Principles of Signals and Systems
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Lecture - 48
Fourier Analysis Examples: Bode Plot for Magnitude/ Phase Response – Simple Example

Hello. Welcome to another module in this massive open online course. So, we are looking at Fourier transforms and example problems for the Fourier transform; this module we are going to start another important utility of the Fourier transform which is known as the bode plot which can be used to visualize the frequency responses visually represent the frequency response of an LTI system ok.

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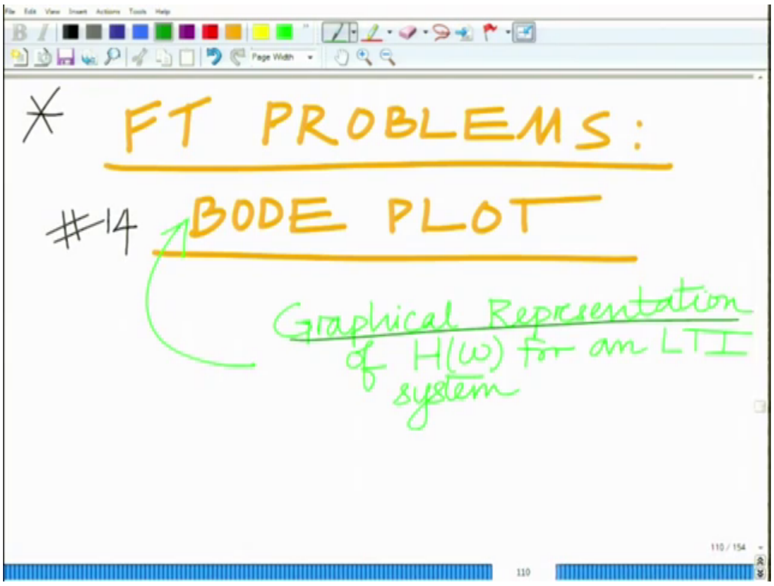

So, we want to look at the Fourier transform example problems correct, in particular in this we want to look at the what is known as the bode plot ok. And this is our problem number 14; we can call this as problem number in continuation of our problem numbering is problem number 14. And what the bode plot does for us remember it gives us a graphical representation, graphical representation of the transfer function H of omega for an LTI.

So, what it does is it uses a graphical representation of the transfer function of an LTI system; it goes as a tool to graphically represent visualize and they better understand the properties of the LTI system under consideration; we will look at this by the aid of an example ok.

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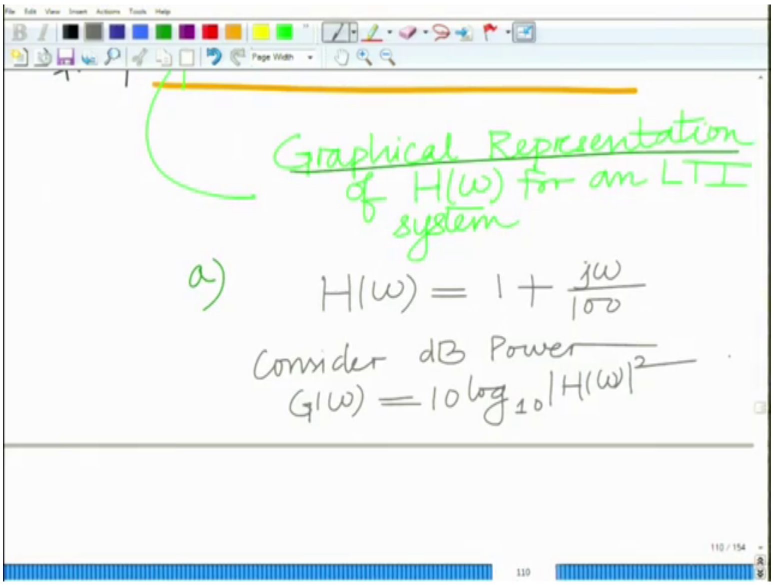

For instance consider, look at the first example a consider the following transfer function H of omega equals 1 plus j omega divided by 100 this is our example transfer function. Now consider the dB power power in dB that is given as follows that is G of omega equals 10 log to the base 10 magnitude H omega square.

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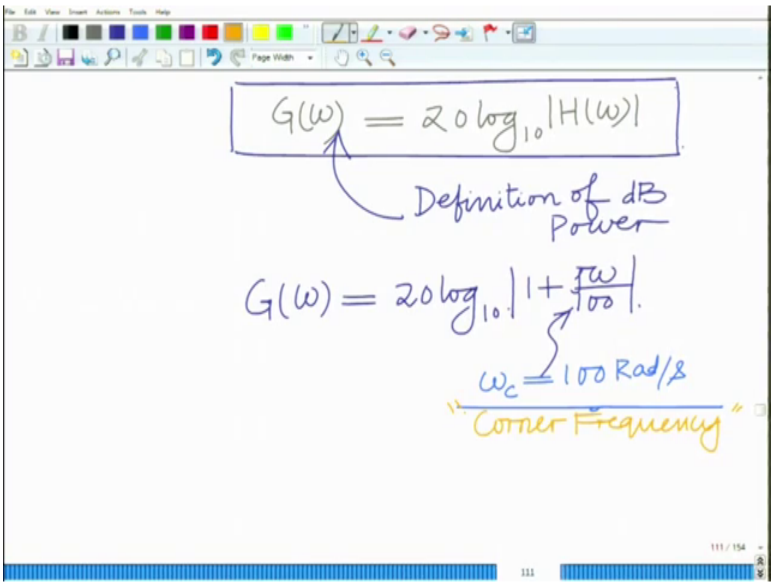

So, magnitude H omega square is the power 10 log to the base 10 of magnitude H omega square is the dB power which is 10 into 2; that is 20 log to the base 10 magnitude H of omega this is or dB power; so this is our definition of dB power. So, this is our definition of dB power that is 20 log to the base 10 magnitude of H of omega ok.

Now, in this case that would be G of omega for the given H of omega is 20 log to the base 10 magnitude 1 plus j omega divided by 100. And this 100 that is omega c equals 100 radians per second this is termed as the corner frequency ok. So, this quantity is termed as your corner frequency and the reason for that will become apparent. So, this is termed as a corner frequency ok.

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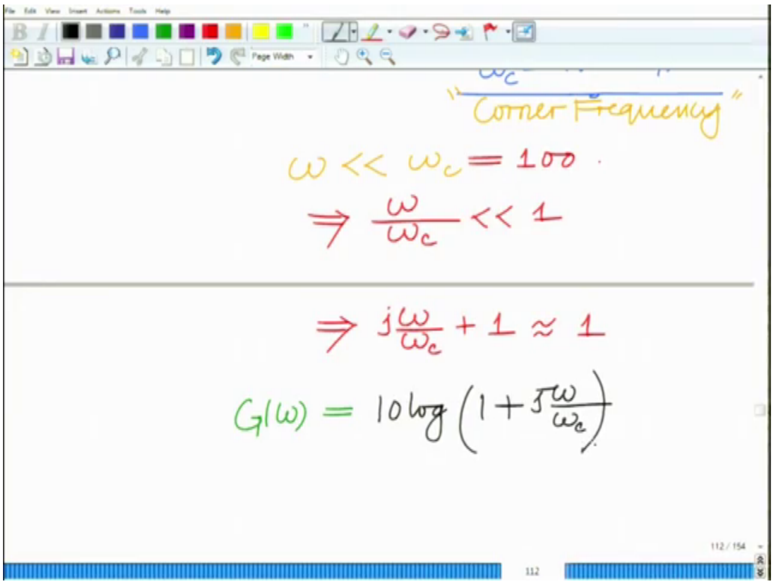

And now to plot this let us realize one thing let us divide this into two regions; one is when omega is much smaller than the corner frequency, this omega is much smaller than omega c and another corresponds to when omega is much larger than the corner frequency that is omega is much larger than omega c ok. So, consider the case when omega is much smaller than omega c.

Now, this obviously, implies omega or omega c is much less than 1 which implies omega over omega c is the 1 j omega or omega c plus 1 is approximately equal to 1. So, I can ignore the j omega over omega c or omega over 100 ok. Now in this case what we will have or omega c in remember the corner frequency omega c in this case is equal to 100 ok.

Now, in this case what you can see is G of omega equals 10 log to the base 10 I will omit the log to the base 10 because since we are going to write it frequently 1 plus j omega by omega c or omega over 100.

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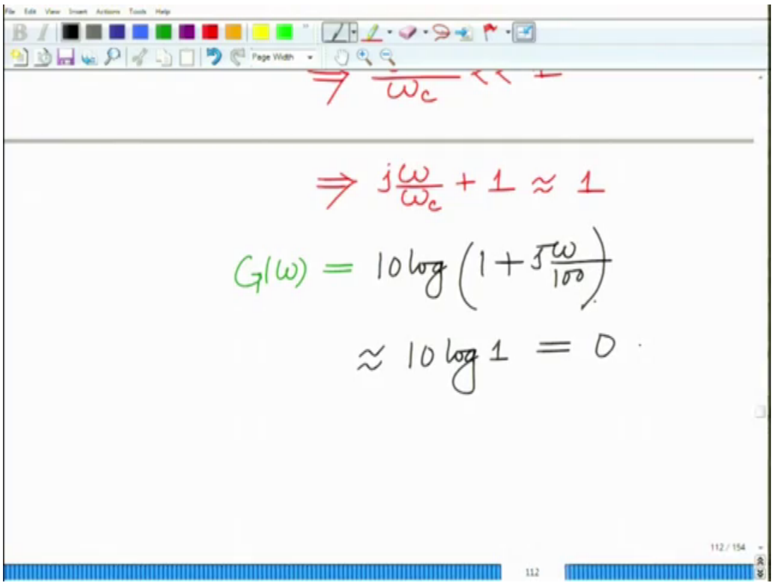

This is approximately equal to since we said is approximately 1. So, this is 10 log to the base 10 of 1 which is equal to 0 ok. So, G of omega for omega is much less than the corner frequency omega; so, that is 100 this is equal to 0.

Now, what happens when omega is much larger than the corner frequency omega c that is omega is much larger than 100 radians per second.

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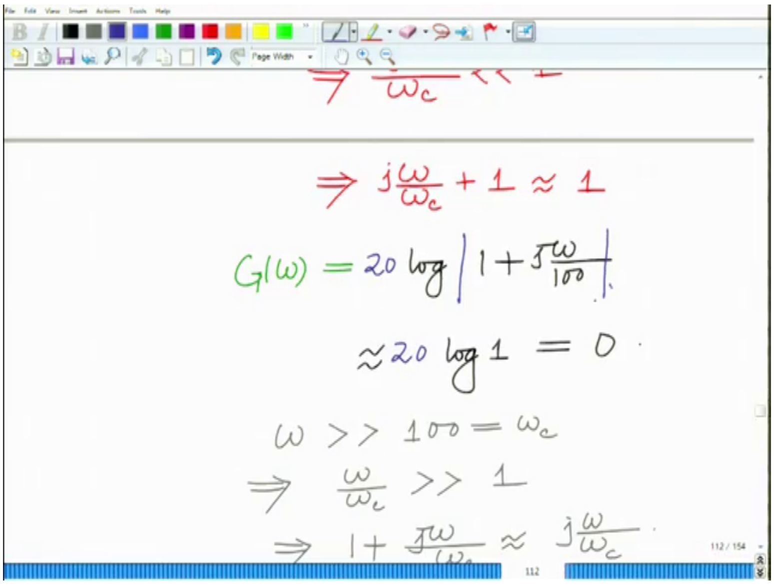

Now, the second case corresponds to when omega is much larger than 100; this implies omega over omega c is much larger than 1, this implies 1 plus j omega or omega c approximately equals j omega or omega c ok.

Now, therefore, G of omega I am sorry here this should be 20 not 10 by mistake this should be 20, this should be 20 and this should be a magnitude because this is a complex quantity ok; so, 1 plus magnitude j omega 100.

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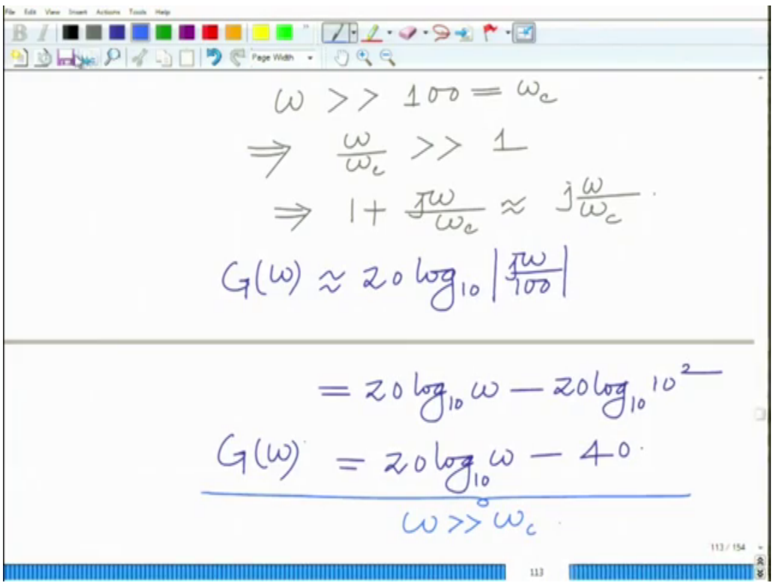

Now, G of omega in this case approximately 20 log to the base 10 magnitude j omega over 100 which is equal to 20 log to the base 10 omega minus 20 log to the base 10; 100 which is 10 square.

So, this is 20 log to the base 10 minus 40 dB ok. So, this is G omega remember this for the region, this is for the region omega is much larger than omega c ok.

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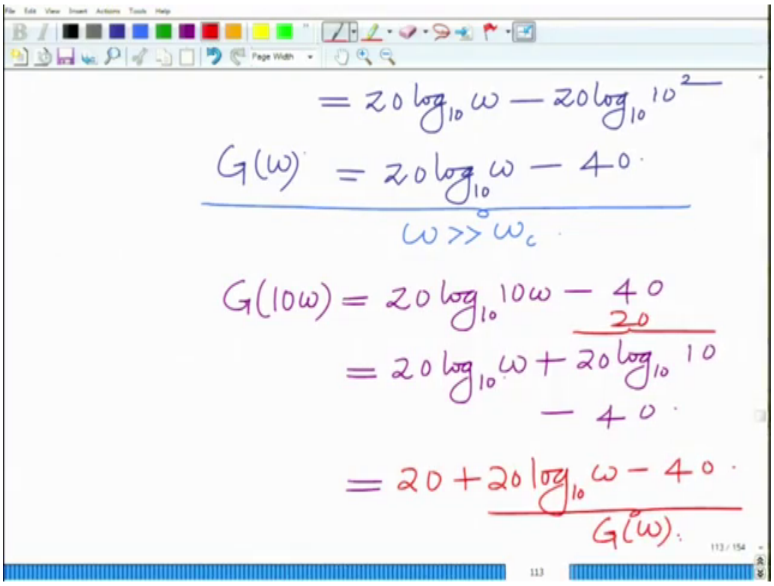

So, now if you look at this you will realize something interesting we are going to simplify this. So, G omega is basically the raw expression is 20 log to the base 10 omega minus 40 dB. Now let us look at what happens if omega increases by a factor of 10; now G of 10 omega equals 20 log to the base 10 10 omega minus 40 which is equal to.

Now, you can see 20 log to the base 10 omega plus 20 log to the base 10 of 10 minus 40. Now 20 log to the base 10 of 10; this is 20, 20 dB. So, this is 20 plus 20 log to the base 10 omega minus 40, but this is nothing, but you can see G of omega.

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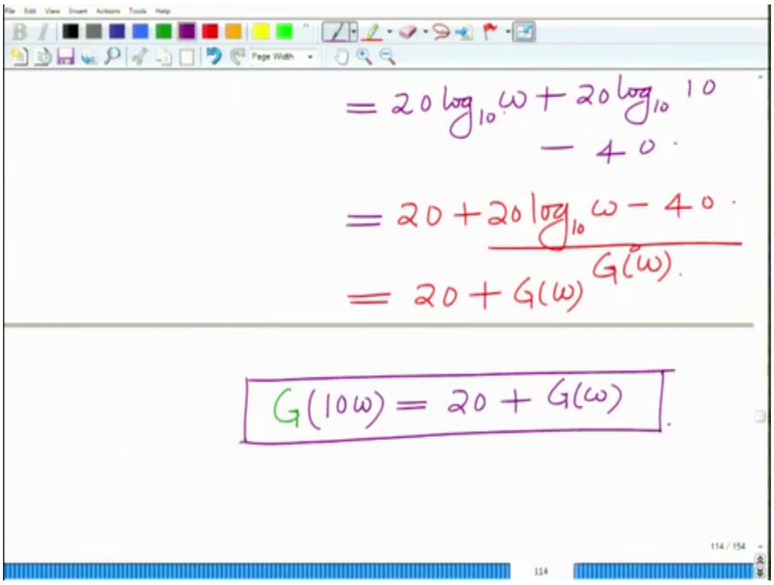

So, G of 10 omega is basically 20 plus G of omega and this is an important relation. So, what we have shown is that G of 10 omega is 20 plus G of omega.

So, what you can see is that gain rises; this gains G of omega raises by 20 dB whenever omega increases by a factor of 10 alright. So, whenever I am again. So, omega becomes 10 omega it rises by a factor of 20 dB 10 omega becomes 100 omega it rises by another factor of 20. So, for every factor of 10 increase in omega it rises by 20 dB this is termed as a 20 dB per decade increase 20 dB per decade increase in the bode plot.

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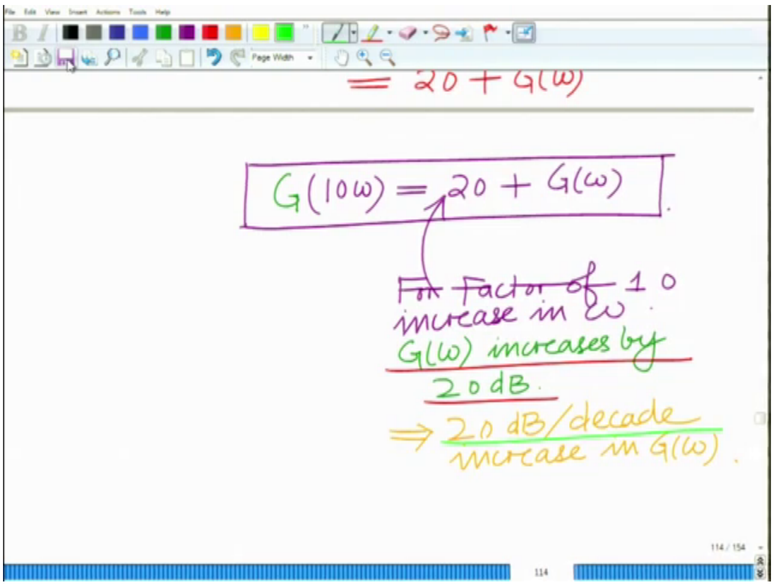

So, for every such term you will have a 20 dB per decade increase after the corner frequency this is something that is important. So, for factor of 10 increase in omega G omega increases by 20 dB this is an important point G omega increases by 20 dB and this implies basically a 20 dB per decade. So, every time G omega increases by omega increases by a factor of 10; this increases by 20 db.

So, 20 dB per decade; so, this is basically your 20 dB per decade increase ok. So, this is termed as a 20 dB per decade increase ok.

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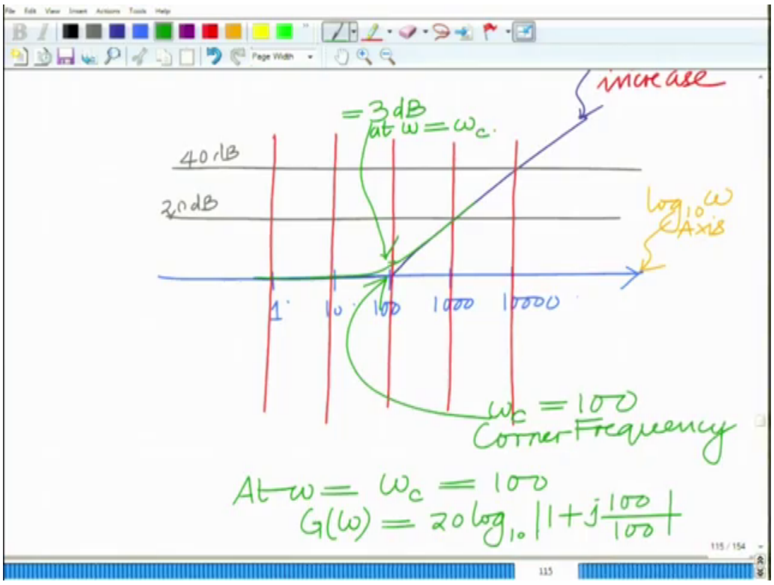

So, I am going to plot the bode plot and the bode plot is best plot on a log axis. So, what we are going to do is rather than conventional plotted on a linear axis, we are going to because it varies logarithmically in omega. So, it is best to plot on a log axis.

So, on a log axis you can see that every decade occupies a every decade occupies a single unit. So, when it goes from 1 to 10 alright remember the log increases from 1 0 to 1, it goes from 10 to hundred log of omega goes from log to the base 10 goes from 1 to 2. So, every decade increase occupies a single unit this is the log omega axis ok. So, this is your log omega axis or log to the rather log to the base 10 omega axis. And here we are going to have the vertical lines and this horizontal lines will be the dB values.

So, this is your let us say 20 dB; 40 dB. So, for omega very small that is omega much smaller than 100 this is. So, we are plotting G of W. So, G of W versus log of omega ok;

so, omega much smaller than 100 this will be basically this is this will be omega much smaller than 100 this is basically 0 dB and omega much larger than 100 it increases as 20 dB per decade.

So, when it goes from 100 to 1000; it goes to 20 dB where it goes from 1000; 1000 to 10000 it goes by another 20 degrees. So, this is the 20 dB per decade increase and so, this is basically your corner frequency you see omega c equal to 100 this is basically your omega c equal to 100, this is your this is your corner frequency.

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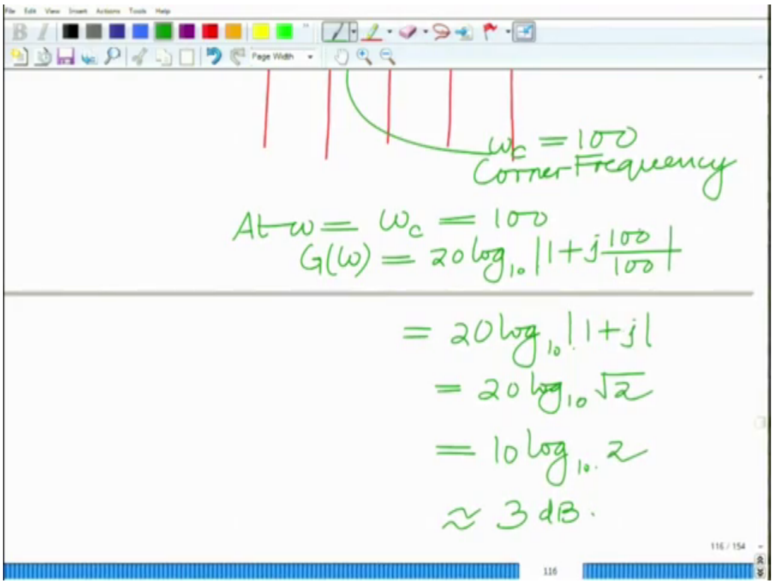

And if you want to compute the value of omega exactly at the corner frequency at omega equal to omega c; G of omega equals 20 log 10 base 1 plus j times 100 by 100 which is 20 log to the base 10 magnitude 1 plus j which is 20 log to the base 10 square root of 2, which is basically equal to you can verify approximately this is 3 dB which is 10 log to the base 10 2 and 10 log to the base 10 2 was approximately equal to 3 dB ok.

So, at the corner frequency it is basically 3 dB. So, what you are drawn is something that is approximate and the corner frequency this will be equal to set omega c there is 100 radians per second this would applaud magnitude will be 3 d ok. So, here it will be rather; so, here it will rather be 3 dB. So, this will be exactly the exact plot will look something like; this it will be something like this at the corner frequency it is three dB and then it will look something like this. So, at corner frequency this is 3 dB and this is basically termed as the bode plot.

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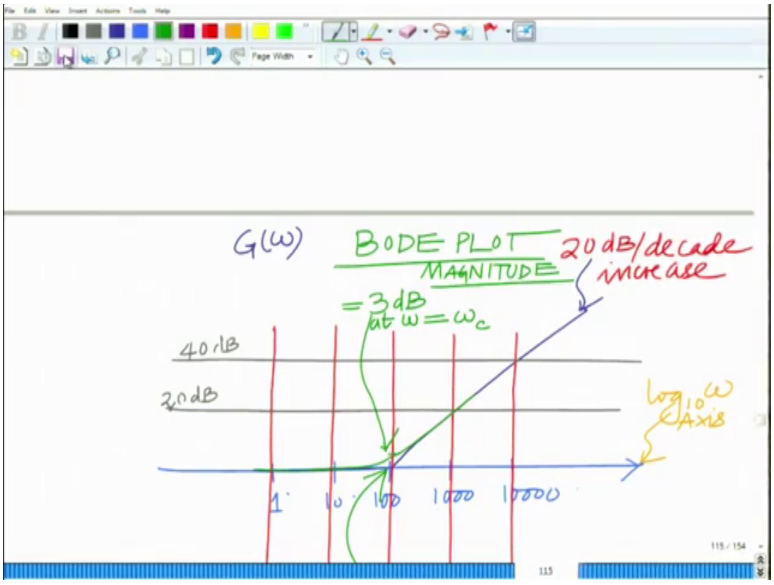

And bode plot for the magnitude to be more specific bode plot for magnitude or power one and the same thing ok. So, this is basically the bode plot representation this is the bode plot representation which omega is a present on the log axis and the gain is represented in the dB scale gain of magnitude H omega square represented on dB scale on the y axis ok.

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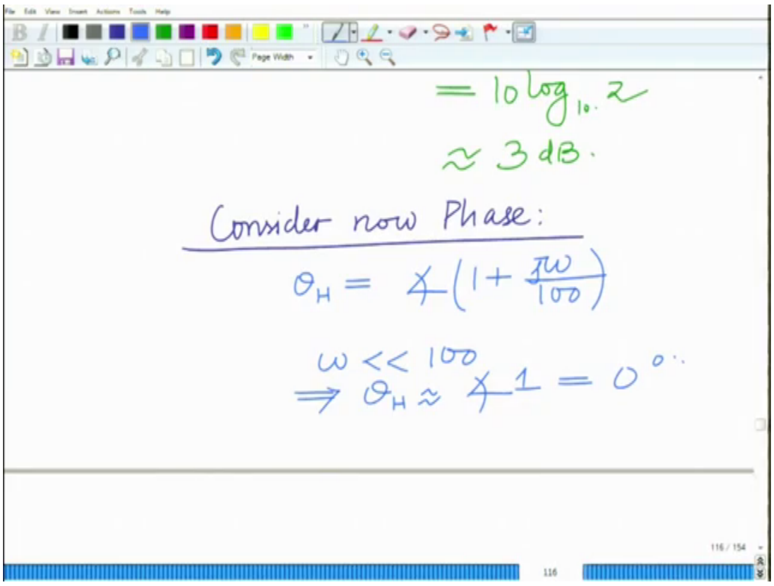

And you can see that after the corner frequency for every such term you have a 20 dB per decade increase. Let us now look at the phase plot corresponding to this which is also interesting. So, if you look at this quantity; so, we have the phase consider now the phase consider now the phase. Now the phase of this is theta H because angle of 1 plus j omega over 100 where 100 is the corner frequency.

Again we consider analyze this similar to what we have done before we consider two cases one is omega is less than less than much less than 100 which is the corner frequency. This implies theta H is approximately equal to angle of 1 because omega over 100 is negligible in comparison to 1 which is equal to 0. So, angle of simply a real number ok.

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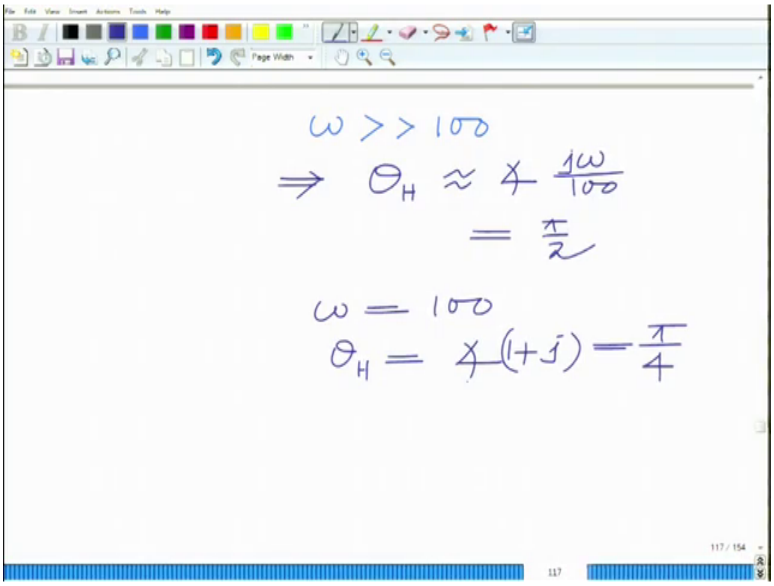

So, for omega a much less than the corner frequency 100 the phase is 0 ok. Now, for omega significantly larger than 100 this will be approximately G omega by 100 whose angle is pi by 2 ok. So, for omega much larger than 100; this implies angle that is theta H approximately equal to j of omega, this is purely imaginary. So, therefore, angle is pi by 2 for theta much greater than 100; now add theta at omega equal to 100 you can see theta H equals angle of 1 plus j and this is equal to pi by 4.

So, angle starts at 0 for omega much less than the corner frequency at the corner frequency its pi by 4 and after a significantly greater than the coordinate of frequency that is when omega is significantly greater than the conference corner frequency it is pi by 2 ok.

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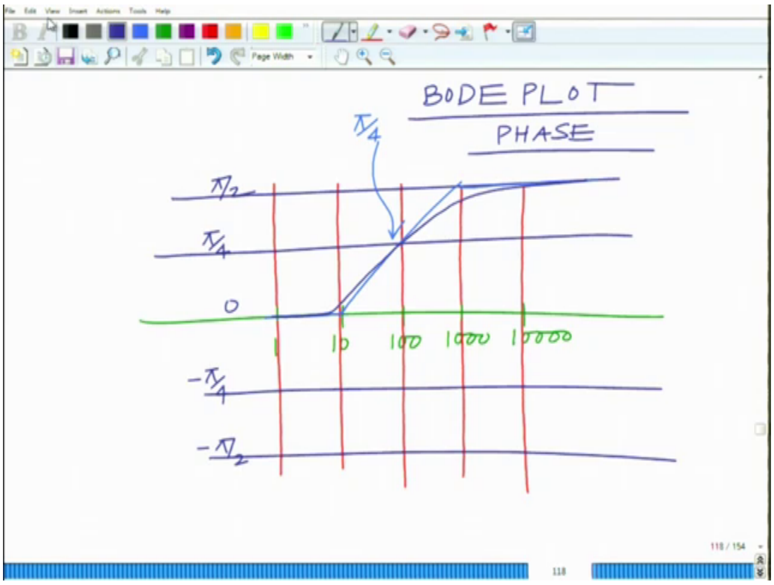

So, if you plot this once again the bode plot for the phase that looks again on the log axis just plot this on the log axis; so, this is 1, 10, 100, 1000.

So, remember once again each unit represents. So, the bode plot each unit represents a factor of 10. So, this is 1 alright and therefore, now if you plot the phase. So, this is 0; this is pi by 4, this is pi by 2 minus pi by 4, minus pi by 2 at much less than 100 let us approximately it tell till 10; it is 0 at 100 it is pi by 4 much greater than 100 the phases basically pi by 2. So, it rises from 0 to pi by 2 at 100 this is the corner frequency this is equal to pi by 4.

And the actual phase plot will look something like it will look something like the following it will look something like this; this is their bode plot for the phase this is the bode plot for the phase ok. So, this is the bode plot for a simple example that is 1 plus j omega there is H omega is 1 plus j omega over 100 ok. Now let us look at a slightly more sophisticated an example for this bode plot ok. So, what we want to do now is look at a slightly more sophisticated example for the bode plot.

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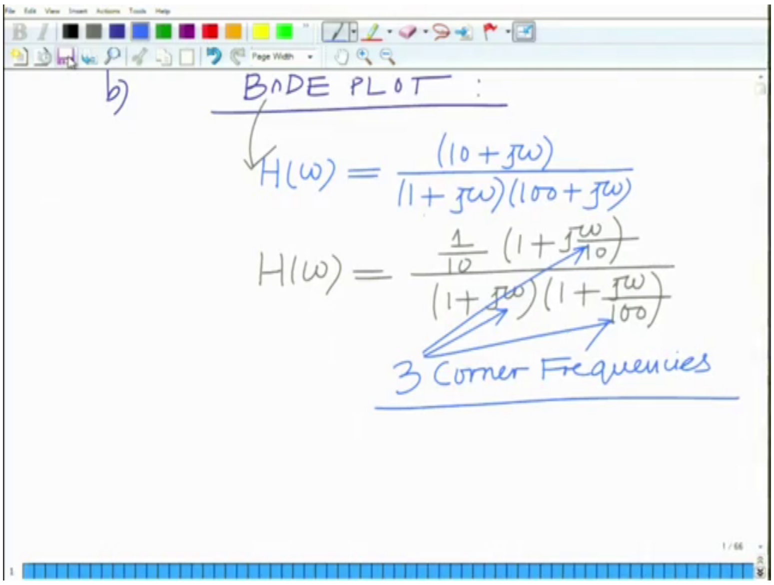

So, we want to start with a slightly more sophisticated example. So, this is our example b for the bode plot and now let us take an example H of omega equals 10 plus j omega over 1 plus j omega into 100 plus j omega; I can write this as now I want to find the bode plot for this now I can simplify this as follows, I can write this as basically H of omega equals 1 over 10 into 1 plus j omega over 10 divided by 1 plus j omega into 1 plus j omega over 100 ok.

You can verify that I can write this as I can simplify it this expression. Now you can see there are three corner frequencies quarter frequency of 1, 10 and 100 corresponding to each of the terms correct each of the terms in the numerator and denominator. So, there are going to be for this there are basically if you look at this there are 3 corner frequencies.

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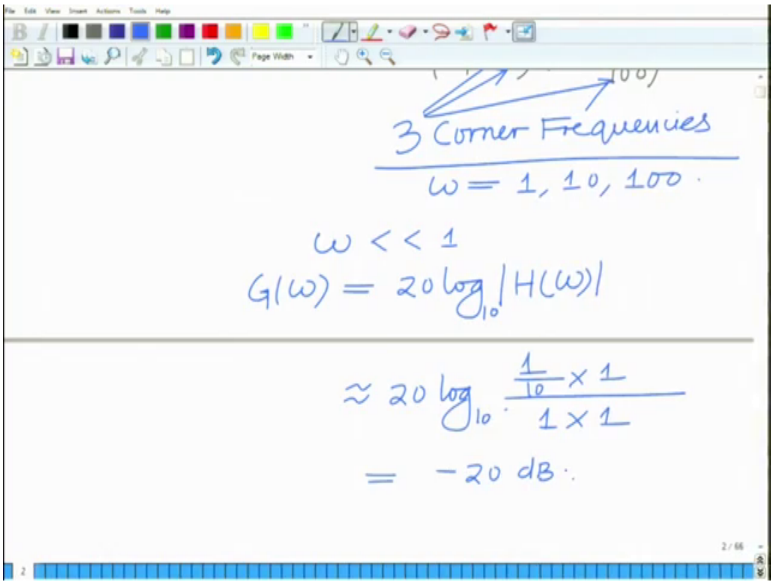

There are 3 corner frequencies and these are basically omega equals 1, 10 comma 100. Now for omega very small let us start with the first corner frequency. So, for omega much smaller than 1 your G of omega which is again 20 log to the base 10 magnitude H of omega; this is approximately equal to 20 log to the base 10. Now remember an omega is much smaller than 1 you will have 1 by 10; the term 1 plus j omega by 10 is approximately 1, 1 plus G omega is also approximately 1, 1 plus j omega by 100 is also approximately 1 because omega is much smaller than 1.

All the terms will simply reduce to the constants ok. So, j of omega simply reduces to 20 log to the base 10; 1 over 10 you can see that. So, this is 20 log to the base 10; 1 over 10 times 1 which is basically minus; so, this is basically your minus 20 dB ok.

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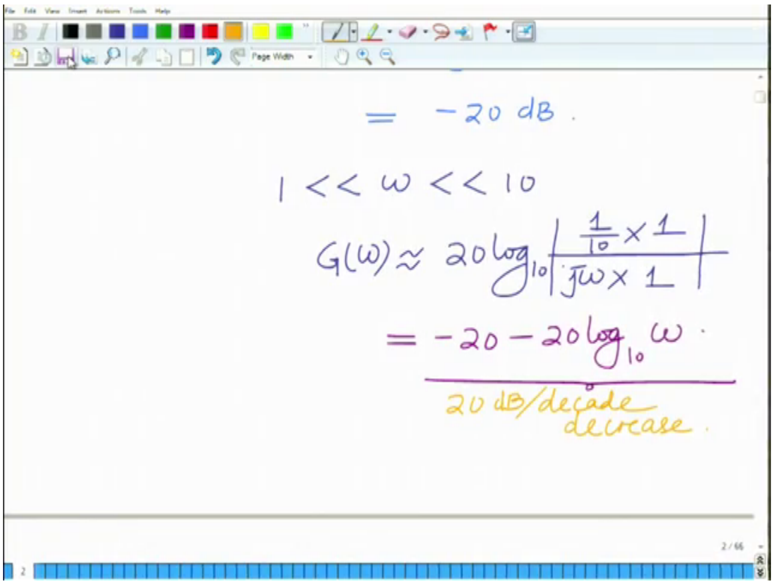

Now, consider another scenario when omega much larger that is 1 is much smaller than omega is in this range 1 is much smaller than omega.

But omega is much smaller than 10; in this case what happens is you can check G of omega is approximately 20 log to the base 10, 1 over 10 omega is much smaller than 10. So, 1 plus j omega or 10 is approximately 1 divided by 1 plus j omega that becomes your 1 plus j omega. Since omega is much larger than 1 that simply becomes j omega into 1 plus j omega by 100 that is approximately equal to 1.

So, the magnitude of this; so, this becomes well minus 20 minus 20 log to the base 10 omega. So, it starts decreasing us now since this term 1 plus j omega is in the denominator, it starts decreasing at 20 dB per decade ok. So, in this term what happens is you can see it basically 20 dB per decade it decreases as 20 dB per decade. So, this term it starts decreasing as 20 dB per decade alright. So, what we will do is we have in the middle of this example.

So, we will stop here in this module and we will continue with this example look at the bode plot of the magnitude and the phase and also other examples in the subsequent module.

Thank you very much.