Principles of Signals and Systems

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Lecture – 59

DTFT: Discrete Time LTI Systems- LTI Systems Characterized by Difference Equations

[noise]

Hello welcome to another module in this massive open online course. So, we are looking at the d d t f t the discrete time fourier transform ah for the analysis of discrete time signals and systems and in this module ah we will look at the d t f t for the analysis of discrete time l t i systems ok. So, we [noise] intend to discuss [noise] the d t f t [noise]

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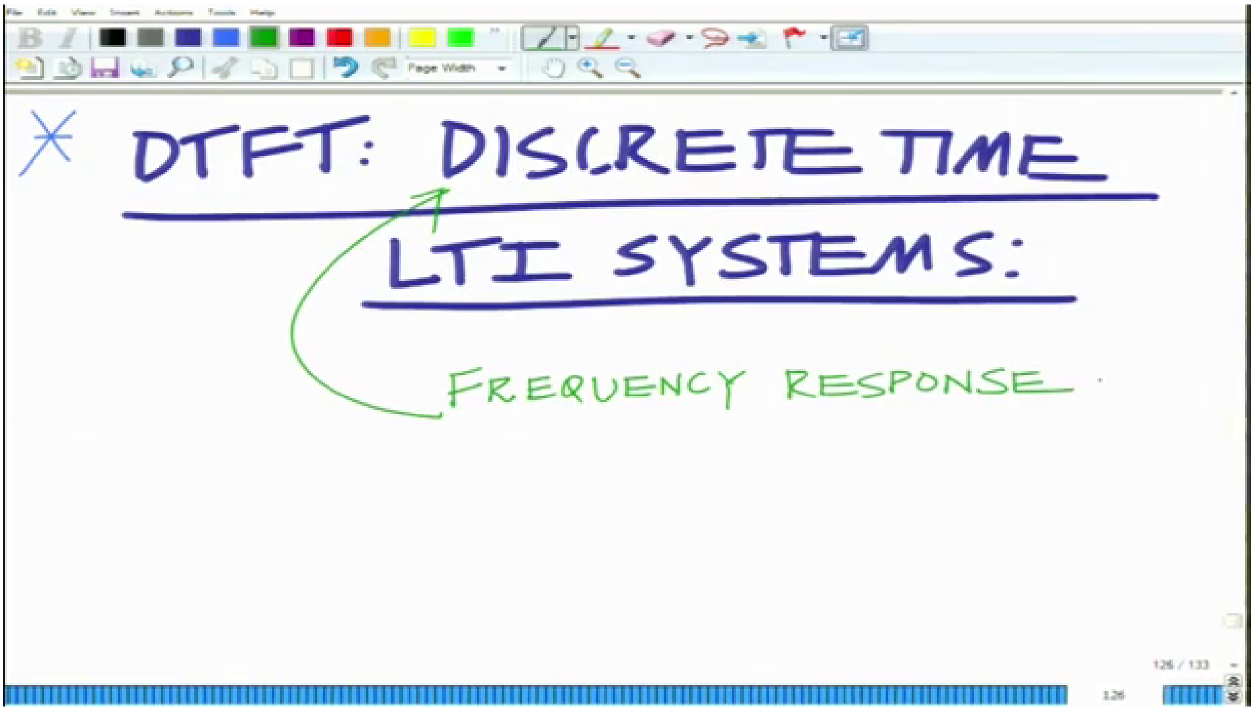

application of the d t f t for ah [noise] for discrete time [noise] [noise] in particular ah we would like to [vocalized-noise] ah look at it in terms of the frequency response.

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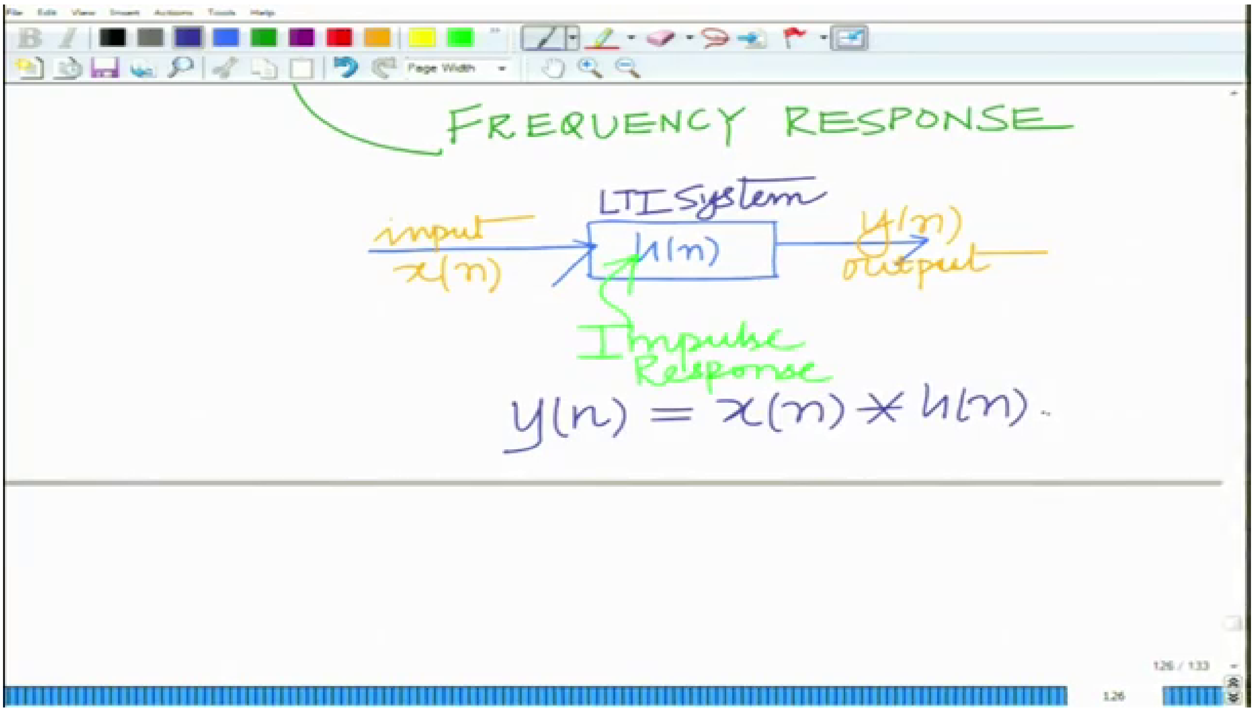

Of these discrete time l t i systems ok what is the [noise] frequency response [noise] ok. So, consider a discrete time l t i system with impulse response given by h n ok [noise]. So, the input [noise] to this [noise] l t i system is x n [noise] output is y n [noise] ok and h n denotes the impulse response of this system [noise] it denotes the impulse response of this system [vocalized-noise] and the input output relationship therefore,. So, this is your l t i system [noise] ok [vocalized-noise] the discrete time l t i system with input given by x n output given by y ok h n denotes the discrete time impulse response of this l t i system ok [vocalized-noise]

And now naturally we know that the output y n is given as the convolution of x n [noise] times [noise] h n or x n is the input h n is the impulse response are now taking.

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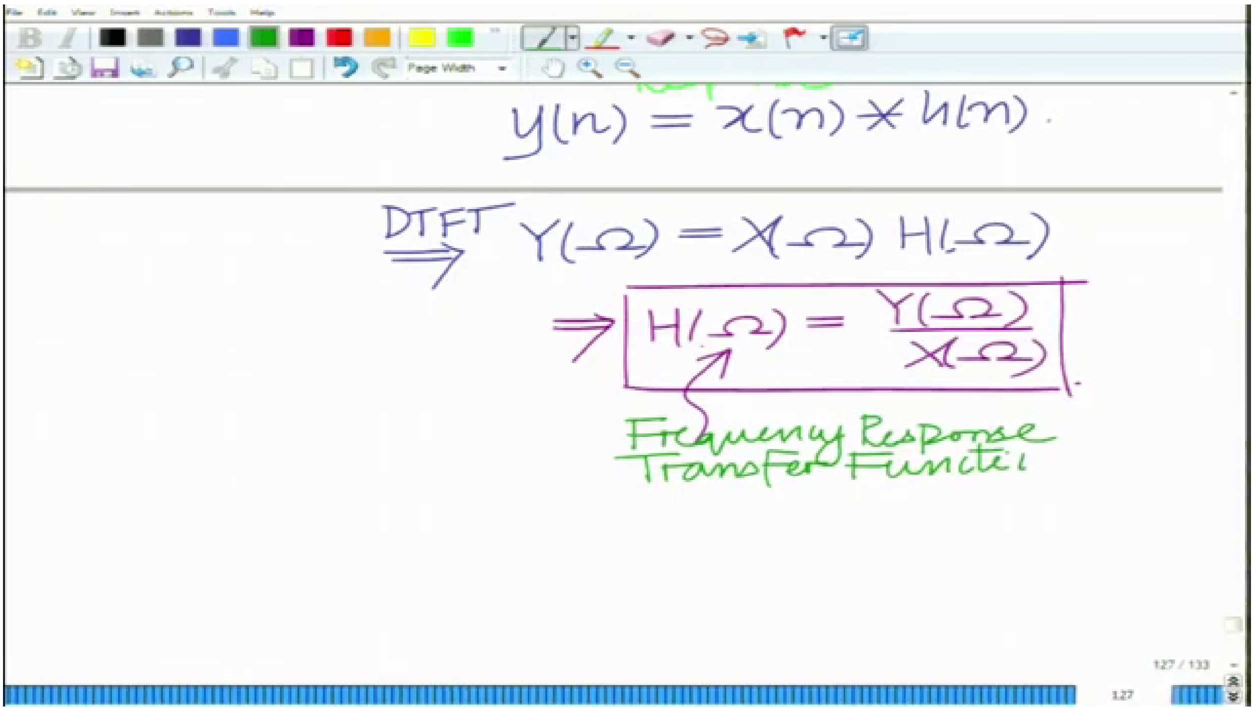

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The d t f t [noise] discrete time fourier transform we have y omega equals x omega [noise] [noise] times h omega which implies [noise] as usual the transfer function h omega [noise] equals y omega [noise] divided by [noise] x omega this is the [noise] transfer function ok there is a frequency response of the l t i system of the transfer function [noise] [vocalized-noise] frequency response of the transfer function [vocalized-noise] and now ah what happens if x n is periodic now

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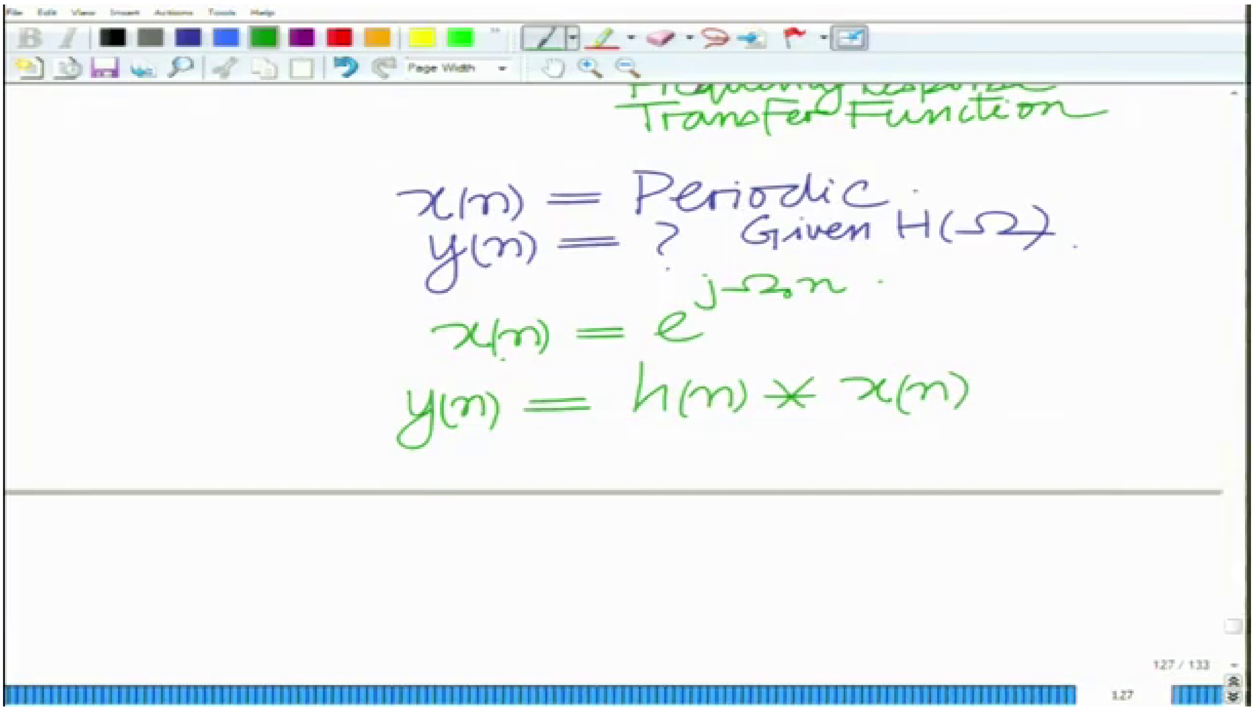

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lets say x n the input signal is [noise] periodic in that case how do you obtain [noise] y n ah given the impulse response that is given the [vocalized-noise] given h of omega [noise] [vocalized-noise].

Now, first observe that if x n is a complex exponential [noise] [vocalized-noise] if x n equals e raised to j omega naught n ok now the output y n equals h [noise] n convolved with x n [noise] ok which is basically equal to

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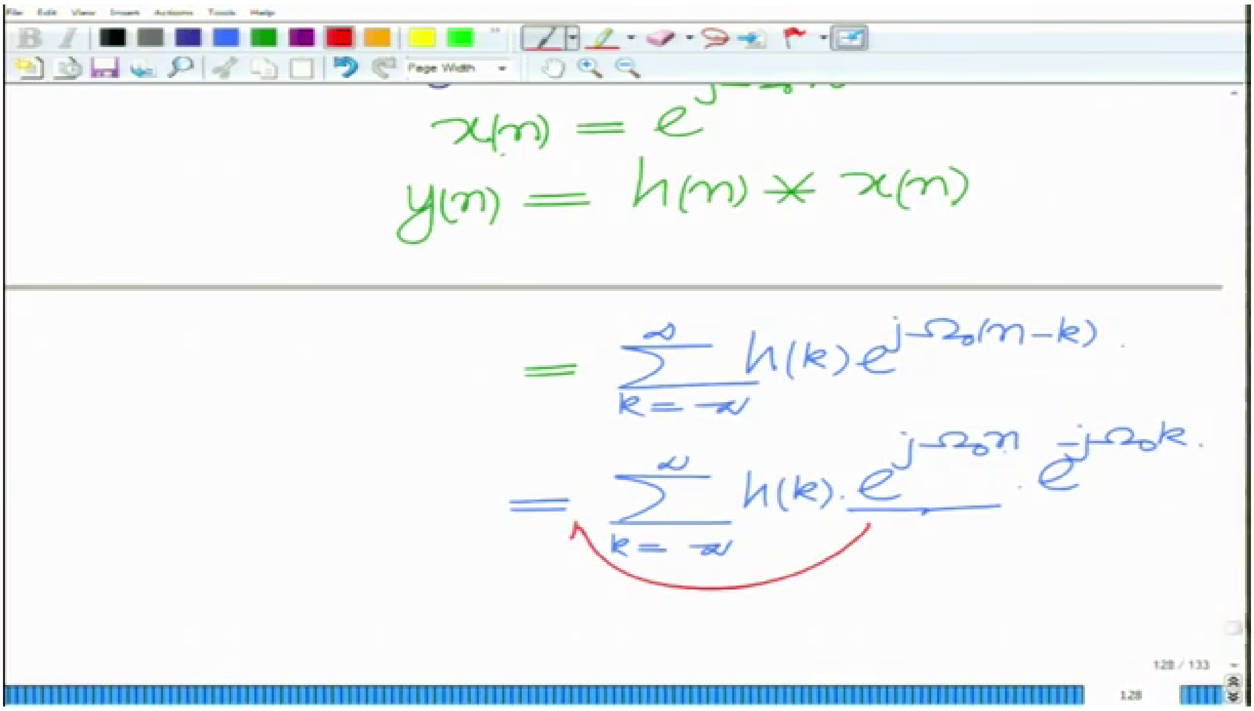

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or which [noise] equals the summation of [noise] k equals [noise] minus infinity to infinity [noise] [noise] h k e raised to j omega naught n minus k which is summation k equals minus infinity to infinity [noise] h k e raised to j omega naught n e raised to j omega naught k now this e raised to j omega naught n which is common i take that outside. So, that becomes

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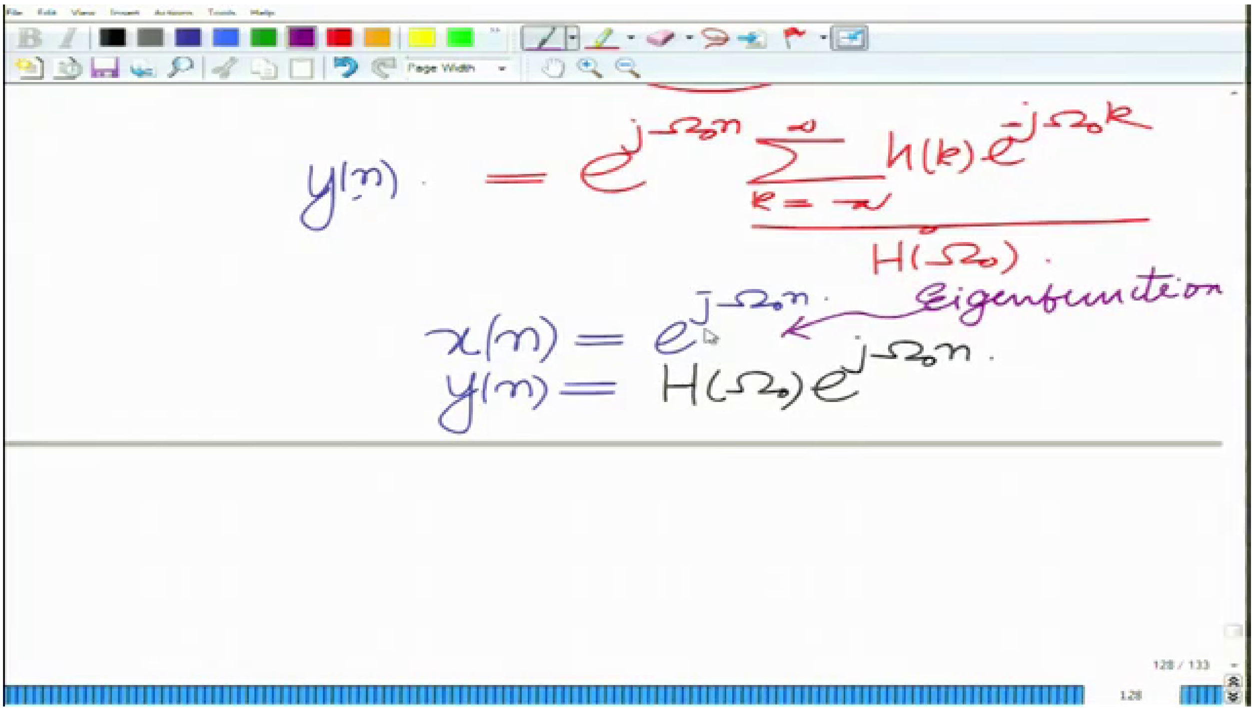

e [noise] raised to j omega naught n [noise] summation k equals minus infinity to infinity [noise] h k [noise] e raised to minus j omega naught k which is simply [noise] h of omega naught that is the d t f t at omega naught that summation h k e raised to minus j omega naught k [vocalized-noise]. So, the output is simply given as a is simply given as h of for the complex exponential at frequency omega naught ok the output is simply h of omega naught times e raised to j omega not n. So, y n [noise]. So, we have [noise] [vocalized-noise].

And this is interesting and we already seen this [noise]. So, if x n [noise] is e raised to j omega naught n then the output signal y n is simply [noise] ah multi plus scaled version of the input signal (( )) h h omega naught times e raised to j omega naught n and therefore, this is also known as the eigen function ok or the eigen signal [noise] [vocalized-noise] this is the eigen function of this l t i system because the output is simply a scaled version of scalar multiple times the input ok [vocalized-noise] now.

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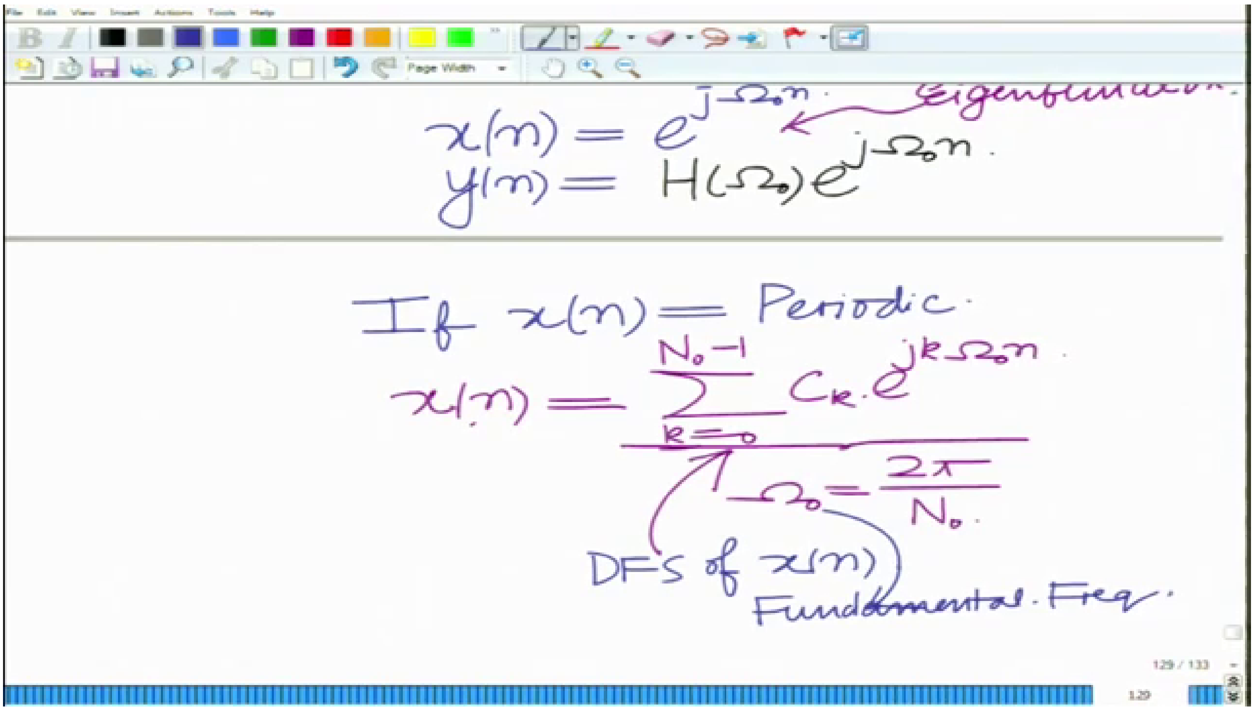

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If x n is periodic [noise] then we realize that x n can be expressed as a discrete fourier series [noise] then we have [noise] x n equal to summation [noise] k equal to zero to n naught minus one c k e raised to [noise] j [noise] k omega naught n [noise] where omega naught equals [noise] two pi over [noise] n naught ok this is c k e raised to [noise] (( )). So, what is this this is basically your discrete fourier series [noise] [noise].

And this omega naught is basically the fundamental frequency [noise] ok [vocalized-noise]. So, it is expressed as a multiple is expressed as a sum of the complex explanation at the fundamental frequency and its harmonics ok k equal to zero to n naught minus one this is a discrete fourier series ok now from linearity now we know that corresponding to each e raised to j k omega naught n the output of the l t i system is simply h of k omega naught times e raised to j k omega naught n ok

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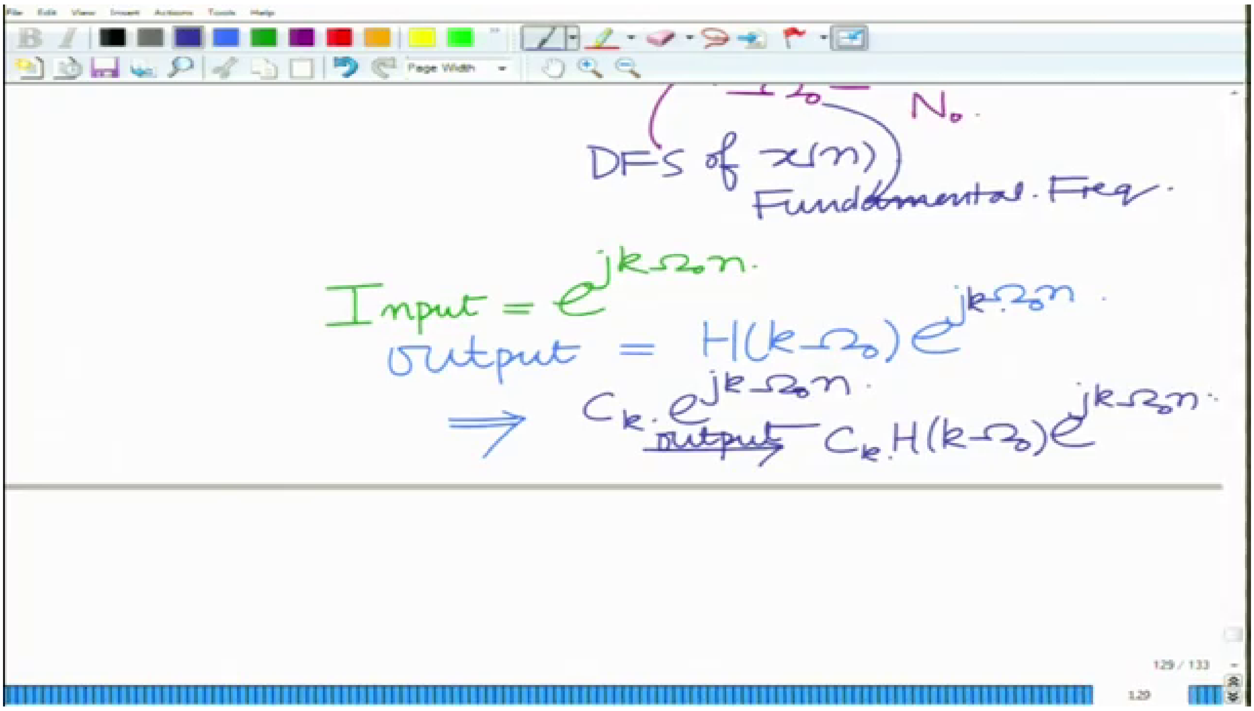

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. So, for each e raised to input [noise] k omega naught n ok now the corresponding output [noise] we have seen [noise] equals h of k omega [noise] naught [noise] times e raised to j [vocalized-noise] k omega naught n which implies for input of c k [noise] times e raised to j omega naught n gives the corresponding output c k. So, the input is scaled by linearity output is also scaled [noise] i am sorry this has to be [noise] k omega naught n [noise] [vocalized-noise] and therefore, if input is x n [noise] equals summation [noise]

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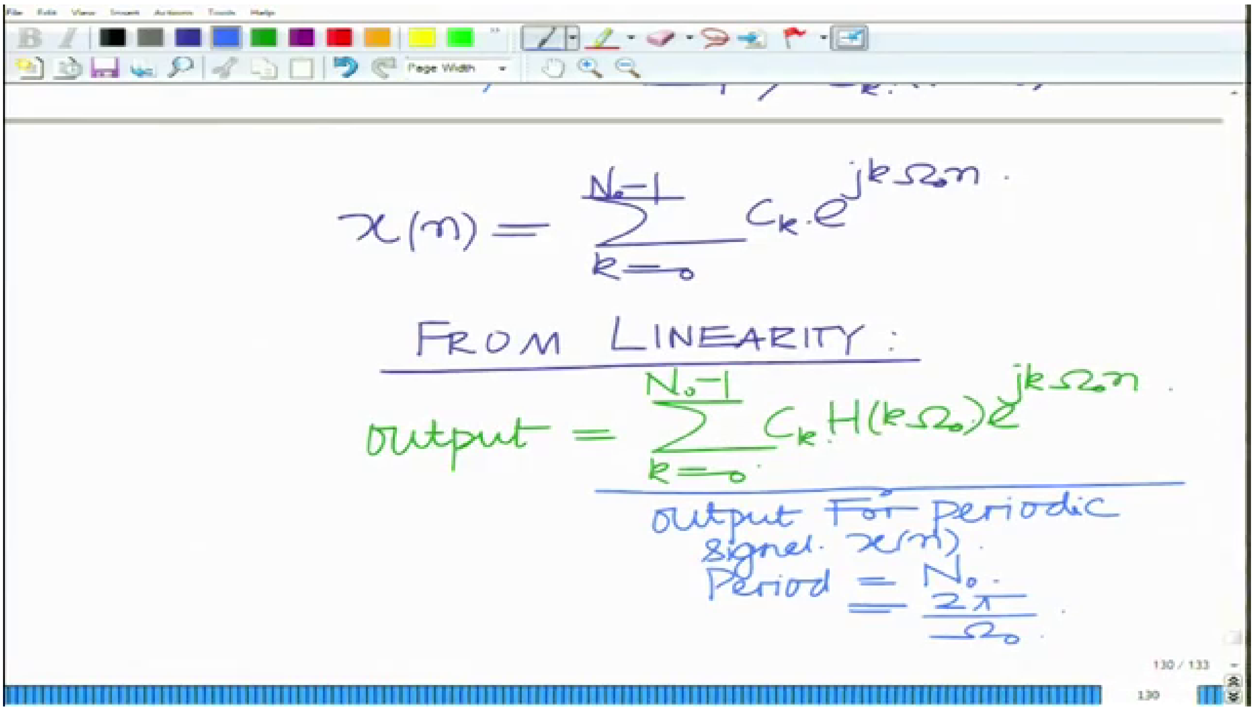

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k equal to zero to n naught n minus k equal to zero to n naught minus one [noise]

C k e raised to j k omega naught n the output by linearity [noise] from linearity [noise] the output is simply [noise] the output is [noise] [vocalized-noise] k equal to zero to n naught minus one c [noise] k h of k omega naught e raised to j k omega naught n ok. So, this is the corresponding output ok. So, this is your output to the [noise] periodic [noise] signal x n ok this is the output for periodic signal x n with period n naught [noise] equals two pi by omega naught all right you can see the if input is periodic the output is also ah the output is also basically periodic ok [vocalized-noise] all right [vocalized-noise] and finally, for l t i systems characterized by difference equations

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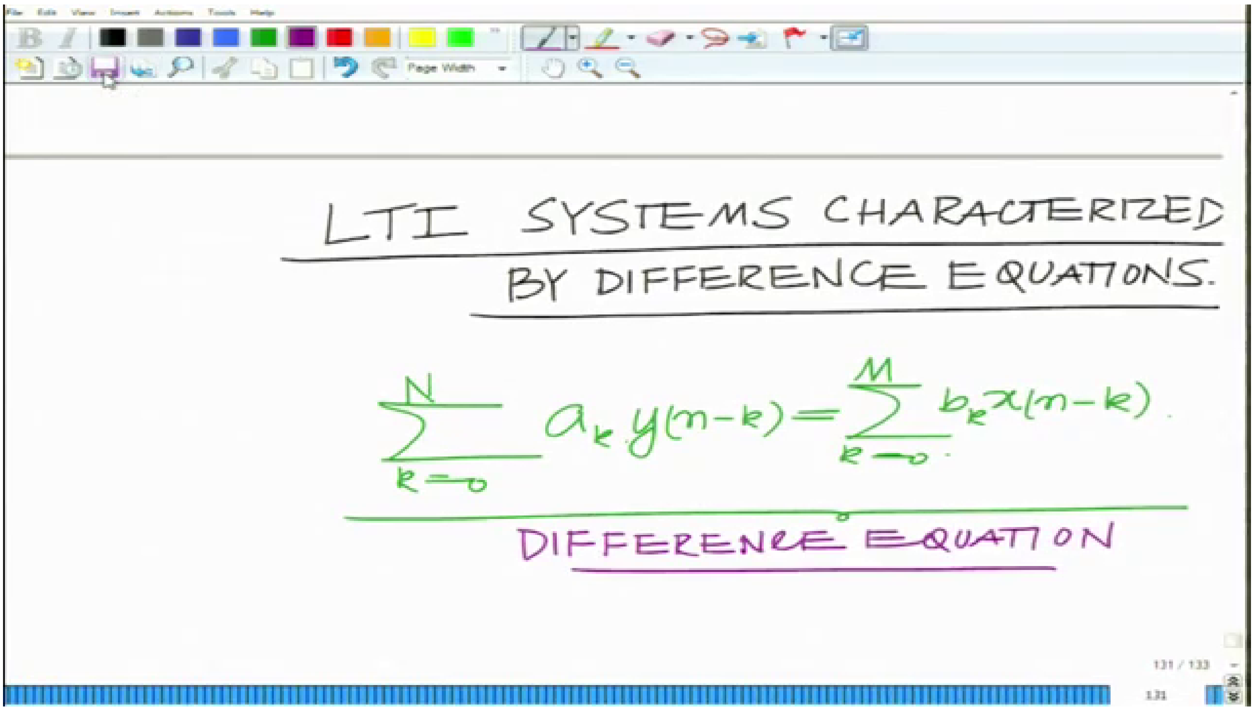

let us now look at similar to what we have seen before [noise] discrete time l t i systems [noise] [noise] characterized [noise] characterized by [noise] difference equations ok we have summation k equal to zero to n minus one lets say the difference equation is summation k equal to zero to n minus one y of n minus k equals summation k equal to zero ah sorry k equal to zero to n equals summation k equal to zero to m [noise] [noise] b k x n minus k this is the difference equation ok [noise]

This is the reference equation now taking the d t f t

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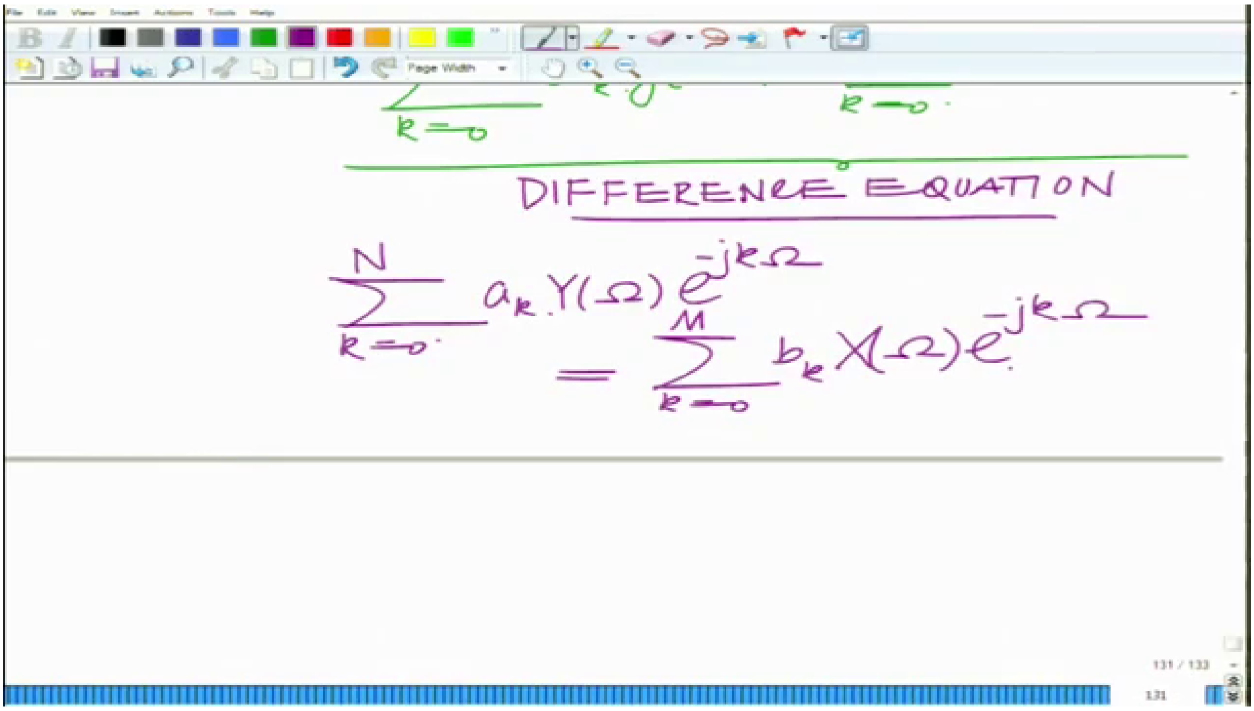

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what we have a summation [noise] k equal to zero to n minus one a k y omega the d t f t of y n minus k is y omega e raised to minus j k omega that must be equal to summation k equal to zero to m b k [noise] x of omega e raised to minus j k omega this means now taking the [noise]

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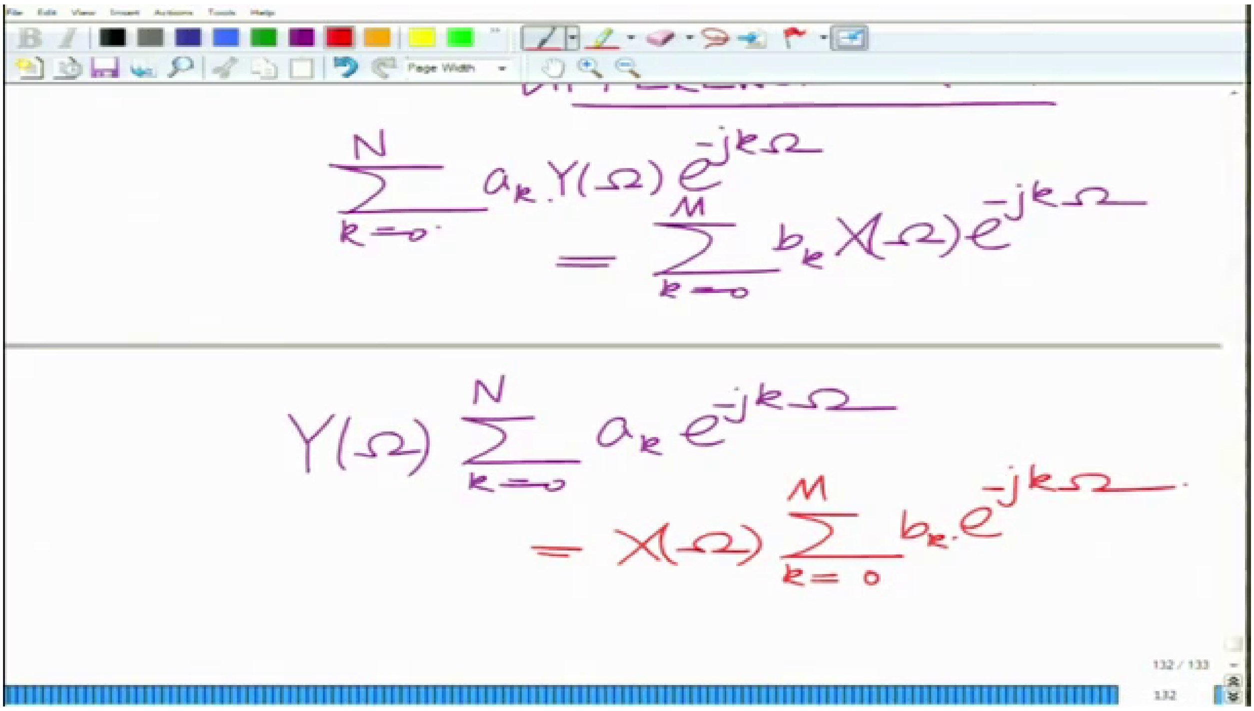

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y of omega common on the left i have y of omega summation k equal to zero to n a k e raised to minus j k omega equals

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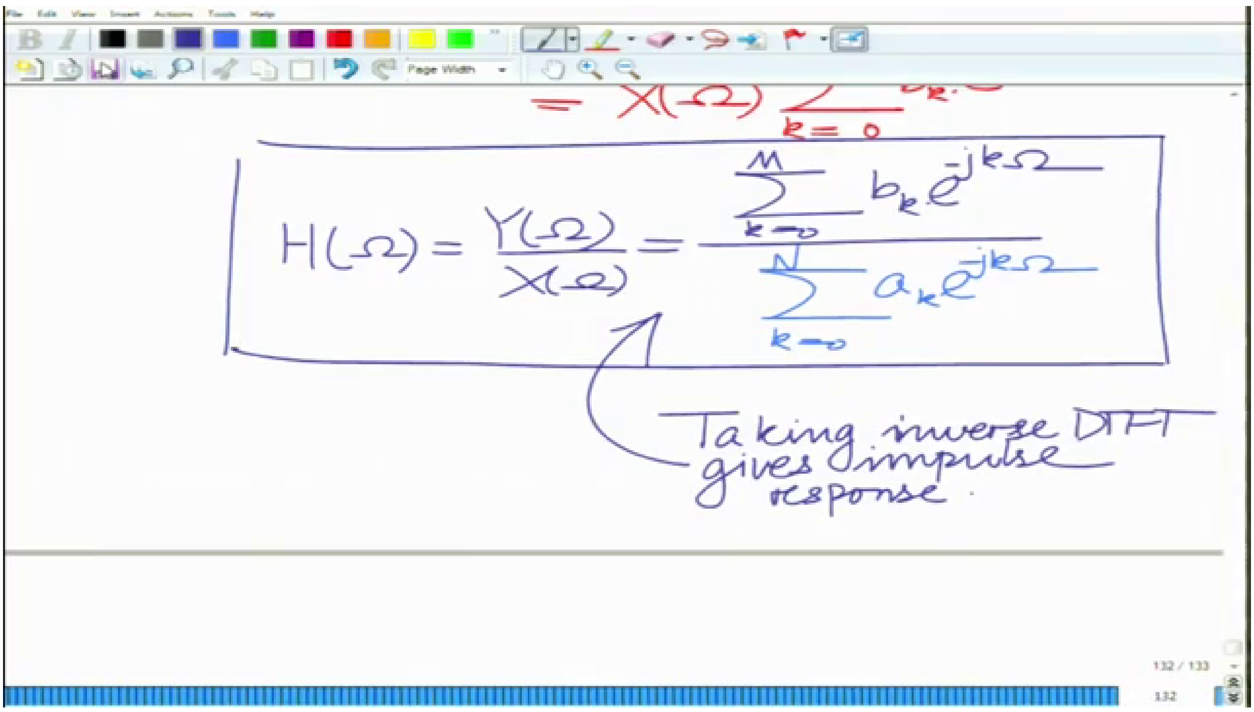

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taking x of omega common on the right x omega k equal to zero to m b k [noise] e raised to minus k omega which means h of omega equals y omega by x omega [noise] equal summation k equal to zero to m [noise] b k e raised to minus j omega divided by summation k equal to zero to n a k [noise] e raised to minus j k omega ok.

So, that is the transfer function [vocalized-noise] and taking the inverse d t f t of this gives the impulse response ok [noise] taking the inverse d t f t of this gives the impulse response [vocalized-noise] all right. So, basically that completes the analysis of the application on the properties of the d t f t that is the discrete time fourier transform for l t i systems with respect to the transfer function the output output for an periodic signal output for an a periodic signal and also ah how to derive the impulse response and the transfer function of a d t of an l t i system [vocalized-noise] described by a constant coefficient difference equation alright. So, well stop here and continue in the subsequent module.

Thank you very much [noise]