Principles of Signals and Systems

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Lecture – 31
Example Problems in z–Transform – Impulse Response of LTI System Described by Difference Equation

Hello welcome to another module in this massive open online course. So, we are looking at example problems in the z transform right to understand the various properties and applications of the z transform.

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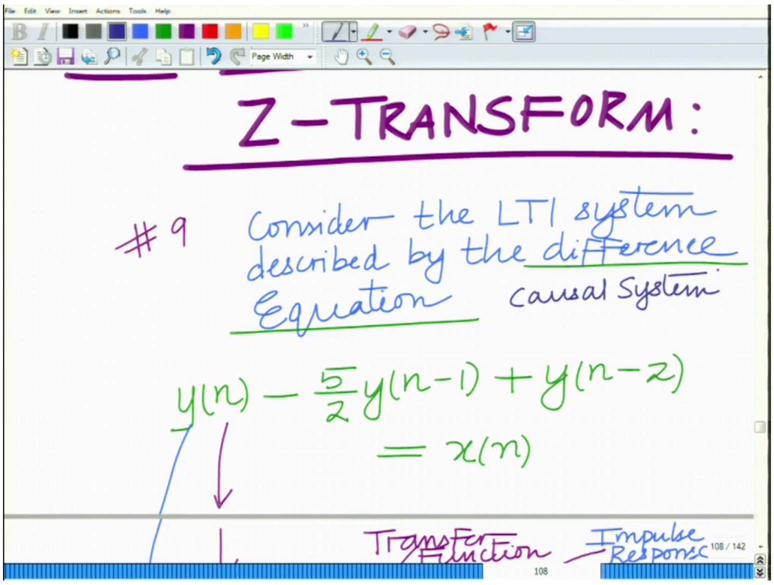

So, let us continue that discussion to do our final example. So, we are looking at example problems.

We are looking at example problems for the z transform, and let us look at the last example number 9 that is consider we are looking a we will look at a difference equation, consider the LTI system described by the constant coefficient differential equation correct different described by the difference equation, we are looking at a difference equation and the difference equation is given as y n minus 5 by 2 y n minus 1 plus y n minus 2 equals x n .

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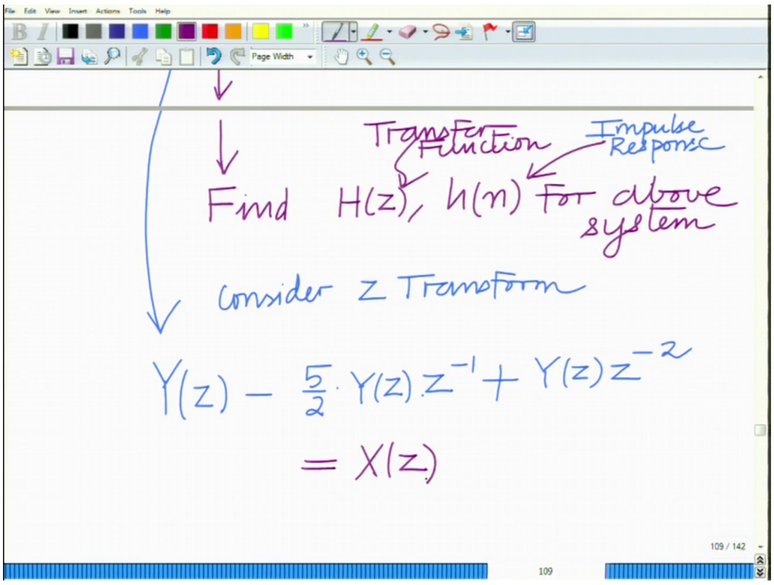

Now, what we are interested in finding is for this we want to find the impulse response, and the transfer function. So, first we want to find the transfer function H z and impulse response for the our system, remember H z this is the transfer function and h n this is the impulse response, and this can be done as follows if we consider the z transform on both sides, and then what we have is if you take the z transform y n lets say we denote the z transform of y n by Y z minus.

Now the z transform of y n minus 1 remember this is the delayed version of y n z for the z transform of y n minus 1 will be Y z times z inverse, that is Y z times z raised to minus 1. So, minus 5 by 2 Y z times z raised to minus 1 plus the z transform of y n minus 2 which is naturally Y z into z raised to minus 2, and this is equal to z transform the right z right hand side is x n so, z transform is x z.

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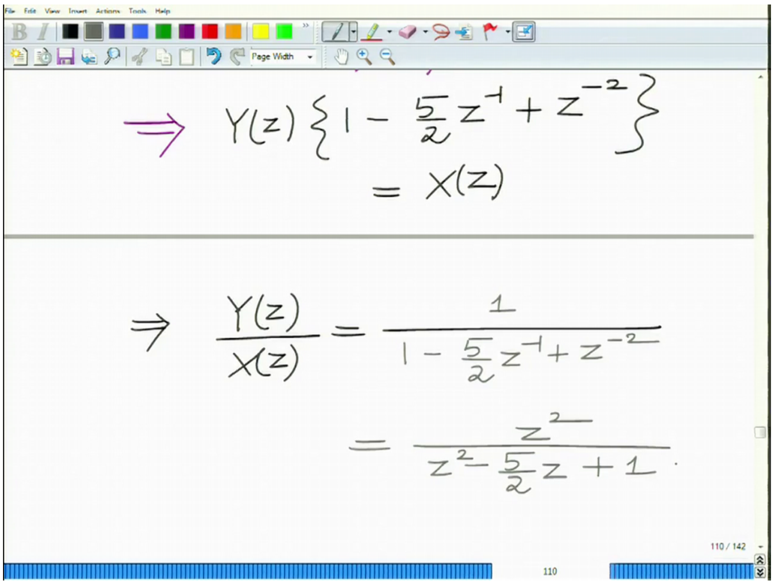

So, this implies that basically we have something very simple we have Y z times 1 minus 5 by 2 z inverse plus z minus 2 equals X z which implies Y z by X z equals 1 divided by 1 minus 5 by 2 z inverse plus z minus 2 which is equal to z square divided by z square minus 5 by 2 z plus 1 all right.

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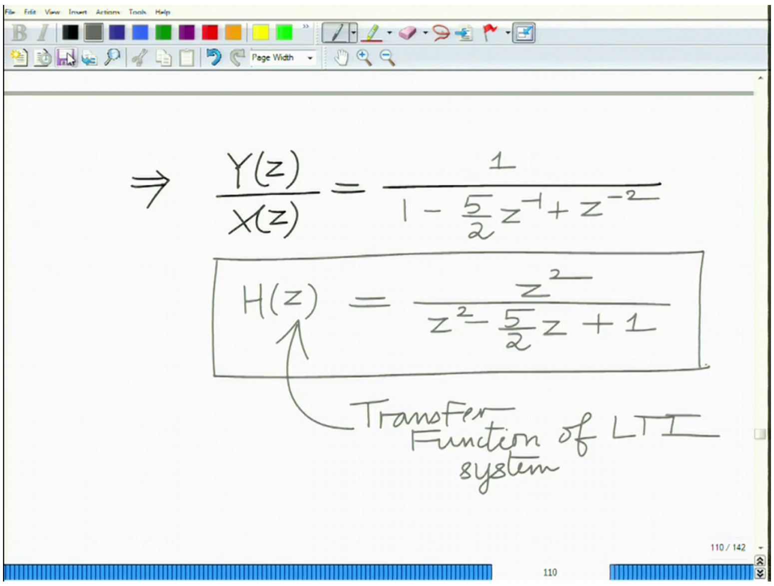

And this is basically your Y z by X z which is the transfer function. So, this is basically the transfer function that is z transpose the impulse response transfer function of the LTI system.

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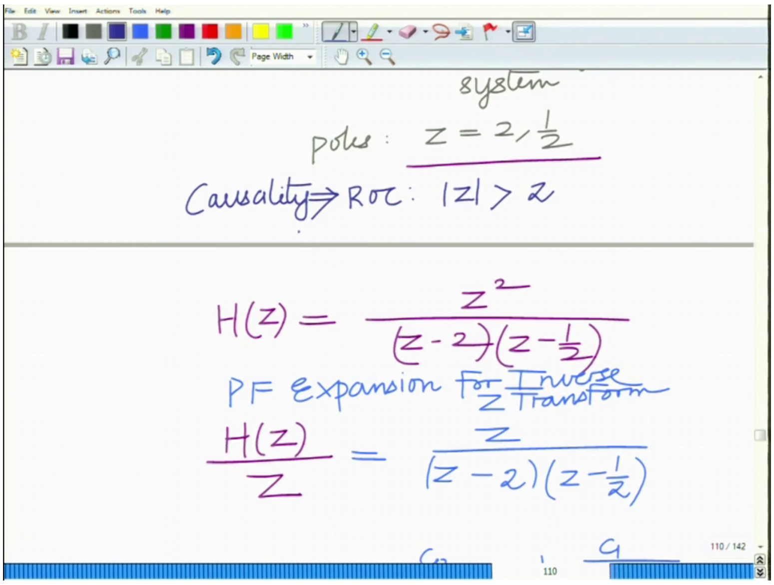

Now also H z now remember H z equals z square by z minus 2 so, the poles are z equals 2 comma half, because we can write this as now H z can also be simplified as observe z square over z minus 2 into z minus half. So, there are 2 poles at z equal to 2 and z equal to half, these are poles each pole has multiplicity 1 all right. So, these are simple poles and the poles are z equal to 2 and z equals half ok.

So, now we are going to perform the partial fraction expansion to find the inverse z transform. So, consider now H z over z. So, we use the partial fraction expansion for the inverse z transform, and this is equal to well z over z minus 2 into z minus half.

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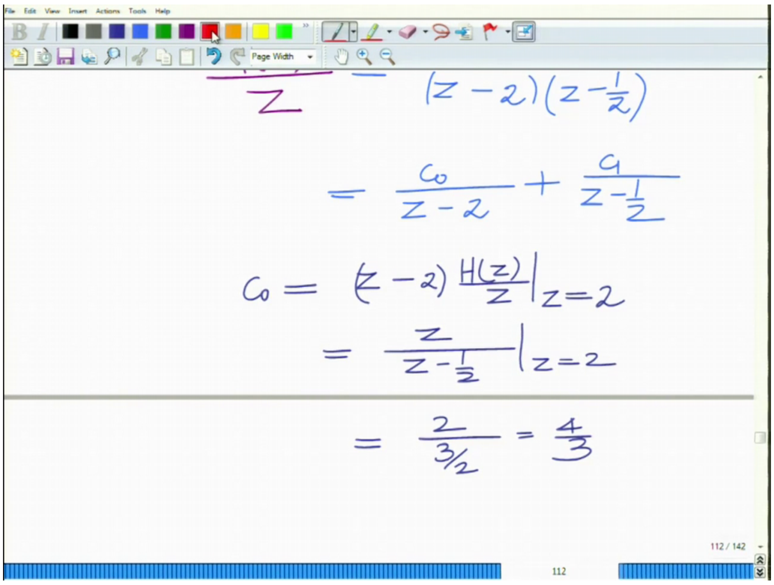

And let us say we express this as C naught mine over z minus 2 plus C 1 over z minus half. Now we consider this let us also be consider a system that is a causal. So, causal system implies that remember that it has to be a right handed signal therefore, ROC is of the form it has to be greater than equal to 2, since 2 is the maximum magnitude of the post correct. So, since we have a causal system. A causal system implies that the impulse response is a right handed signal, for a right handed signal the ROC will be of the some magnitude z is greater than r max, where r max is the maximum magnitude of the poles of the transformation all right.

So, causality implies that the ROC is of the form magnitude c greater than 2, and which is basically also equal to r max remember. So, this is equal to r max maximum of the amplitude of the poles of the transfer function.

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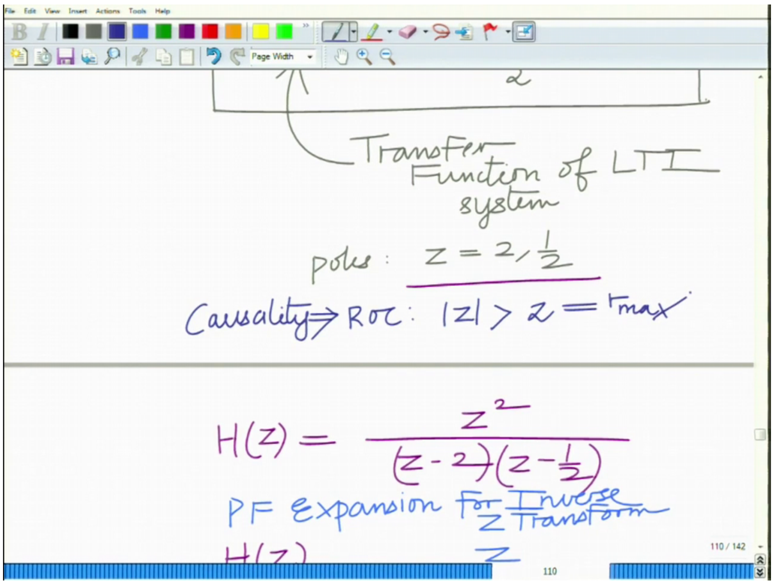

Now, here now once we have done that now it is easy to relatively easy to find C naught we have done this several times before so, C naught it is a simple pole. So, z minus 2 times its z over z evaluated at z equal to 2 which is basically your z over z minus half evaluated at z equal to 2 which is basically 2 divided by 2 minus half that is 3 by 2 so, this is equal to 4 by 3.

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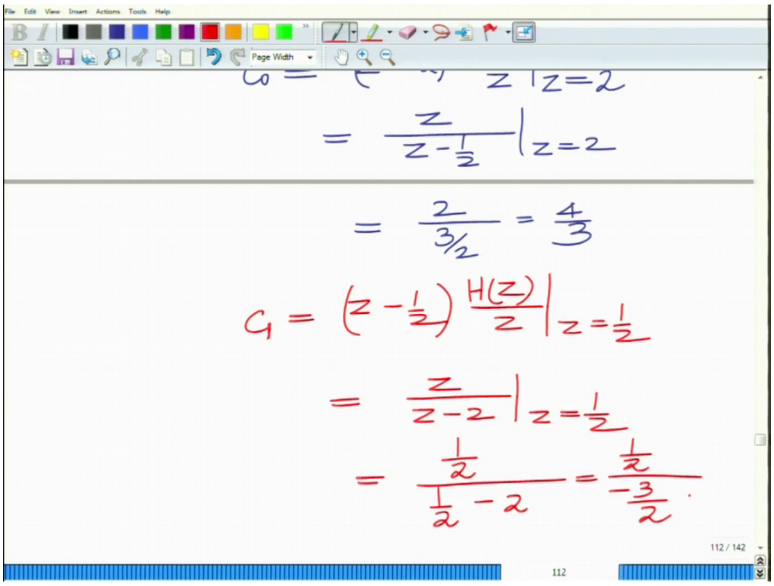

Now, similarly C 1 equals z minus half into H z over z evaluated at z equal to half, which is equal to z divided by z minus 2 evaluated at z equal to half. So, that is basically half divided by half minus 2 equals half divided by minus 3 over 2.

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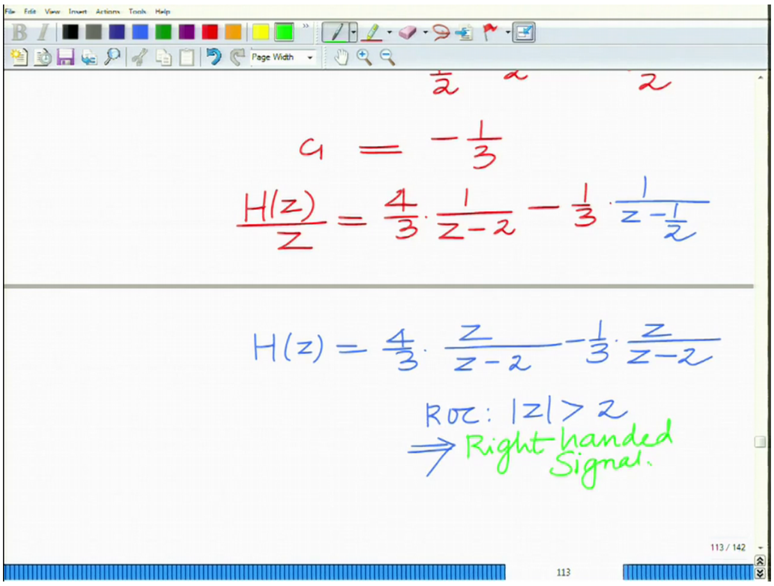

So, that is basically so C 1 equals minus 1 by 3, and therefore what we have is H z over z equals you have 4 over 3 divided by 1 over z minus 2 minus 1 over 3 times 1 over z minus half and therefore, H z itself equals 4 over 3 into z over z minus 2 minus 1 over 3 into z over z minus 2, and we have the ROC magnitude z greater than 2 implies this causal of course, it implies this is a right handed signal.

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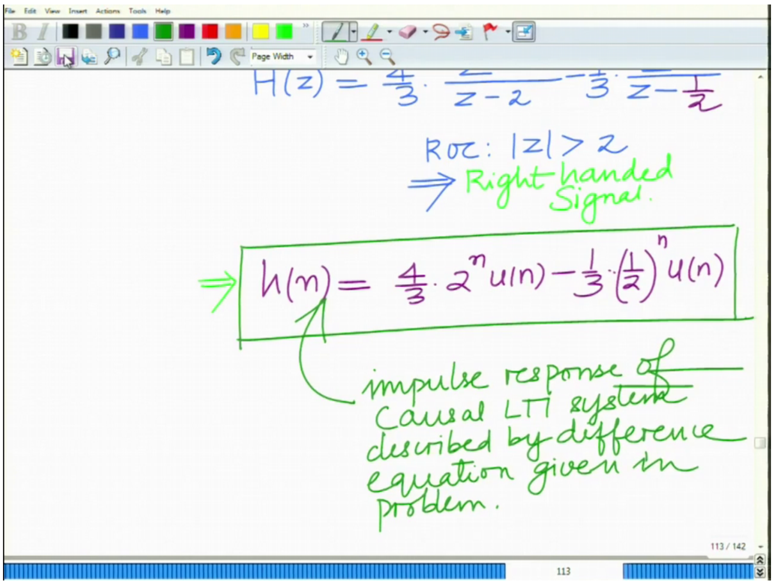

Which implies that this quantity h of n that is impulse response h of n is simply 4 by 3, 2 raise to n u n minus 1 by 3 of course this is z over z minus half, minus 1 over 3 times half raised to n un that is basically your impulse response of the LTI system described by the of the causal LTI system described by the differential equation given in the problem all right.

So, basically what we have seen in this problem is we have seen a difference equation all right, an LTI system described by the difference equation, how to find the transfer function from that, and how to invert the transfer function correct to find the impulse response, that is the response of the LTI system to an impulse in that time all right. So, we will stop here and look at other aspects in the subsequent modules.

Thank you.