Principles of Signals and Systems
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Lecture - 62
Example Problems: DFS Analysis of Discrete Time Signals, Problems on DTFT

Hello, welcome to another module in this massive open online course. So, we are looking at example problems right, we have started looking at example problems for the Fourier analysis the Fourier analysis of discrete time signals and we have started with example problems for the discrete Fourier series, all right.

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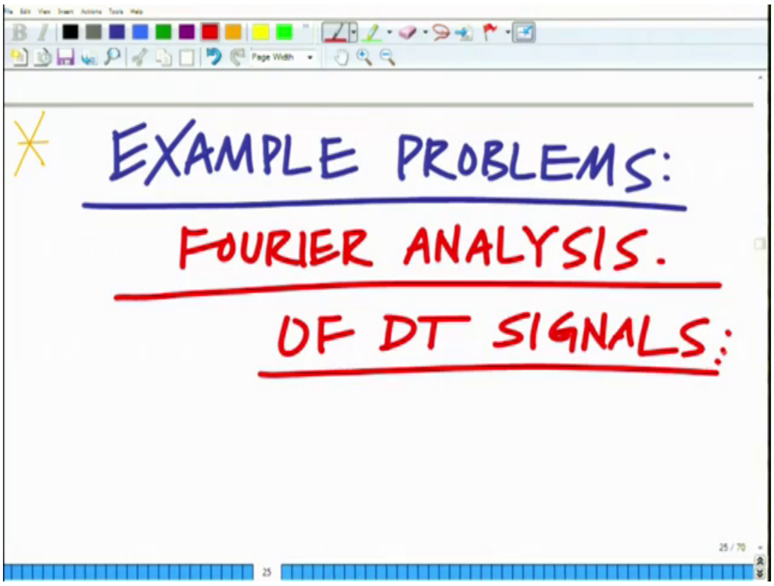

So, let us continue our discussion. We are looking at example problems for the Fourier analysis the Fourier analysis of DT signal discrete type signal and in particular we have started with the discrete Fourier series all right, for discrete time periodic signals all right.

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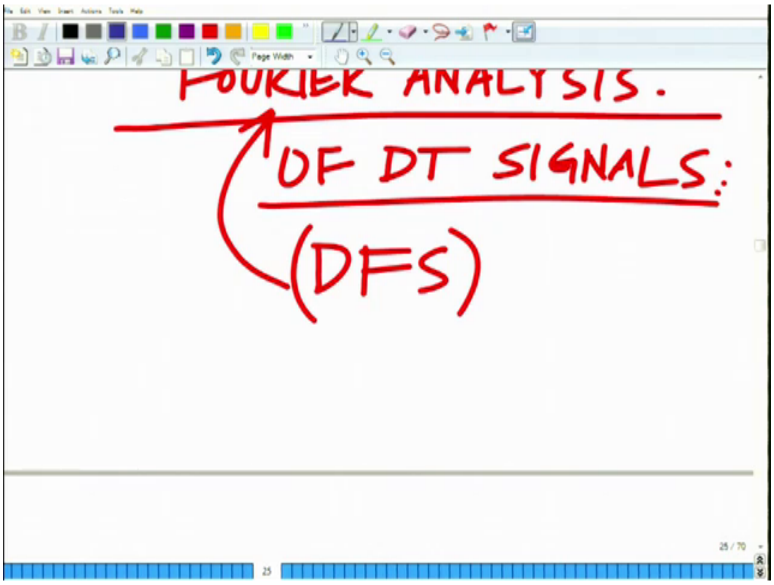

So, let us continue our discussion and let us look at the next problem. So, if you looked at problem number 1 yesterday if I remember correctly in the previous module. So, let us look at the second problem that is problem number 2 which is the following.

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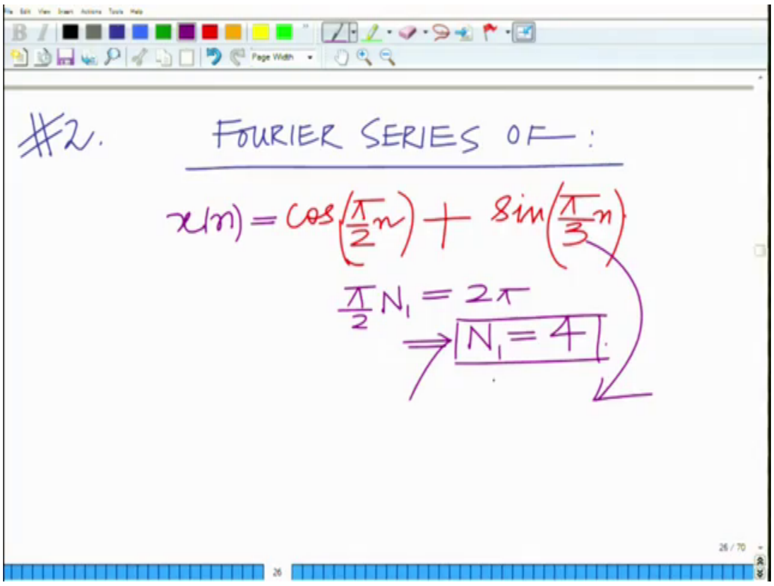

So, we want to find the Fourier series the Fourier series of cosine pi by 2 n plus cosine pi by 2 n plus sin pi by 3 and naturally you can see that these are periodic in the period of this. So, this is a periodic signal all right.

So, this is your x n and as you let cosine pi by 2 n the period of this can be found as follows pi by 2 into k or pi by 2 into N 1 let us call this N 1 equals 2 pi implies because cosine x is cosine x plus 2 pi. So, N 1 equals basically 4 that is the period of the first signal. Similarly for the sin signal we have pi by 3 N 2 equals 2 pi which implies N 2 equals 6.

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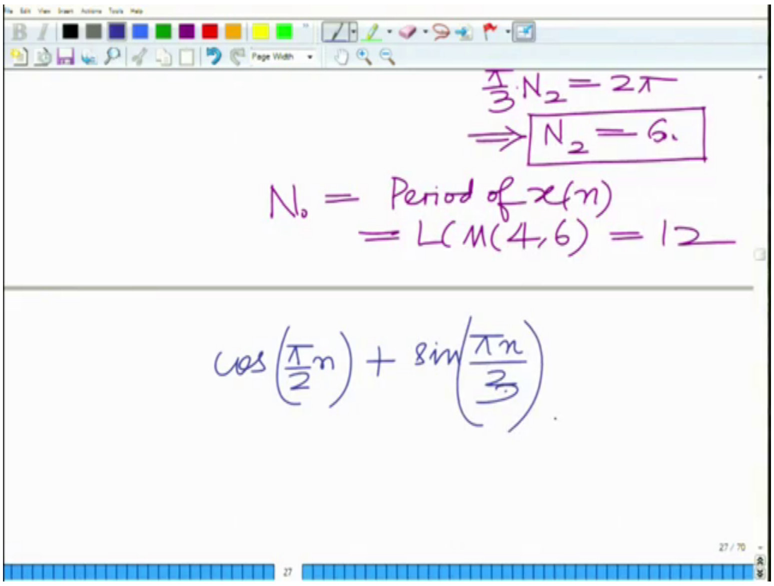

So, period is of the first component that is cosine pi by 2 n is 4 and period of the second component that is sin pi by 3 n is 6 and the period of the sum correct is therefore, the least common multiple because it has to be a multiple of both and therefore, the minimum possible size length is the least common multiple of these.

So, therefore, the period fundamental period of this sum signal is N naught which is the least cover LCM of 4 and 6 which is 12. So, N naught equals period of x m equals LCM of 4 comma 6 the least comma multiple of 4 comma 6 which is 12 ok.

And now, I can write e raise to now, I can write using the properties of cosine pi by 2 n using the properties of complex exponentials pi n by 3. I can write this as this can be written as well e power j pi by 2 n plus e power minus j pi by 2 n divided by 2 plus e power j pi by 3 and sin pi by 3 n is e power j pi by 3 and minus e power minus j pi by 3 n divided by 2 j.

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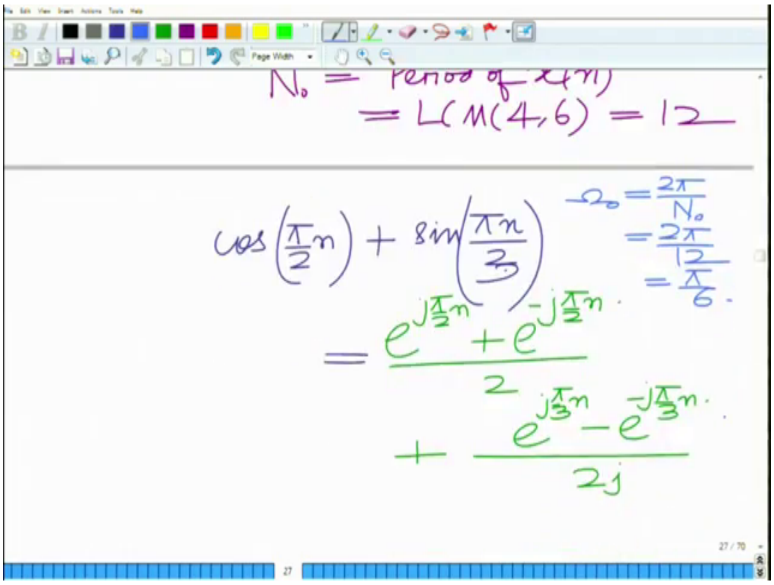

So, therefore, this is equal to well half. Now, look at this pi by so omega naught, so if you look at omega naught. Now, the lcm N naught equals 12. So, we have omega naught equals 2 pi over N naught equals 2 pi over 12 equals pi by 6. So, pi by 2 is basically 3 omega naught. So, omega naught is basically pi by 6. So, pi by 2 is 3 omega naught and pi by 3 is 2 omega naught ok.

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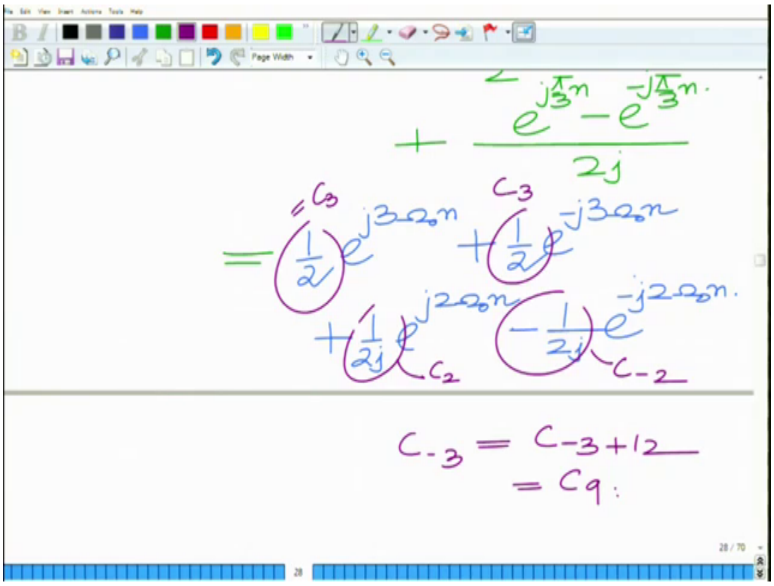

So, I can write this as half e power j 3 omega naught n plus half e power minus j 3 omega naught n plus 1 over 2 j e power j 2 omega naught n minus 1 over 2 j e power minus j 2 omega naught.

And clearly you can see this is the coefficient of 3 omega naught that is half. So, this is equal to C 3, this is C of minus 3, this is C of 2 the DFS and this is C of basically minus 2, but C of minus 3 everything is modulo n. So, C of minus 3 C of minus 3 plus 12 that is n minus k so that is basically C of 9 and C of 2 minus 2 equals C of minus 2 plus minus 2 plus 12 that is equal to C of 10.

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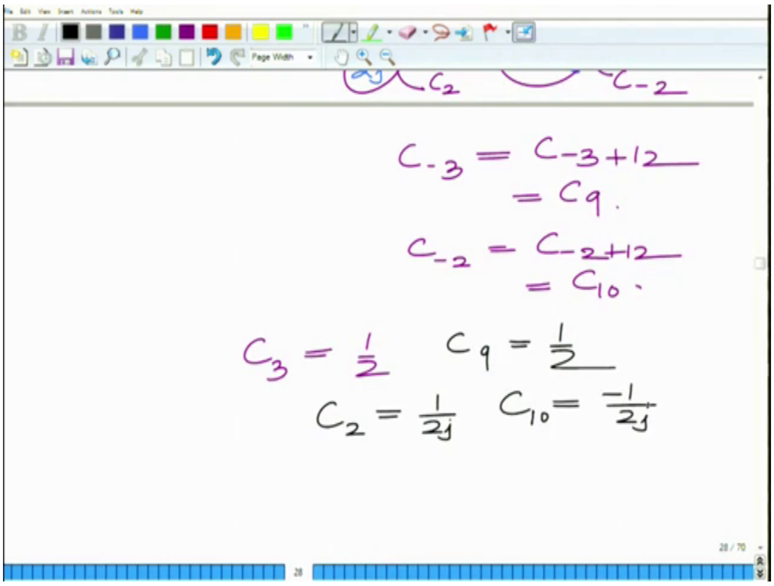

So, this is basically. So, what we have is from the above discrete Fourier series is C of 3 equals half C of minus 3 that is C of 9 is also equal to half C of 2 equals 1 over 2 j that is 1 over 2 j which is also you can write it as minus j over 2 and C of 10 is equal to minus 1 over minus 1 over 2 j.

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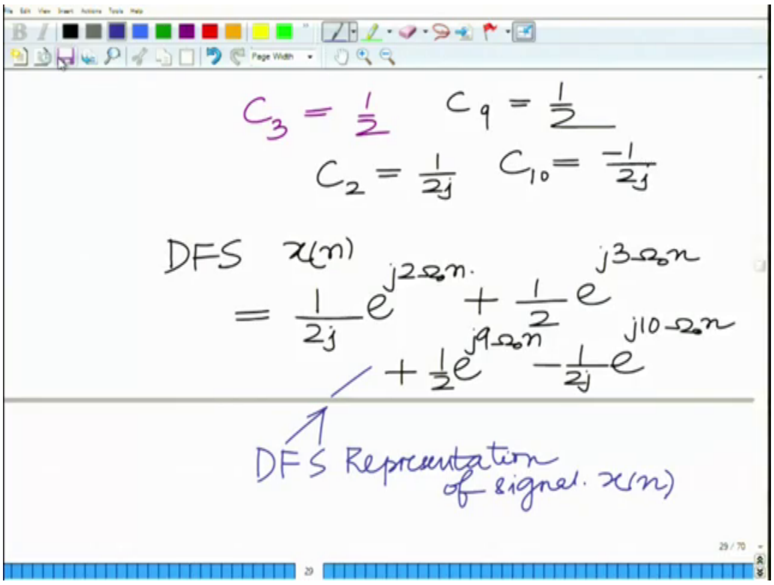

And therefore, I can write the signal as the discrete Fourier series representation x n as C 3 e raise to j 2 omega naught n as our C 2 e raise to j over 2 omega naught n e raise to j 2 omega naught n plus C 3. That is half e raise to j 3 omega naught n plus C 9 which is half e raise to j 9 omega naught n C 9 e raise to 9 j 9 omega naught n minus C that is plus C 10 which is minus 2 j e raise to j 10 omega naught n.

So, this is the discrete Fourier series representation of the. So, this is the DFS,. This is the DFS representation of the signal x n all right. Discrete Fourier series representation of the signal x n so that completes problem number 2 let us look at problem number 3.

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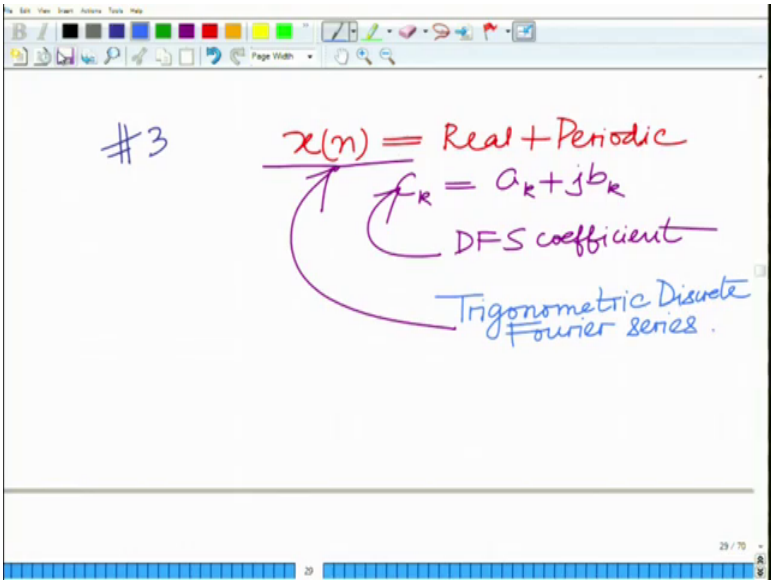

And problem number 3 as follows let x n be a real periodic sequence. So, let us write it as this let us write it in compact form, x n is real plus periodic ok. So, it is a real periodic sequence.

And the DFS coefficient, so given that the DFS coefficient is a k plus j b k DFS coefficient C k can be expressed as a k plus j j b k and we want to find the trigonometric discrete Fourier series. Now, for this we want to find the trignom given the discrete Fourier series, the discrete Fourier series coefficients of C k we want to find the trigonometric discrete.

So, we want to find the trigonometric Fourier discrete Fourier series given the discrete Fourier series the complex discrete Fourier series all though you can also see the complex exponential discrete Fourier series coefficients C k ok. And let us also consider the scenario for simplicity where N naught is odd ok. So, let us consider the scenario where N naught equals odd which also implies that N naught minus 1 equals pi ok.

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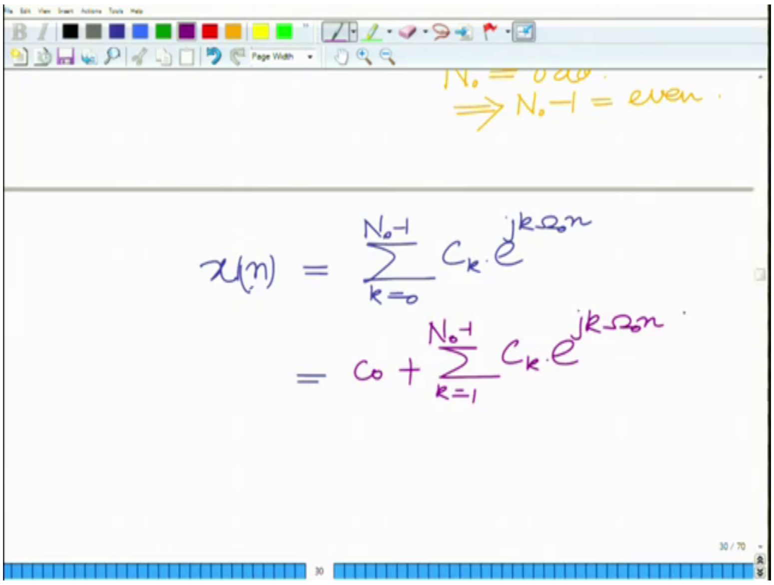

Now, the discrete Fourier series is given as x n equals k equal to 0 to N naught minus 1 C k e raise to j k omega naught n which is remember N naught is odd, so N naught minus 1 is even..

So, this is C naught plus summation k equal to 0 or k equal to 1 to N naught minus 1 C k e raise to j k omega naught n. So, N naught is odd N naught minus 1 is even.

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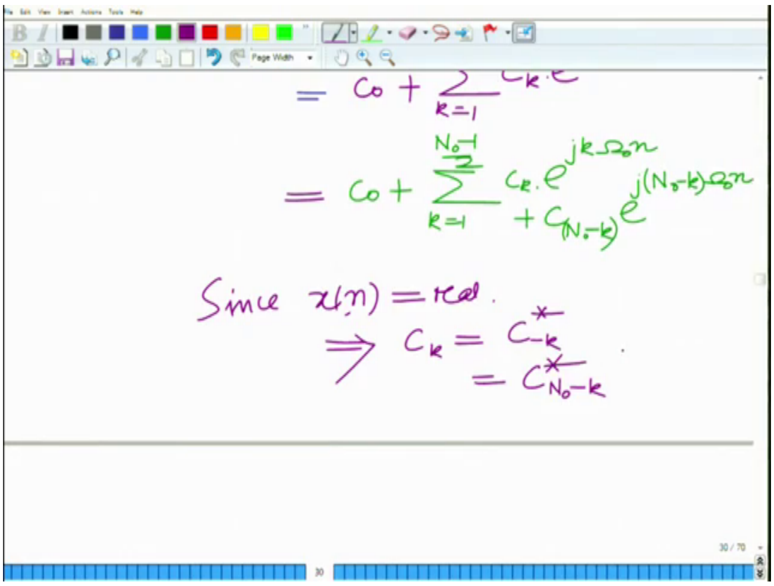

So, I can write this as C naught plus summation k equal to 1 to N naught minus 1 by 2 C k e raise to j k omega naught n plus C of N naught minus k e raise to j N naught minus k omega naught n. So, I am splitting it into 2. I am writing at and varying the index only from k equal to 1 to N naught minus 1 by 2 and writing it as a sum of C k e raise to j k omega naught n plus C n minus k e raise to j k n N naught minus k times n.

Now, if you look at this since this is a real signal this implies C k and C naught n minus k which is C of minus k or conjugates of each other. So, C k equals C minus k conjugate equals C N naught minus k conjugate, ok.

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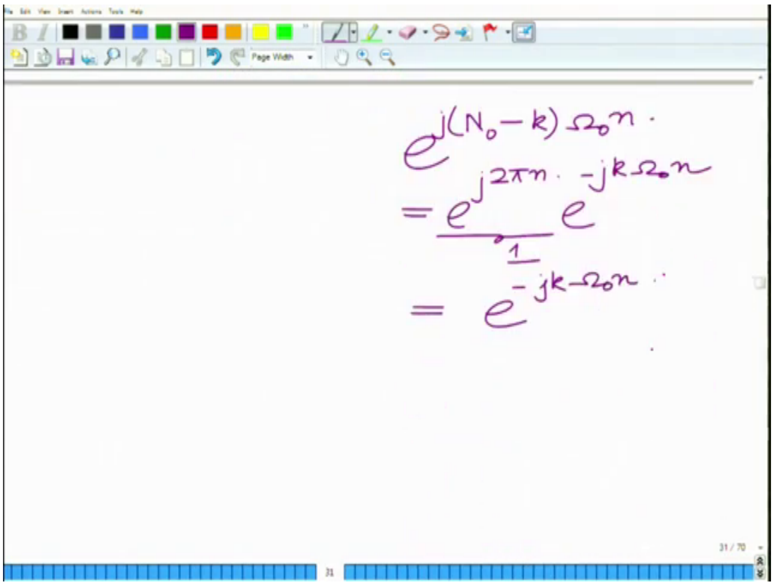

And further if you look at e raise to j N naught e raise to j N naught minus k omega naught n. This is e raise to j N naught into omega naught is 2 pi e raise to j 2 pi n into e raise to minus j k omega naught n and e raise to j 2 pi n is basically unity ok. So, this is unity and therefore, this is e raise to minus j. This is e raise to minus j k omega naught n, ok.

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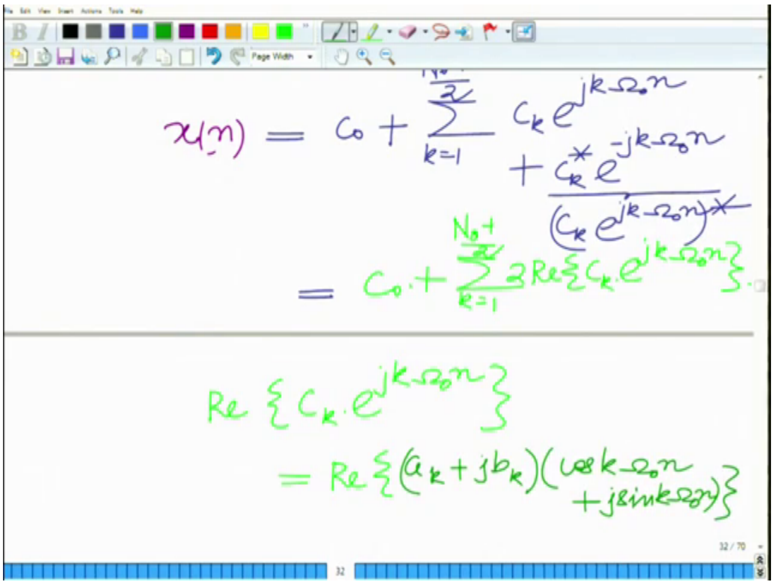

So, basically what this tells us is that this is equal to, so I can write rather recast x n as C naught plus summation k equal to 1 to N naught minus 1 divided by 2, C k e raise to j k omega naught n plus C k conjugate e raise to minus j k omega naught n, ok.

Now, C k, now, you can see this C k conjugate I raise to minus j k omega naught n is nothing but C k e raise to j k omega naught n conjugate ok. So, C k conjugate e raise to minus j k omega naught n is basically C k e raise to j k omega naught n conjugate ok. So, now, we have C k e raise to j k omega naught n plus its conjugate. So, basically what remains is what it what is this leads to its twice its real part twice the real part of C k e raise to j k omega naught n ok.

So, this will be C naught just simplify this will be C naught plus summation k equal to 1 to N naught minus 1 by 2 twice real part of C k e raised to j k omega naught n. And the real part of C k e raise to j k k omega naught n equals basically the real part of a k plus j b k cosine k omega naught n plus j sin j omega naught n. Which is equal to the real part is well a k cosine k omega naught n minus b k sin k omega naught n ok.

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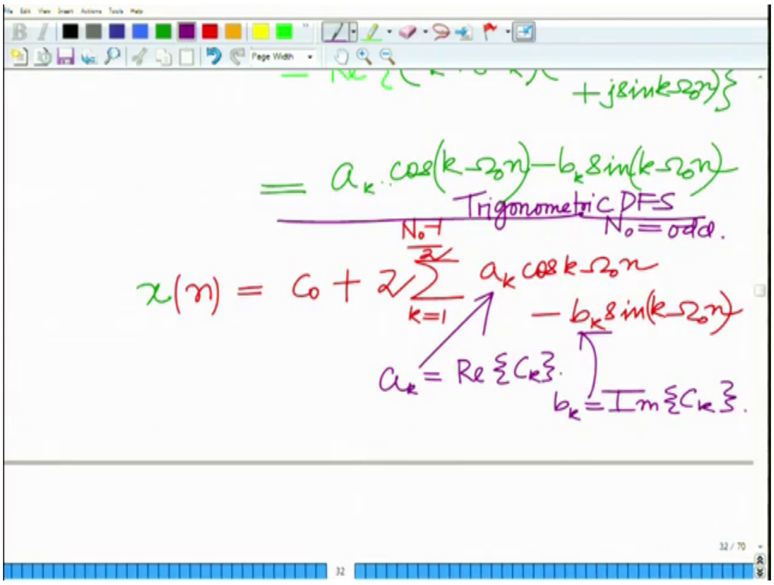

And therefore, substituting this now, you have x n the trigonometric discrete Fourier series can be obtained as C naught plus twice summation k equals 1 to N naught minus 1 by 2, a k cosine k omega naught n minus b k sin k omega naught n. Whereas I already told you that this a k and b k are obtained from the DFS coefficient C k such that C k is a k plus j b k.

So, these are given from the these are basically the real part a k is the real part and b k is the imaginary part of the DFS coefficient C k, ok. So, a k equal to real part of C k and b k equals basically the imaginary part of C k all right, ok. And this is basically your trigonometric Fourier series.

But remember when also remember this is the trigonometric discrete Fourier series, but remember this is when N naught equals N naught is even sorry or N naught is odd sorry N naught is odd all right. Similarly you can find the discrete Fourier series the trigonometric discrete Fourier series when N naught is even in that case N naught minus one be odd. So, you cannot apply the same technique all right.

So, basically that brings us to the next problem which is problem number 4, if I remember correctly. So, this brings, this brings us to problem number 4.

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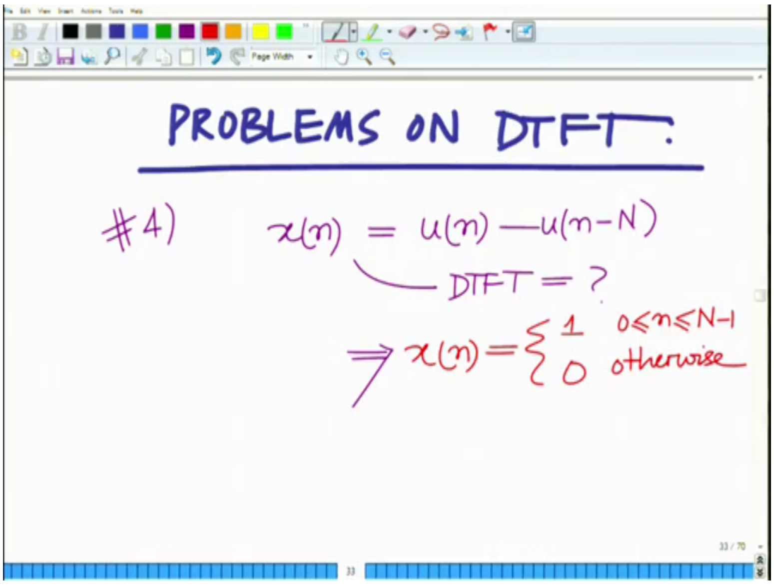

Find the discrete time Fourier transform. So, now, we are entering problems on discrete time Fourier transform that you just you completed the Fourier series these are the problems on DTFT ok, the problem on DTFT. This is number 4. Let us say x n equals un minus u n minus n what is the corresponding DTFT of this signal.

So, this is a u n minus u n minus N minus 1, this implies if you can see this is clearly x n equals 1, 0 less than n less than equal to n minus 1 and 0 and 0 otherwise ok.

This is your x n that is 1 only in a window of 0 to capital N minus 1. And outside this window that is for n less than 0 strictly less than 0 and n strictly greater than n minus one or basically n greater than or equal to capital N this is 0 ok.

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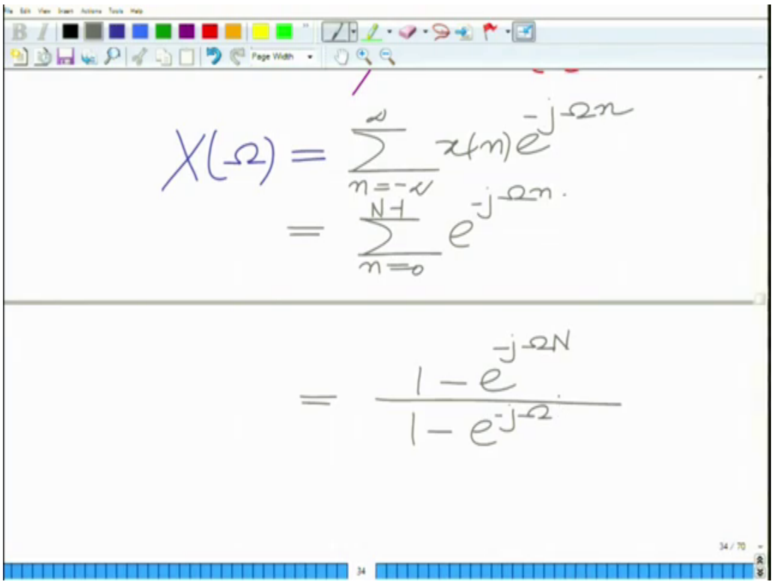

And therefore, x of omega, now, if you find the DTFT that is given as x of omega equals summation n equals minus infinity to infinity x n e raise to minus j omega n.

Now, there is only nonzero from 0 to n minus one. So, this is and in that window it is one. So, this is simply summation n equal to 0 to n minus 1 e raise to minus j omega m omega n which is a geometric series and the sum of this is 1 minus e raise to minus j omega n divided by 1 minus e raise to minus j omega. Which is now, if you write take e raise to minus j omega N by 2 common in the numerator and erased to minus j omega by 2 common in the denominator.

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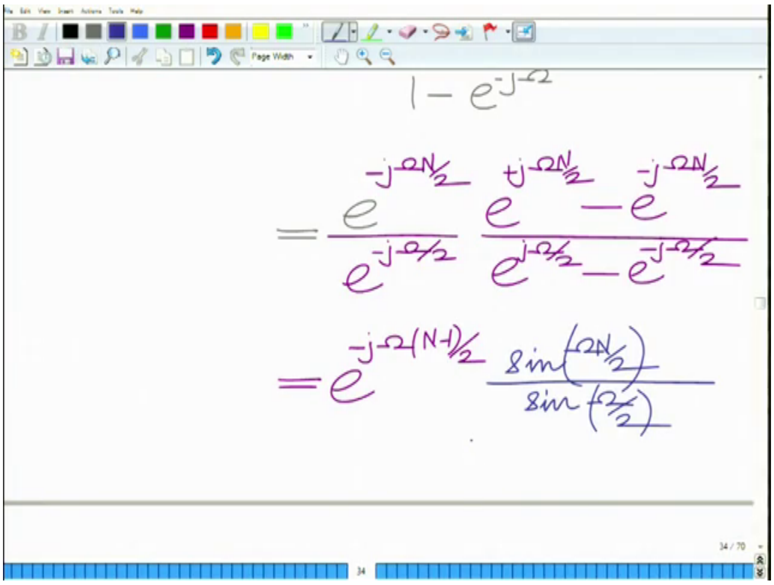

So, this will be e raise to minus j omega N by 2 minus or in that e raise to plus j omega N by 2 minus e raise to minus j omega N by 2.

In the numerator and the denominator it will be e raise to j omega by 2 minus e raise to minus j omega by 2 which is equal to e raise to minus j omega n minus 1 divided by 2 times e raise to j omega n by 2 minus e raise to minus j omega capital N over 2 this is twice j sin omega N over 2.

So, and you can see that the 2 j factor in the numerator and denominator cancels. So, what you are left with a sin n omega by 2 in the numerator and sin omega by 2 in the denominator ok, sin omega 2 by 2 in the denominator.

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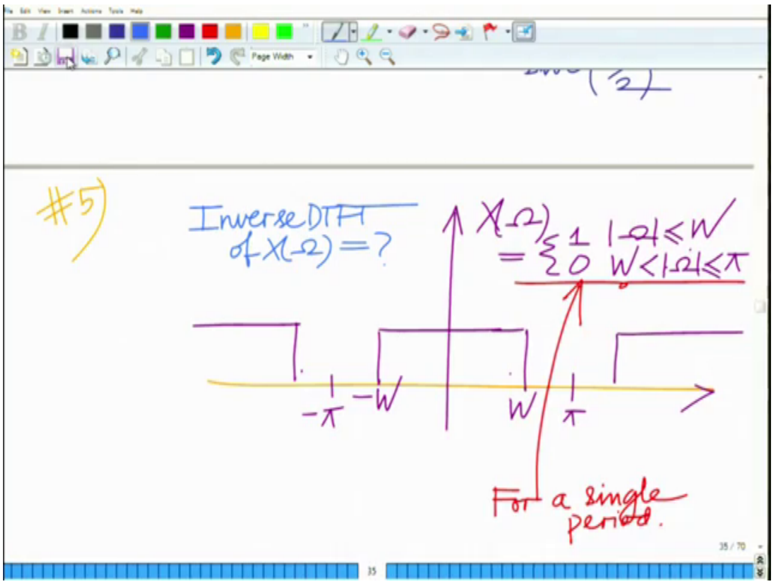

Let us find the DTFT of the inverse DTFT rather of another signal. So, this is problem number 5, you have x of omega.

So, this is remember the DTFT is periodic. So, this is your pi periodic with periodicity 2 pi. So, we have x of omega this is your x of omega which is equal to 1, as shown magnitude of omega less than or equal to W and 0 0 otherwise. This is in one period or let us write it is 0 W less than more omega less than less than or equal to less than or equal to less than or equal to pi, ok.

So, basically this is this thing this describes it in one period, this is remember this is also because this is always a periodic signal. So, this is for a single period this is for a single period. So, this is x of omega the inverse DTFT.

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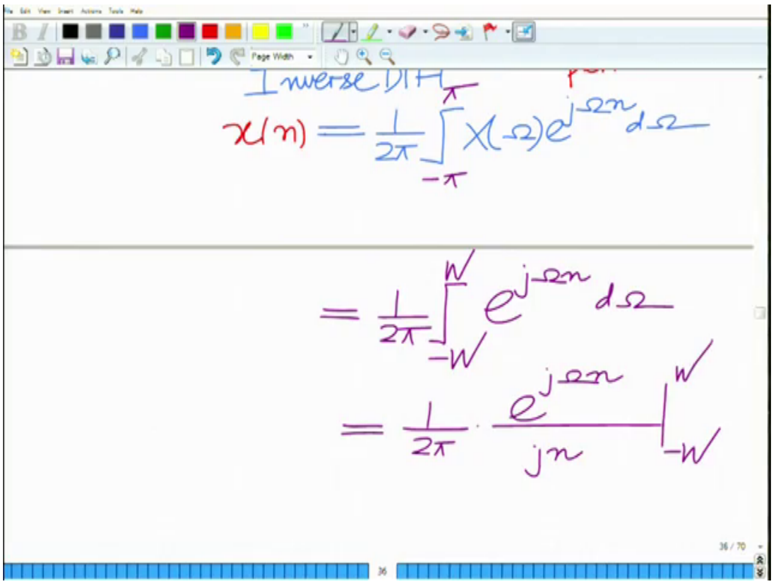

And what we need to do is we have to find the inverse DTFT of x of omega. The problem is; what is the inverse DT of this DTFT of this x omega which is 1 in this interval of width minus omega to omega.

So, you can say this is discrete Fourier time Fourier transform which is the bandwidth of capital W all right. It is one in this band W to W, ok. And the DTFT is given as inverse DTFT is given as remember the inverse DTFT formula is x of n equals minus one over 2 pi minus W to W.

Student: Omega.

X of omega e raise to j omega t omega which is equal to 1 over 2 pi. In fact, this is simply minus pi to pi therefore, this is one only in the interval minus W to W. So, this reduces to integral minus W to W and in this interval it is 1.

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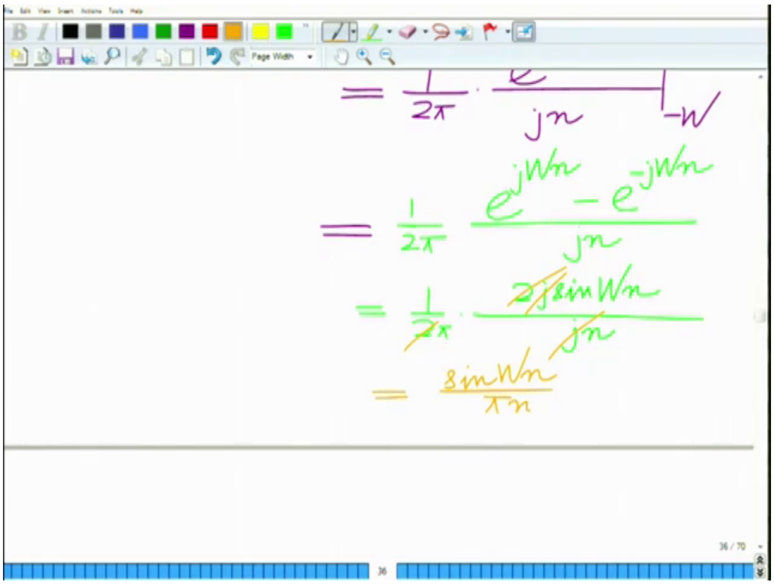

So, this is raise to j omega n d omega which is one over 2 pi e raised to j omega n over j n evaluated between the limits minus W to W equals 1 over 2 pi e raise to j omega W minus e raise to or e raise to I am sorry e raise to e raise to j W n minus e raise to minus j W n divided by j W n which is 1 over 2 pi e raise to j W n. So, this is 2 j sin W n divided by j n. So, removing the j is the factor of 2 is also go away. So, what we have is sin W n by pi n. So, basically that gives us x n the inverse DTFT, x n equals.

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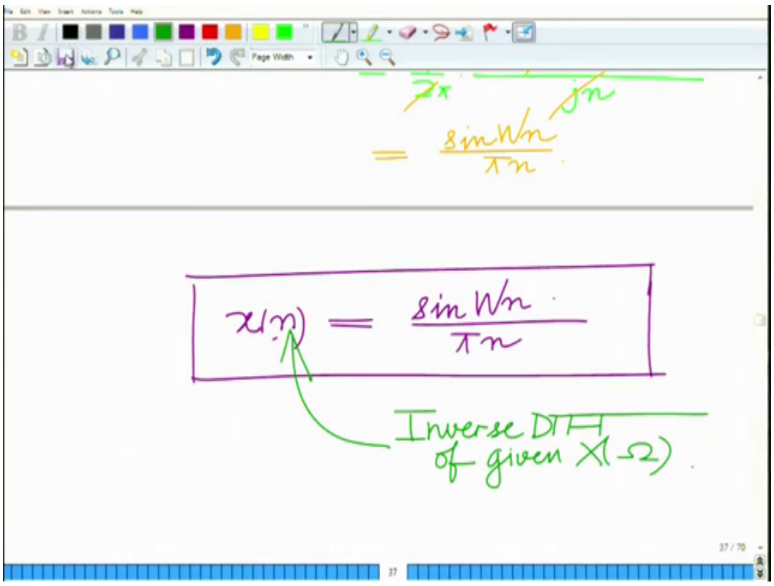

So, this is basically the inverse DTFT of the, inverse DTFT of the inverse DTFT of the given signal ok, the corresponding in fact, a periodic time domain signals, all right.

So, we have looked at problems pertaining to the pertaining to the discrete Fourier series and also the DTFT started looking at problems pertaining to the DTFT in this module. Let us continue this discussion in the subsequent modules.

Thank you very much.