Principles of Signals and Systems
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Lecture - 49
Fourier Analysis Examples: Bode Plot for Magnitude/ Phase Response – Second Example, Fourier Transform of Hilbert Transformer

Hello, Welcome to another module in this massive open online course. So, we are looking at the example problems for the Fourier transform and particularly focusing on the Bode plot to represent them, both the magnitude as well as the phase response of an LTI system ok.

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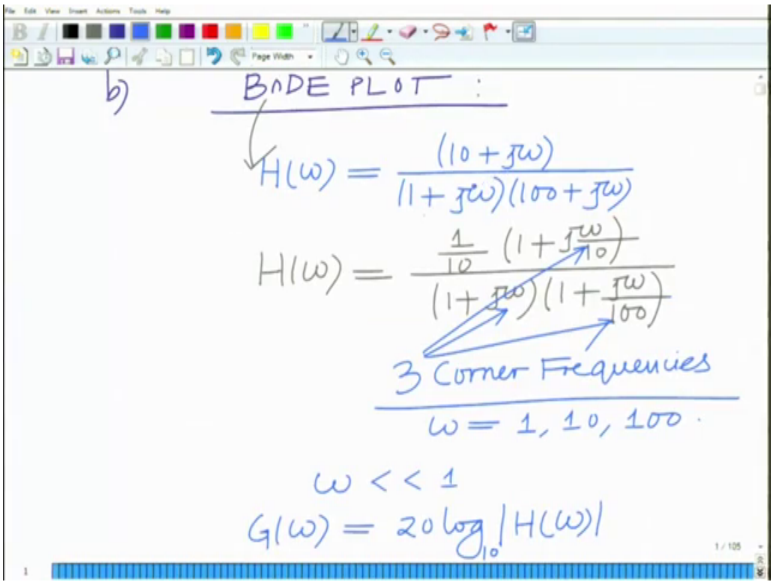

So, we are looking at the Bode plot specifically and in particular, we are looking at this transfer function one 10 plus j omega over 1 plus j omega into 100 plus j omega, correct.

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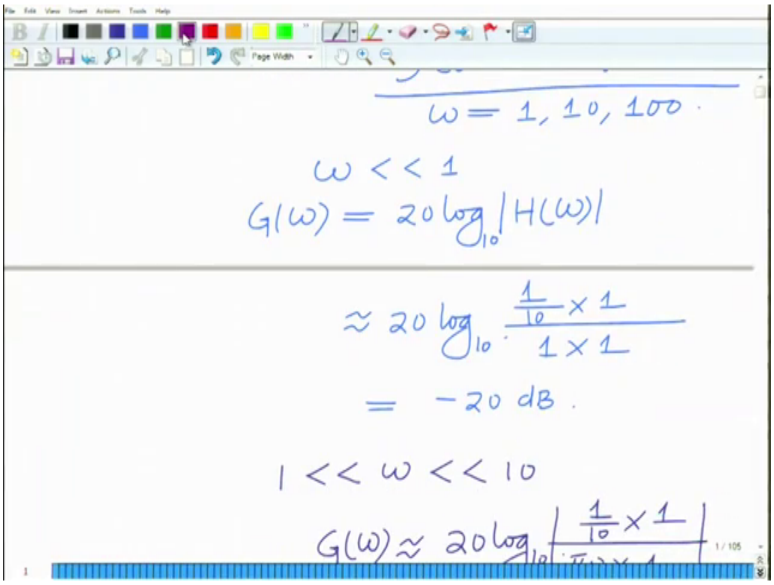

And we have derived the Bode plot for 2 regions that is for omega less than much less than 1 and omega lies between 1 and 10 radians per second.

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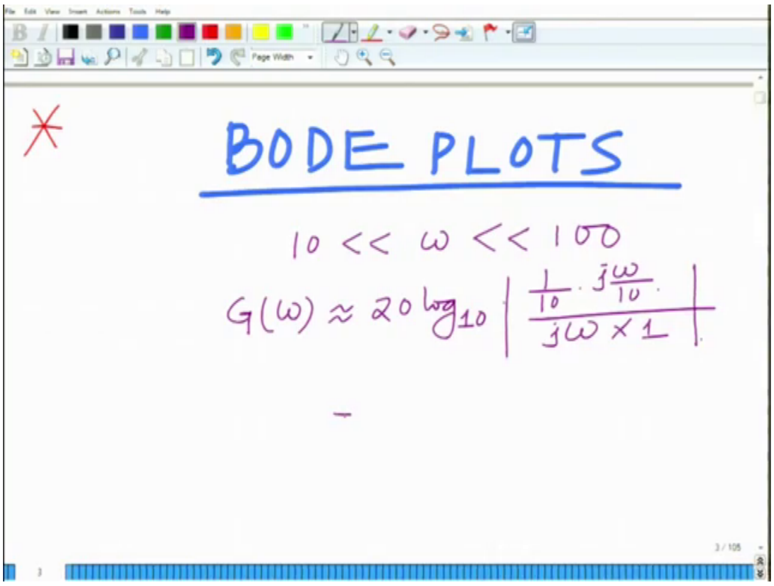

Now, for omega lie when omega lies between 10; Now, we can treat this as another case, omega lies between 10 and 100 radians per second, you can see that in this case G of omega is approximately equal to 20 log to the base 10, 20 log to the base 10, this would be 20 log to the base 10. Now, 1 plus j omega over 10, since this is great much greater than 10, this will be approximately j omega or 10, 1 plus j omega the denominator will approximately be j omega and 1 plus j omega over 100 will be plus since this is omega is much less than 100.

So, this is approximately equal to 20 log to the base 10 magnitude of 1 over 10 to j omega over 10 by omega or rather j omega into 1 which if you look at this will be equal to 20 log to the base 10 and the omegas cancel.

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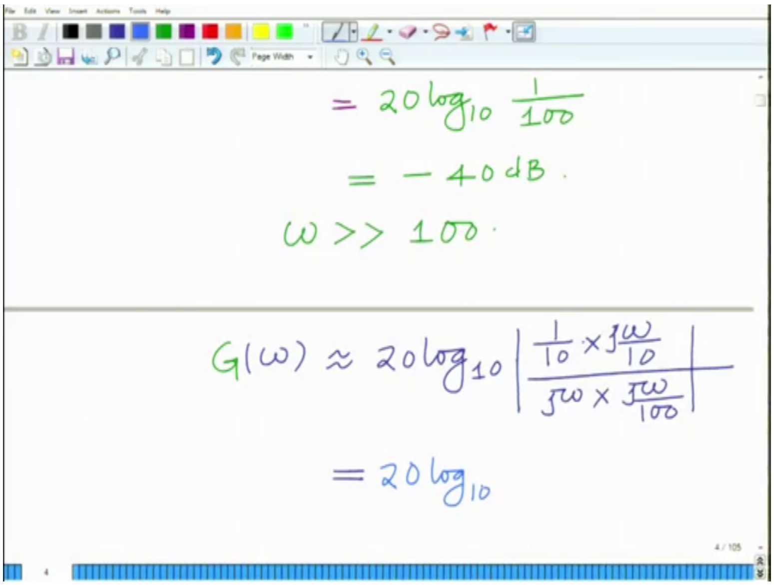

So, 20 log to the base 10 plus over 100 which is equal to your constant minus 40 dB. So, when omega is much greater than 10 and less than omega is much greater than 10 really much greater than 10 radians per second and much less than 100 radians per second in that decade, it is flat it is minus 40 dB.

And then finally, now when omega is much greater than 100 radiations per second omega is much greater than 100 radians per second, you have G of omega approximately equals 20 log to the base 10 1 over 10, 1 plus j omega over 10 is approximately that will be j omega over 10 times 1 plus j omega will be j omega times 1 plus j omega or 100 will be j omega over 100. So, this is equal to 20 log to the base 10 correct 20 log to the base 10 1 over 1 over.

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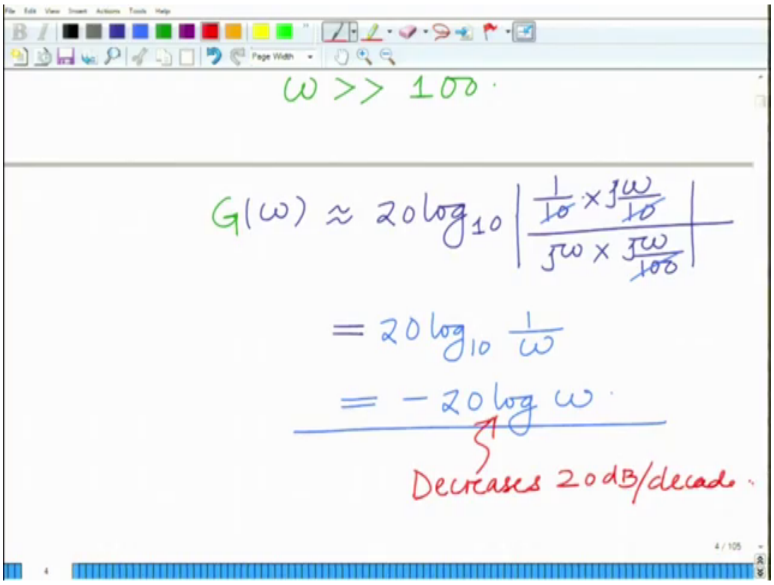

So, this would be 20 log to the base 10, I would have 20 log to the base 10, 1 over omega which is equal to minus 20 log to the base 10 omega which basically, this again decreases as 20 dB per decade this decreases 20 dB per decade.

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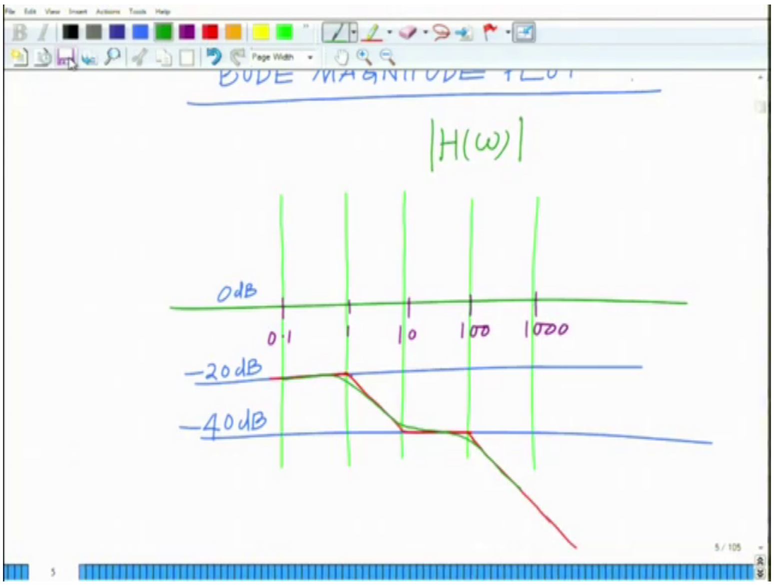

And therefore, the Bode magnitude plot looks something like the Bode magnitude plot looks as follows. So, this will be correct starting from point 1 to plus to 10 to 100 to 1000 radians per second and this is your 0 dB minus 20 dB minus 40 dB. So, this starts with for it is when omega is much less than plus; you can see as we have derived previously when omega is significantly small this would be minus 20 dB. So, over omega much less than 1 or omega much less than for omega much smaller than 1, you can see this is the region that we are talking about for omega much smaller than 1 this is minus 20 dB.

So, for omega much smaller than plus this is minus 20 dB. So, it would look something like this when omega is between 1 and 10, it decreases 20 dB per decade, correct between 10 and a hundred it is constant minus 40 dB and when omega is greater than 100, it again starts decreasing as 20 dB per decade. So, that would be something like this ok. So, this is your 20 dB per decade and the actual Bode plot will look something like ok.

So, this is the actual Bode plot, all right. So, this is basically your magnitude of this is basically the Bode plot for the magnitude of H omega ok. So, it looks something like this. Now, when you look at the phase, it becomes a little bit more complicated. So, that completes our discussion for the Bode plot of the magnitude now to look at the Bode plot of the phase, it is slightly complicated. So, I am going to explain the various cases ok.

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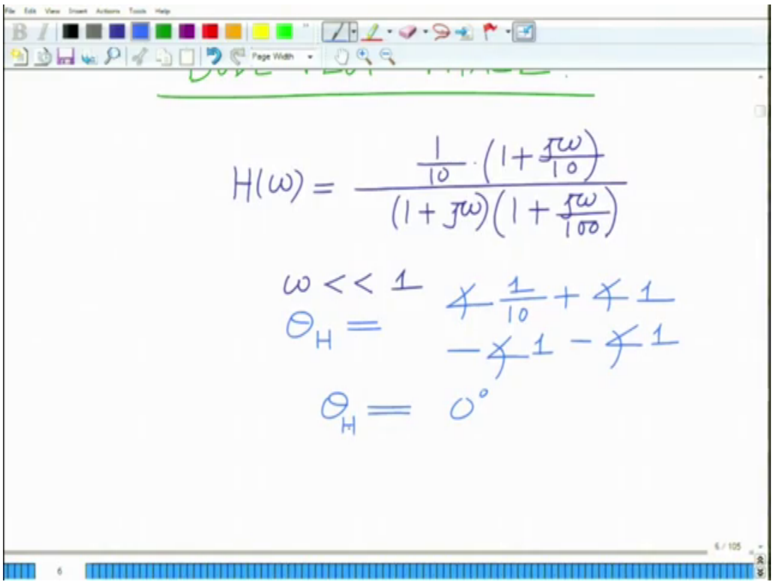

So, let me now look at the Bode plot of the phase. Now, if you look at the Bode plot of the phase that is basically, again let me write down this transfer function that is H of j omega H of omega equals plus over 10, 1 plus j omega divided by 10 into 1 plus j omega into 1 plus j omega over 100.

Now, consider the first case that is omega much less than the first corner frequency that is plus when away is much less than plus the phase theta H that is a angle of H omega, you can clearly see is basically 1 plus, this is you can see this corresponds to. So, angle of 1 over 10; so, this is basically the angle of the constant plus over 10 in the denominator numerator plus angle of 1 in the denominator numerator because j omega is much smaller than 1 minus angle of in the denominator 1 plus j omega is approximated by 1 minus angle of 1 minus angle of again 1 because omega is much smaller than 1.

Now, I know all these angles are 0 because these are real numbers. So, theta H is basically this is 0 degrees ok. So, all these are constants it is 1 over 10 basically it is for omega much smaller than 1, we can see that its H of j omega is approximately 1 over 10 all right and therefore, all the angles are 0 because it is a real number correct and positive real number therefore, the phase is 0 degrees ok.

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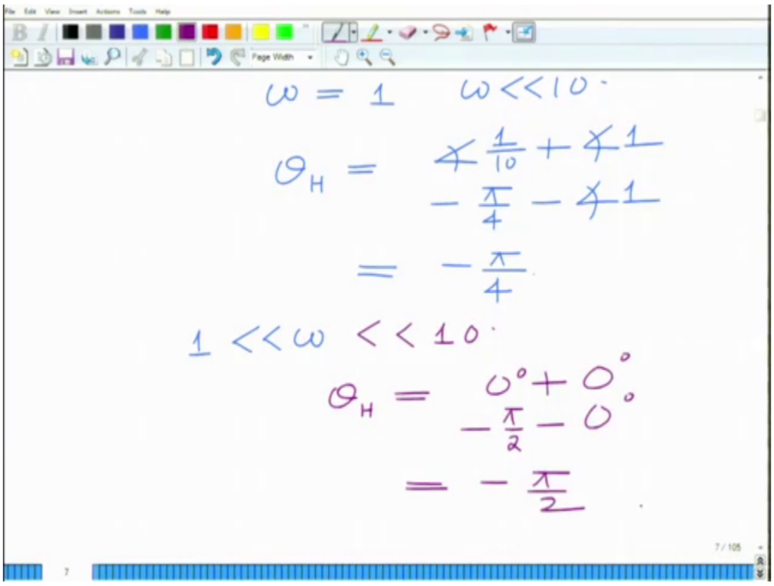

Now, when omega becomes now becomes comes the interesting cases when omega equals one, but omega is much smaller than 10 omega equals 1 now theta H equals no angle of 1 that is plus over 10 in the numerator plus angle of 1 plus j omega over 10 is still the angle of plus approximated by 1 minus. Now the angle of 1 plus j omega when omega is plus, this is pi by 4. So, minus pi by 4 minus angle of 1 plus j omega over 100 when omega is 1 is again angle of 1 because 1 plus j omega 100 is well approximated by 1. So, this is equal to minus pi by 4.

Now, proceeding so forth when omega is much greater than 1 , but much smaller than 10 theta H is angle of plus over 10 in the numerator that is 0 degrees plus the angle of 1 plus j omega or 10, but since omega is much smaller than 10, this is also 0 degree, but since omega is much greater than plus 1 plus j omega is well approximated by j omega, therefore, the angle corresponding to that is pi by 2, correct and it is in the denominator. So, that contributes minus pi by 2. So, minus pi by 2 minus 1 plus j omega by 100; that is again 0 degrees; so, this will be minus pi by 2.

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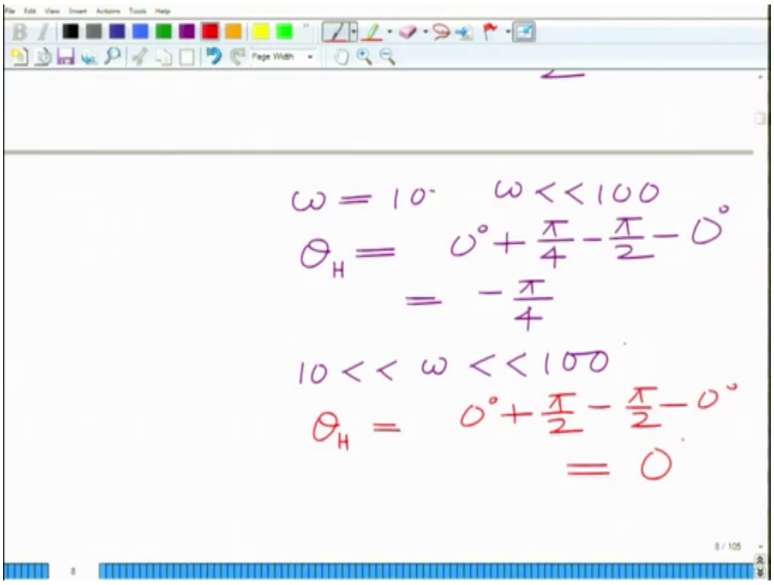

When omega equals 10 and still again omega is much smaller than 100 degrees then theta H equals, once again 0 degrees plus.

Now, since omega equals 10 in the numerator 1 plus j omega by 10; that is 5 by 4 denominator, again j 1 plus j omega that is pi by 2 because omega is much greater than 1 and since omega is much less than 100, it is 0 1 plus j omega by 100; that contributes 0. So, this is minus pi by 4 and since and fine and also now this is omega equal to 10.

Now, omega is much greater than 10 still much less than 100 or 100 radians per second, I am sorry, then we have theta H equals numerator again plus over 10 contribute 0 degrees, the j omega over 10 since omega is much greater than 10 j omega over 10 contributes pi by 2 denominator j omega also contributes pi by 2, these 2 cancel, but omega is much less than 100. So, from that component there is there is still 0 1 plus j omega over 100.

So, this is 0 plus pi by 2 minus pi by 2 minus 0 degrees which is equal to 0.

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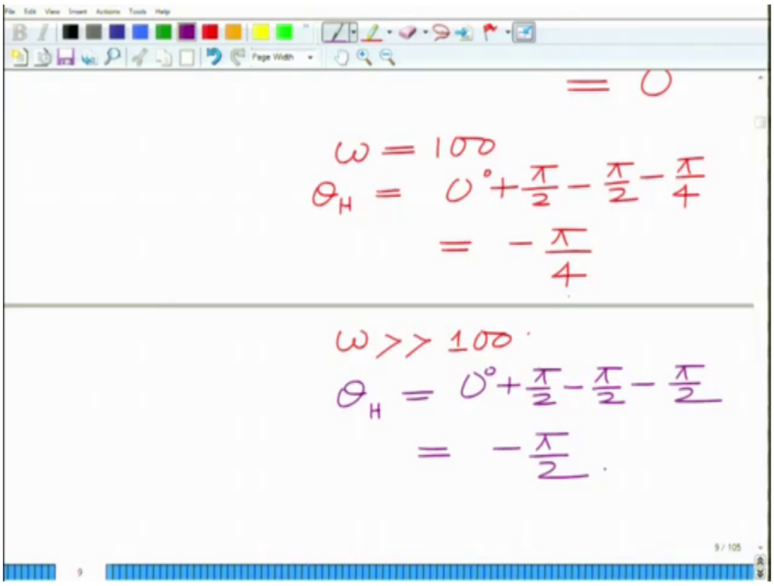

And finally, when omega equal to hundred when omega equals hundred theta H equals 0 degrees plus 1 plus j omega over 10 pi by 2 denominator 1 plus j omega that is minus pi by 2 and 1 plus j omega 100 that corresponds to minus pi by 4 ok. So, this will be minus pi by 4.

And finally, when omega is much greater than the corner frequency of the last corner frequency 100, we have theta H equals 0 degrees plus pi by 2 minus pi by 2 1 plus j omega by 100 is approximately j omega by 100. So, that contributes minus pi by 2. So, finally, it will settle at some point it will settle at minus pi by 2. So, if you take all these cases and try to plot the Bode plot for the magnitude, you take these various cases and you plot the Bode plot for the magnitude.

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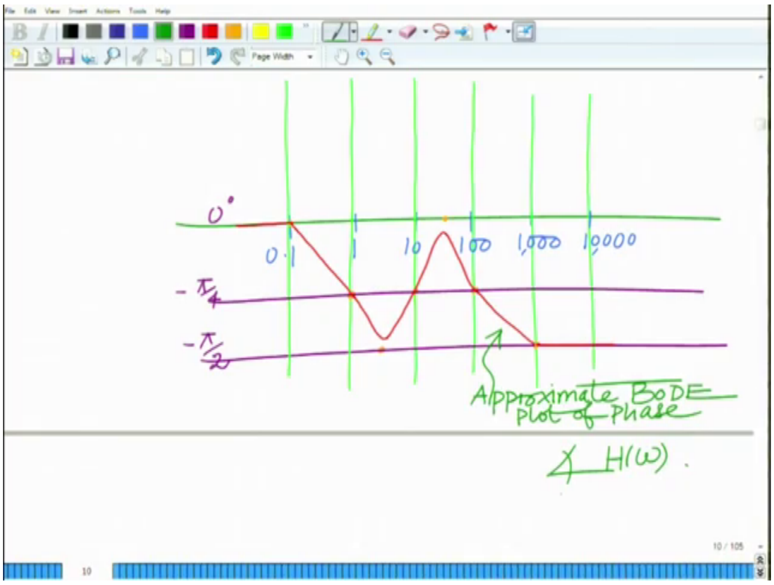

So, once again you have 0.1, 1, 10, 100, 1000, 10,000 and now if you plot the Bode plot. So, let us. So, this is a little bit tricky and it course is going to be very approximate because plotting the phase is more difficult. So, this is let us say minus pi by 4, this is 0 degrees this is pi by 2 minus pi by 2. So, this is minus pi by 2 and now at of course, when it is much less than 1 for very small omega, this is 0 degrees. So, you come from here which is 0 degrees when it is very is much smaller than plus it is 0 degrees ok.

So, much smaller than 1, it is 0 degrees at 1 it is minus pi by 4 as we have seen above and omega equals 1, this is minus pi by 4 at omega. So, this is at omega equal to 1, I am just going to draw the dots a little bit bigger this is minus pi by 4 at omega equals 10 when omega equals 10, this is again you can see minus pi by 4 and between 1 and 10 when omega is much greater than 1 and omega is much smaller than 10 it somehow gets close to minus pi by 2.

Now, at 100 omega equals 100, this again minus pi by 4 between 10 and 100 when omega is much greater than 10 much smaller than 100, it becomes close to 0 and when omega is much greater than 100, it becomes it becomes pi by 2. So, this is how the plot looks like. So, it starts from here which is 0 when it is much less than plus. So, if you join these by a line, now it looks something like it looks something like, this is how it looks basically approximately this is the Bode magnitude of the Bode plot of the phase ok.

It has something like this is approximate notice that is not exact because it is much more difficult to plot the phase when compared approximate the Bode plot of phase that is you have your angle of the transfer function as it varies from very small value it starts from 0 finally, you can see two things one is when it is very small, it is 0 degrees because it is just a constant when its much larger than 1 hundred all right all the terms become; so, numerator because 1 over 10 into j omega over 10 divided by j omega into j omega over 100. So, basically it will basically be plus over j which will correspond to phase of minus pi by 2.

So, you start from 0 ends up in minus pi by 2 and in between it has this kind of a phase characteristic ok. So, that basically completes our discussion on the Bode plot right as we already try to impress upon you, the Bode plots Bode for the magnitude and phase are very important together and they are very convenient tools to plot and derive a visual a visual representation of both the magnitude and the phase response of a linear time invariant system from the frequency response ok.

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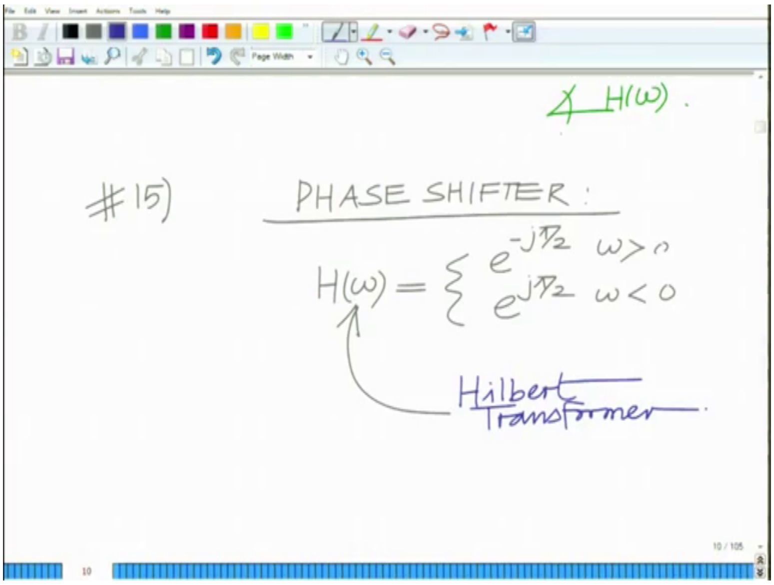

Let us now do another problem moving from the Bode plots now let us consider a phase shifter system now in a phase shifter basically the frequency response the frequency response of the system looks like this H of omega equals e power minus j pi by 2 for omega greater than 0 and e power j pi by 2 for omega less than 0.

Now, this is also termed as a phase shifter or also termed as a Hilbert transform or another name for this is also. In fact, this has a lot of applications in communication; this is used to generate a type of amplitude modulation known as single sideband modulation instead of transmitting both the sidebands one can transmit only a single sideband ok. So, this is a very important system which is practically relevant, all right.

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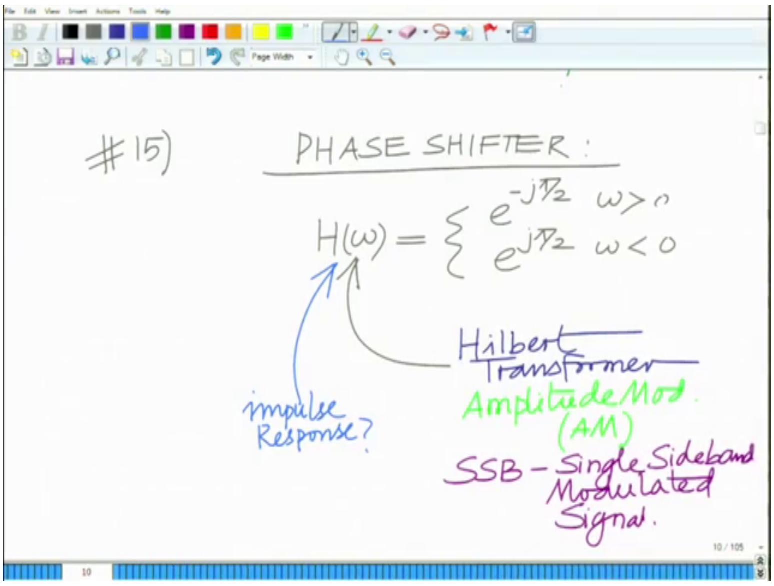

So, let me just note that this is used in amplitude modulation.

More specifically a m to generate what is known as an SSB that is single sideband to generate a single sideband modulated signal now H of j. Now you can see H of omega basically that corresponds to H of omega.

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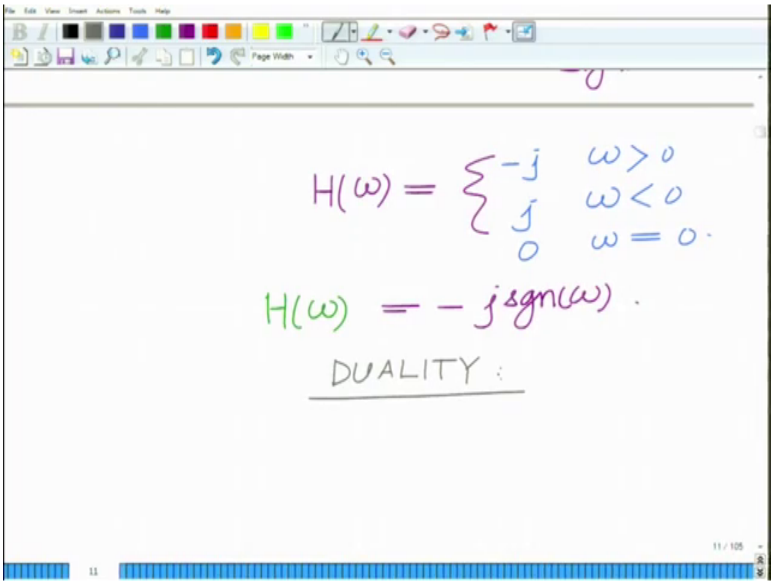

Now, what we want to do is we want to derive the impulse response of this system, ok, the phase shifter. Now what the question that we want to ask is what is the impulse what is the impulse response of this system corresponding to this frequency response what is the impulse response ok.

And you can clearly see all this is doing is all the positive frequency components is shifting the phase by minus pi by 2 all the negative frequency components is shifting the phase by pi by 2 ok. So, H of omega equals minus j omega less than omega greater than 0, it is j omega less than 0 it was 0 you can set it as equal to 0 although we do not explicitly mention it.

And therefore, you can write this as equal to minus j sin omega. So, this is basically your frequency response. Now we will use the principle of duality.

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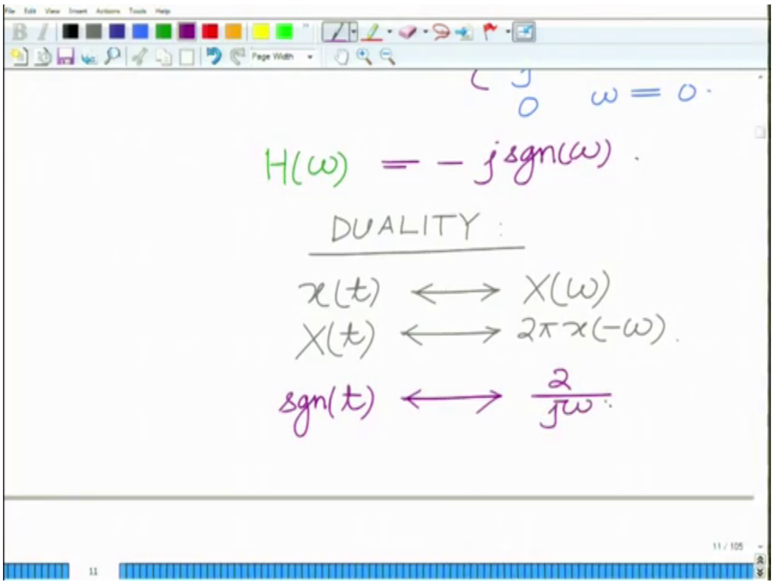

Which states that if xt has Fourier transform you can recall xt as Fourier transform capital X of omega, then capital Xt as Fourier transform x of or capital X is 2 pi X of minus omega now using. Now, we know that sign of t has Fourier transform 1 byte or 2 by rather this is something that we have derived 2 over j omega.

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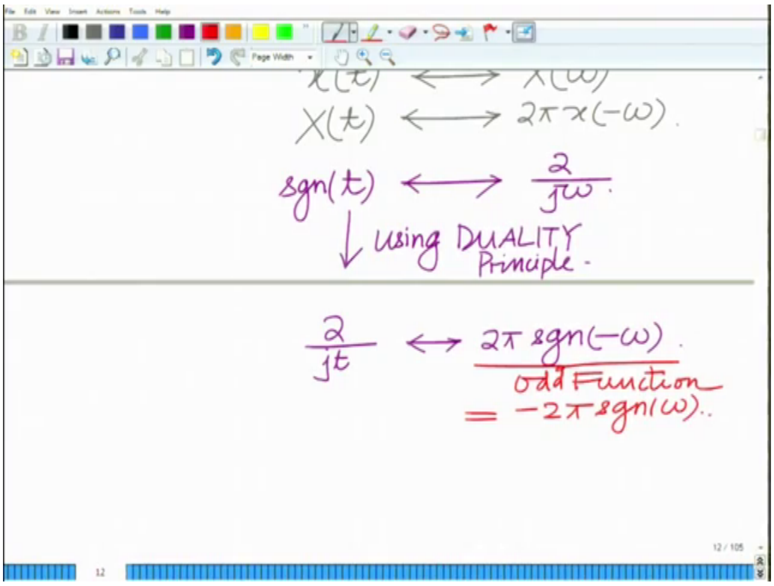

Which means now using duality or using the using the duality principle.

What we have is basically 2 or 2 over jt has Fourier transform 2 pi sign of minus omega, but real is sin of t is an odd function therefore, sign of omega is minus sign of omega ok.

So, this is an odd function. So, this is equal to minus two pi sign omega which implies basically that basically that sign of omega.

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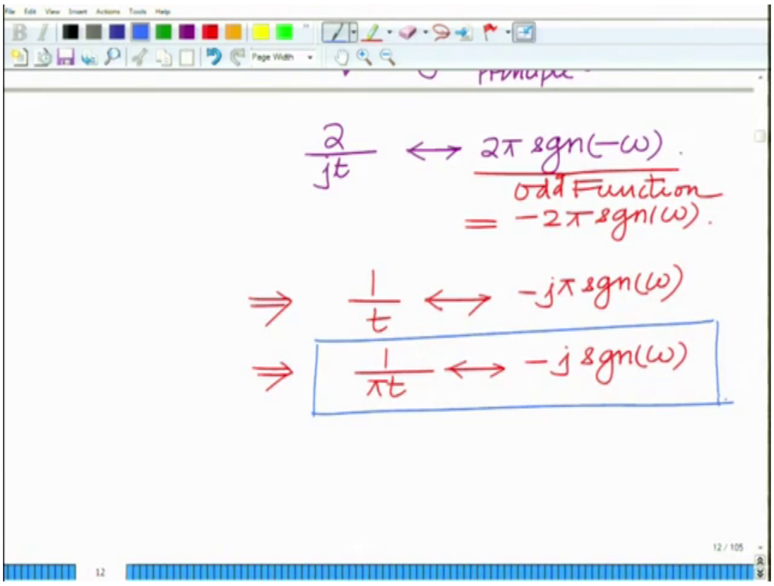

Which you can which implies basically that now that basically implies that 1 over t has Hilbert has the Fourier transform.

Now, the 2s cancel minus j pi which implies that 1 over multiple dividing both sides by pi 1 over pi t has Fourier transform minus j sign omega which is the impulse response that we set out to derive minus j sign omega is 1 over pi t. So, this is the corresponding impulse response.

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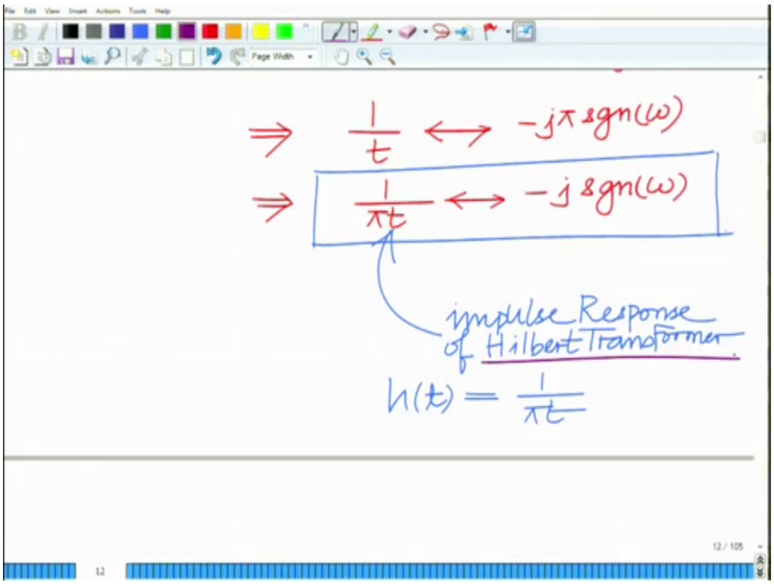

So, this is the corresponding impulse response as I have already told you, this is the impulse response of the Hilbert transform Hilbert transformer that is ht equals 1 over pi t ok, this is the response of the Hilbert transformer.

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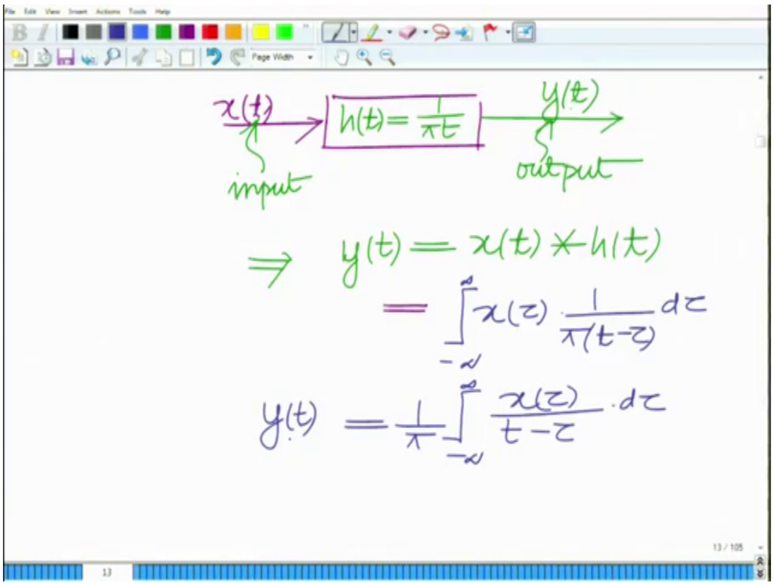

Now, if I represent this as a system, basically what I would have is I would have input signal xt output signal yt and the impulse response is 1 over pi t which implies. Now, this is the input, this is the output this implies that remember y t equals xt convolve with ht which is basically equal to now we know the impulse response. So, this is basically equal to minus infinity to infinity x tau 1 over pi t minus tau d tau taking the constant 1 over pi outside.

So, in the time domain, this is 1 of minus into 1 over pi integral minus infinity to infinity x tau divided by t minus tau d tau ok, this is the output of the Hilbert transformer.

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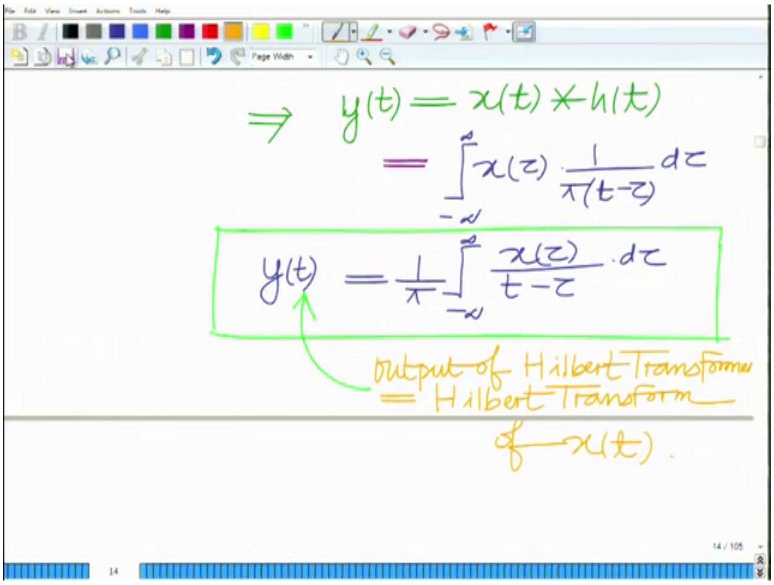

So, this yt is basically if you realize this is the this is the output of the Hilbert transformer is also known as the Hilbert transform of the signal xt ok. So, this yt also known as Hilbert transform of also known as the Hilbert transform of x t ok, this is basically the Hilbert transform of x t ok. So, all right; so, we have looked at a few more interesting problems, we will stop this module here and continue with other aspects in subsequent modules.

Thank you very much.