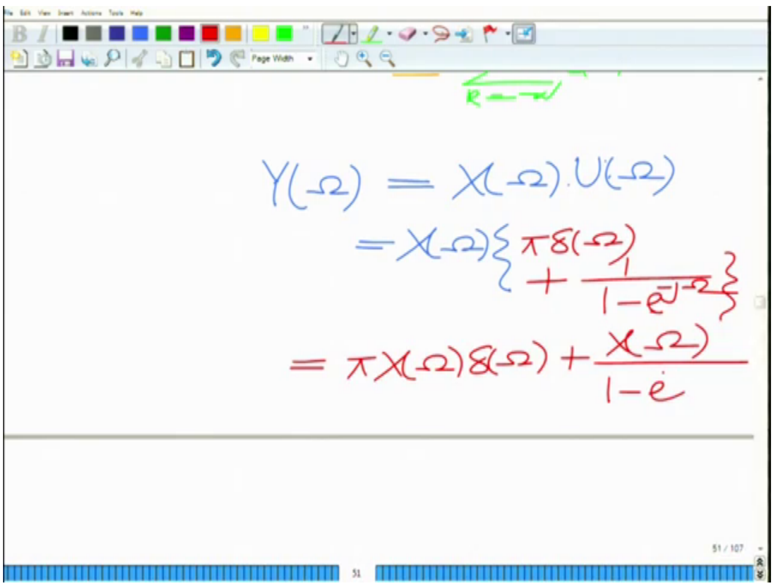

Which is pi times X of omega times d of omega plus X of omega over 1 minus e raise to minus g omega.

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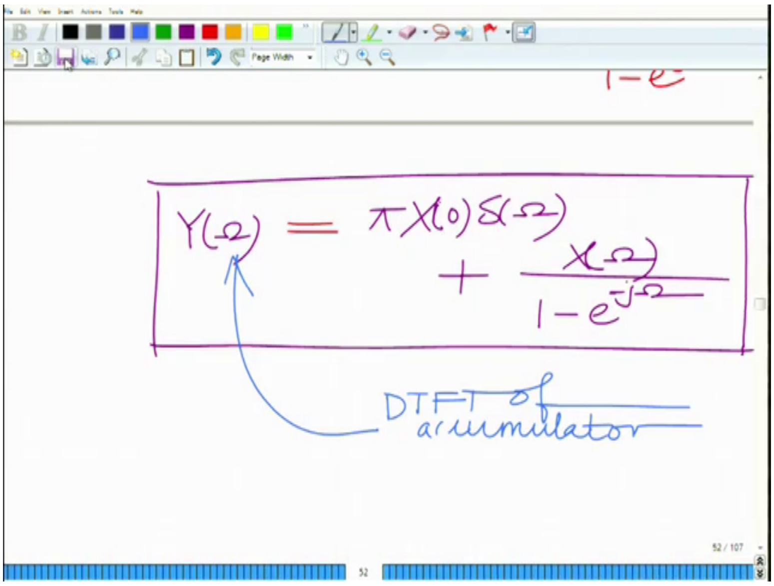


And now you can see that X of omega into d of omega that is nothing, but x of 0 from the properties of the unit impulse function all right, that digital the discrete time impulse function, this is simply x of 0 into delta of omega. So, this is also pi of x of 0 delta omega plus X of omega over 1 minus e raised to minus j omega all right. So, basically that is basically your accumulator. So, that basically gives the DTFT of the accumulator. So, that basically gives the DTFT of the accumulator all right.

So, that let us stop this module here and we will continue looking at similar other problems in the subsequent modules.

Thank you very much.