Principles of Signals and Systems
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 36
Fourier Transform (FT): Definition, Inverse Fourier Transform, Fourier Spectrum, Dirichlet Conditions, Relation to Laplace Transform, FT of Unit Impules

Hello welcome to another module in this massive open online course. So, we are looking at the Fourier series representation and it is various properties in this module. Let us start looking at a different topic that is the Fourier transforms ok. So, we want to start looking at.

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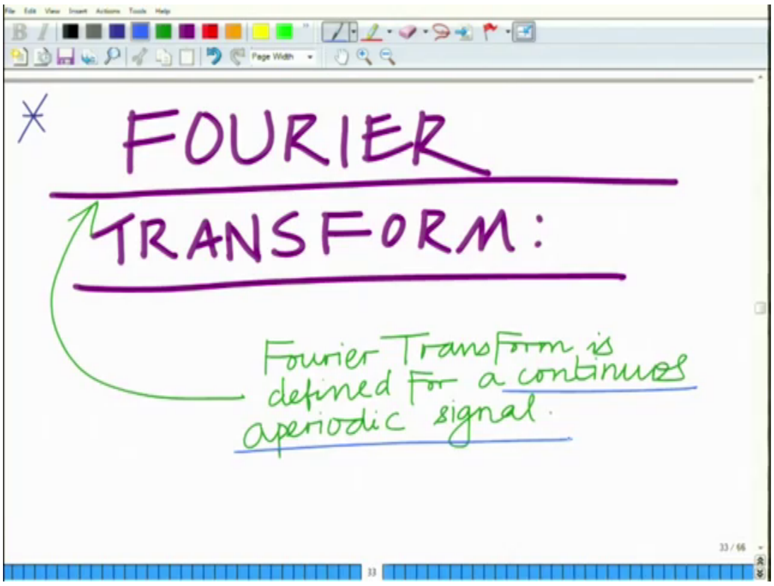

The concept of the Fourier transform and this Fourier transform remember the Fourier series is defined for a continuous periodic signal this is for a continuous a periodic signal. So, the Fourier transform is defined for a continuous is defined for a continuous and then aperiodic signal.

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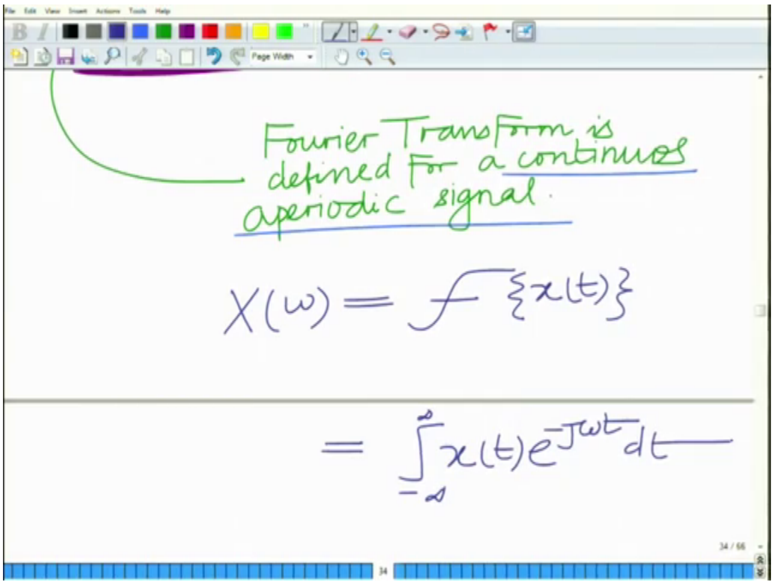

Now, the Fourier transform the definition of the Fourier transform is given as follows that is X of I can define the Fourier transform as a function of the angular frequency X of the omega equals the Fourier transform of x t which is equal to represented as the Fourier transform of x t which is integral minus infinity to infinity x of t e raised to minus J omega t d t ok. So, the Fourier transform let me just write it again a bit more prominently.

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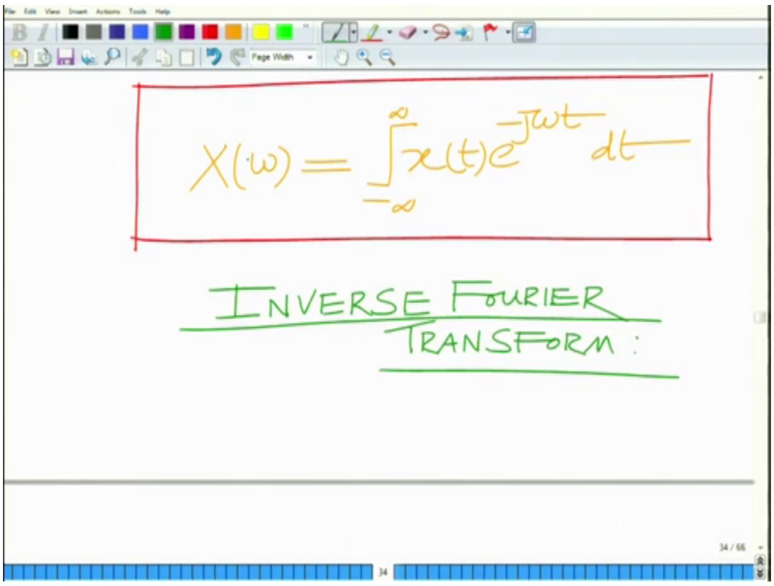

So, this is your X of omega equals minus infinity to infinity x of d t e raised to minus J omega t d t. This is one of the most important transforms and as I have already told you the Fourier transform or the Fourier transform framework is one of the most commonly used frameworks or those commonly used transforms in a to understand and analyze the previous properties and the behavior of signals and systems all right.

And, similarly the inverse Fourier transform that is this relates the signal to it is Fourier transform the inverse Fourier transforms. The inverse Fourier transform is given as follows, that is; which relates gives the signal from the Fourier transform that is your.

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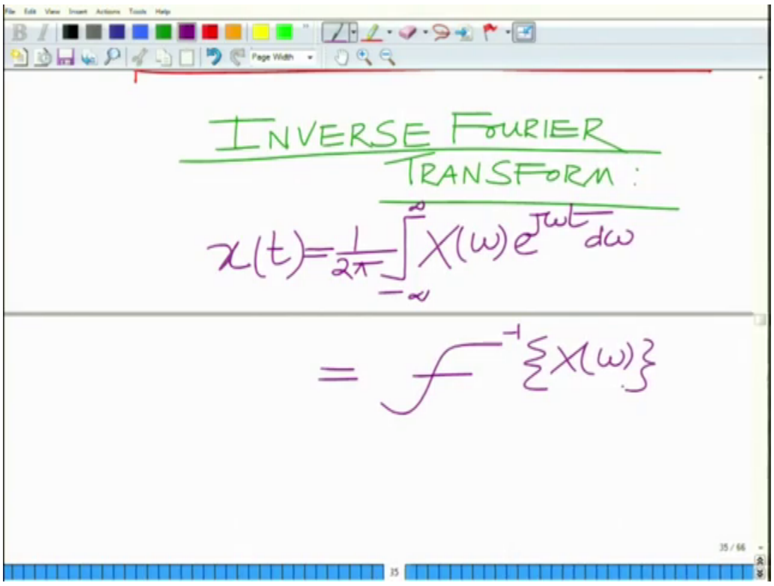

X of t equals minus infinity to infinity of well X of omega e raised to ZJ omega t d t and there is a factor of 1 over 2 pi, sorry; this 1 over 2 pi integral minus infinity to infinity X omega e raised to J omega t d omega and this is simply represented as f inverse of X of omega that is the inverse Fourier transform.

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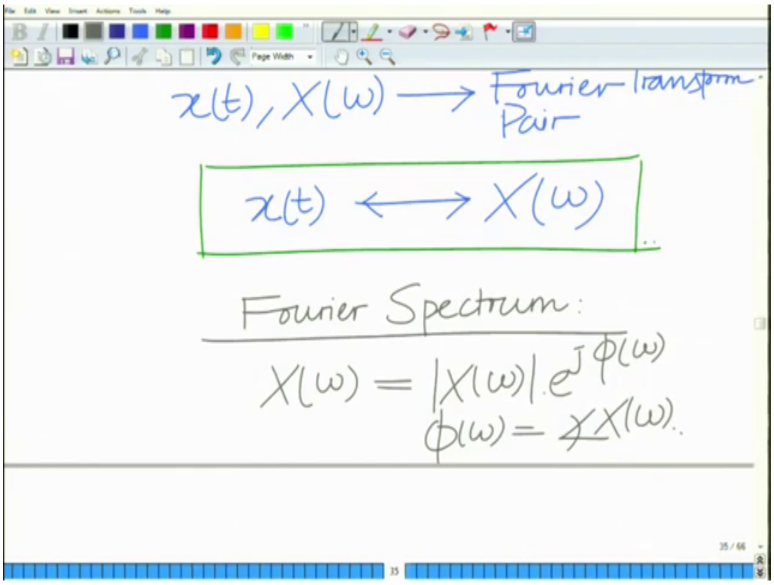

And x t and X of omega constitute a Fourier transform pair this form what are known as the Fourier transform. These from the Fourier transform pair and represented simply using a double headed arrow that is x t and X omega this form a Fourier transform these form a Fourier transform pair ok. It is form a Fourier transform pair and the Fourier spectrum.

The Fourier spectrum can be obtained as follows, that is; again from the Fourier transform all right from the signal x t once you obtain the Fourier the Fourier transform the Fourier spectrum can be obtained as follows; that is I can write X of omega equals magnitude X of omega e raised to J phi of omega where phi of omega is the angle or phase of omega.

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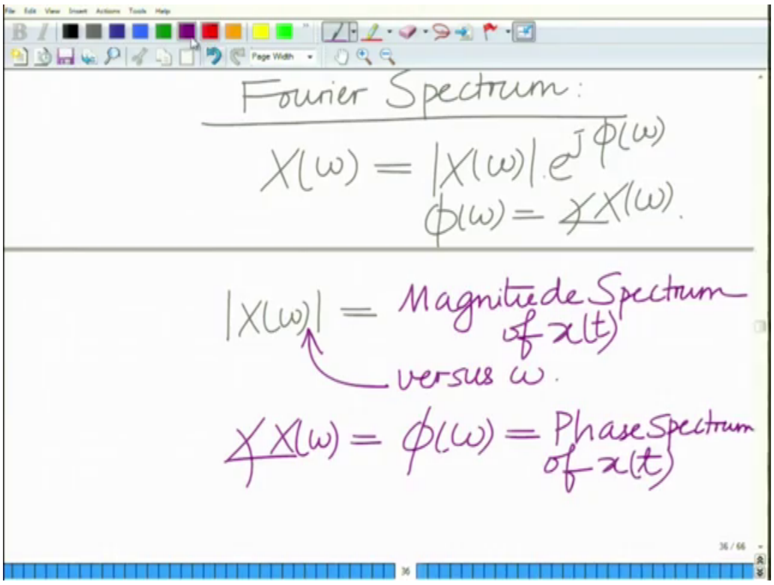

And this magnitude X of omega again this is the magnitude spectrum of x of t that is versus omega versus angular frequency omega and the angle of X of omega or the phase of phi of omega this is the phase spectrum.

So, here the magnitude spectrum of x of t and you are the phase spectrum of the signal x of t now for real signals x of t.

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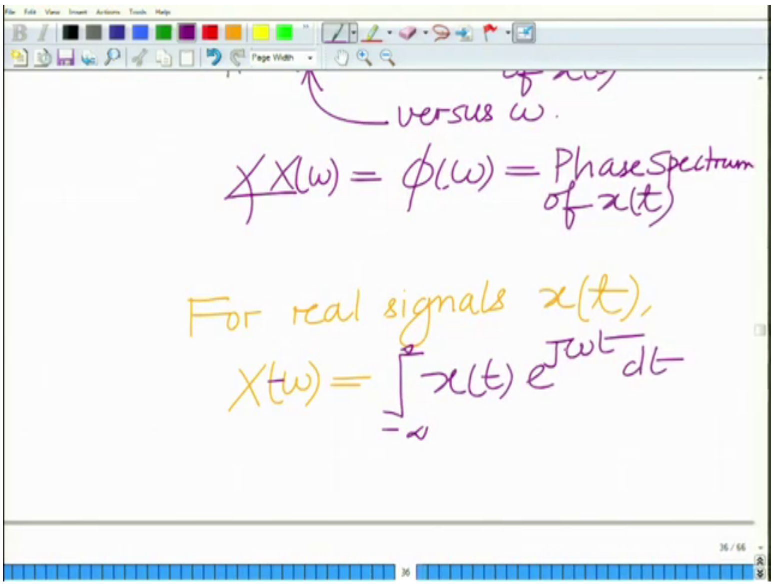

For real signals x of t we have X of omega well if you look at x of minus omega well; that is minus infinity to infinity. Now for real signals x of t of X of minus omega is minus infinity to infinity x of t e raised to J omega t because you are considering X of minus omega.

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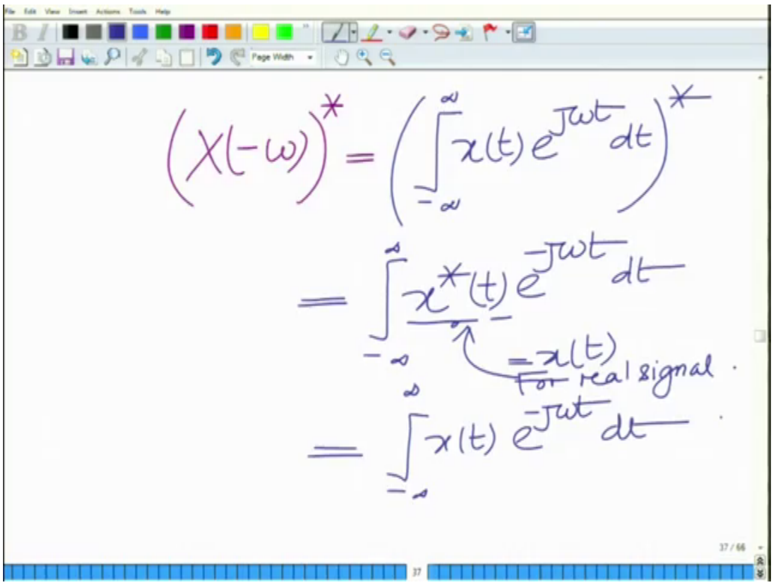

And now, therefore, if you consider X of minus omega conjugate this is basically integral minus infinity to infinity x of t e raised to J omega t conjugate which is equal to integral minus infinity to infinity x conjugate of t e raised to minus J omega t d t, but we know x conjugate of t this is equal to x of t for a real signal implies this is equal to minus infinity to infinity x of t e raised to minus J omega t dt, but this is nothing else, but X of omega this is again equal to.

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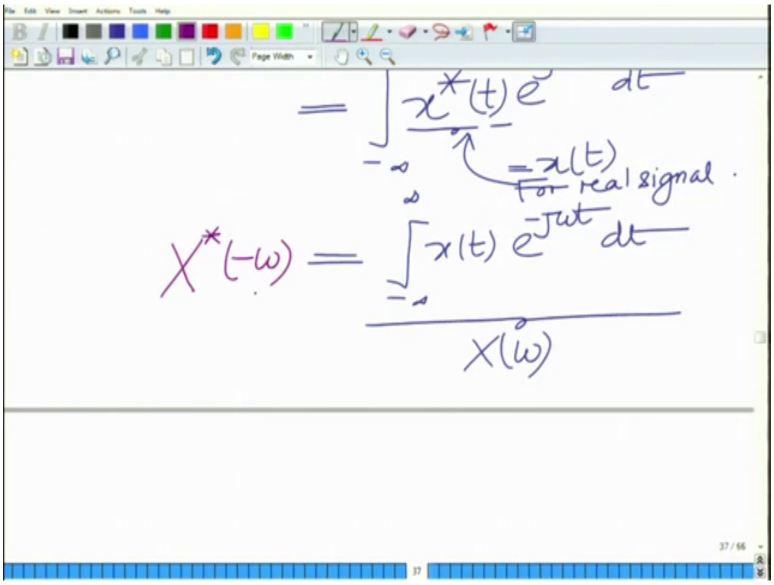

X of omega. So, therefore, what we have is X conjugate of minus omega equal to X omega. So, of real signal that is the Fourier transform at minus omega is the conjugate of the Fourier transform at omega.

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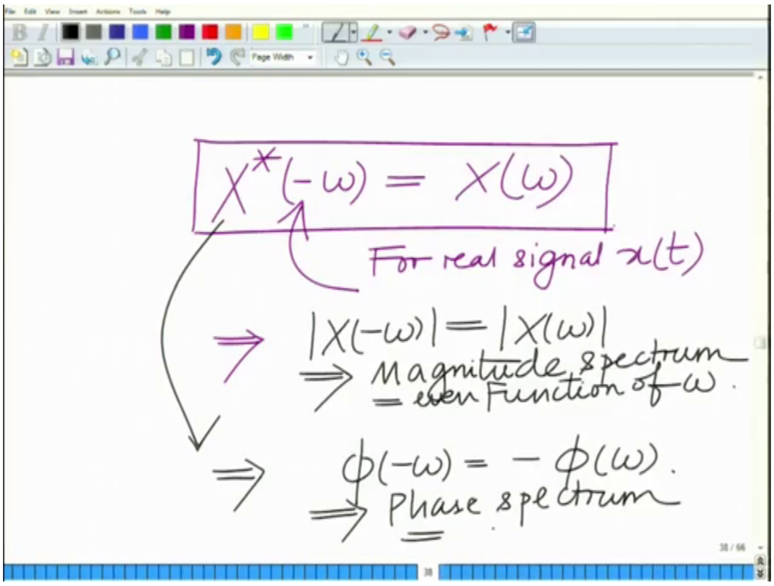

So, we have X conjugate of minus omega equals X of omega for a real signal for a real signal x of t and therefore, this implies well what does this imply you can readily say that this implies that magnitude, because the conjugate quantity and it is complex conjugate have the same magnitude X minus omega equal to magnitude X omega which basically implies that magnitude spectrum is even function of omega; that is for a real signal and this also implies that, because the conjugate has a negative phase. So, f of minus omega equals minus f of omega which implies that phase spectrum is equal to the phase spectrum is equal to odd function of omega the phase spectrum is an odd function of omega all right.

So, what we have shown is that; similar again once again to the Fourier series. What we have shown is that for a real signal xt; we have the property that is X conjugate of minus omega equals X of omega which means the magnitude X of minus omega equals magnitude of X of omega. So, the magnitude spectrum is even and the phase at minus omega is minus of the phase at omega. So, the phase spectrum is basically an odd function of omega all right.

So, we will now look at of course, another important aspect is the conditions for convergence of X of omega.

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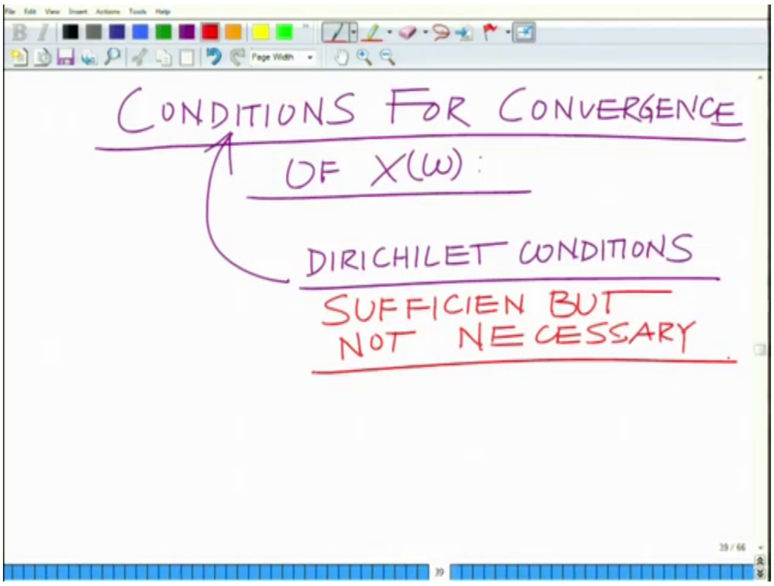

Again similar to the Dirichlet conditions the conditions for the convergence of X omega remember; again these are the Dirichlet conditions Dirichlet conditions these are also sufficient, but not necessary conditions remember if the signal x t satisfies the Dirichlet conditions the Dirichlet conditions, then the Fourier transform exists. What is the Fourier transform exists it does not automatically; that it does not imply that the Dirichlet conditions are satisfied remember these are sufficient, but not necessary.

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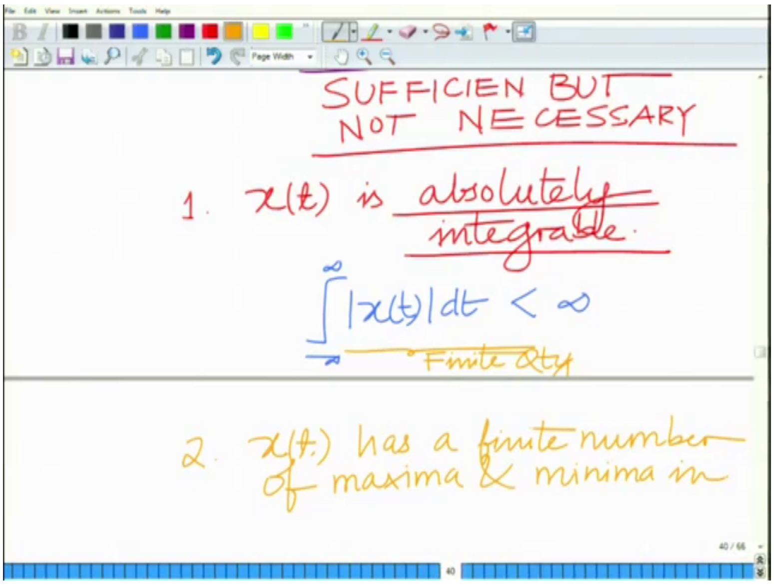

And again similar to the discrete case these are as follows that is your x t is absolutely integrable which implies that integral minus infinity to infinity magnitude x t d t is less than infinity which implies; that this basically has to be of finite this basically has to be a finite quantity 2, we must have the x t has a finite number of maxima and minima finite number of maxima minima in any finite interval.

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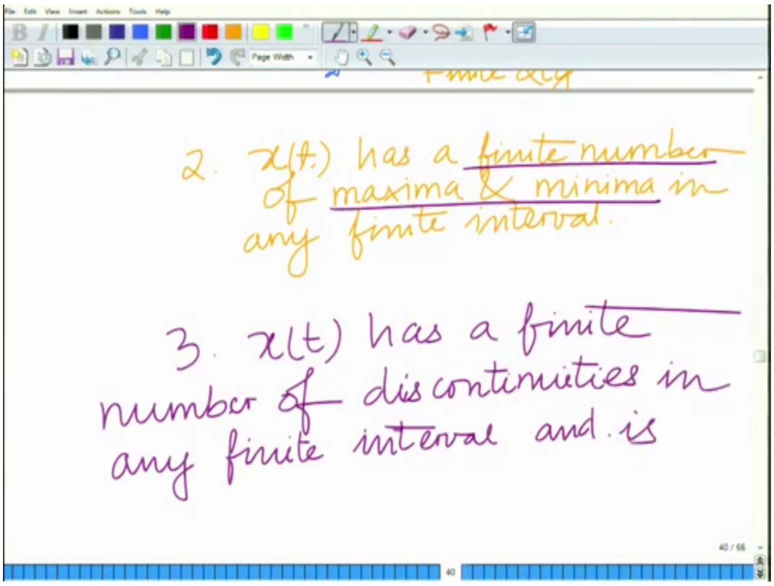

It has a finite number of maxima and minima finite number of maxima and minima in any interval of a finite duration.

And 3 x t is a finite number of discontinuities; that is this signal x t in any finite interval has a finite number of discontinuities and at each of these discontinuities it has a finite value is x t has a finite number of discontinuities in any finite interval and is finite at each of these discontinuity and is finite at each of this discontinuities.

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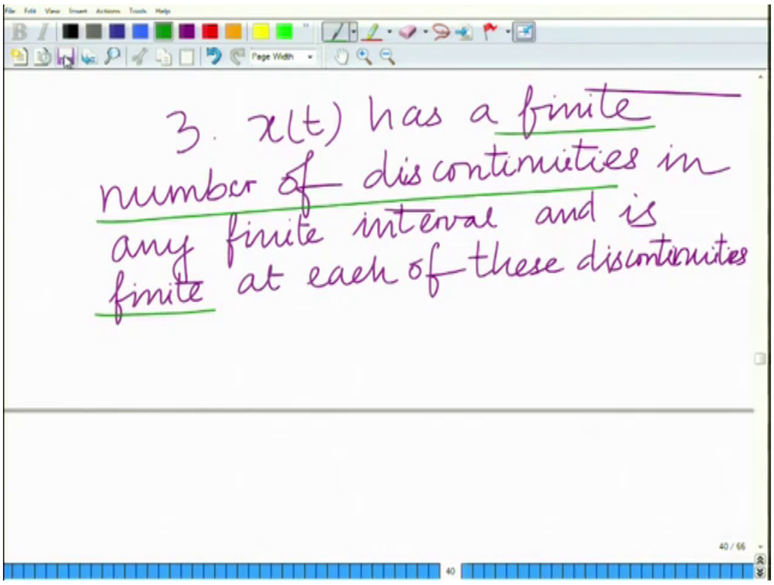

It has a finite number of discontinuities and it is finite at each of these discontinuities ok. So, these are this Dirichlet that these are the Dirichlet conditions for the existence of the convergence of the Fourier integral and remember these are sufficient, but not necessary. In fact, you will see examples of signals which do not satisfy; these are special signals which do not satisfy the Dirichlet conditions, but the Fourier transform still exists will see a couple of signals.

Now, the relation between the Fourier and the Laplace transforms.

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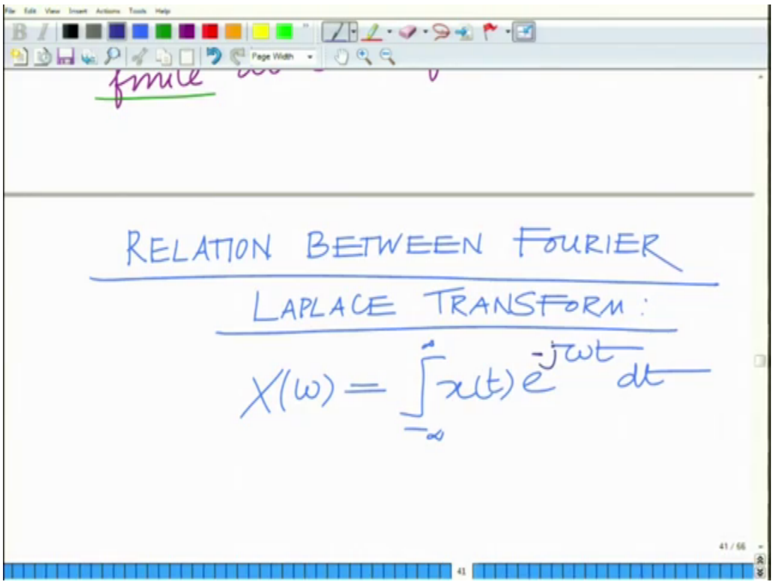

So, we have seen the Laplace transform previously. So, what we want to now see is the relation between the Fourier and the Laplace relation between the Fourier and the Laplace transform and remember the Laplace transform remember the Fourier transform we have integral minus infinity to infinity x t e raised to minus J omega t d t e raised to minus J omega t d t there is a Fourier transform.

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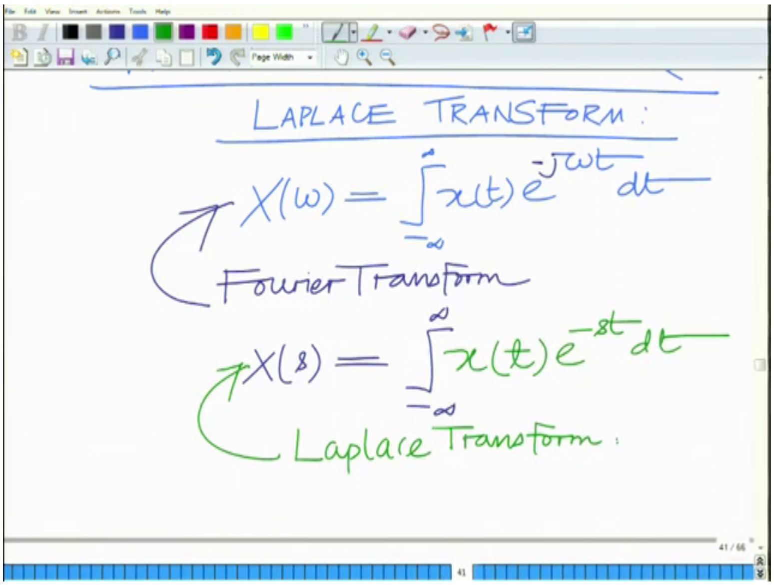

And so, this is your Fourier transform and remember the Laplace transform is given as integral minus infinity to infinity x t e raised to minus x t d t there is a Laplace transform ok.

So, the Laplace transform is integral minus infinity to infinity x t e raised to minus x t d t this is the Laplace transform ok.

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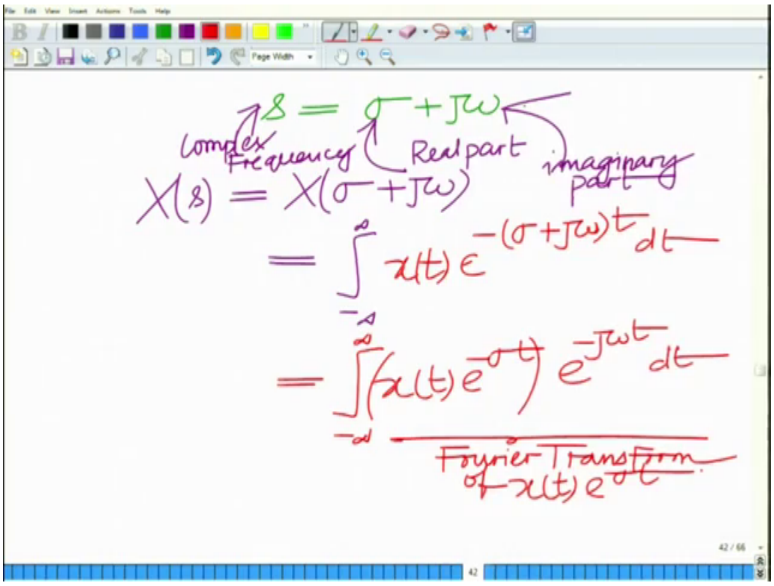

And I can set remember s is a complex number. So, s equals sigma plus J omega which means X of s equals X of sigma plus J omega which is equal to sigma is the real part remember of this complex number this is the real part this is the imaginary part and you can think of this as a complex frequency. .

So, X has both the real and imaginary parts. So, this is complex frequency. So, this X of sigma plus J omega which is integral minus infinity to infinity x of t e raised to minus x t that is e raised to minus sigma plus J omega into t dt which is nothing, but integral minus infinity to infinity x t e raised to minus sigma t e raised to minus J omega t d t.

And now if you see this is nothing, but the Fourier transform of x t e raised to minus sigma. So, this is Fourier transform of x t e raised to minus sigma t. So, what we have is that; the

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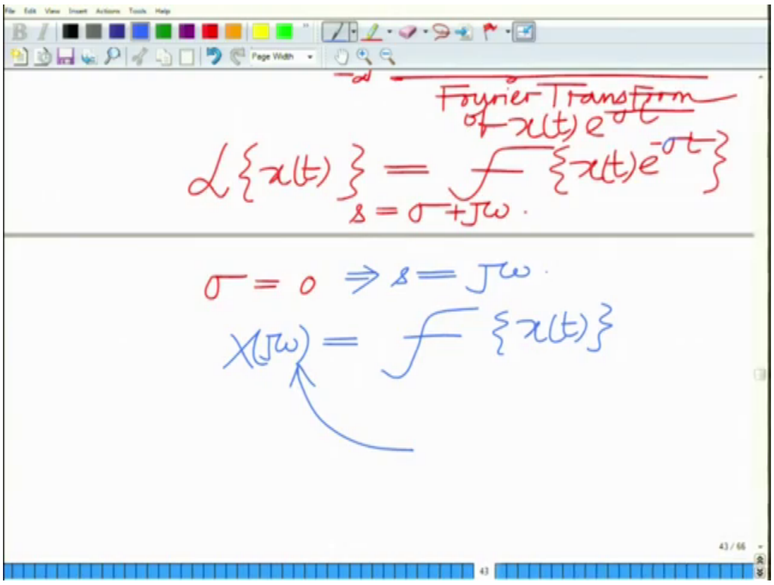

Laplace transform of x of x t a value at s equal to at basically s equal to sigma plus J omega this is equal to basically the Fourier transform of xt e raised to minus sigma t ok. So, Laplace transform where we have s equal to sigma plus J omega remember s is the complex frequency.

So, the Laplace transform of xt at s equal to sigma plus J omega is the Fourier transform of xt of Fourier transform of xt e raised to minus sigma t where sigma is the real part of the this complex frequency that is s and now you can see that if s equal to 0 that is not sorry if sigma equal to 0 all right which implies which basically implies that s is equal to J omega then we have x of J omega there is a Laplace transform x of J omega is nothing, but the Fourier transform of xt e power minus sigma t, but sigma is 0. So, this is nothing, but Fourier transform of xt.

So, the Fourier transform of xt. So, the Laplace transforms. So, you can simply deduce. So, you can say that the Fourier transform. So, you can say that the Fourier transform.

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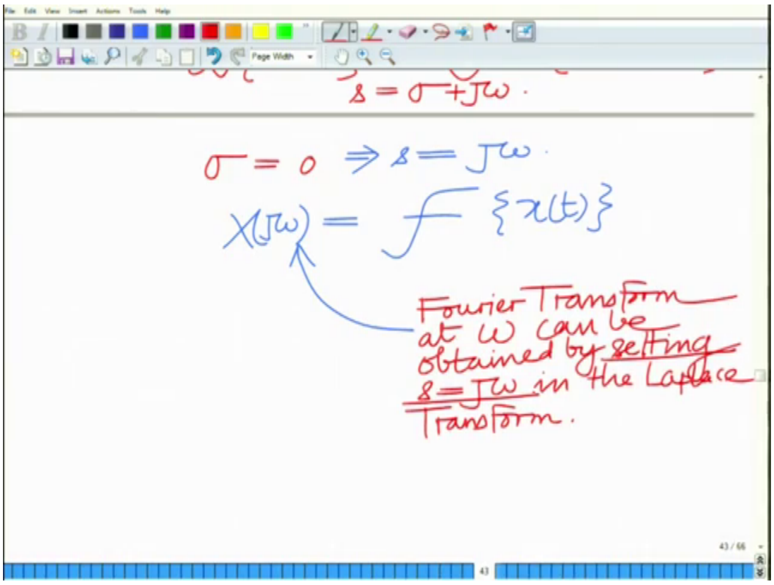

At omega can be obtained by setting s equal to J omega in the in the Laplace transform ok. So, you can say that the Fourier transform can be obtained by setting s equal to J omega in the Laplace transform; however, note that this is only true under some conditions caution ok.

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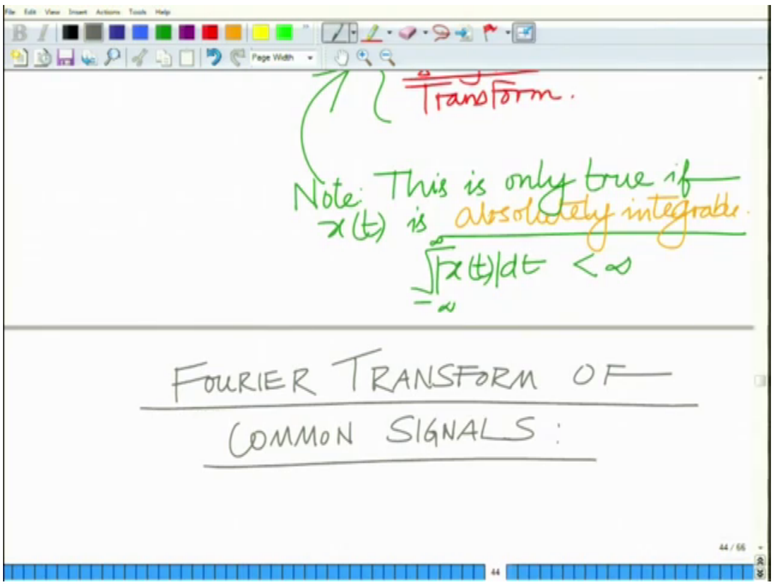

Note this is only true if the signal xt is absolutely integrable is only true if the signal xt is absolutely integral; that is minus infinity to infinity xt dt is that is magnitude of xt dt is less than infinity only if the signal is absolutely integrable ok.

And so, this is only true, but; however, not all signals. So, and this not true the signals are is not absolutely integrable not now all signals we are going to several we are going to say examples in which the signals do not satisfy this property. Hence, in those scenarios you cannot simply obtain the Fourier transform from the Laplace from the Laplace transform by setting s equal to J omega ok. So, there is a word of caution that you can use this technique to obtain the Fourier transform from the Laplace transform only if the signal is absolutely it all right with that let us proceed to look at the Fourier transform of some common signals.

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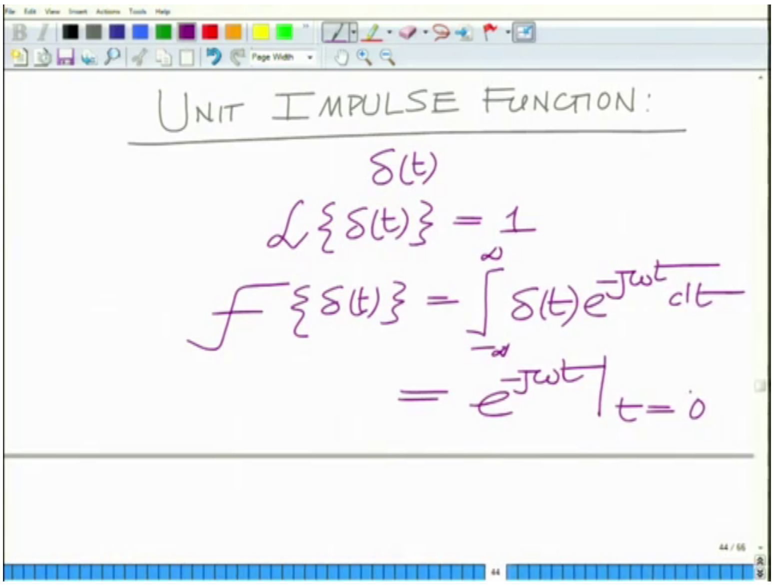

Now, the Fourier transform of some commonly some common signals and one of the most prominent signals is the unit impulse function which is the direct delta function that is delta t and we know that the Laplace transform of the direct delta is unity. Now let us look at the Fourier transform of this that is delta t this is equal to integral minus infinity to infinity xt e raised to minus J minus J omega t dt, but xt is delta t. So, e raised to minus J omega t dt and remember this is e raised to minus J omega t evaluated from the property of the delta function this is e raised to minus J omega t evaluated at t equal to 0.

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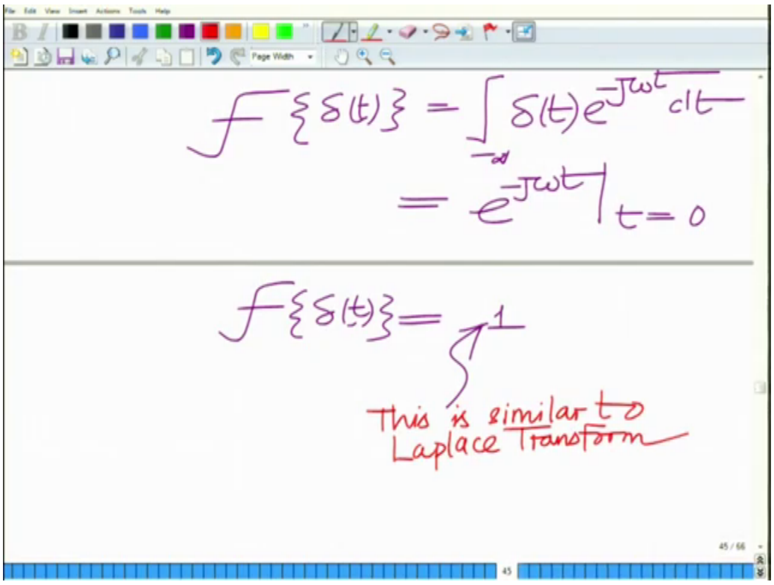

This is simply one and therefore, the Fourier transform of the delta function is the unit impulse function is one and this is similar to the Laplace transform and you can also see this similar to the Laplace transform the Laplace transform is unity there is no s all right.

So, you do not need to replace s by J omega. So, the Laplace transform is unity the Fourier transform also unity and also note that the delta function is absolutely integrable it is always it is it is non negative function all right and integral minus infinity to infinity delta t equal to 1 all right. So, we have the delta if function is non negative integral minus infinity to infinity delta t dt equals 1 and this is an absolutely integrable function absolutely integrable and therefore, the Fourier transform in this case is basically similar to that of the Laplace similar to that of the Laplace transform all right.

So, what we have done in this module is; basically we introduced the concept of the Fourier transform ok. We looked at the properties of Fourier transform the magnitude in the phase spectrum the magnitude of the phase spectra and the magnitude on the phase spectra for the specific class of real signals also the conditions for the existence of the Fourier transform and the relation between the Fourier transform and the Laplace transform right. So, we will stop here and continue with other aspects in the subsequent modules.

Thank you very much.