Principles of Signals and Systems
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Lecture - 54

Fourier Analysis of Discrete Time Signals and Systems – Introduction

Hello. Welcome to another module in this massive open online course. In this module, we are going to look at, start looking at the Fourier analysis for discrete time signals and systems.

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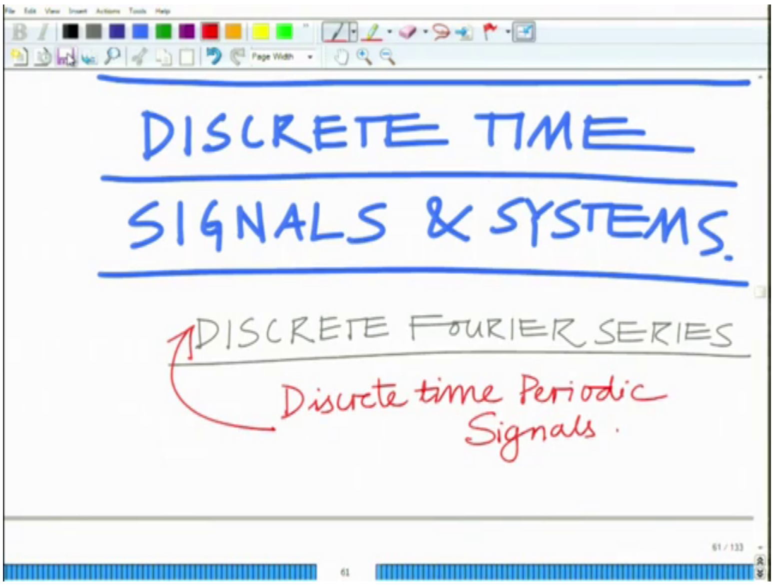

So,. So, far we have looked at the Fourier analysis for continuous time periodic as well as a periodic signal. A periodic signals starting with this module, we are going to start looking at discrete time periodic as well as periodic signals, alright. So, we are going to start looking at the Fourier analysis, the Fourier analysis for discrete time ok, for discrete time signals and discrete time signals and systems for that matter; so the discrete Fourier series.

Now, we are going to start with, what is known as the discrete Fourier series? Now, the discrete Fourier series, this is defined for a discrete time periodic signal. This is design for discrete time periodic signals, similar to the complex exponential Fourier series and the trigonometric Fourier series that are defined for continuous time periodic signals. The discrete Fourier series is defined for a discrete time periodic signal ok.

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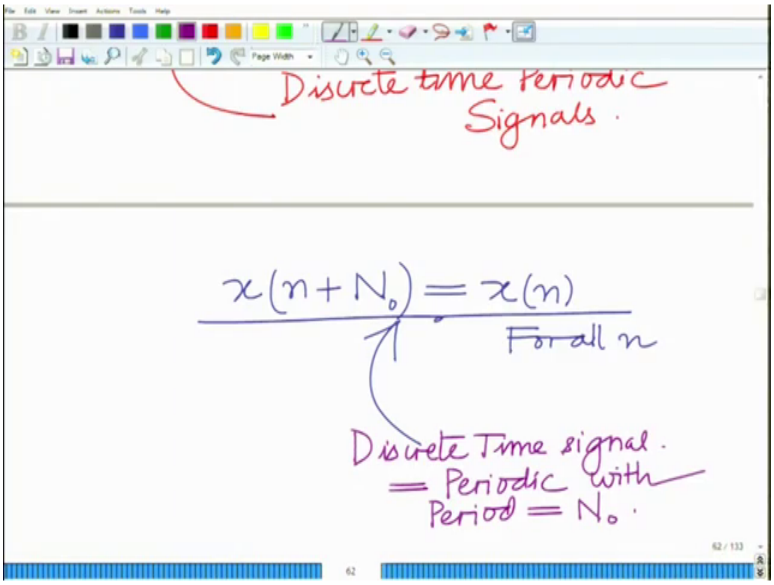

And so, we consider a discrete time periodic signal with period N naught such that X of n plus N naught equals X of n for all n, then we say the discrete time periodic sequence, discrete time periodic signal equals is periodic with period equals N naught all right.

So, when there exists at N naught, for a discrete time signal n such that X of n plus N naught equals X of n for all n all right. We say all right, the discrete time signal is periodic and it is period is N naught ok.

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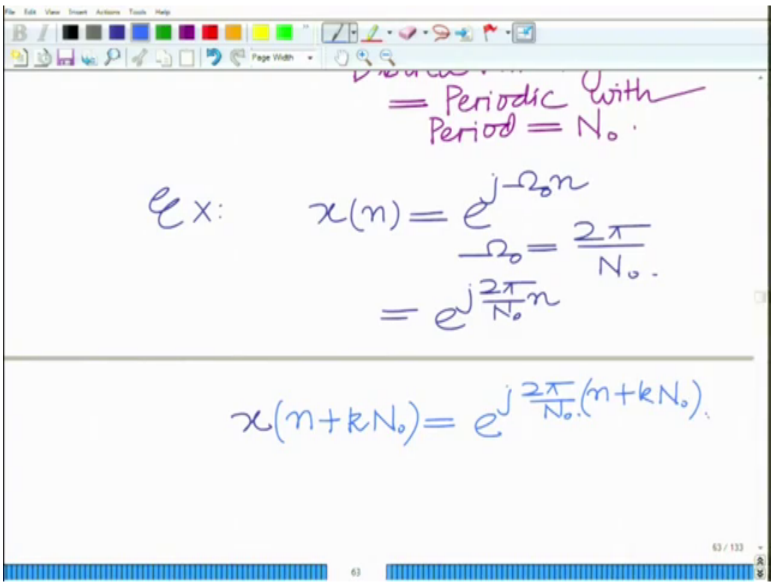

All right and for example, we have seen that the exponential signal, we have x of n equals e raised to the complex exponential signal e raised to j omega naught n, where omega naught equals is of the form 2 pi over N naught. This is a periodic signal with period N naught that is you have x of e raise x n equals e raised to j n not 2 pi by N naught of n and if you looked at x of n plus N naught in for that matrix of n plus k N naught, where k is any integer.

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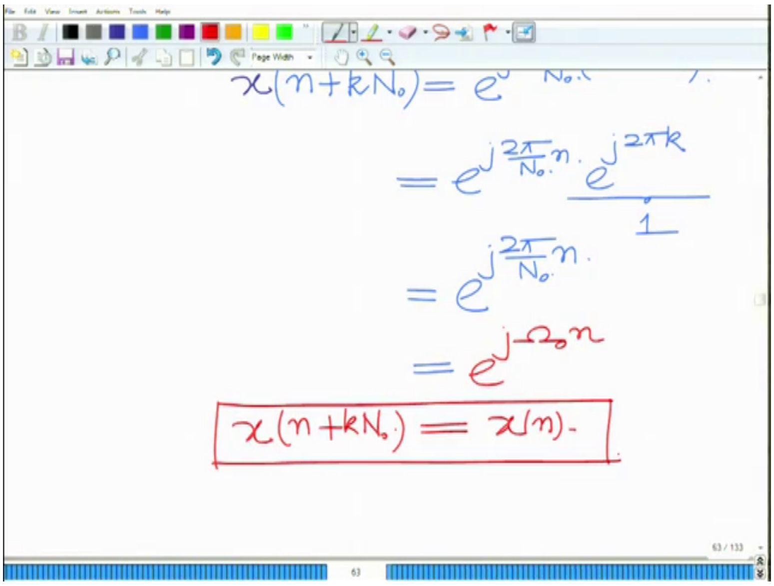

This is e raised to j 2 pi over N naught n plus k N naught, which is equal to e raised to j 2 pi over N naught n into e raised to j 2 pi N naught. So, 2 pi k and e raised to j 2 pi k. This is equal to 1 for all k.

So, this is simply e raised to j 2 pi over N naught times n, which is nothing, but e raised to j omega naught n this is e raised to j omega naught n which is basically x of n. So, we have x of n. In fact, we have x of n plus k N naught equals x of n. So, we have x of k n plus k N naught equals x of n for all n.

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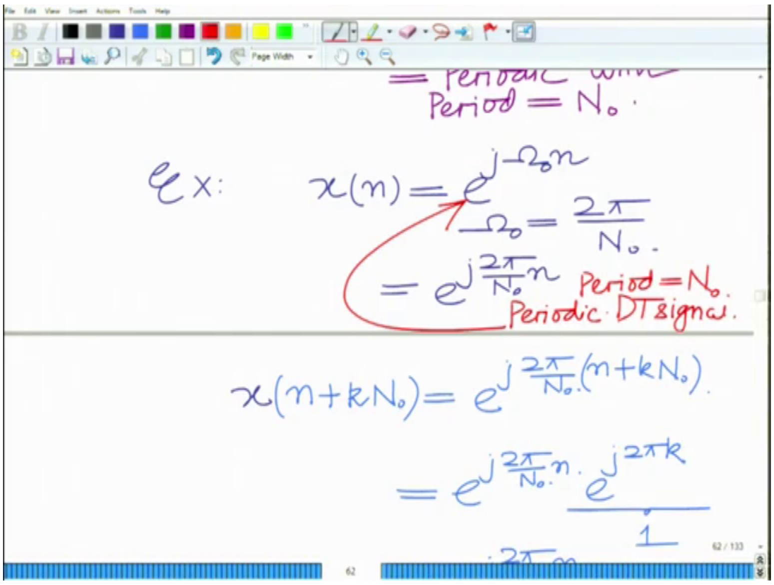

So, which means this is a periodic signal, periodic discrete time signal and the period equals N naught ok. So, e raise to j omega naught n, where omega naught is 2 pi N naught and not being an integer is a periodic signal is a discrete time periodic signal and the period is n ok.

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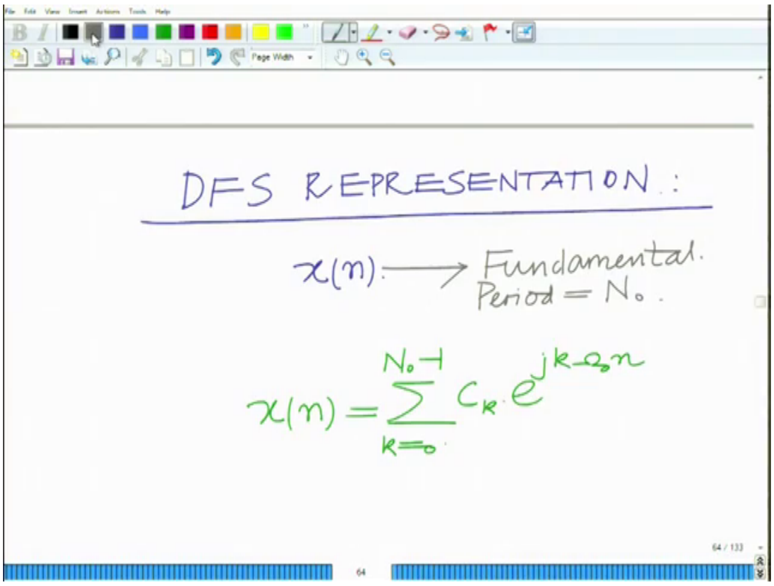

Now, the Discrete Fourier Series representation is given as follows; we can, the Discrete Fourier Series representation is given as follows. So, consider a signal x n and the fundamental period that is a smallest period. Fundamental period equals N naught then this can be expressed as x of n can be expressed as the sum of the complex exponential signals k equal to 0 to N naught minus 1 C k e raise to j k omega naught n and these are, these C k's are basically the coefficients of the Discrete Fourier Series.

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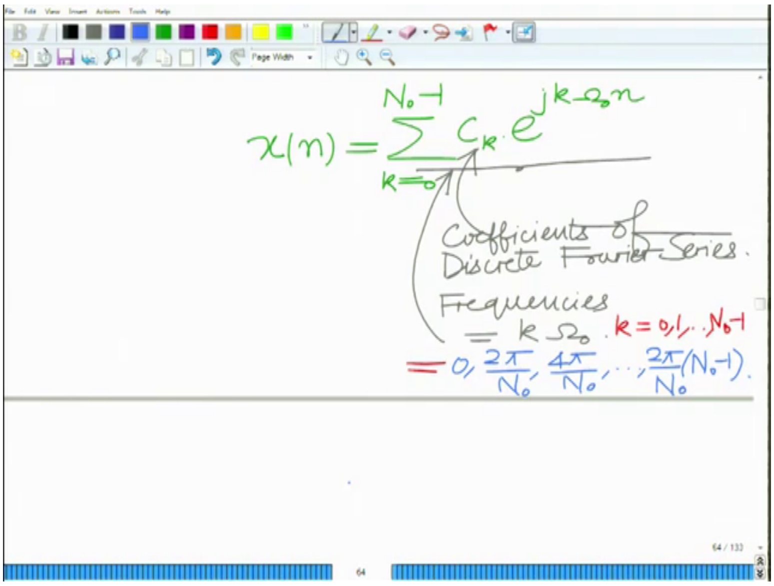

These are the coefficients of the Discrete Fourier Series or DFS. So, you can see that, this can be expressed as a sum of a finite number of complexes, unlike the complex Fourier. The complex Fourier series, which is the sum of an infinite number of complex exponentials corresponding to the fundamental frequency and it is harmonics, this can be, that is a periodic discrete time signal can be expressed as a sum of a finite number of complex exponential with, with frequencies 0 to pi over N naught 2 pi over N naught times 2, so on and so forth. Up to 2 pi over N naught times N naught minus 1.

So, the frequencies are frequencies of the complex exponentials are k times omega naught k equals 0 1 up to N naught minus 1. So, the frequencies are basically 0 2 pi over N naught 4 pi over N naught so on. 2 pi over N naught into N naught minus 1. So, these are the discrete set of frequencies all right. The set of discrete is the set of frequencies ok.

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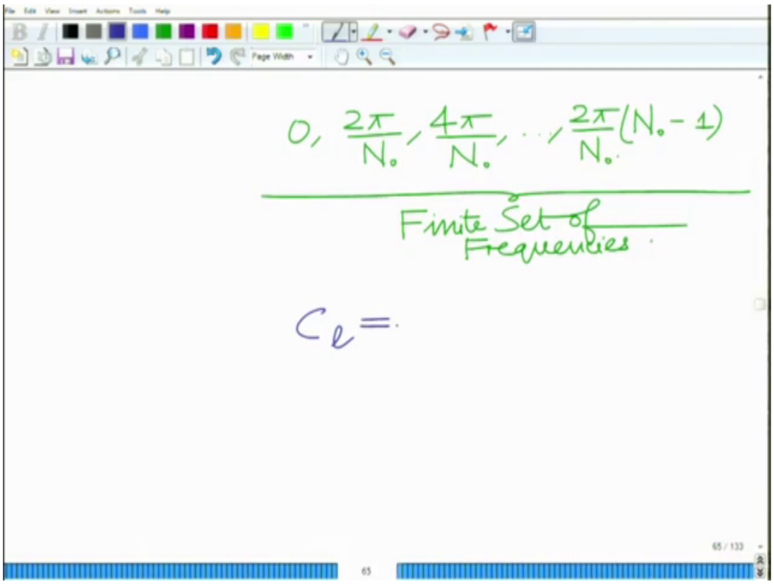

So, 4 pi over N naught 2 pi over N naught N naught minus 1. These are the finite set of frequencies. So, the complex; so basically, so the periodic sequence, the periodic discrete time signal with fundamental period N naught can be expressed as the sum of a finite set of complex exponential signals all right. Corresponding to the frequencies that are given as multiples of 2 pi over N naught all right, multiples integer, multiples integers ranging from 0 to N naught minus 1 of the fundamental frequency, 2 pi over N naught ok.

And now, how to find these coefficients of the DFS, the discrete Fourier series, that is C l, what is the coefficient the l coefficient of the DFS.

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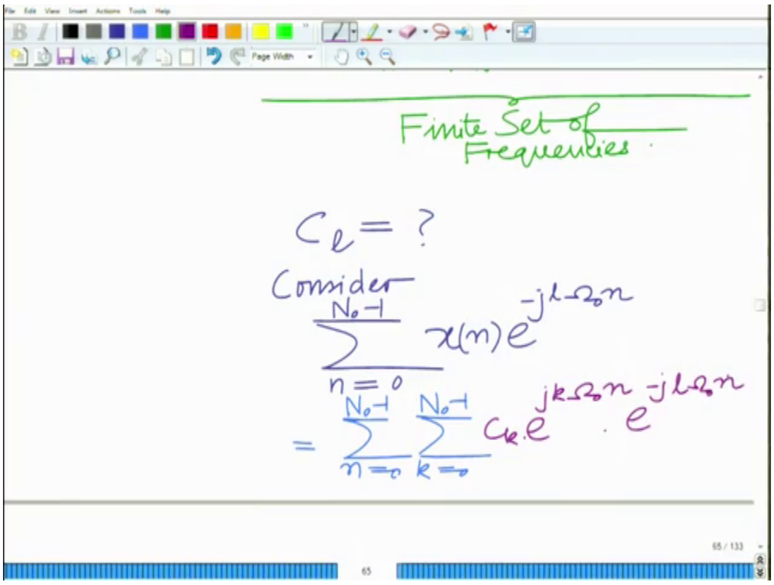

And you can see that is given as follows. Now, consider summation n equals 0 to N naught minus 1 x n e raised to j minus l omega naught n, this is equal to substituting. Now, the expression for the DFS equal to 0 to N naught minus 1 C k e raised to j k omega naught n into e raised to minus j l omega naught e raised to minus j l omega naught n.

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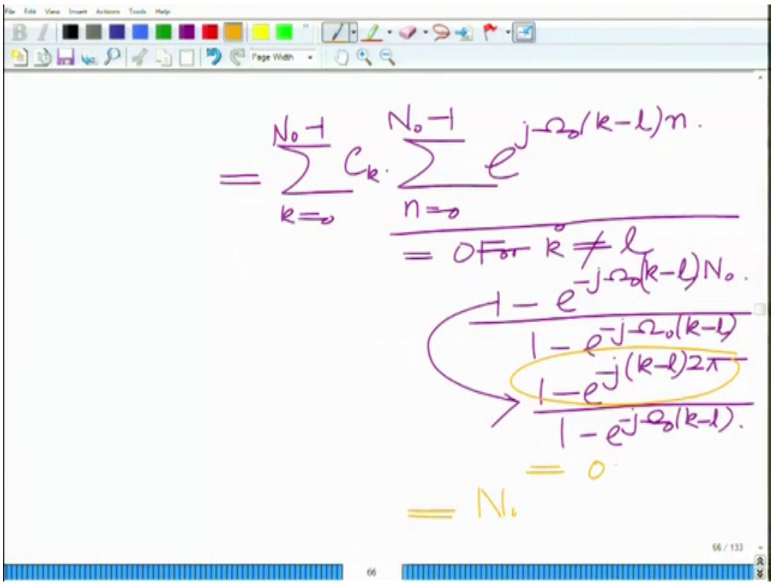

And now, interchanging the summation; so I have summation k equal to 0 to N naught minus 1 can take C k out. Since, that depends only on k summation n equal to 0 e raise to j k e raised to j omega naught k minus l into n ok. Now, you can see for k not equal to l. In fact, this sum is equal to equal to 0 for k naught equal to. In fact, for k naught equal to l. This is 1 minus e raise to minus j omega naught k minus l times, N naught by 1 minus e raise to minus j omega naught e raise to minus j omega naught k minus l ok.

And e raise to minus j omega omega naught N naught that is 2 pi. So, this quantity. Here is 1 minus e raise to minus j k minus l in k minus l and this quantity is 0. The numerator is 0, because I raised to minus j k minus l 2 pi is 1. So, this is missing, when k is not equal to l, this is 0. So, the only case that is remaining is, when k equal to l, when k equal to l k equals l, this is e raised to j omega naught 0, which is 1; so summation of 1 n equal to 0 to N naught minus 1 that is N naught ok.

So, what we are left with is basically a naught, if k is equal to l at 0, if p is k k is not equal to l.

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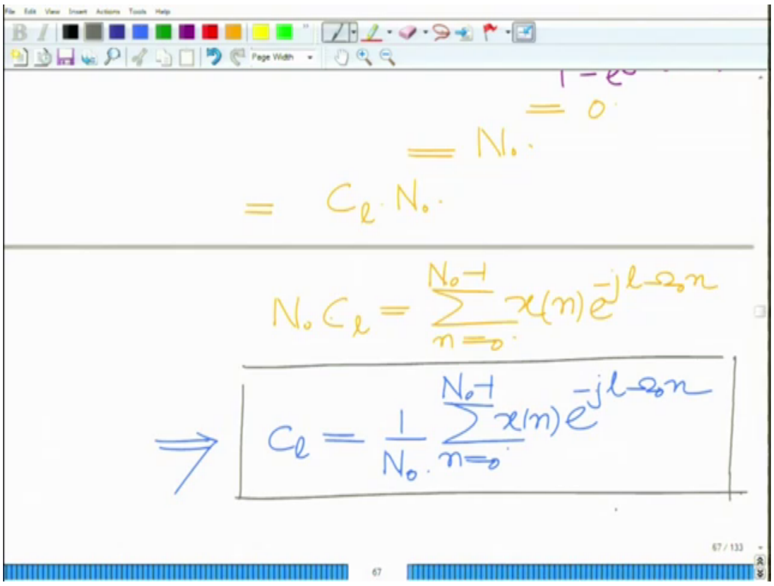

And therefore, only the Ck, the coefficient C k corresponding to k equal to l survives. So, that will be C l times N naught, which means, we have summation n equal to 0, which means we have N naught into C l equals summation n equal to 0 to n naught minus 1 x n e raise to minus j omega naught e raised to minus j l omega naught n, which basically, implies that your C l the l th coefficient of the discrete Fourier series is 1 over N naught, n equal to 0 to N naught minus 1 x n e raised to minus j l omega naught n ok.

So, that is basically your discrete Fourier series coefficients. So, this is and you can also verify that these summations need not be over N naught equals 0 to n minus 1 in fact.

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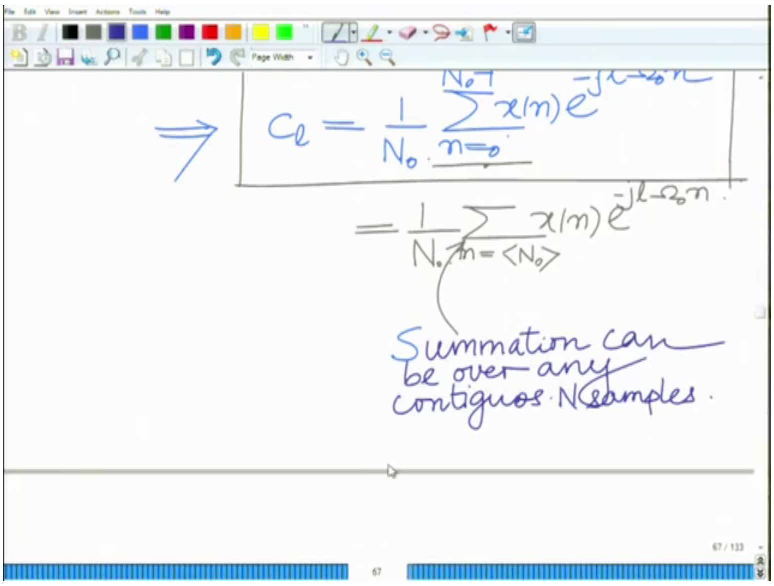

This can be over any N naught contiguous samples. So, you can have this from n equals N naught right. So, this can be from 0 to n minus 1 or this can be from n equals 1 to N naught or this can be from 2 to N naught plus 1. So, it, it can be or the summation can be taken over any contiguous and not samples and that I am representing it using this notation.

So, summation can be over any contiguous N N naught samples the DC coefficient and finally, you can see the DC coefficient, l equal to 0 C naught equals 1 over N naught summation, n equal to 0 to N naught minus 1 x n.

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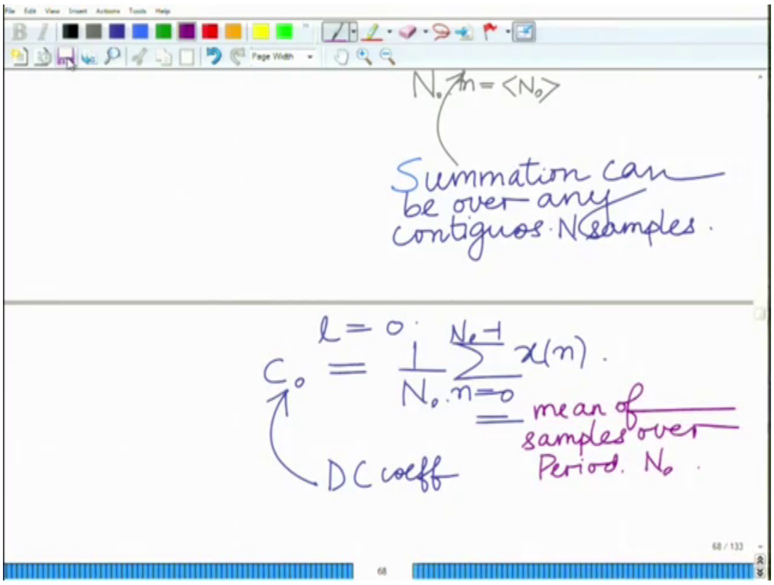

So, this is the DC coefficient simply, the mean of the samples over a period, over the single period N naught all right.

So, this is the mean of the samples. Sample mean of the samples over a single period N naught ok. So, this is basically the expression of; so, this is basically the expression for the inverse discrete Fourier series that is to extract the coefficients CK's of the coefficient C l from the periodic discrete time signal x n ok.

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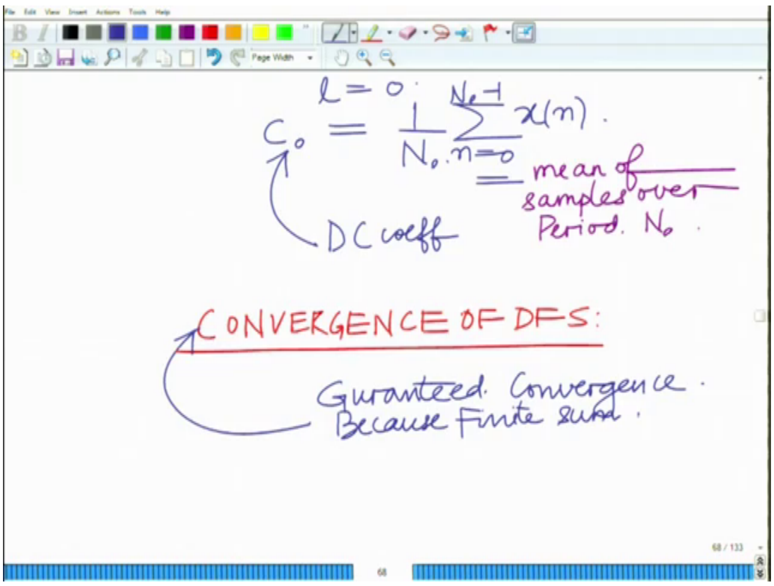

And now, coming to converge as the discussion on convergence is going to be very brief. Now, the convergence there are typically no issues. The convergence is guaranteed, because it is a finite sub. Now, you can see basically, both the discrete Fourier series correct if you can look at this expression for the discrete Fourier series. So, this is a finite sum.

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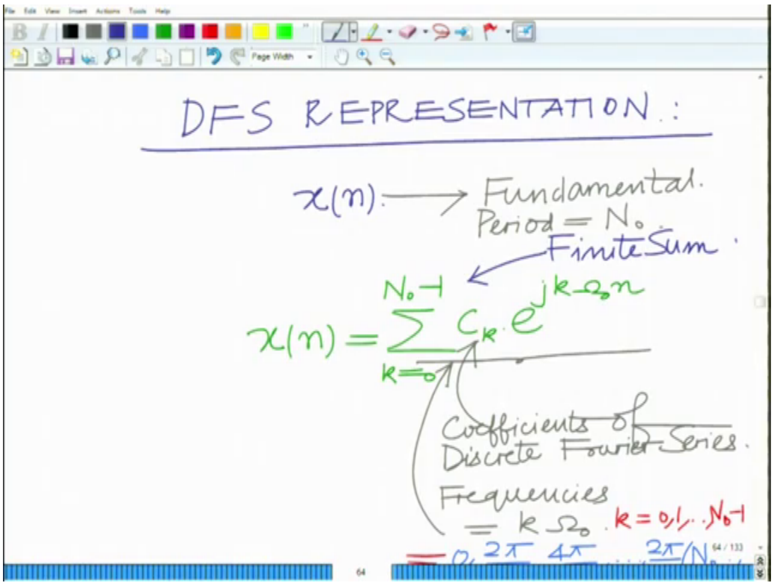

For finite sum, we mean the summation over a finite number of elements and as well as the inverse discrete Fourier series to relate the coefficients C l, this is also a finite sum right.

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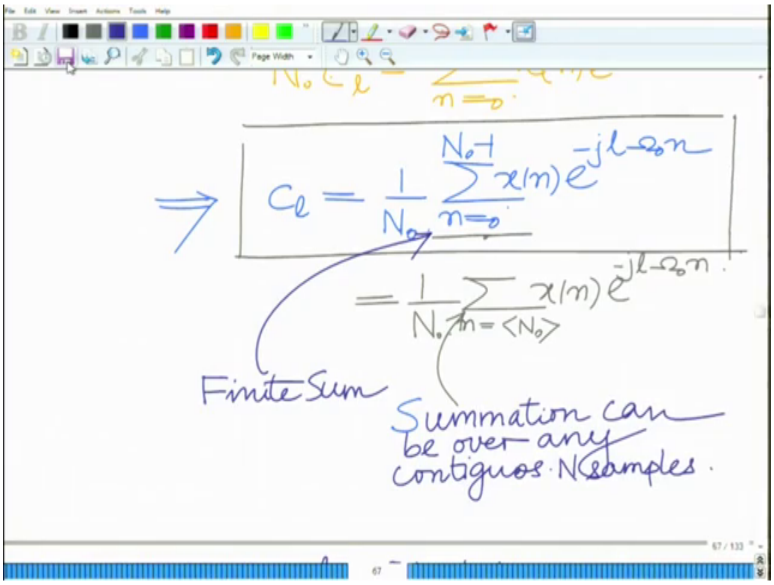

So, this is also a finite sum. So, because both the sums are finite, the convergence is guaranteed, all right. The convergence is currently under certain, very mild conditions ok. So, the convergence is there typically, because unlike both the continuous time periodic, the continuous time periodic signal and there as well as the continuous time, a periodic signals, because in the continuous. In the case of the continuous time periodic signals, it is a summation over infinite series, there is a complex exponential Fourier series and in the case of the continuous time, a periodic signals, it is integral over an infinite duration and it is in both those scenarios. It is important to ensure that the relevant summations or the integrals converts.

However, since in this case both the summations are finite summations that is a summation over a finite number of terms, the convergence is guaranteed ok, all right. The periodicity of the Fourier series, Fourier series coefficients then you will also note that these Fourier series coefficients.

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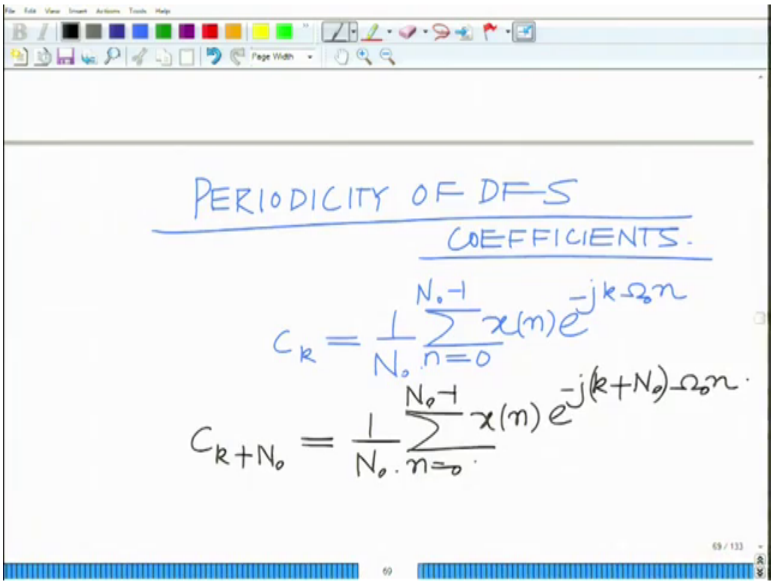

It is coefficients are periodic that is very easy to see, you can see that C k is 1 over N naught summation n equal to 0 to N naught minus 1 x n e raise to minus j k omega naught n.

Now, if you consider C of k plus N naught that is equal to 1 over N naught summation n equal to 0 to n minus 1 x n e raise to minus j k plus N naught omega naught n, which is a 1 over N naught.

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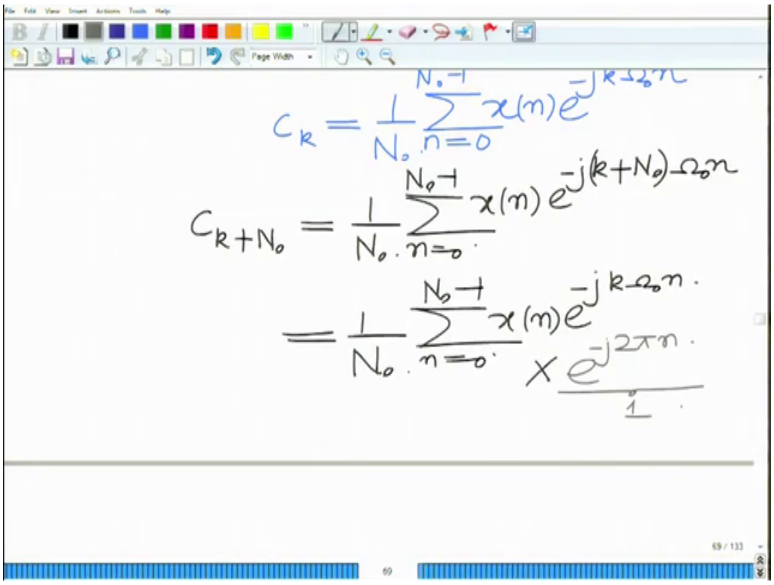

Summation n equal to 0 to N naught minus 1 x n e raised to minus j k omega naught n times e raised to minus j n naught omega naught is 2 pi. So, this is 2 pi n and e raised to minus j 2 pi n is 1.

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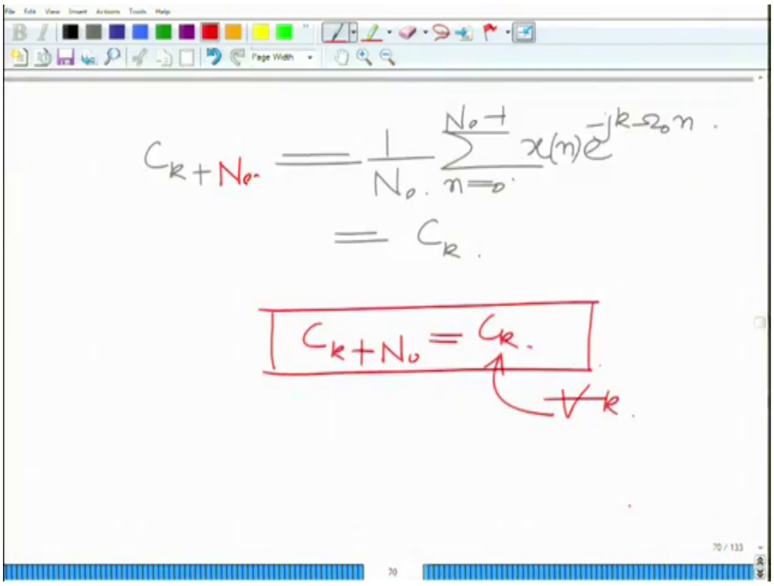

And therefore, this again reduces to C k plus N naught reduces to 1 over N naught n equal to 0 to N naught minus 1 x n e raise to minus jk omega naught n, which is nothing, but C K.

So, we have C k plus N naught equals C k sorry, this is capital N naught. This is for any general k implies. The discrete Fourier series is periodic implies the DFS coefficients are periodic.

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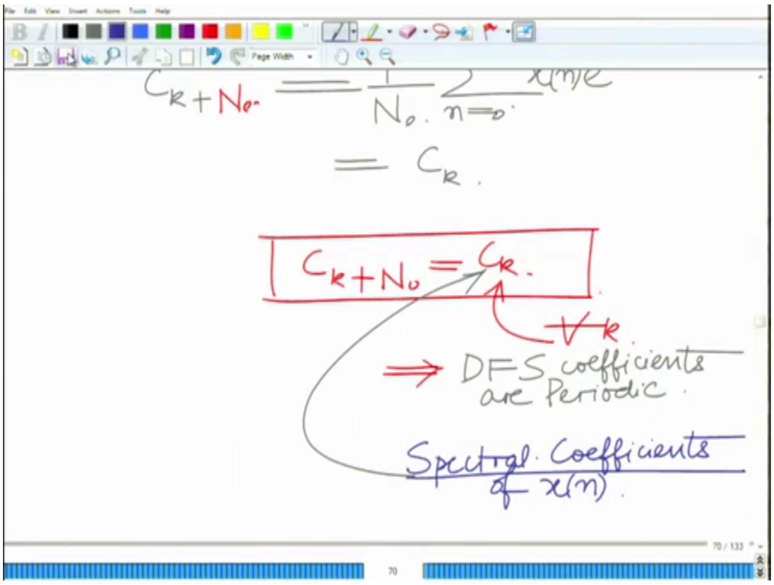

So, we have a discrete time periodic signal in time and the corresponding discrete time, Fourier series corresponding the discrete Fourier series coefficients in the frequency domain are also periodic and the period incidentally is the same that is a period is N naught for both the time, as well as the spectral domain.

And. In fact, these quantities, these are known as the spectral coefficients these quantities. The C k s C k plus N naught, these are known as the spectral coefficients of x n. These are known as the spectral coefficients of, these are known as the spectral coefficients of x n, all right.

So, in this mode you will started looking at the discrete Fourier series or the Fourier analysis for discrete time discrete time signals and we have started with the discrete time, discrete Fourier series for a periodic discrete time signals, alright.

So, we will stop here and continue in the subsequent modules.

Thank you very much.