Principles of Signals and Systems
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Lecture – 40
Fourier Transform – Distortionless Transmission, LTI Systems Characterized by Differential Equations, Ideal Low Pass and High Pass filters

Hello, welcome to another module in this massive open online course. So, we are looking at the Fourier transform and its properties, all right. So, let us continue this discussion. Today let us look at another new aspect that is distortionless transmission how do you characterize a system with distortionless transmission ok.

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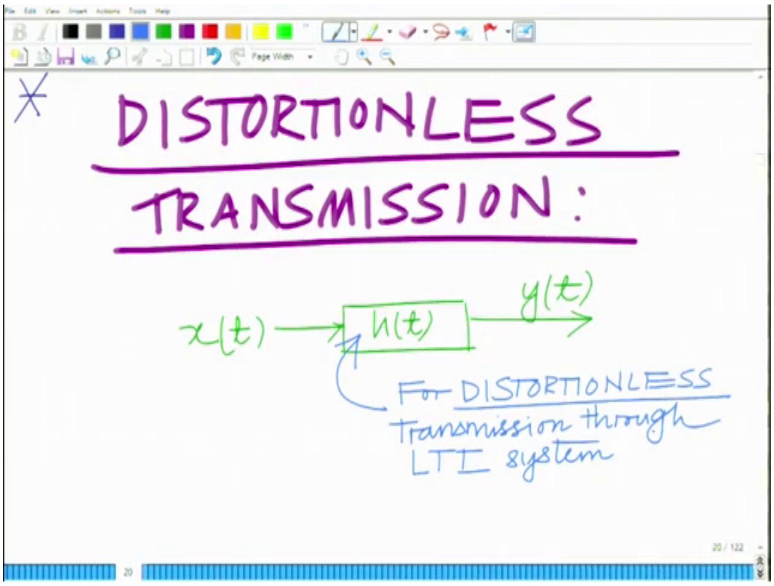

So, what we want to look at today is the notion of distortionless transmission, the notion of distortionless transmission through an LTI system.

Now, let us say again let us consider an LTI system that is with input x t impulse response given by h t and output given by y t. Now, for this system to be distortionless right for distortionless transmission distortionless let me just write it that is no distortion for, so for distortionless transmission through LTI system, for distortionless transmission through LTI system ok. The signal x t we call it distortionless that is transmission term distortionless if y t is a delayed and scaled that is amplified or attenuated version of x t.

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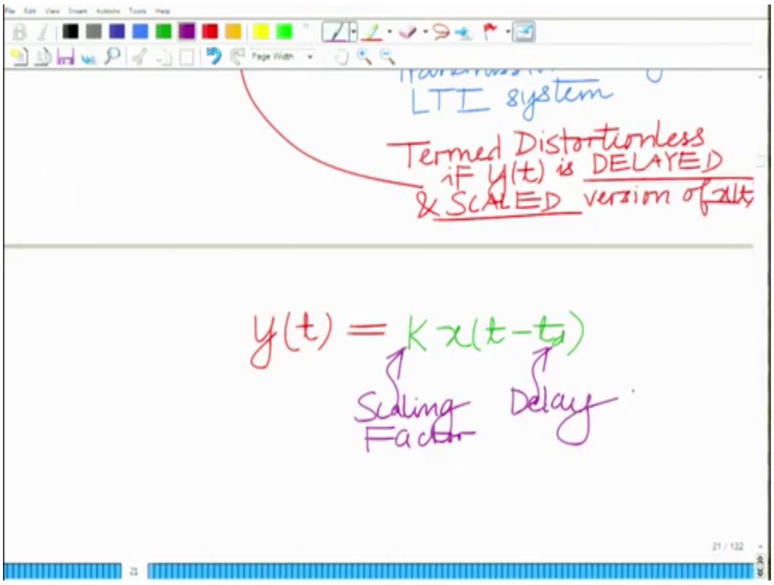

So, what we mean to say by this is that is we if you transmit x t through this LTI system we call this LTI system distortion left distortionless if the output y t is simply a scaled and possibly delayed version of and or possibly a delayed version of x t. So, what we want is that y t should not distort x t, but y t can differ from x t only to the extent that it is a scaling factor K times x minus t d. So, we have this scaling this is your K which is your scaling and this is the delay.

So, let us say we have x t which is consider a simple example for your signal x t ok.

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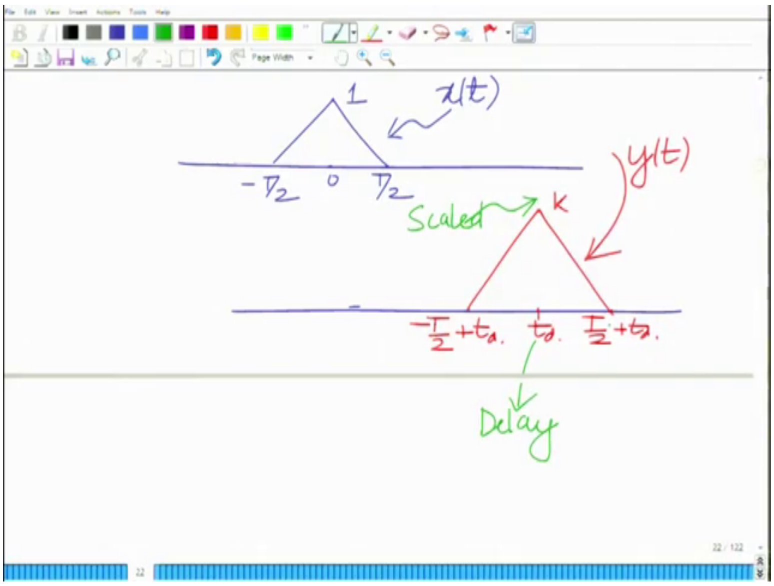

Let us say we have a triangular pulse of height one center is at 0 from T by 2 to minus T by 2. So, this is let us say your original signal x t. Now, you pass it through the LTI system if the LTI system is distortionless all it can do is it can simply scale correct and delay. So, from minus, so it delays this by t d. So, which means. So, we have this will go from. So, this is shifted to t d. So, this is shifted to minus t by 2 plus t d t by 2 plus t d and the height is k. So, it is scaled. So, this is a this is your y t which is delayed by. So, scaled by scaled and this is your.

What you can see basically that the shape remains intact that is x t and y t are similar to each other in the sense that y t is simply delayed and a scaled that is its either amplified or attenuated version of x t we call such an LTI system as a as a distortionless transmission system. And typically we only consider a delay because if a cause if a system is causal then it can only a it can only delay the signal right if its advances the signal or the technically that will also be distortionless, but the system would be non causal which is not practically which is not practically feasible right.

So, we consider only an amplification or attenuation that is scaling by a scaling factor K and also a delay by t ok.

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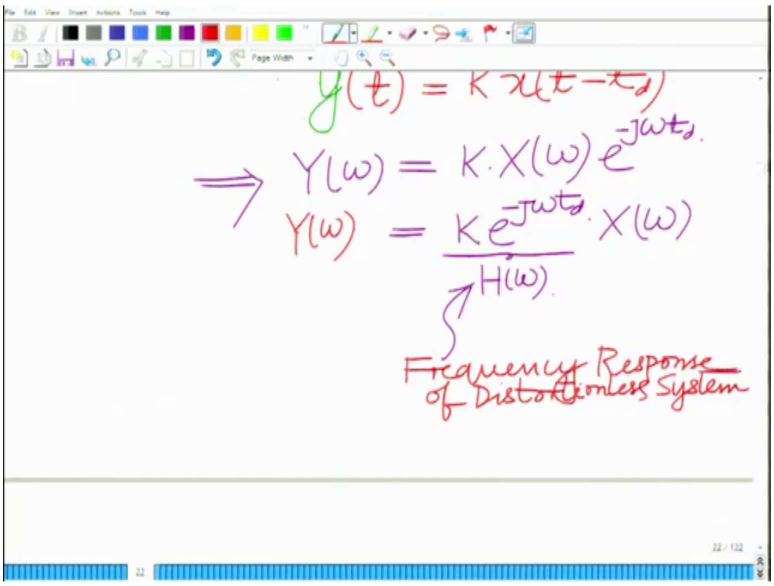

Now, therefore, we have basically if you look at such a system you can naturally see we have; y t equals K times x minus x of t minus t d implies if you take the Fourier transform you have y of omega equals K times. Now, delayed version of, so this is delayed version of x delayed by t d. So, the Fourier transform of x of omega e raise to minus omega t d which is basically K e raise to minus J omega t d times x of omega and now, you can see this is nothing, but your frequency response H of omega of this distortionless system ok.

So, we can characterize since the output frequency response output response the Fourier transform of the output is the frequency response of the system times the Fourier transform of the input we have H of omega is K e raised to minus J omega t d which is basically the characterizes the frequency response of characterizes a frequency response of this distortionless system. And therefore we have H of omega is e raised to minus g omega t d which or K e raised to minus g omega t d which means if you look at the magnitude spectrum we have magnitude H of omega equals K that is a constant.

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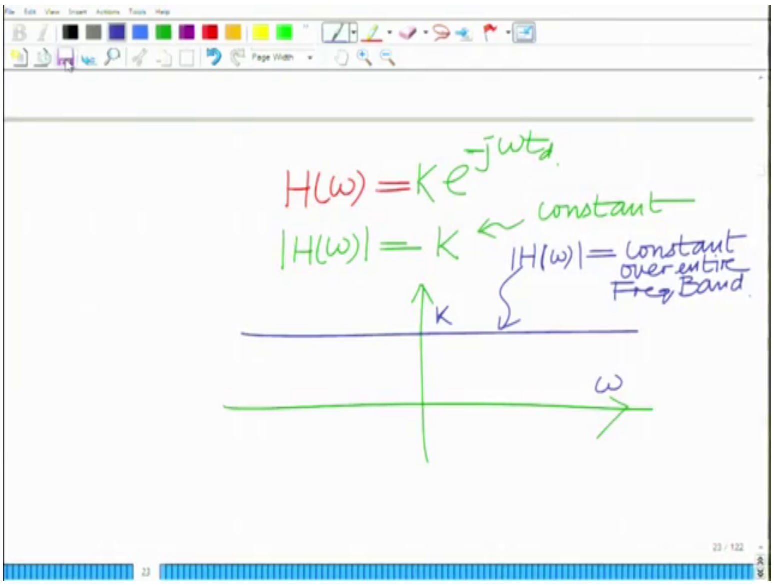

So, for a distortionless spectrum the magnitude spectrum looks as follows it is simply it has to be constant over the entire frequency band. So, this is your magnitude H of omega equals constant over entire frequency band.

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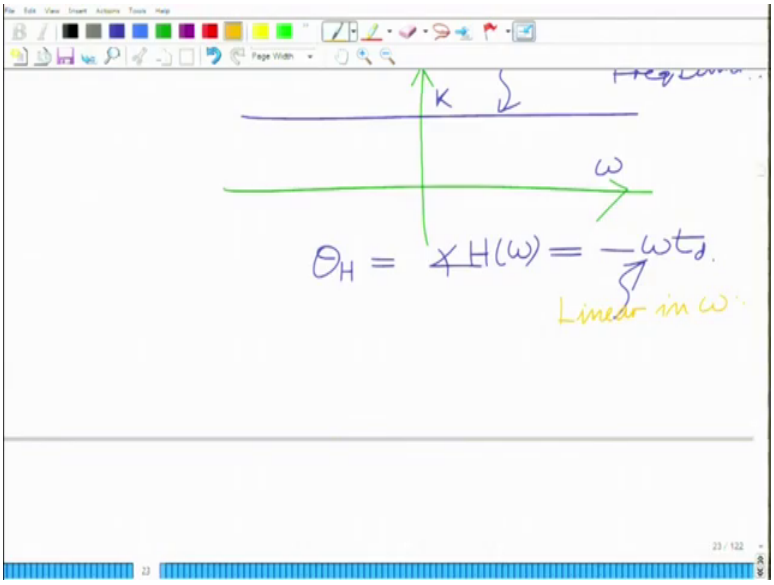

And if you look at the face of omega if you look at the angle of omega or if you look at theta H that is the angle of H of omega you can see this is minus omega t d and if you look at this quantity this has a linear characteristic this is linear in omega ok.

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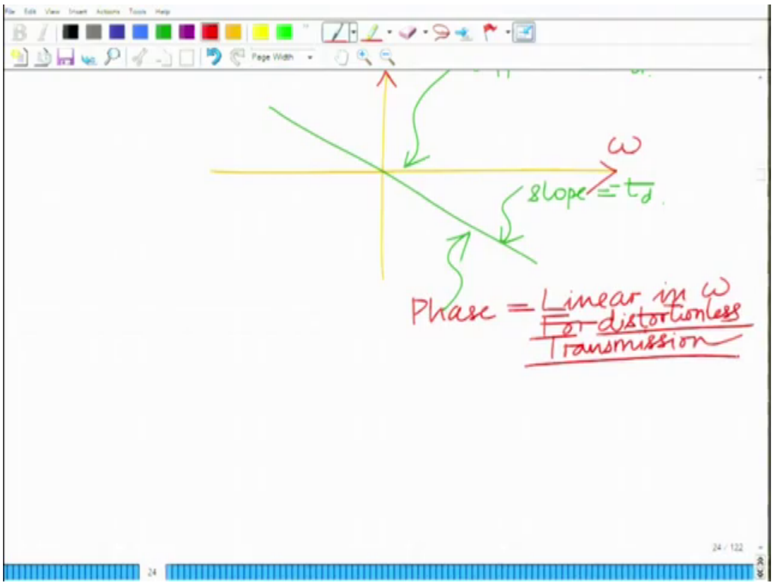

So, if you look at this will be something like its characteristic will be this slope equals minus t d ok. So, this is passes through the origin and this is your theta H equals minus omega t d. So, this is the phase characteristic the phase is linear in omega ok. So, this phase is linear in omega for distortionless transmission, for distortionless transmission ok. So, basically what this shows is this characterizes the amplitude and phase characteristics of an LTIs or the frequency response of an LTI system for distortionless transmission.

The magnitude or the amplitude of the frequency response has to be constant over the entire frequency band and the phase has to be linear in the frequency it has to have a linear phase response ok, all right. So, that is an important property because distortionless systems are frequently encountered and very important in the analysis of signals and systems.

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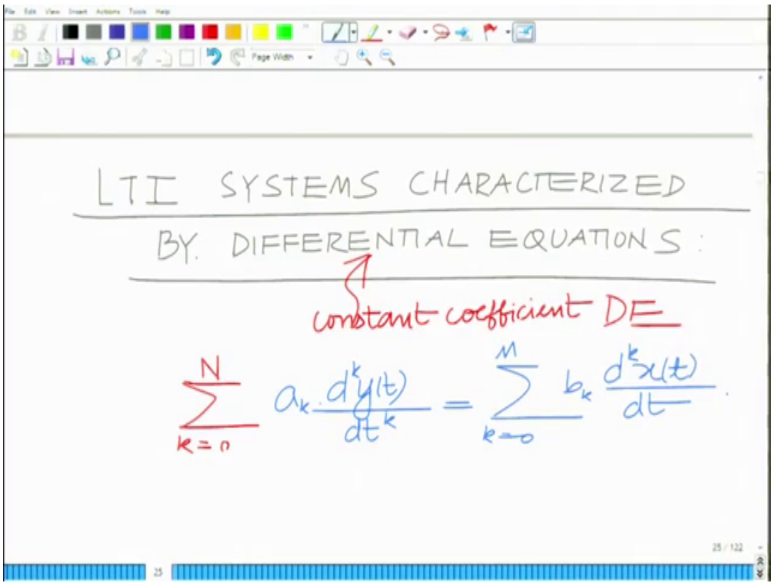

Now, let us now, look at LTI systems characterized by differential equations. So, we have LTI systems that are characterized by that are characterized by differential equations. So, this by this we mean a constant coefficient differential equation typically we meets a constant coefficient differential equation and that is given as follows.

Summation K equal to 0 to N a k d to the k y t by d t k equals summation K equal to 0 to M, b to the K b k d to the K x t that is the kth order derivative of x t and these a k and b k are these are the constant coefficients ok, these are the constant coefficients.

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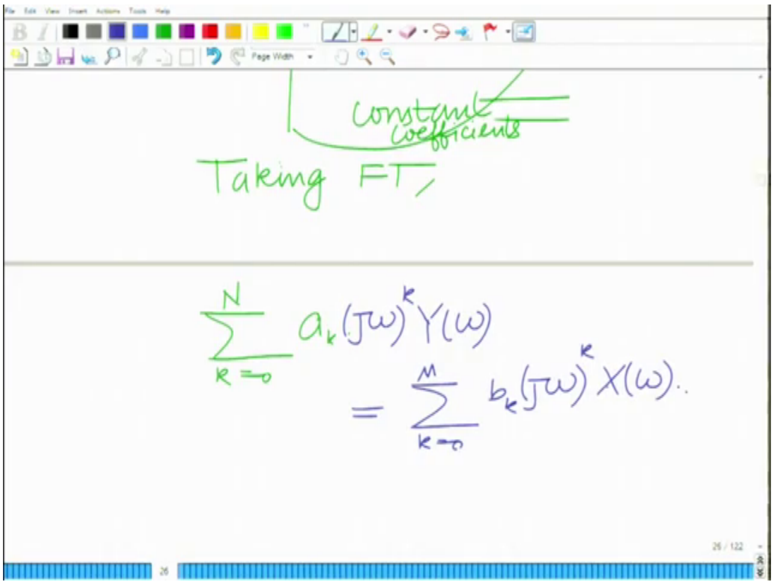

Now, if you take the Fourier transform on both sides taking the Fourier transform we have summation K equal to 0 to N Fourier transform of the kth order derivative we know is J omega raised to the k.

So, the Fourier transfer of the kth order derivative d raise d to the k y t or d t k that is J omega to the power of K y omega and the Fourier transform on the right is well that is summation K equal to 0 to M, b k again the kth order derivative of x t as Fourier transform J omega raised to K X omega.

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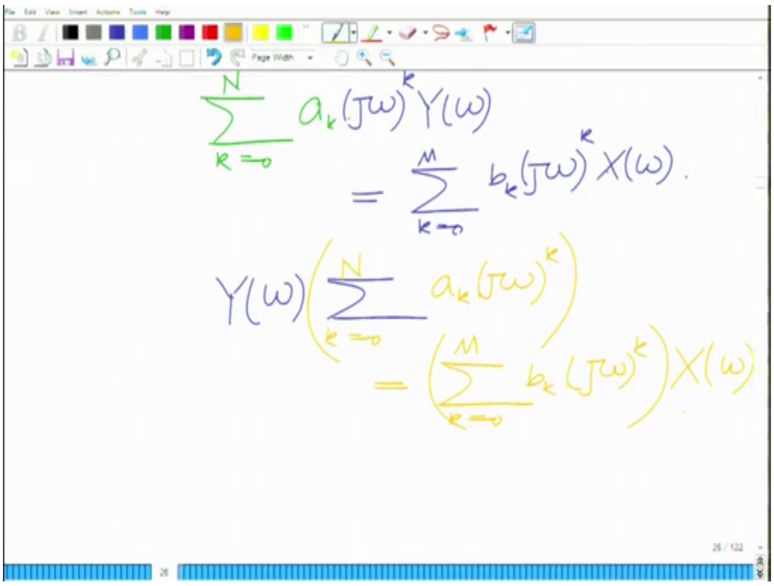

Taking omega common you have Y omega summation K equal to 0 to N, J omega power K equals summation K equal to 0 to M bk J omega power K into X omega which basically implies.

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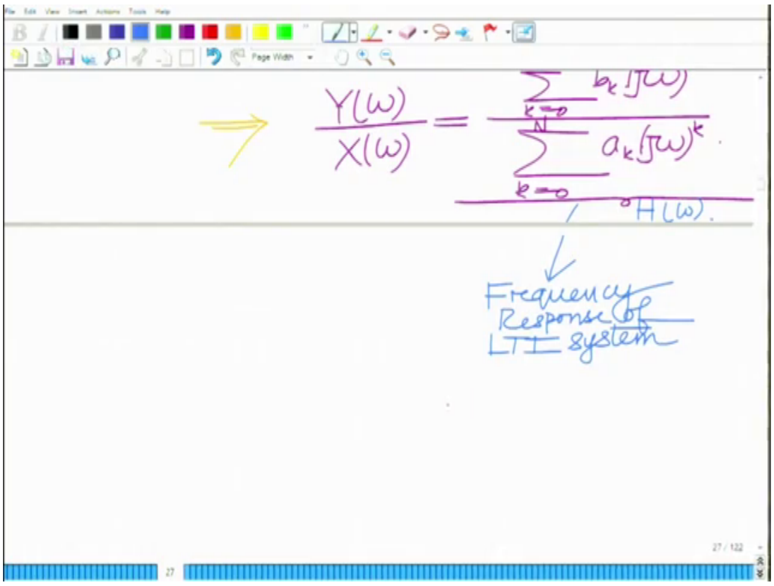

Now, that Y omega over X omega which is nothing but the transfer function H omega this is summation K equal to 0 to N or K equal to 0 to N b k j omega K divided by K equal to 0 to N, a k J omega K and this is your H of this is your transfer this is your frequency response of the LTI system. This is the frequency response of the LTI system.

So, that characterizes the frequency response. So, that characterizes the frequency response of the LTI system which is characterized by the constant coefficient differential equation ok. And now, let us look at the ideal filters the frequency response characterization of ideal filters.

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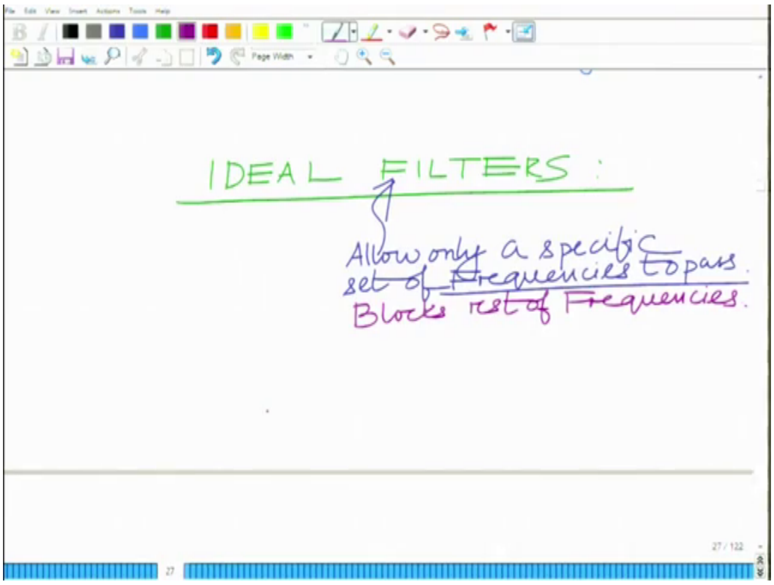

Now, ideal filters are the following. Now, filters basically allow only a specific set of frequencies to pass ok. So, what we have what we have is we have a system if you input a certain signal to that system x t, it only allows a specific set of frequencies that is specific frequency components of x t to pass through and it blocks the rest of the frequencies ok.

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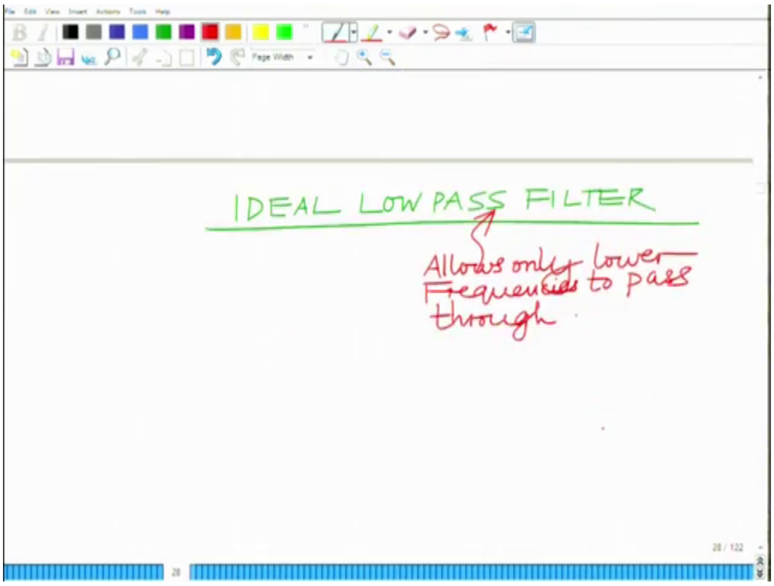

So, allows only specific set of frequencies to pass and blocks ideally blocks the rest of the frequencies ok. Example for instance we have an ideal low pass filter. Now, as the name implies ideal low pass filter allows only the lower frequencies pass through.

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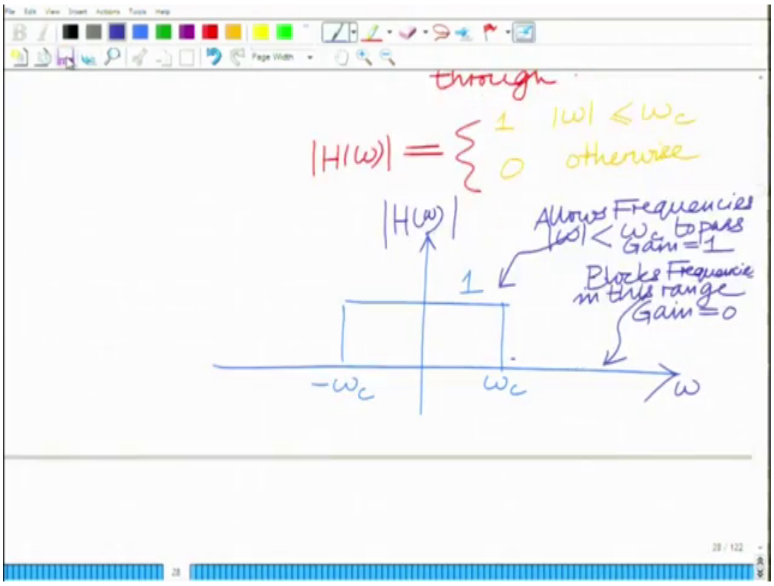

So, for instance we have the response magnitude response magnitude H of omega equals 1 for mod omega less than equal to omega c and this is 0 otherwise ok.

So, what we have is a filter that looks like this. So, we have in this band for all frequencies less than omega or greater than omega minus omega c and less than omega c its response is unity ok. So, it basically allows these frequencies to pass ok, because in this band minus omega to c to omega c the frequency response is unity when it multi when its multiplied by the frequency response of the input you can see all these frequencies all the components correct in this band are multiplied by unity gain therefore, it allows these components to pass.

But the gain outside this band of omega c and minus omega c that is frequency is less than minus omega c and frequency is greater than omega c you can see that the gain is 0 which means the input frequencies are completely blocked ok. Because the input frequency components corresponding to this that is band greater than omega c and less than minus omega c is multiplied by a gain of 0.

So, this is the pass band this is the this blocks. So, it blocks all frequencies in this range. So, this is your blocks frequencies in this range gain equal to 0 and in the pass in minus omega c to omega c the gain is equal to 1 ok, and this quantity omega c this is termed as a cutoff frequency. For a low pass filter all frequencies of the input signal greater than this cutoff frequency omega c are basically cutoff or these are blocked ok.

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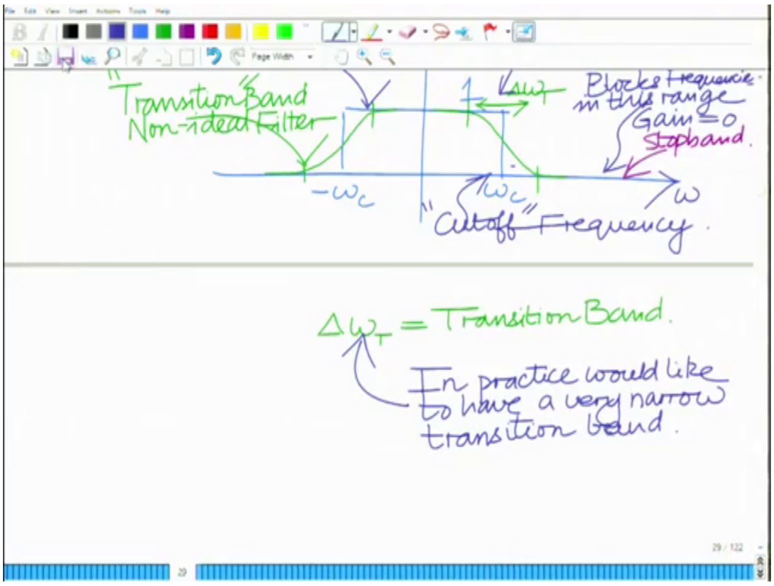

And this characterizes the ideal filter the ideal filter has very sharp cutoff frequency. You can see for omega less than omega c, you have a gain of unity for omega greater than omega c you have a gain of 0. In practice it is difficult to design such sharp filters and typically what you have is more of a transition ok. So, you have this is the pass band, this is the stop band is what is known as the and in practice you will have what is known as a transition band where it is transitioning ok.

So, in a non ideal filter this you will have in a, so in a non ideal filter you will also have a transition band where the frequency response is transitioning from the pass band to the stop band stop band to the pass band ok, but it is not exactly 1 or 0, but it takes gains which are between 1 and 0. So, this is basically you will have a transition band you can call this as a transition band in a non ideal filter. So, and we would like to obviously, design filters which are as close to ideal as possible which means the transition band which means the cutoff is as sharp as possible. The transition band is as small as possible.

So, in practice we would like to have this delta w transition we can call this delta w trans, delta w in practice would like to have a very narrow, in practice we would like to have very narrow transition band that is a narrow trans which means we want to have a very sharp cutoff, very sharp edges of the filter, ideal filters have very sharp edges. So, ideal filters have a very sharp implies they are difficult to design. So, the ideal filters which are very sharp edges these are difficult to design.

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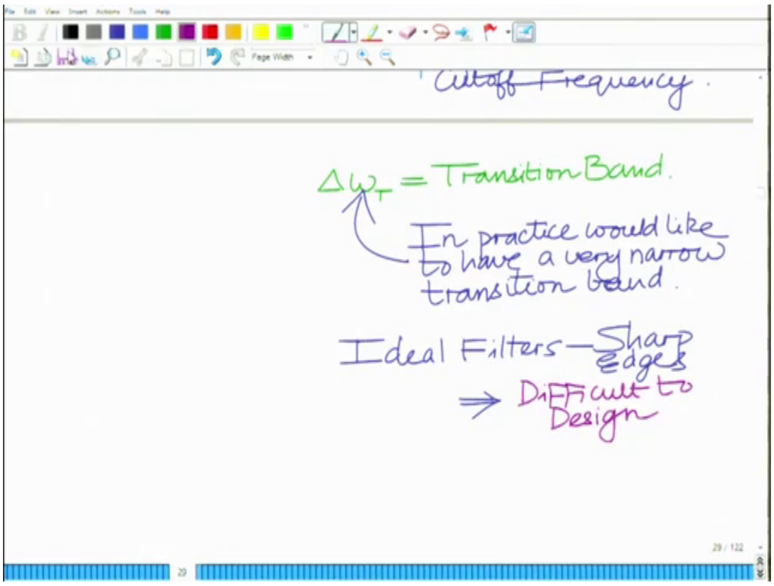

Similarly one can have an ideal high pass filter ok. So, one can have an ideal high pass filter. Now, in an ideal high pass filter what you have is you basically have something that looks like this. So, in an ideal high pass filter let me just write it on a new page. So, we have an ideal pass filter.

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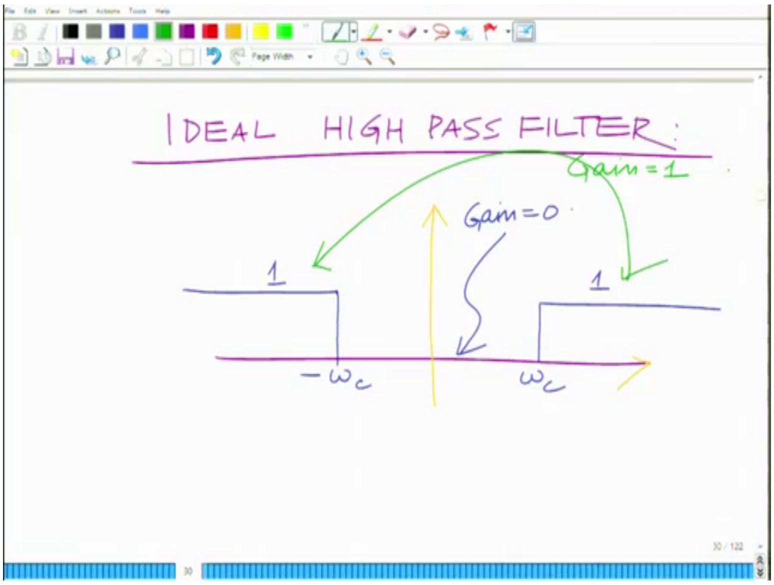

And in an ideal high pass filter as the name implies again opposite it is a exact opposite of a low pass filter. So, instead of blocking frequencies greater than a cutoff frequency omega c, you allow the frequencies greater than a cutoff frequency omega c to pass and so the gain outside minus that is greater the freq frequency is greater than omega c or frequency less than minus omega c is 1 and in this band minus omega c to omega c the gain is 0 and the gain here in these outside gain equals 1.

So, blocks all frequencies components. So, this blocks all frequency components in the frequency band minus omega c to omega c.

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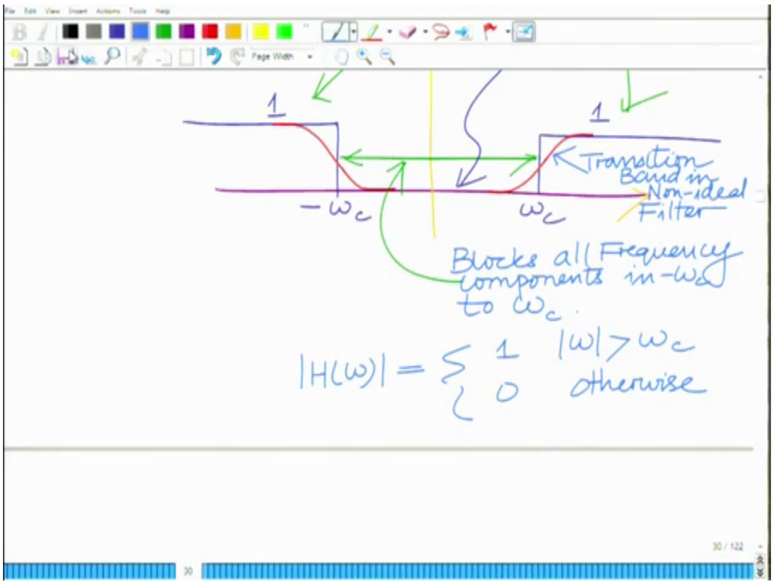

So, naturally its magnitude response is magnitude H of omega equals 1, mod omega greater than omega c this is equal to 0 otherwise ok. So, this is an ideal high pass filter. It blocks all the signals in the lower all the components in the low frequency region that is from minus omega c to omega c and allows only those frequency components which are greater than omega c or less than minus omega c all right to pass, all right has unity gain in the bands which are frequency bands which are greater than omega c and less than minus omega c. And obviously, once again it is very difficult to design an high pass filter high pass filters with such sharp edges.

So, once again you will have transition bands from the stop bands to the pass band. So, again once again you will have a transition. So, these are your these are your transition bands in a non ideal filter and we would like to make this transition bands once again we would like to make this transition bands very narrow all right.

So, in this module we have looked at the frequency response of a distortionless LTI system, its magnitude of a face characteristics, and also we have also started looking at ideal filters in particular the ideal low pass and the ideal high pass filter all right. So, let us stop here and look at other aspects in the subsequent modules.

Thank you very much.