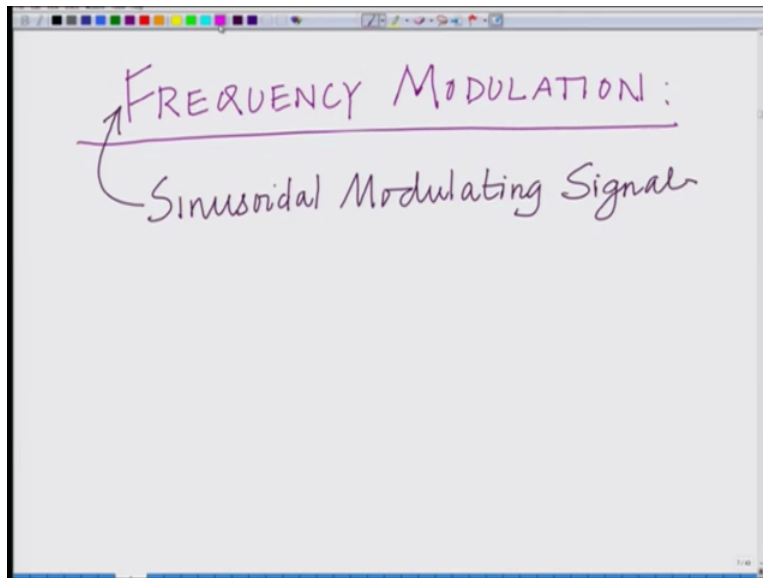


Principles of Communication- Part I
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Module No 5
Lecture 29

**Frequency Modulation (FM) with Sinusoidal Modulating Signal and Pictorial Examples,
Insights of PM and FM Signals**

Hello welcome to another module in this massive open online course, so we are looking at and modulation of which phase modulation and frequency modulation are particular types of modulation, alright. So we have also looked at phase modulation with Sinusoidal modulation, so in this module let us look at frequency modulation with a Sinusoidal modulating signal, okay.

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So what we want to start looking at is frequency modulation that is F_m , frequency modulation with our Sinusoidal modulating signal, okay. So with a Sinusoidal modulating signal okay.

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Sinusoidal Modulating Signal

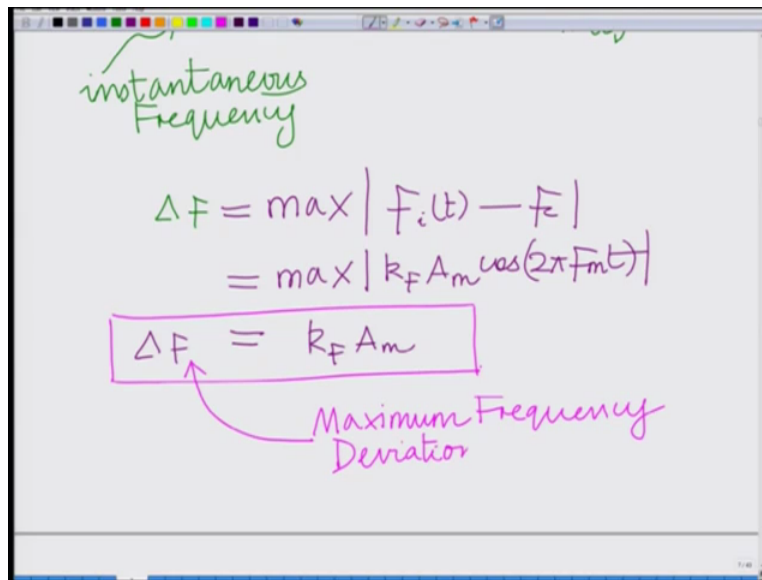
$$m(t) = A_m \cos(2\pi f_m t)$$
$$f_i(t) = f_c + k_f m(t) = f_c + k_f A_m \cos(2\pi f_m t)$$

instantaneous Frequency

So let $m(t)$, alright. So let the message signal $m(t)$ be $m(t)$ equal to $A_m \cos(2\pi f_m t)$ then our frequency $f_i(t)$ instantaneous frequency, remember $f_i(t)$ is the instantaneous frequency this is the instantaneous frequency and this is equal to your f_c the frequency of the carrier plus the component from the message that is k_f times $A_m \cos(2\pi f_m t)$ this is your instantaneous frequency, okay.

So the frequency is basically the frequency of the carrier unmodulated carrier f_c to that you are adding a component, so that the frequency is now modulated by the message signal, so instantaneous frequency becomes $f_c + k_f A_m \cos(2\pi f_m t)$, okay because $A_m \cos(2\pi f_m t)$ is of message signal, so this is $m(t)$ so this is remember this is our message signal, alright. So what we are doing is we have $f_i(t)$ is $f_c + k_f m(t)$ where $m(t)$ is the message signal, okay.

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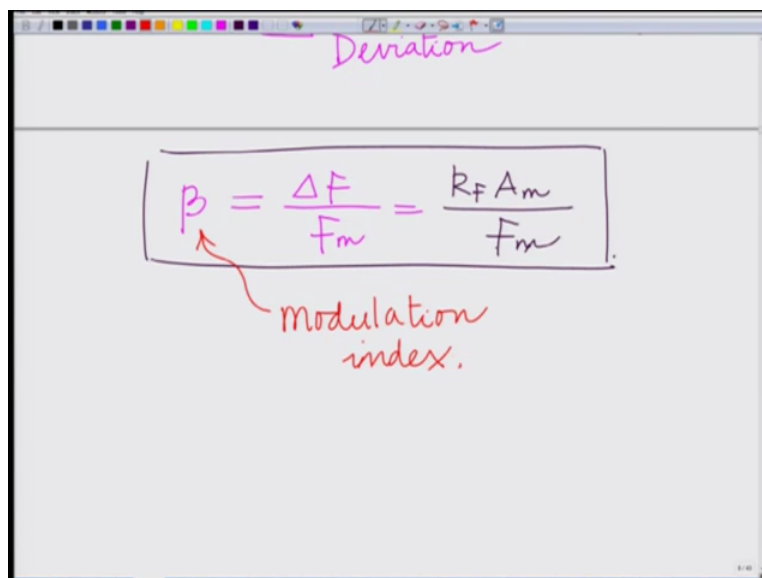
A handwritten derivation on a whiteboard. At the top, the words "instantaneous Frequency" are written in green. Below this, the equation $\Delta F = \max |f_i(t) - f_c|$ is written in purple. This is followed by $= \max |k_f A_m \cos(2\pi f_m t)|$. The final result, $\Delta F = k_f A_m$, is enclosed in a purple rectangular box. A purple arrow points from the text "Maximum Frequency Deviation" below to the boxed equation.

$$\Delta F = \max |f_i(t) - f_c|$$
$$= \max |k_f A_m \cos(2\pi f_m t)|$$
$$\Delta F = k_f A_m$$

Maximum Frequency Deviation

And therefore now if you look at the maximum frequency deviation of frequency deviation Delta F equals this is given as the maximum of $f_i(t)$ minus f_c where f_c is the carrier frequency that is equal to the maximum of well, $k_f A_m \cos$ of $2\pi f_m t$ and the maximum of this is nothing but k_f times A_m .

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A handwritten definition on a whiteboard. The word "Deviation" is written in purple at the top. Below it, the equation $\beta = \frac{\Delta F}{f_m} = \frac{k_f A_m}{f_m}$ is written in purple and enclosed in a purple rectangular box. A red arrow points from the text "modulation index." below to the boxed equation.

$$\beta = \frac{\Delta F}{f_m} = \frac{k_f A_m}{f_m}$$

modulation index.

So delta F the maximum frequency deviation the maximum frequency deviation equals k_f times A_m and beta that is modulation index. Therefore equals the frequency deviation divided by f_m

the message frequency ΔF divided by F_m which is equal to $k_f A_m$ divided by F_m , okay. So this is β your modulation index. So β is basically the, this is the modulation index. This is the modulation index, okay.

So we have shown that the maximum so we have been the modulating signal is a sinusoidal modulating signal that is $m(t)$ is $A_m \cos(2\pi F_m t)$ we have shown that the frequency deviation is k_f times A_m the modulation modulation index β is $k_f A_m$ divided by F_m there is a frequency deviation peak (freq) peak frequency deviation divided by the frequency of the message signal, okay.

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index.

$$\theta_i(t) = 2\pi \int_0^t F_i(t) dt$$

instantaneous phase

$$= 2\pi \int_0^t (F_c + k_f A_m \cos(2\pi F_m \tau)) d\tau$$

Now $\theta_i(t)$ coming to $\Theta_i(t)$ you can see that $\theta_i(t)$ is the phase remember this is your instantaneous phase. We have frequencies the rate of change of phase, so phase is the integral of the frequency in fact $2\pi \int_0^t F_i(t) dt$ which is equal to $2\pi \int_0^t (F_c + k_f A_m \cos(2\pi F_m \tau)) d\tau$.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is:
$$\text{phase} = 2\pi \int_0^t (f_c + k_f A_m \cos(2\pi f_m \tau)) d\tau$$
 The second equation is:
$$= 2\pi f_c t + \cancel{2\pi} k_f A_m \cdot \frac{1}{\cancel{2\pi} f_m} \sin(2\pi f_m \tau) \Big|_0^t$$
 The final equation, enclosed in a box, is:
$$\theta_i(t) = 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

Which is equal to well $2\pi f_c t$ plus $2\pi k_f A_m$ times integral of cosine $2\pi f_m \tau$ $d\tau$ that is 1 over, well $2\pi f_m$ times integral of cosine $2\pi f_m \tau$ is $\sin 2\pi f_m \tau$ evaluated between the limits 0 to τ and therefore now if you can see we have this 2π factor cancelling, so we have $2\pi f_c t$ plus $k_f A_m$ divided by f_m $\sin 2\pi f_m t$, okay. This is basically the expression for your phase $\theta_i(t)$, okay. So this is basically the expression for the phase the instantaneous phase $\theta_i(t)$. And therefore the modulated signal, correct? This is the expression for the phase $\theta_i(t)$ which is given as the integral of the instantaneous frequency $f_i(t)$ and therefore the modulated signal $s(t)$ is $A_c \cos \theta_i(t)$, alright.

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Handwritten notes on a whiteboard. At the top, there is a bracket labeled f_m . Below it, the FM signal equation is written:

$$S(t) \leftarrow \begin{aligned} & \text{FM Signal.} \\ & A_c \cos(\theta_i(t)) \\ & = A_c \cos\left(2\pi f_c t + \frac{k_f A_m \sin(2\pi f_m t)}{f_m}\right) \end{aligned}$$

Below the equation, the phrase "Max phase Deviation" is written and underlined.

So the signal the FM signal is $A_c \cos(\theta_i(t))$ which is $A_c \cos(2\pi f_c t + k_f A_m \sin(2\pi f_m t) / f_m)$, so this is basically your this is basically the modulated or this is basically your FM signal this is basically we can say this is the FM signal, okay. This is basically the FM signal $A_c \cos(\theta_i(t))$ that is $A_c \cos(2\pi f_c t + k_f A_m \sin(2\pi f_m t) / f_m)$.

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Handwritten notes on a whiteboard showing the derivation of maximum phase deviation:

$$\Delta\phi = \max |\theta_i(t) - 2\pi f_c t|$$
$$= \max \left| \frac{k_f A_m \sin(2\pi f_m t)}{f_m} \right|$$

The final result is boxed:

$$\Delta\phi = \frac{k_f A_m}{f_m} = \beta$$

Arrows point from the boxed equation to the labels "Maximum Phase Deviation" and "Modulation index".

Now therefore now let us find the maximum phase deviation the maximum phase deviation will be $\Delta\phi$ is maximum of $\theta_i(t) - 2\pi f_c t$ which is maximum of well, you can see

$k_f A_m$ by F_m $k_f A_m$ by $F_m \sin 2\pi F_m t$ which is equal to $k_f A_m$ by F_m which is again equal to you can say this is nothing but your modulation index beta.

So again the modulation index beta again for the frequency modulation also is equal to your phase deviation. So this is nothing but your beta which is basically your modulation index and this is the maximum phase deviation again it can be seen that even for the case of frequency modulation the modulation index is basically (eq) also equal to the maximum phase deviation, okay.

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The diagram shows a whiteboard with the following content:

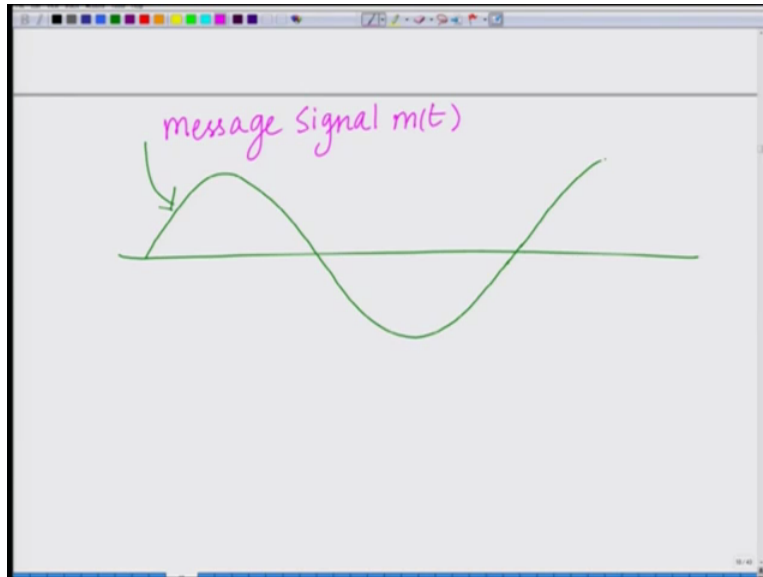
- At the top, a boxed equation: $\Delta\phi = \frac{k_f A_m}{F_m} = \beta$. An arrow points from $\Delta\phi$ to the text "Maximum Phase Deviation". Another arrow points from β to the text "Modulation index".
- Below this, the text "Sinusoidal Modulating Signal $m(t)$ " is written in purple.
- Two lines branch out from "Sinusoidal Modulating Signal $m(t)$ ":
 - The upper line points to the equation: $\text{PM } \beta = k_p A_m = \Delta\phi$.
 - The lower line points to the equation: $\text{FM } \beta = \frac{k_f A_m}{F_m} = \Delta\phi$.

So we have looked at both your Sinusoidal module that is the frequency modulation that is if you look at a Sinusoidal modulating signal that is when you look at a Sinusoidal modulating signal $m(t)$ we have the modulation index beta equals k_p times A_m for phase modulation and for frequency modulation we have beta equals k_f times A_m divided by F_m and in both cases this is also equal to your delta phi which is the maximum phase deviation, okay.

So for phase modulation it is beta equals k_p times A_m we have for frequency modulation beta equals k_f times A_m divided by F_m , okay. So that is what we have for our Sinusoidal modulating signal, alright. So we have considered both phase modulation and frequency modulation with a Sinusoidal modulating signal and we have derived the modulating index beta and the maximum phase the maximum frequency deviation modulation index and also the maximum phase

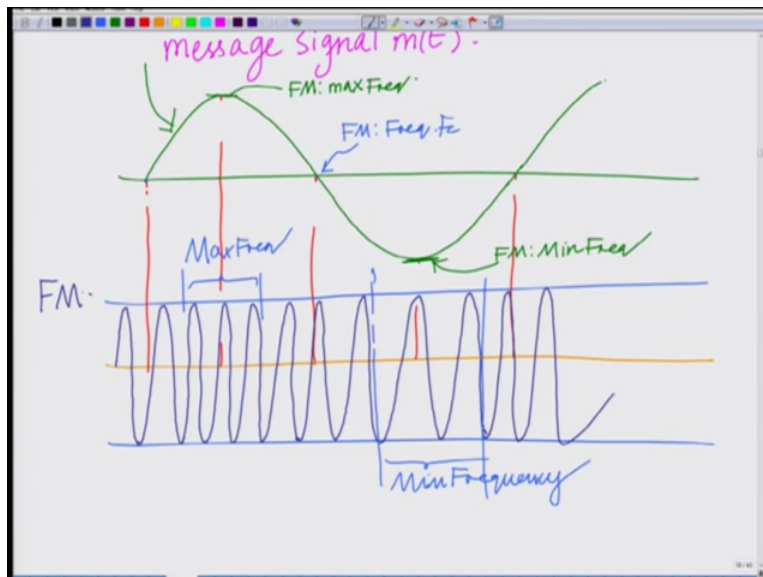
deviation both the for both the scenarios and also the modulated signal, okay. So now what I want to like to do is like to demonstrate an example of a or basically pictorial example of a frequency and phase modulated signal, alright.

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So let us consider a simple Sinusoidal modulating signal so let us say we have a simple Sinusoidal modulating signal, okay. So let us say we have a simple Sinusoidal modulating signal, okay. So this is your modulating signal or message signal let us call this the message signal $m(t)$.

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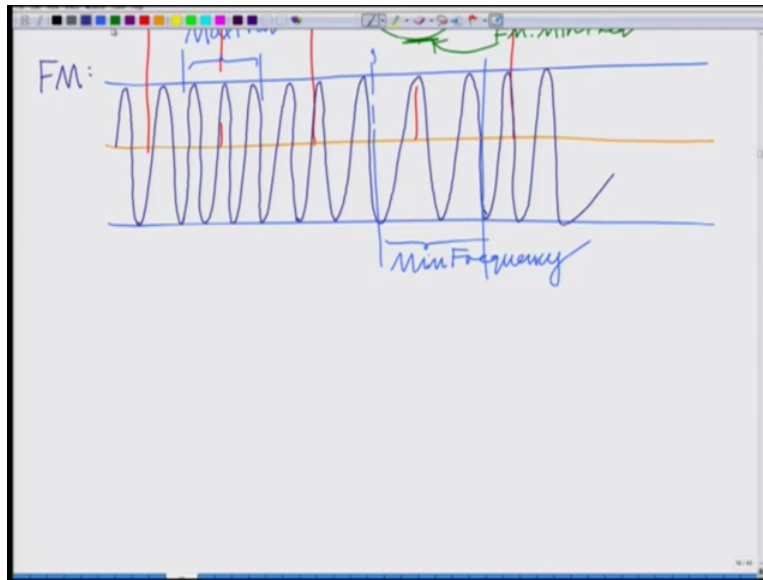
This is your message signal $m(t)$ and now let us say we have a carrier which is modulated by this message signal $m(t)$, so let us say I have a carrier which is modulated by this message signal $m(t)$, now if you look at this there are a couple of points which are of special interest, correct? So this is your, okay. So I have a carrier which is modulated by the message signal $m(t)$ and now you can clearly see that in this part if you can look at this, so this part where the message is high remember the message is modulating the frequency.

So this part corresponds to follow the Fm this part will be maximum frequency this part the lower the lowest part of the function will be for Fm this will be the minimum frequency and therefore here let us say we have a carrier at this part of course message is 0, so for Fm at this part frequency will be simply the frequency of the carrier because the message signal is 0 at the 0 crossing since message signal is 0, the message with a frequency of the carrier the instantaneous frequency will be simply the frequency of the modulated carrier that is FC.

So if I draw this it will look something like this at this point I will have the frequency of the modulated carrier slowly as it comes to words the as it comes towards the peak of the signal the frequency increases and once again it starts decreasing it starts decreasing to the frequency of the unmodulated carrier, okay. And then it becomes the frequency of the unmodulated carrier and frequency further decreases at this part which is the minimum this part corresponds to the lowest frequency and again it starts at when the 0 crossing it again it starts becoming equal to the frequency of the unmodulated carrier.

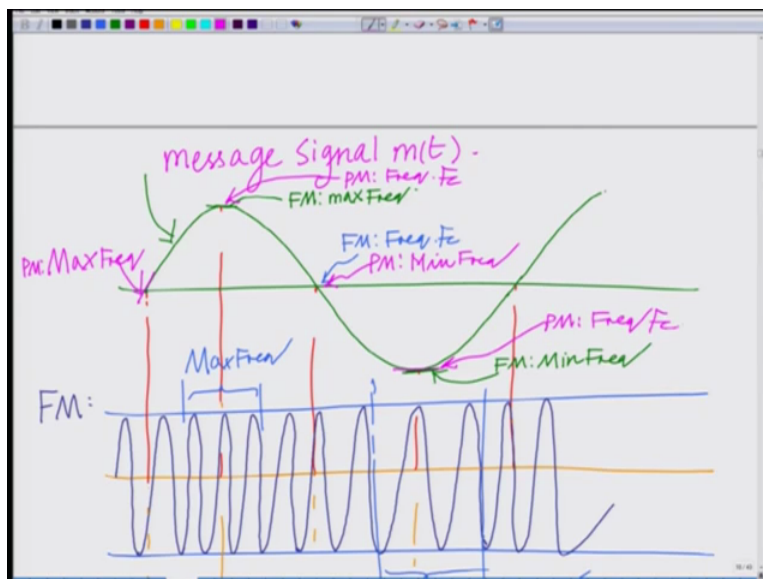
So here you can see that this portion somewhere around this portion corresponds to the maximum frequency this portion corresponds to the somewhere around this portion corresponds to the maximum frequency for frequency modulation and this portion somewhere around this portion corresponds to the minimum frequency this portion corresponds to minimum frequency and this is for a frequency modulated signal this is for a frequency modulated signal.

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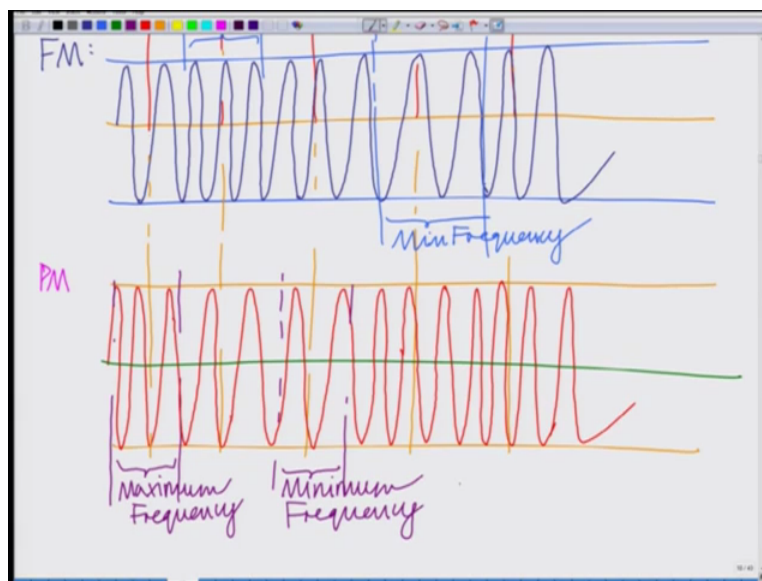
Now for phase modulation if ever to draw the same diagram for phase modulation we will observe something interesting, now for phase modulation, okay. Let us say this is envelope, remember the envelope is always going to be constant because the amplitude is not changing, right? So the envelope is always going to be constant. Now if you look at the phase modulation the message modulates the phase and the frequency is given by the derivative of the phase. So the frequency is maximum where the derivative of the phase is maximum.

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Now if you can see the message signal at this point we have maximum derivative that is the maximum derivative occurs at this point, so here that is at the 0 crossing the maximum (cross) that is at the 0 crossing with increasing amplitude the derivative is maximum, so this corresponds to maximum frequency phase modulation and this point here the 0 crossing where the (phase) amplitude is decreasing this corresponds to the minimum frequency. So this corresponds to minimum frequency for the and these 2 points where the derivative is maximum there function is maximum the derivative is 0. So for phase modulation this corresponds to simply the free carrier frequency that is F_c and also the minimum where the derivative is 0 for phase modulation corresponds to frequency corresponds to frequency simply F_c .

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So if I draw this phase modulation at this point you will have maximum derivative, so at this point you will have frequency that is you will have frequency that is that is maximum slowly at where the function reaches the message reaches the maximum the frequency will become the free running carrier frequency that will become that will become F_c that will become F_c and at that point where the derivative is negative maximum therefore it will have the lowest frequency before again becoming the free running carrier frequency when the function reaches the minimum since the derivative is 0 and then again it reaches the maximum frequency when the derivative is maximum and so on.

So if you look at this function, this is your phase modulated signal, okay. If you look at the phase modulated signal the maximum will occur where the derivative, so this corresponds to the maximum frequency and where the derivative is where the (deri) where the derivative where the derivative is negative maximum that corresponds to, for instance this region this part corresponds to the corresponds to the corresponds to the corresponds to the minimum frequency.

So this is a simple example or a simple pictorial illustration or a good intuitive good insight into this frequency and into the free nature of frequency and phase modulated signal remember the message is modulating the frequency in a frequency (modulati) frequency modulated signal. So the frequency is maximum when the message is maximum and the frequency is minimum and the message is minimum. On the other hand in a phase modulated signal since the message is modulating the phase the frequency is maximum when the derivative of the message signal is maximum the derivative of the modulating signal is maximum and the frequency is minimum when the derivative of the modulating signal is minimum that is the negative peak, the negative peak of the modulating signal the frequency of the modulated signal is minimum and at the peaks of the modulating signal.

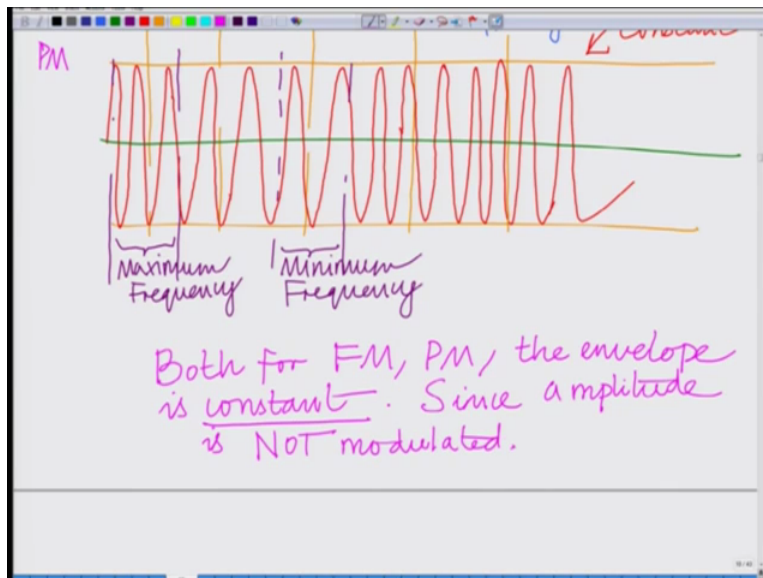
At the extrema of the modulating message signal since that derivative vanishes therefore in those points the frequency of the modulated signal is same as that of the frequency of the unmodulated carrier that is F_c , alright. So these are some things these are (su) these are interesting insights into the nature of both frequency and phase modulated signals.

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And also you will not that the envelope, note that the envelope is constant both for frequency modulation and phase modulation envelope.

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So both for frequency and phase modulation the envelope is constant because amplitude is not, both for FM and PM there frequency modulation and phase modulation the envelope the envelope is constant this is because amplitude is not modulated. Since amplitude is not so the

envelope is constant it is just that the frequency or the phase varies according to the message signal.

And also the other thing that we have seen is that in phase modulation in particular if there are discontinuities in message if there are discontinuities in the message signal that results in discontinuities in the phase of the modulated carrier and that causes problems because that gives rise to high frequency components which increases the bandwidth which drastically increases the bandwidth of the signal, alright.

So frequency modulation is preferred in comparison to phase modulation because frequency modulation does not even if even when there (discontinuis) discontinuities in the message signal the phase the resulting phase of the frequency modulated signal is still continuous because frequency (modul) because the phase of the frequency modulated signal is given by the integral of the integral of the instantaneous frequency, alright.

So even if there is discontinuities in the message signal they will be smoothen out by the integration operation, alright. So that avoids the high frequency components, alright. So these so in this module we have looked at frequency modulation with Sinusoidal modulating signal derived the modulating index the frequency deviation and the phase deviation and also we have looked at summary presentations or (())(24:41) insights into the nature of this frequency and phase modulated signal by giving a pictorial description of some simple frequency and a phase modulated signal, alright. So we will end this module here and look at other aspects in the subsequent modules, thank you.