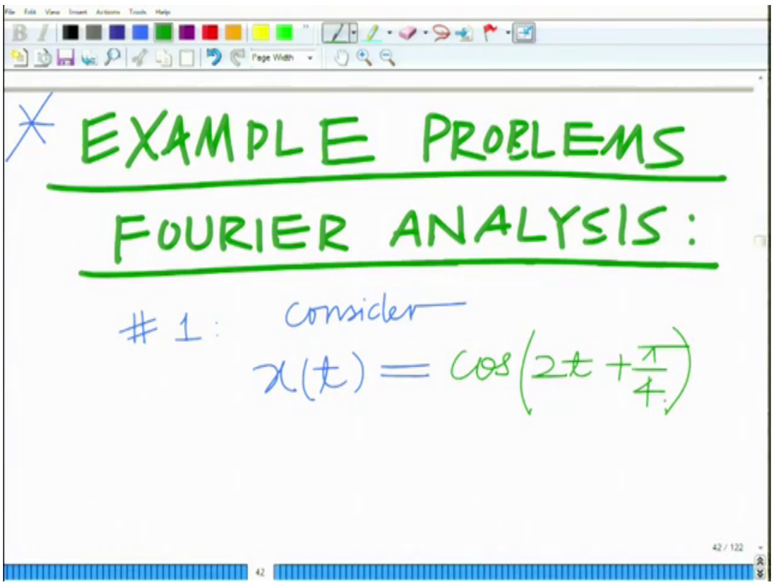
Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology Kanpur

Lecture - 42
Fourier Analysis Examples - Complex Exponential Fourier Series of Periodic Square Wave

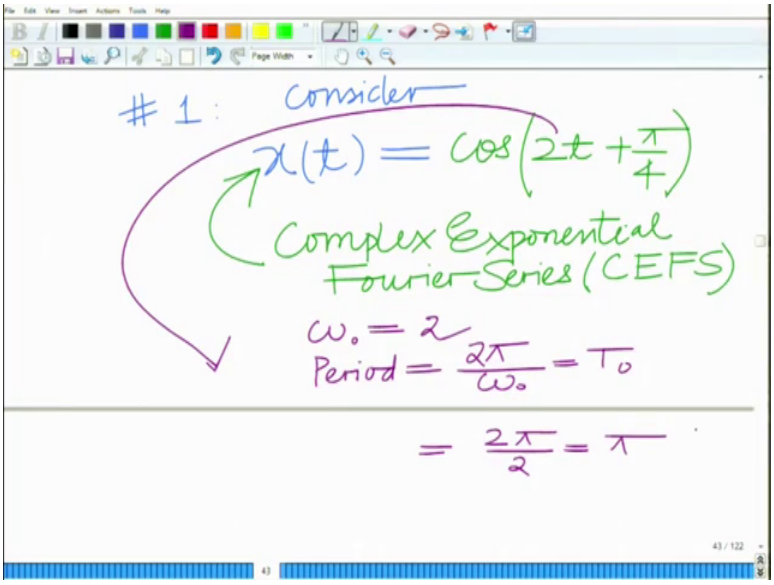
Hello welcome to another module in this massive of online course. So, we are looking at the Fourier analysis of both discrete time and as well as continuous time; Fourier analysis for periodic as well as aperiodic signals alright. And we looked at the theory corresponding to that and now let us start at this module, we are going to start looking at various problems to better understand the implications and the applications of the Fourier analysis for continuous time signal that we have seen so far.

(Refer Slide Time: 00:45)



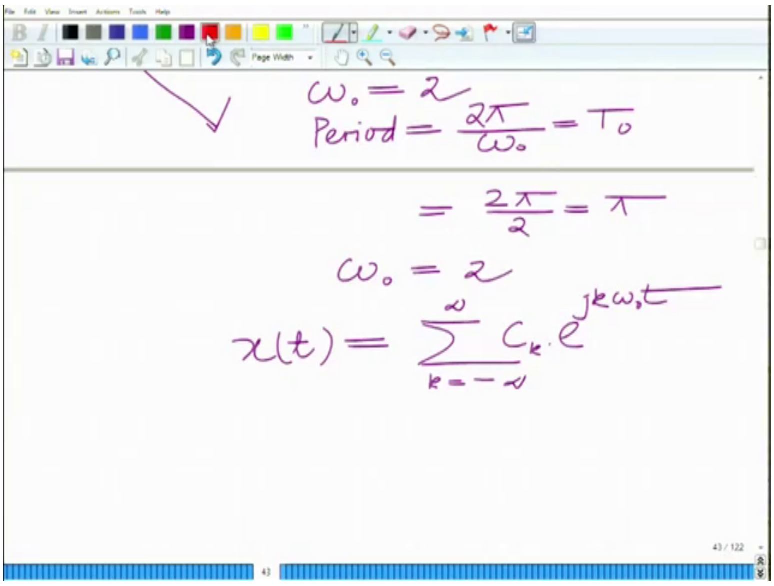
So, let us start by our looking at example problems for Fourier analysis that is the Fourier analysis of continuous time signals and systems alright; and let us start with problem number 1 which is basically consider the signal x t equals cosine 2 t plus pi by 4.

(Refer Slide Time: 01:50)



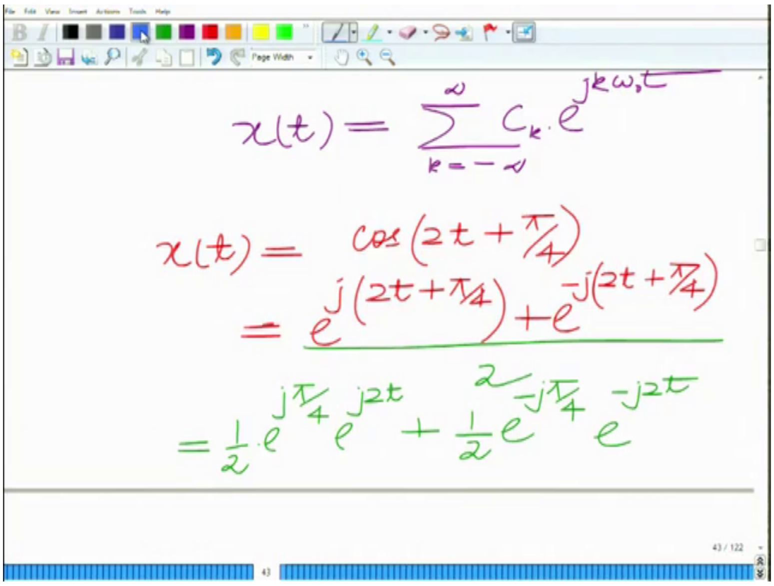
We want to obtain its complex exponential Fourier series or what we can abbreviate as the CEFS we want to obtain CEFS for this given signal that is x t equals cosine 2 t plus pi by 4. Now what we can observe here is that from this you can observe that the period ok. So, this is the cosine 2 t alright. So, omega naught equals 2 which means the period the fundamental period of this equals; well 2 pi by omega naught, that is equal to t naught which is equal to 2 pi by 2 which is equal to pi and omega naught equals 2.

(Refer Slide Time: 03:01)



So, I can express this as x t this is CEFS representation k equals minus infinity to infinity C k e raised to j k summation k equals minus infinity to infinity C k e raised to j k omega naught t this is the complex exponential Fourier series of the CEFS C k denotes the coefficient the complex coefficient corresponding to the k th harmonic correct that is the frequency component, which is k times omega naught where omega naught is the fundamental frequency t naught is the fundamental period of this periodic signal.

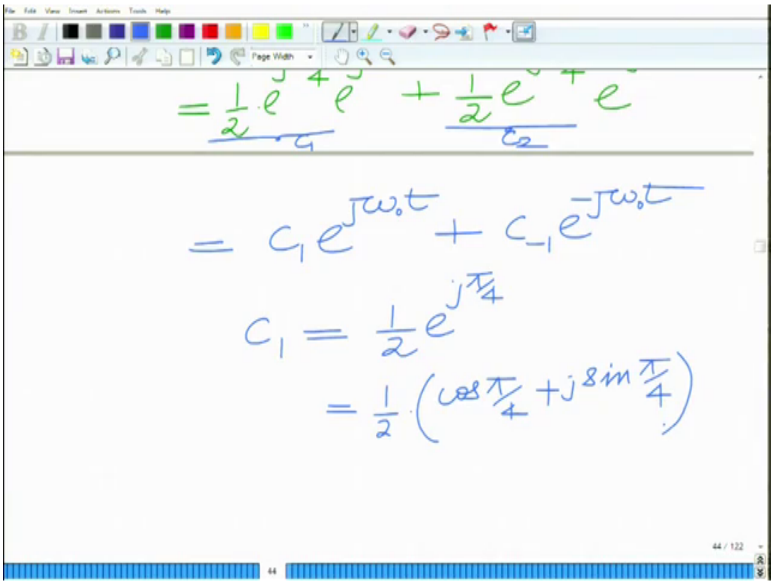
(Refer Slide Time: 03:52)



And now rather than evaluate this directly evaluate the coefficients directly, we can see that I can evaluate this as x t equals I can directly evaluate this as as x t equals cosine 2 t plus pi by 4 which is e raised to j cosine x cosine of well cosine theta cosine theta e raised j theta plus e raised to minus j theta by 2. So, this is e raised to j 2 t plus pi by 4 plus e raised to minus j 2 t plus pi by 4 divided by 2.

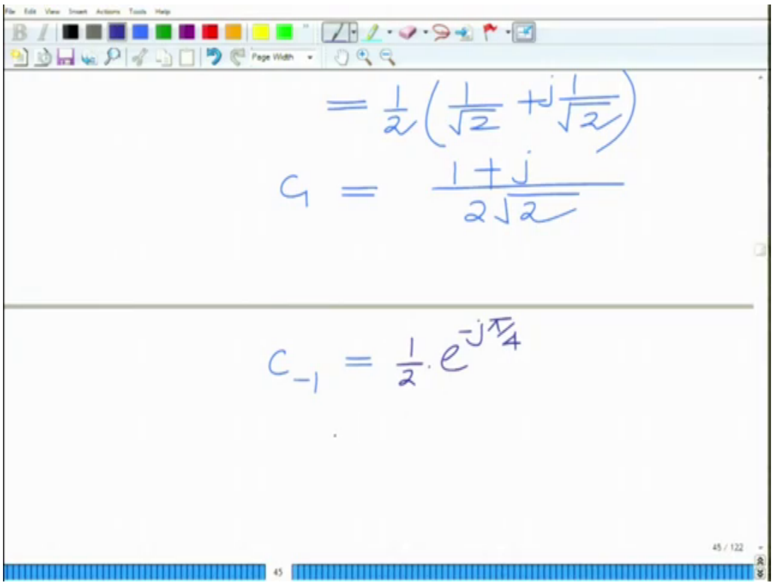
Which I can write as half e raised to j pi by 4 e raised to j 2 t plus half e raised to minus j pi by 4 e raised to minus j 2 t.

(Refer Slide Time: 05:06)



Which is equal to C 1 e raised to j omega naught t plus you can see this C minus 1 e raised to minus j omega naught t where this coefficients C 1 you can see this is basically your C 1 and this is basically your C 2. So, C 1 is basically this coefficients. So, C 1 equals well half e raised to j pi by 4 equals half well e power j theta is cosine theta plus j sin theta. So, e raised to j power pi by for is half cosine pi by 4 plus j sin pi by 4.

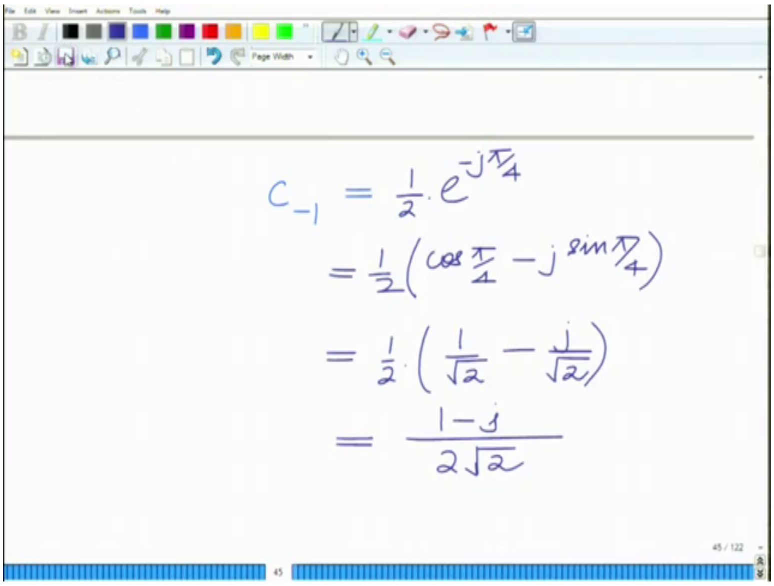
(Refer Slide Time: 06:02)



Which is half 1 over root 2 plus 1 over root 2 which is 1 plus of course,1 plus j over 2 root 2 ok. So, this is c 1 equals 1 plus j over. So, the coefficients C 1 in the CEFS is one plus j over twice square root of 2.

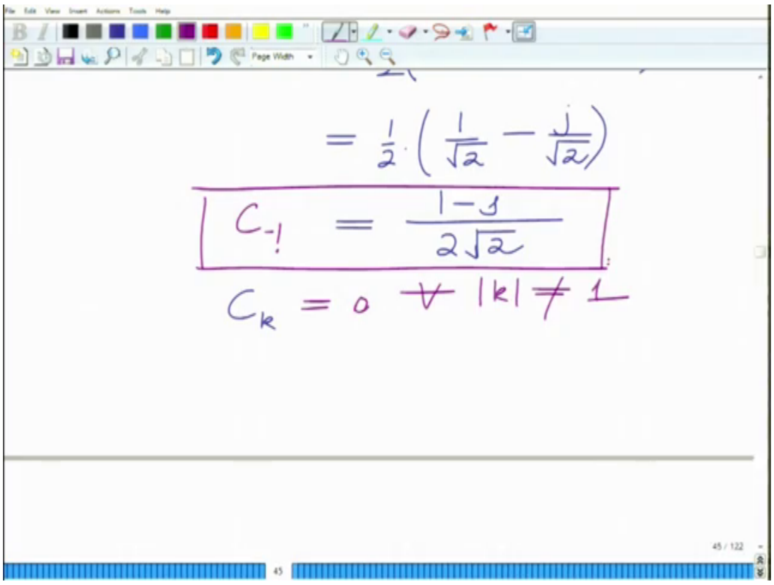
Similarly, C minus 1 that is the coefficient of e raised to minus j omega naught t or minus j 2 t in this case is minus 1 equals well half e raised to minus j pi by 4.

(Refer Slide Time: 06:53)



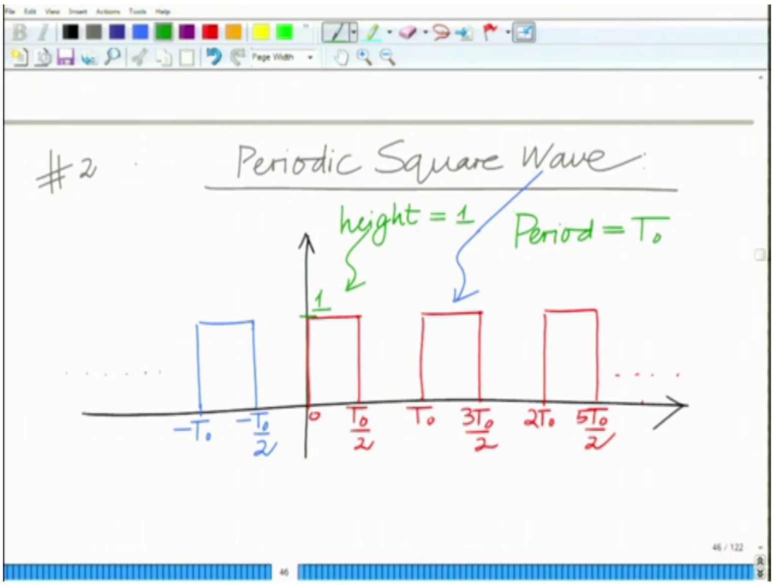
Which is half cosine pi by 4 minus j sin pi by 4 which is basically half one over root 2 minus j over root 2 equals 1 minus j over 2 square root of 2 ok. And you can clearly see from this is that, from this you can clearly see that all C k that is because since this is expressed simply as a combination of e raised to j 2 t and e raised to minus j 2 t all the term C k for k not equal to 1, that is C 0 as well as C 2 C minus C 3 minus etcetera. So, on as it is C k for any k not equal to either plus 1 or minus 1 is basically 0 ok. So, all the coefficients. So, C k the rest of the C k.

(Refer Slide Time: 08:01)



Equal to 0 for all mod k not equal to 1, that is either k equal to plus 1 or minus 1. That is the complex exponential Fourier series and these are the coefficients. So, this is coefficient C minus 1 in the complex exponential Fourier series and this is the coefficients C 1 in the complex exponential Fourier series now let us look at a periodic sequence or periodic square wave.

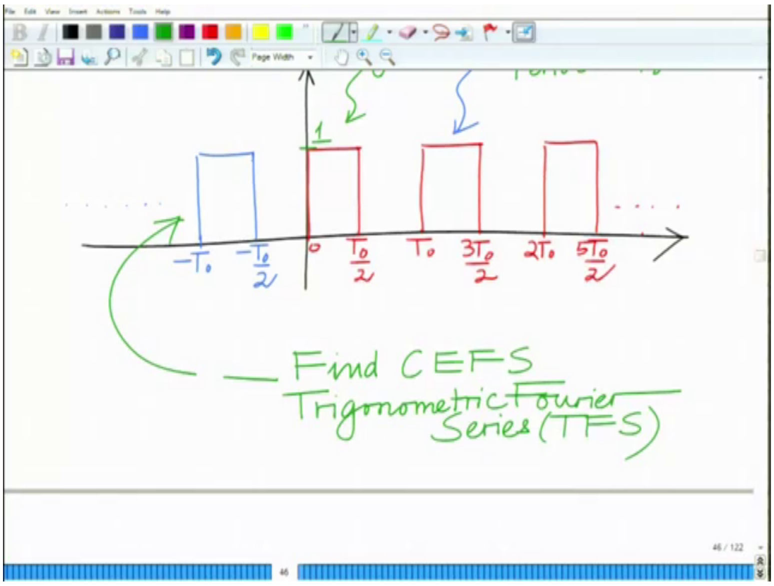
(Refer Slide Time: 08:40)



So, let us look at this is example number 2 which we want to look at a periodic square wave and periodic square wave can be represented as follows. So, you have a square wave which is periodic. So, this is of width T naught by 2 and this has a period of. So, this has a period of T naught. So, this is minus t naught over 2.

So, this is your periodic square wave correct this is your periodic square wave and the time period here equals the time period here equals T naught and let us say the height is equal to height of each square pulse or the amplitude of each square pulse is 1 and what we want to do is for this periodic square wave.

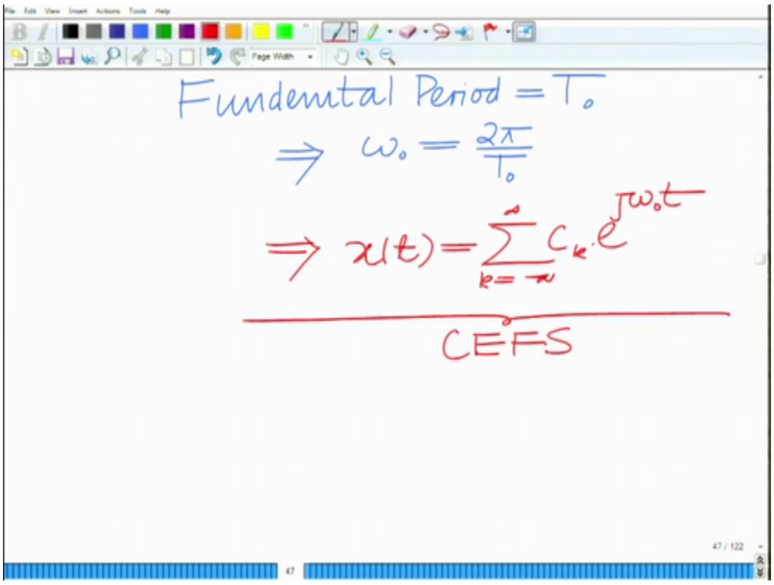
(Refer Slide Time: 10:37)



We want to find the complex exponential Fourier series and also the trigonometric; remember this is an alternative Fourier series representation, which is known as the trigonometric Fourier series we want to denote this by TFS. So, want to find the complex exponential Fourier series and as well as the trigonometric Fourier series representation Fourier series representation for this periodic square wave, which are square pulses of height 1 width T naught over 2 and fundamental period is T naught.

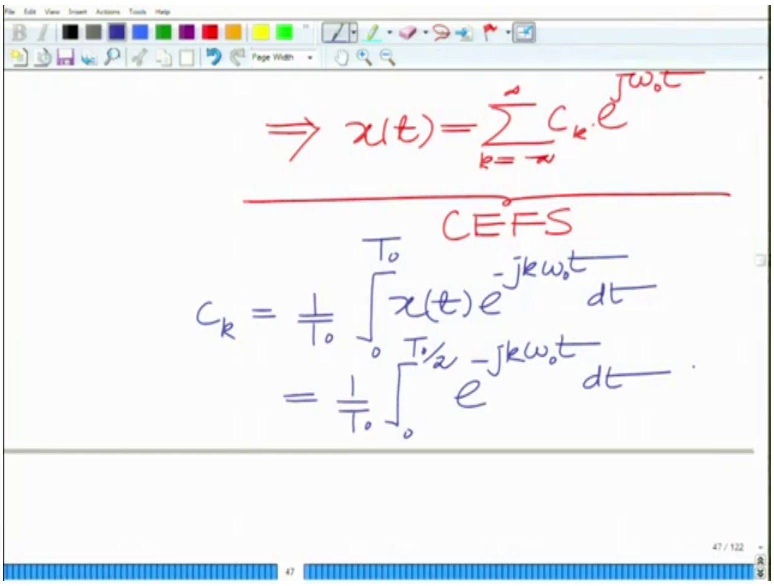
Alright and this can be done as follows now first observe.

(Refer Slide Time: 11:27)



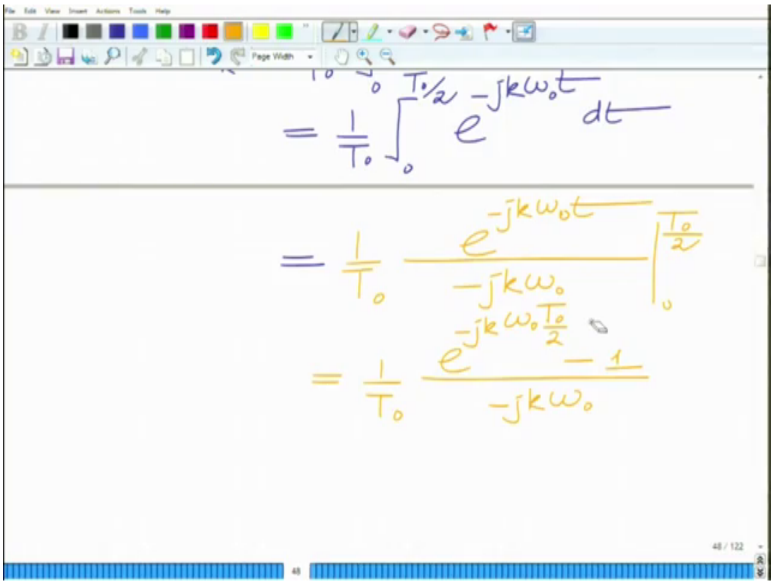
So, first observe that the fundamental period, the fundamental period equals t naught which implies omega naught equals 2 pi over T naught and this implies that x t equals summation. So, I can write the CEFS x t equals equals minus infinity to infinity C k e raised to j k omega naught t. So, this is your CEFS representation the complex exponential Fourier series because this is periodic with T naught omega naught equals 2 pi by T naught this is the CEFS representation.

(Refer Slide Time: 12:30)



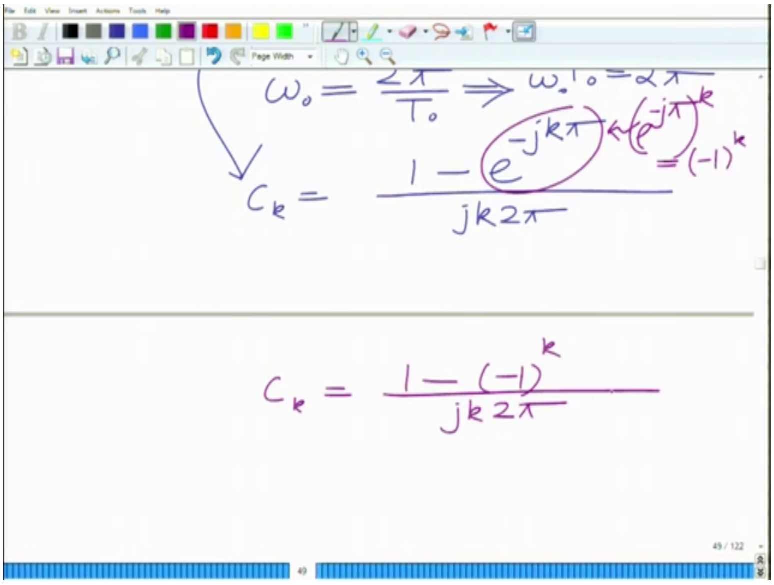
And now C k the coefficients C k can be found as remember is 1 over T naught integral you can find it as 0 to T naught for any period T it of duration T naught integrals period of duration T naught in fact, x t e raised to minus j k omega T naught omega naught t d t, which is basically we know that in the period 0 to T naught x is a simply a pulse of height one in the period its non-zero in the period T naught 0 to T naught over 2 in that pulse of height one. So, this is x t will be 1, 0 to T naught over 2 e raised to minus j k omega naught t d t.

(Refer Slide Time: 13:22)



Which you can see is basically 1 over T naught e raised to minus j k omega naught t by minus j k omega naught t evaluated between the limits, 0 to t naught over 2 which is 1 over T naught. E raised to minus j k e raised to minus j omega naught t naught over 2 minus 1 divided by minus j k omega naught im sorry this has to be minus j k omega naught.

(Refer Slide Time: 14:08)



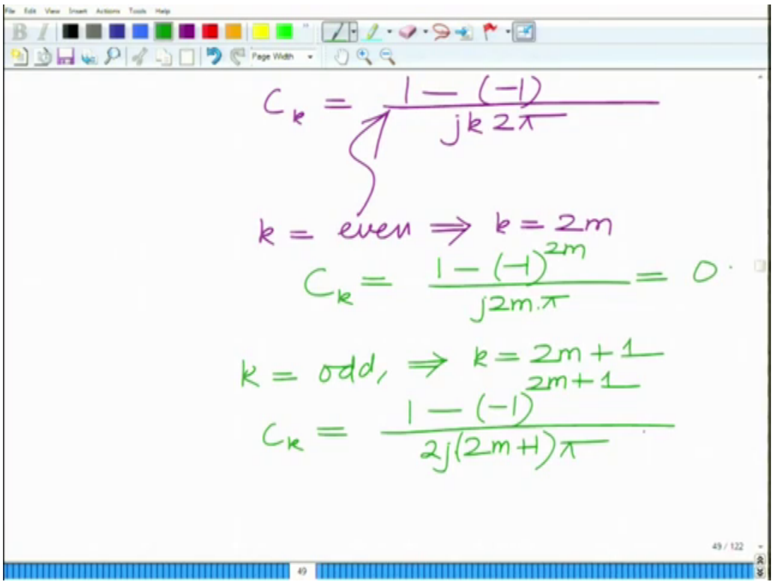
And now realize that we have omega naught equals 2 pi by T naught which implies omega naught T naught equals 2 pi and therefore, this form this expression for C k can be simplified as look at this in the numerator we have omega naught into T naught by 2. So, omega naught into T naught by 2 will be pi and the denominator we have T naught into omega naught that will be 2 pi. So, this will be 1 minus e raised to minus j k omega naught into T naught pi divided by j k omega naught into T naught equals 2 pi.

So, this is the expression form the coefficients C k which is a complex exponential Fourier series coefficient all right this is C k is 1 minus e raised to minus j k pi divided by j k into 2 pi all tight. And we have used the principle that omega naught T naught equal 2 pi; omega naught T naught over 2 is simply pi.

And now you can observe that e raised to minus minus e raised to minus j k 2 times pi is nothing, but minus 1 raised to the power of k ok. So, this is e raised to minus j k pi is basically e raised to minus j pi raised to the power of k which is nothing, but minus 1 raised to the power of k. So, this if you see this is e raised to minus j pi this is e raised to minus j pi raised to the power of k, which is basically minus 1 which is basically minus 1 raised to the power of k.

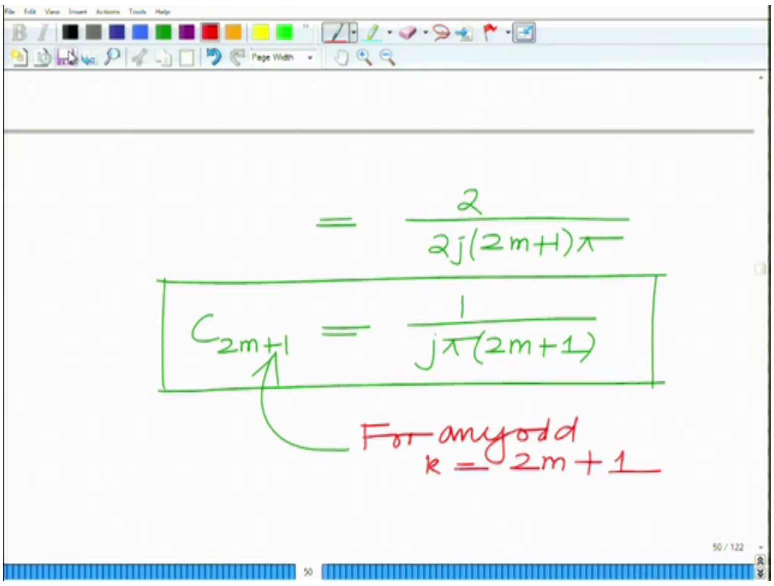
So, C k equals 1 minus minus 1 raised to the power of k divided by j k into 2 pi and now you can observe that if k equal to if k is even.

(Refer Slide Time: 16:19)



Implies if k is equal to 2 m this is equal to c of k equals 1 minus minus 1 to the power of 2 m over j into 2 m. Now you can see minus 1 minus one to the power of minus 1 raised to the power 2 m this is simply 0. So, C k is 0 if k is even and if k is odd implies k equals 2 m plus 1, then C k equals 1 minus minus 1 raised to 2 m plus 1 divided by j 2 m plus 1 times pi j or in fact, j twice 2 m plus 1 times pi j or twice j into 2 m plus 1 times pi equals 2.

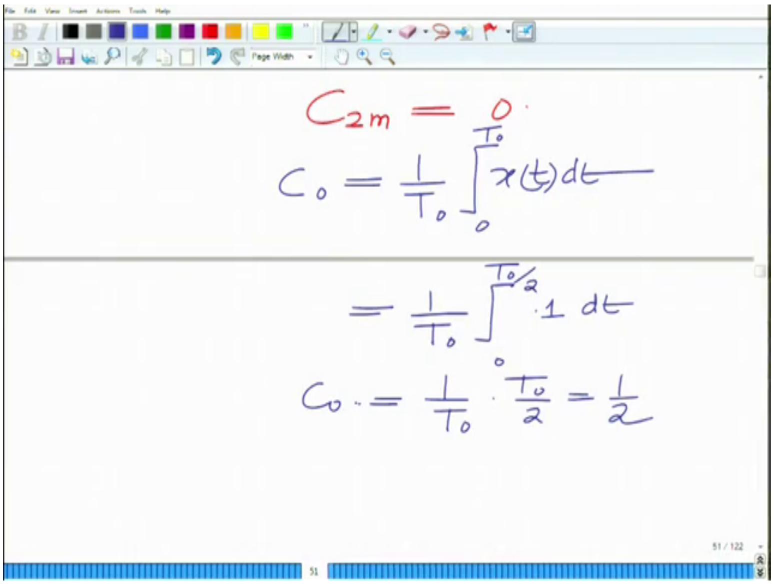
(Refer Slide Time: 17:32)



Over 2 j into 2 m plus 1 pi, which is nothing, but in fact, this is C of 2 m plus 1 k equals 2 m plus 1 see one over j times pi into 2 m plus 1.

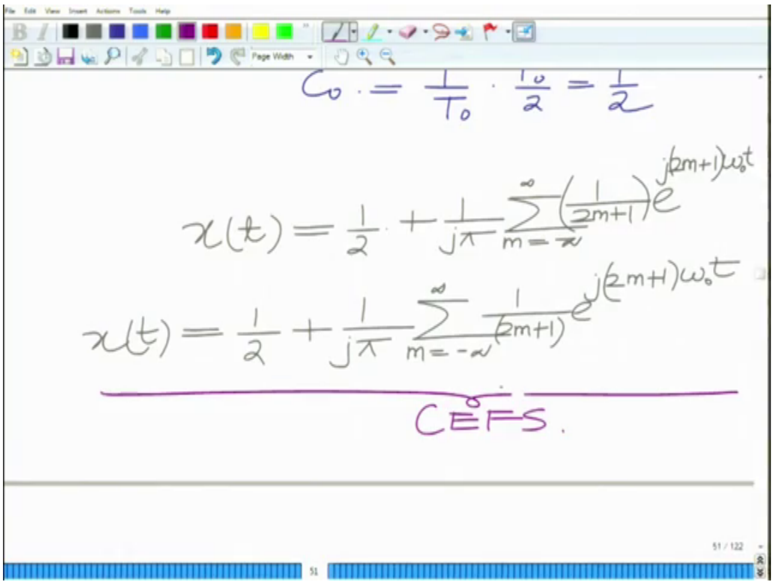
For any therefore, any odd k for any odd k equal to 2 m plus 1 for any odd k equals 2 m plus 1 ok. So, for even k it is 0.

(Refer Slide Time: 18:15)



For odd k it is one over j pi 2 m plus 1 and the d c coefficient remember C naught C naught c naught that corresponding to k equal to 0 that is termed as d c coefficient, which is simply 1 over T naught times the integral of the function over the interval of duration t naught. So, this is 1 of the over T naught that is the average time average of the signal 0 to T naught x t d t which is basically your 1 over T naught again this is non-zero only in 0 to T naught over 2 times 1 d t, which is basically 1 over t naught into T naught over 2. So, this t naught is equal to half and therefore, finally, what we have is basically we have x t equals half plus 1 over j pi summation m equal to minus infinity to infinity 1 over.

(Refer Slide Time: 19:11)



So, this exists only for even k that is of the form 2 m plus 1. So, 1 over 2 m plus 1 I have taken the j pi outside since it is common e raised to j 2 m plus 1 k omega naught t or I can just write it just write it once more a little clearly. So, x of t equals half plus 1 over j pi summation m equal to minus infinity to infinity, 1 over 2 m plus 1 e raised to j 2 m plus 1 omega naught T. So, this is your complex exponential, this is basically the complex exponential Fourier series representation.

This is the complex exponential Fourier series representation of the signal all right. So, basically what we have done in this module is we have started looking at the problems for the Fourier analysis of continuous signals, and systems all right. In particular, we have looked at an example which basically finds the Fourier complex exponential Fourier series of a simple signal followed by the complex exponential Fourier series of periodic square wave of a square wave. And we are also going to the subsequent module we also find the trigonometric Fourier series representation of this all right. So, we stop here.

Thank you very much.