Principles of Signals and Systems
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Lecture – 21
Laplace Transform Example Problems – Evaluation of Laplace Transform, Inverse LT through Partial Fraction

Hello welcome to another module in this massive open online course. So, you are looking at the Laplace transform and the properties and applications of Laplace transform.

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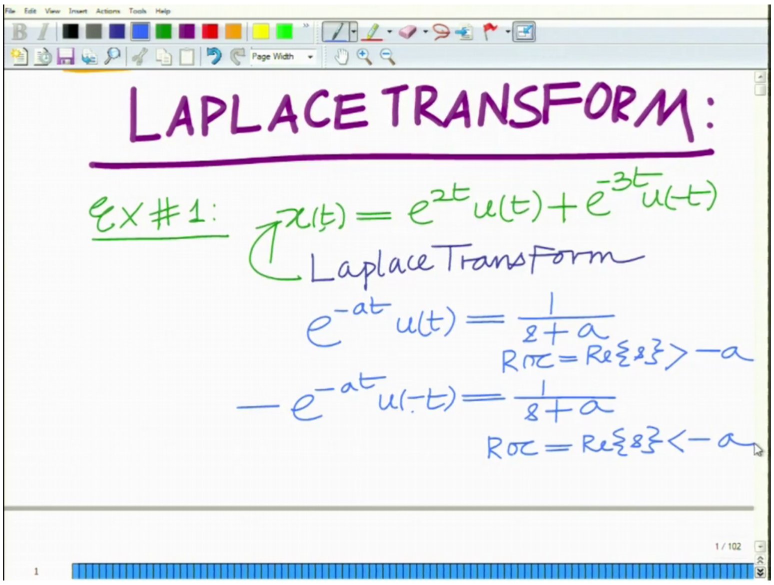

Let us now look at example problems to better understand the Laplace transform. So, what you want to now do in this module and also probably. The next few modules is look at example problems for the properties and application of the Laplace transform ok.

And so, let us look at example problem number 1 let us start with something simple. So, I have a signal x t equals e raised to 2 t u t where you need you t is the unit step function, plus e raise to minus 3 u minus t, now we want to compute the Laplace transform or evaluate the Laplace transform of this signal correct. So, we want evaluate a Laplace transform of e raised to 2 t u t plus e raise to minus 3 t u minus 3 and we also want to find the ROC of course, the Laplace transform is incomplete without the ROC.

So, now let me use the following properties we already know that the Laplace transform of e raised to minus a t u t; this is 1 over s plus a and ROC is real part of s greater than minus a and the Laplace transform of minus e raise to the power of minus e raise to minus a t u minus t is also equals also equals 1 over s plus a, but the ROC equals real part of s less than minus a.

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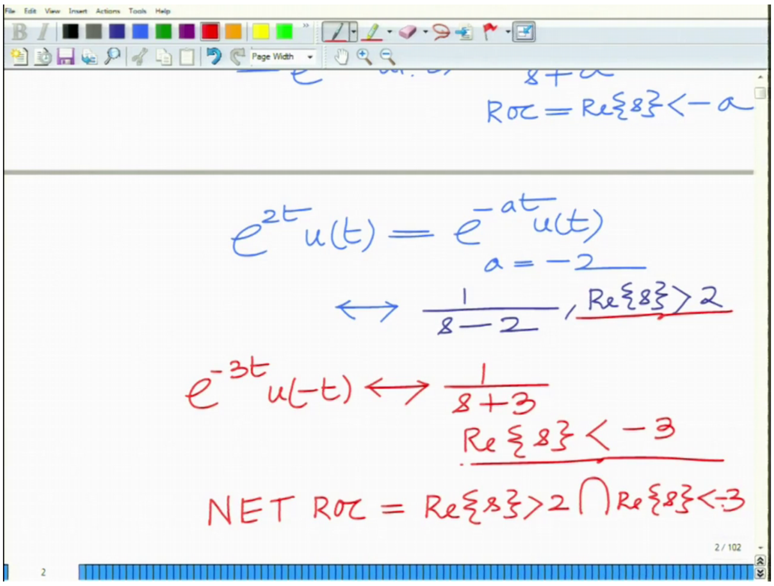

And therefore now, if you look at this signal e raise to 2 t u t this is equal to e raised to minus a t u t a equals minus 2 ok.

So, implies the Laplace transform of this signal can be obtained as 1 over s plus a where a equals minus 2 which means the Laplace transform is 1 over minus 2 the ROC is real part of s greater than minus a that is real part of s greater than a real part of s greater than minus a that is real part of s greater than 2. On the other hand, let us look at the other signal that is e raised to minus 3 t u minus t which is equal to e raised to minus a t with s equals with a equals e raised to minus 80 with a equals 3.

So, the Laplace transform is 1 over s plus 3 the real part of s greater than or real part of s less than, because this is a left handed signal remember u minus t is equal to 0 for t greater than 0 therefore, this is a left handed signal. So, the region of convergence is of the form ROC that is real part of s is less than something.

So, here the ROC is of the form real part of s is less than minus 3, ROC is less than minus a equals 3 here. And therefore, ROC is real part of s less than minus 3. Now you see the net ROC of the signal will be the intersection of these 2 ROCs therefore, the NET ROC equals real part of s greater than 2 intersection with real part of s less than minus 3, and you can see that the intersection of these 2 signals there is real part of s greater than 2 and real part of s less than minus 3 is basically 5 this is an empty set. There is no value of s where both the Laplace transform of both these signals that is e raise to 2 t u t, and e raise to minus 3 t u minus 3 converge. And therefore, correct e raise 2 to t u t and e raise to the power minus 3 t u minus t converge and therefore, which implies that the ROC is the empty set which means that the Laplace transform of this signal does not exist ok.

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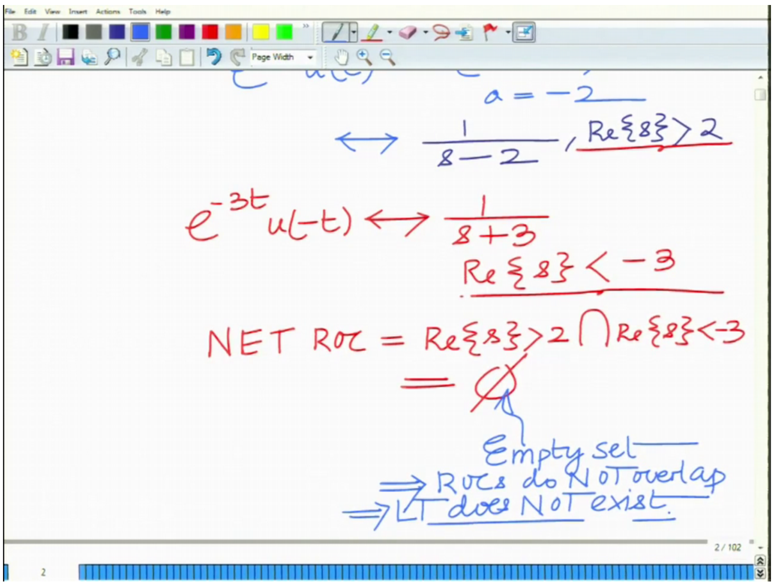

So, this intersection correct this intersection equals phi which is basically empty set correct, this is the empty set implies that implies basically ROCs do not over overlapped implies that Laplace transform exist does not exist for this signal.

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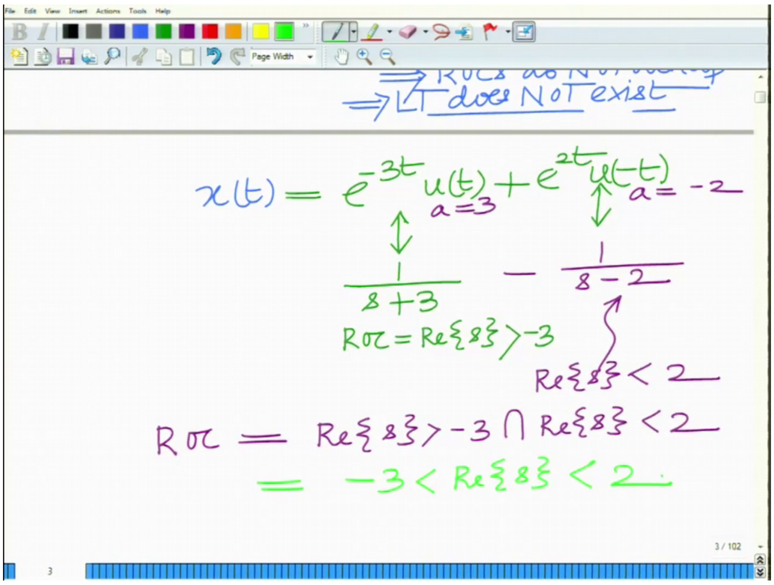

Let us look at another example for instance the same thing. Let us now look at the signal just modify that a little bit, let us now look at the signal x t equals e raise to minus 3 t u t plus e raise to 2 t u minus t. Now the Laplace transform of e raised to minus 3 t u t is 1 over, now this is e raise to minus at u t with a equal to 3. So, this is 1 over s plus 3 ROC is a real part of s greater than real part of s this is a right handed signal, because you u t is basically 0 for t less than zero.

So, this is a right handed signal. So, ROC of the is of the form real part of s greater than minus 3 real part of s greater than minus a equal to 3, and e raised to 2 t u minus t this is a left handed signal with basically with Laplace transform, I think here e raised to minus 3 t the Laplace transform is minus 1 over s plus 3. So, I will just correct this this is minus 1 over s plus 3 ok.

And therefore, the Laplace transform e raise to 2 t u minus t is basically minus e raised to 2 t u minus t is minus 1 over s plus a a is minus 2. So, this is 1 over s minus 2 and minus 1 over s minus 2 and the ROC is real part of s basically real part of s less than minus a that is real part of s less than, so this corresponds this corresponds to a equals minus 2 this corresponds to a equals 3.

So, real part of s less than minus a which means real part of s less than 2, which implies the net ROC equals the intersection of these 2 ROCs that is a real part of s greater than minus 3 intersection, real part of s less than 2 which is equal to well this is equal to minus. So, you can clearly see intersection is minus 3 less than real part of s less than 2, and the NET Laplace transform x s equals 1 over s plus 3 minus 1 over s minus 2.

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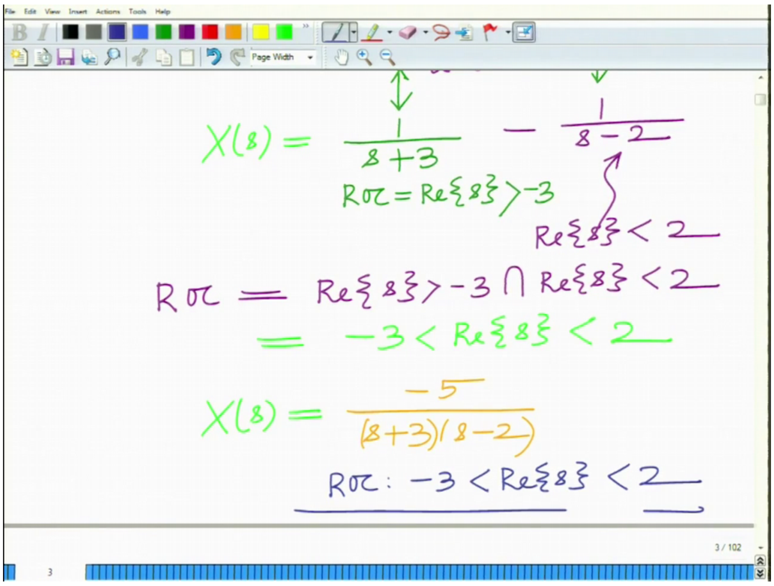

So, therefore, x s is equal to well s minus 2 minus s minus 3 that is minus 5 divided by s plus 3 into s minus 2. And the ROC for this is minus 3 less than real part of s less than that is the corresponding ROC region of convergence with this Laplace transform.

So, the Laplace transform is minus 5 or s plus 3 into s minus through s minus 2, and the corresponding ROC is minus 3 less than real part of this less than 2 all right.

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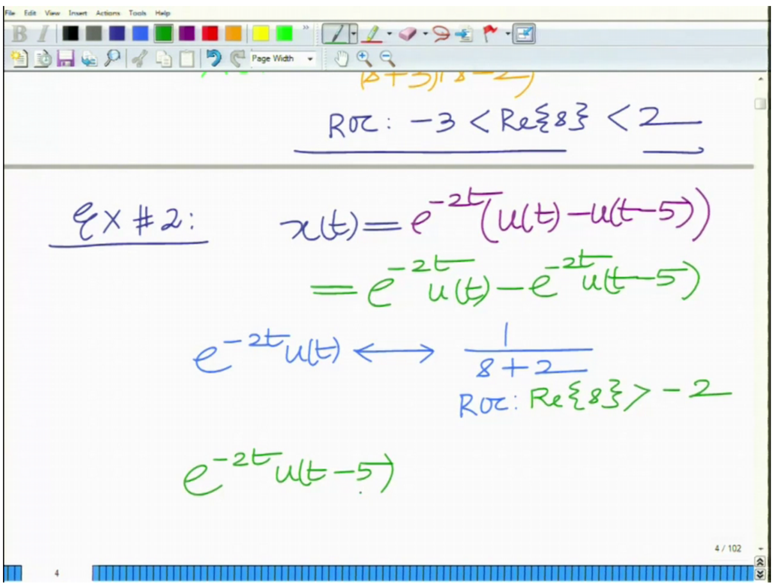

Now, let us look at another example this is a example number the second example problem to understand how to evaluate the Laplace transform. So, I have x t equals e raised to minus 2 t u t minus 5 this is equal to now. Let us look at this let me simply write this as e raise to minus 2 t u t minus e raise to minus 2 t u t minus 5.

Now, e raise to minus 2 t u t this has the Laplace transform this is a right handed signal. So, this has the Laplace transform well a equals, this is e raise to the power minus at u t with a equal to 2. So, this is of has the Laplace transform 1 over s plus 2 and the ROCs of the form real part of s greater than minus 2 because this is a right handed signal.

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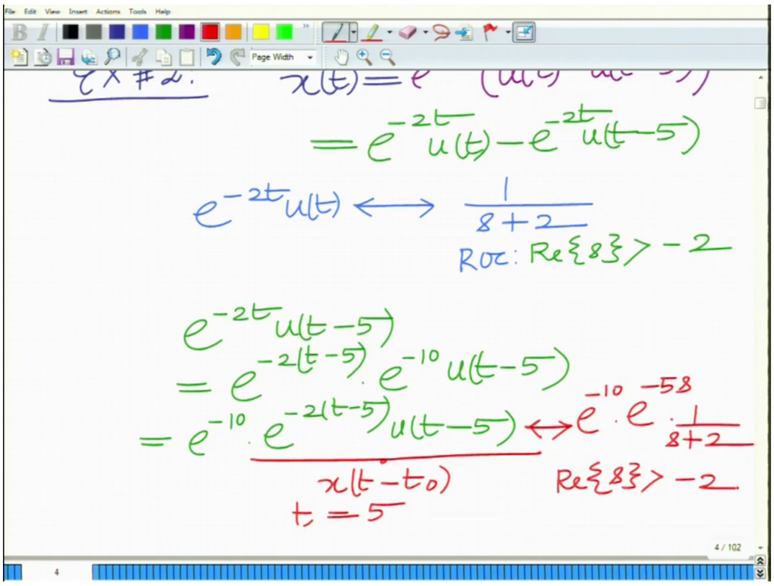

Now, consider this signal e raise to minus 2 t u t minus 5, now this can be equivalently written as e raise to minus 2 t minus 5 correct e raise to minus 2 t minus 5 times e raised to minus 10 into u t minus 5, which is e raise to minus 10 into e raise to minus 2 t minus 5 into u t minus 5.

Now, if you look at this signal e raise to 2 minus 2 t minus 5 u t minus y, this is of the form x t minus t naught where t naught equals 5. And x t is e raise to minus 2 t into u t. So, this is x t this is a signal x, this is a signal x t delayed this is a signal x t where x t is e raise to the power minus 2 t u t delayed by t naught where t naught equals 5.

Hence we know that the Laplace transform x t minus t naught is e raised to minus s t naught into x s therefore, the Laplace transform of this will be, well first we have the constant e raise to minus 10 into e raise to minus s t naught t naught is 5. So, this will be e raise to minus 5 s divided by into 1 over s plus 2 is a Laplace transform of e raise to 2 minus 2 t u t and the ROC will be again real part of s greater than minus 2.

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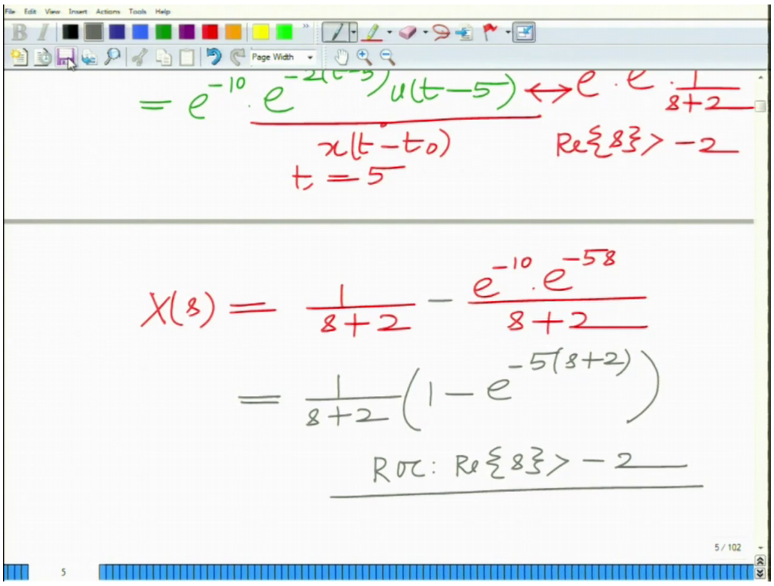

Therefore the final Laplace transform will be that is x s will be the sum of the 2 Laplace transforms 1 over s plus 2 plus e raise to minus 10 e raise to minus 5 s divided by s plus 2, which is equal to 1 over s plus 2, I am sorry this will be a minus 1 minus e raise to minus 5 into s plus 2. And the ROC will be real part of s greater than minus 2 this is the final region of convergence for this problem this is the ROC for this problem ok.

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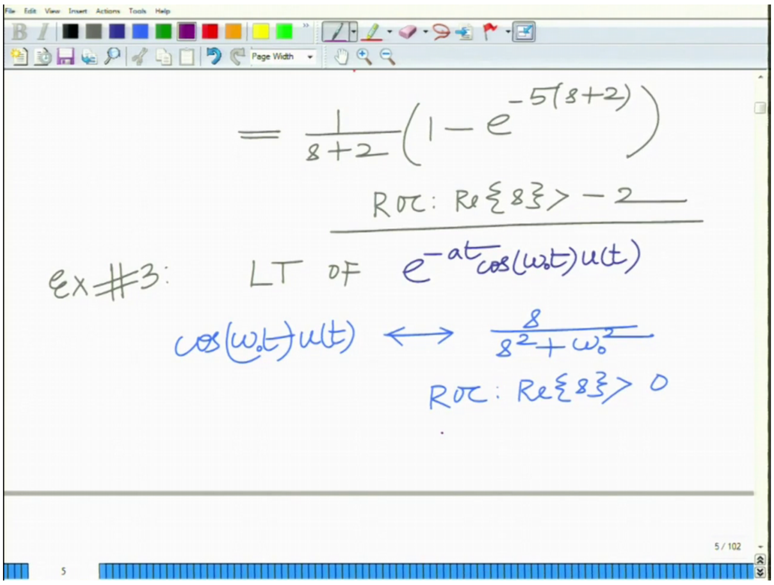

Now, let us do another example we want to evaluate the Laplace transform of the following function, that is LT of e to the e raise to minus a t cosine omega naught t u t, and remember we already know that the Laplace transform of cosine omega naught t u t is s divided by s square plus omega naught square. So, we already know that the Laplace transform of cosine omega naught t u t is s divided by s square plus omega naught square and the ROC for this will be or ROC for this is that the real part of s is greater than 0 is the right handed signal.

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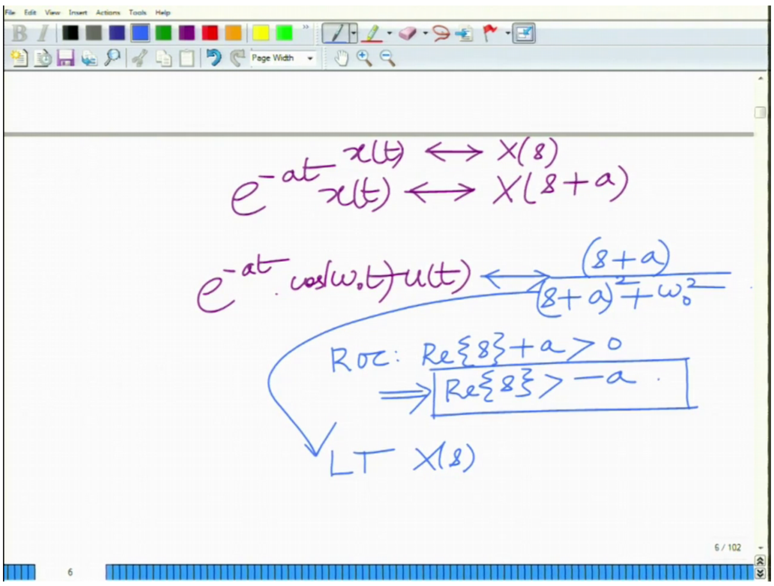

Now, remember we have the property recall we have the property e raise to minus a t x t has the Laplace transform x of s plus x of s plus a correct. So, where X s is a Laplace transform, that is if x t has the Laplace transform X s correct, then e raise to minus at x t has a Laplace transform X s plus a therefore, e raise to minus a t cosine omega naught t u t will naturally have the Laplace transform s plus a that is replace s by s plus a s plus a whole square plus omega naught square.

And the region of convergence will be naturally real part of s plus a greater than, 0 real part of s implies real part of s greater than minus a, this is the region of a. So, this is the Laplace transform that is x s and this is the region of convergence that is real part of s is greater than minus a the Laplace transform is x of s plus a that is s plus a divided by s plus a whole square plus omega naught square all right.

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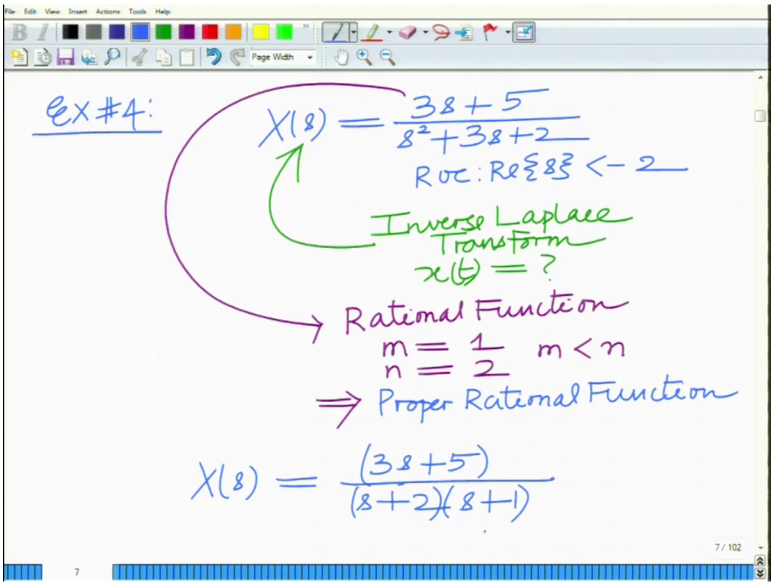

Let us now do another problem, we have the Laplace transform equals given the Laplace transform X s equals 3 s plus 4 divided by s square plus 3 s plus 2 the ROC is real part of s less than minus 2, now we want to evaluate the inverse Laplace transform that is what is the signal x t.

Now, the inverse Laplace now we want to evaluate the inverse Laplace transform, that is corresponding to X s the given Laplace transform the signal the Laplace transform X s, we want to find that what is the inverse Laplace transform that is we want to reconstruct the corresponding signal x t. Now observe first that this is a rational function, that is it is a numerator polynomial and divided by denominator polynomial, further degree of numerator polynomial is 1 degree of denominator polynomial is 2 therefore, we have m less than n which implies as a proper implies as a proper rational function therefore, I express this as partial fractions.

Now, if you look at this I can write X s as 3 s plus 4 divide sorry 3 s plus let me just change this slightly there is 3 s plus 5. So, I can write x s as 3 s plus 5 divided by s square plus 3 s plus 2 the roots are minus 2 and minus 1. So, I can write this as s plus 2 remember the factors are s plus 2 and s plus 1 so, I can write this as plus 2 and s plus 1 and therefore, now the poles p 1 equals minus 2 p 2 equals minus 1.

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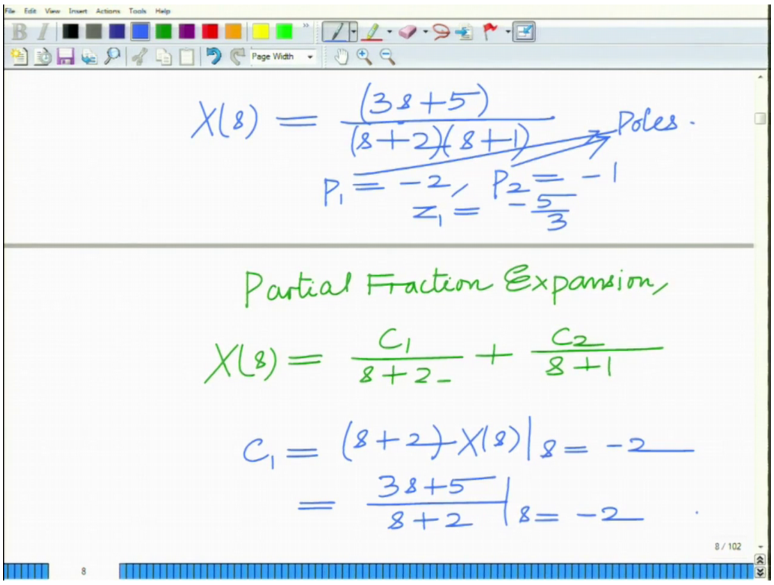

In fact, 0 minus 5 by 3, 3 s plus y equal to 0 so, the 0 is minus 5 by 3. So, these are the poles correct, and these are the poles of the rational transfer function therefore, I can expressing using partial fractions therefore, using the partial fraction expansion I can write X s equals 1 sum constant C 1 divided by s plus 2 plus sum constant C 2 divided by s plus 1. Now the way to evaluate C 1 we already know this is well s plus 2 that is s minus p 1 times X s evaluated at s equal to p 1 evaluated at s equal to minus, 2 which is basically x plus s plus 2 into x s is 3 s plus 5 divided by s plus 2.

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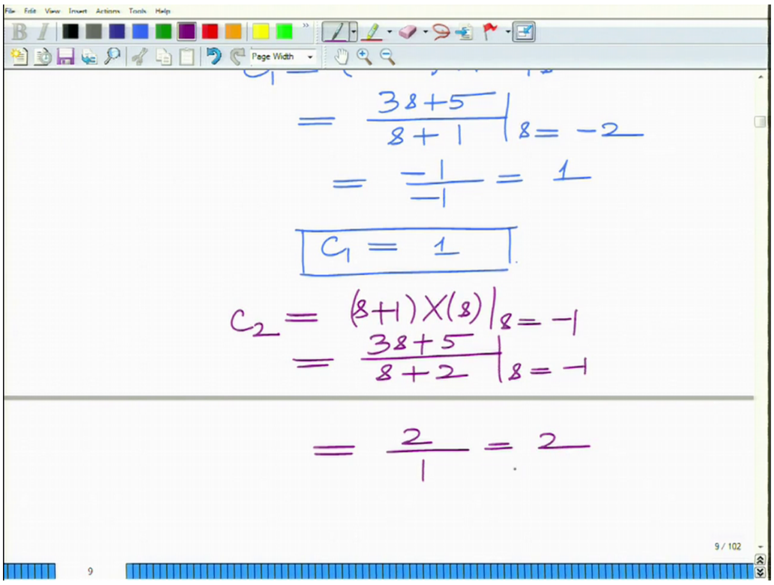

Evaluated at s equal to 3 s plus 5 divided by s plus 1 evaluated at s equal to minus 2 that is minus 1 divided by minus 1 which is equal to 1.

Therefore C 1 is equal to 1, and C that C 1 is equal to 1 the coefficient C 1 corresponding to the pole p 1 equals minus 2 partial fraction expansion of x s is C 1 equals 1. Now C 2 similarly can be evaluated in a similar fashion 1 can evaluate C 2 as s plus 1 into X s evaluated at s equal to minus 1 s plus 1 into X s. Now you can see this is equal to 3 s plus 5 divided by s plus 2 that is evaluated at s equal to minus 1 which is basically a 2 divided by 1 which is equal to 2 ok.

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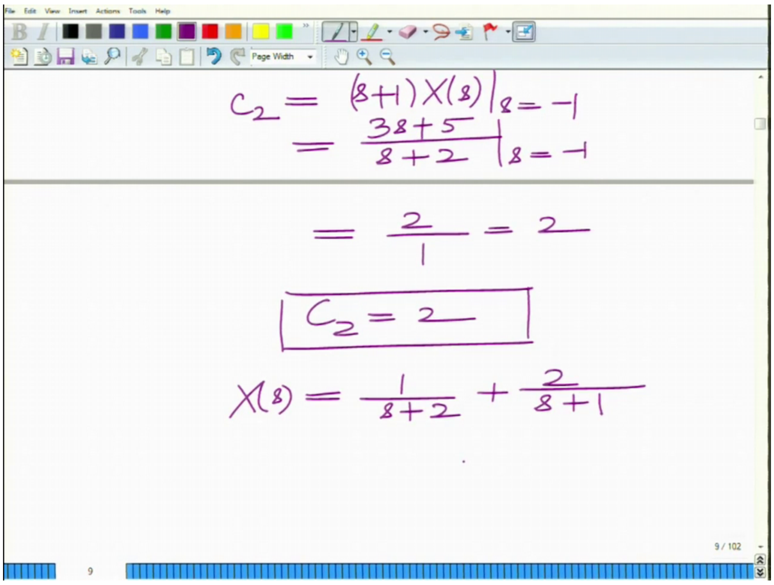

And therefore, C 2 equal to 2 and therefore, from the partial fraction expansion we have X s equals 1 over s plus 2 plus 2 over s plus 1, now you can see the ROC is real part of s less than minus 2, which basically implies that it is a left handed signal ok.

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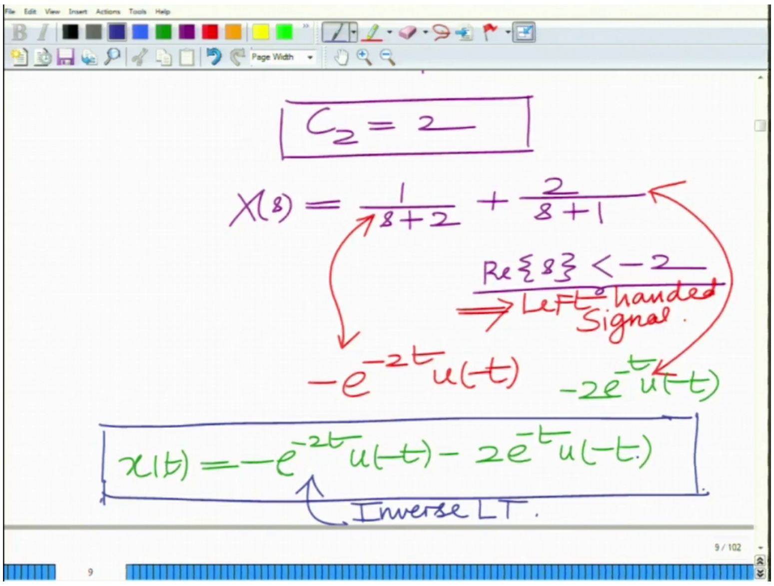

When ROC is of the form real part of s less than sigma max; so this will be of the form minus e raise to minus 2 t u minus t this is of the form that is the time domain signal corresponding 2 over s plus 1 is minus 2 e raised to minus t u minus t therefore NET signal x of t, will be minus e raise 2 minus t or let me just write it clearly x t is minus e raise to minus t u minus t minus 2 e minus t u minus t and that is the NET. So, that is the net time domain signal or the inverse Laplace.

That is the expression for the inverse Laplace transform. So, all right in this module we have seen some examples, the example problems with the Laplace transform how to evaluate the Laplace transform, as well as the inverse Laplace transform, we will continue looking at these examples other examples in the subsequent modules.

Thank you very much.