Advanced Control Systems

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Module No. # 03
Time Domain Based Identification
Lecture No. # 17
DF Based Analytical Expressions for On-line Identification

Welcome to lecture titled, D F based analytical expressions for on-line identification. In this lecture, we shall discuss about the describing function based analysis of a relay control system. Now, the system can be stable or unstable, a relay will be connected in parallel with the controller to induce limit cycle output or the sustained oscillatory output.

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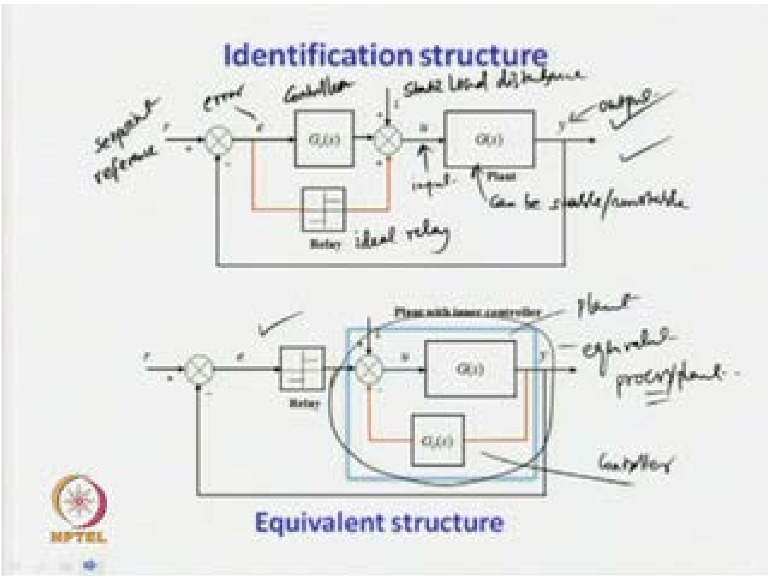

The scheme for the identification is given or shown as as shown over here. Now, R is the set point or reference input, G(c) is the controller connected in parallel with the relay; a relay is assumed to be an ideal relay. l is the static load disturbance and G(s) is the plant or process under relay control or during the relay feedback test. Now, the plant can be stable or unstable, and the remaining signal shown in this block diagram are y is the output, u is the input, and e is the error. Now, this identification structure is known as an on-line identification structure. Why it is an online identification structure? The controller remains in action, during the operation of the system or plant. Now, the controller is never disconnected, even during the relay experiment; that is why, the scheme is known as the on-line identification scheme.

Now, this online identification scheme or structure can be converted in equivalent form, where the plant will get connected with the controller in this fashion. So, this is our plant and this is the controller. Now, the controller appears in the inner feedback path and thus giving us some resulting stabilized process model I can say or an equivalent process. some equivalent process. So, the relay sees now, an equivalent process in place of an open loop stable or unstable process. So, the relay is subjected to an equivalent process or plant and this shows that, I can draw some equivalent circuit also; that I shall do later on, while doing the analysis D F based analysis.

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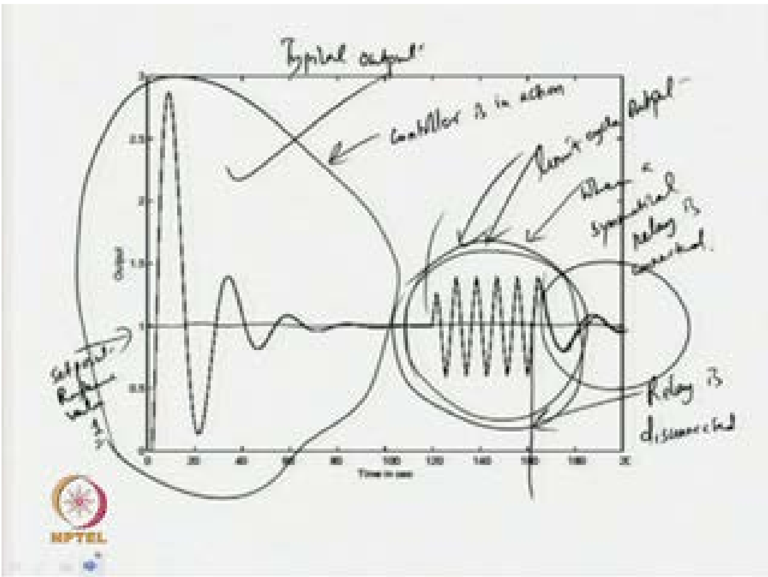

Now, some typical typical output from the plant is shown; where the plant, this part of the output shows that, the controller is in action is in action and normal operation of the controlled system is on or is going on. Now, when the relay is invoked or connected at time T equal to 120 seconds, then the output assumes this form. Now, when the output assumes this form as you can see. If you draw a line like this, the output is symmetrical now, with respect to the reference value 1. This is the set value set point value or reference value of 1.

So, it is symmetrical with respect to the set value, when a symmetrical symmetrical relay is connected. Again at about 160 seconds, the relay is switched off. So, at this time the relay is disconnected, then the system comes back to its normal operation and thus we are getting a dynamic response of this form. So, the complete figure, the typical output is made up of 3 parts now. The part one showing the dynamics of the control system. The one, when you have got the relay in the loop and when the relay is switched off.

So, this type of typical waveform shows that, it is possible to estimate the dynamics of the plant or process based on the measurements made on this sustained oscillatory output, known as limit cycle output. It is possible and the beauty of this scheme is that, the relay is in action throughout and it is never disconnected, during the relay experiment or relay test. ok. So, the controller is in action throughout; not the relay. The controller is in action throughout the operation of the system, whereas the relay can be connected as and when necessary and as and when it is felt that, you need to change the tuning parameters of the controller.

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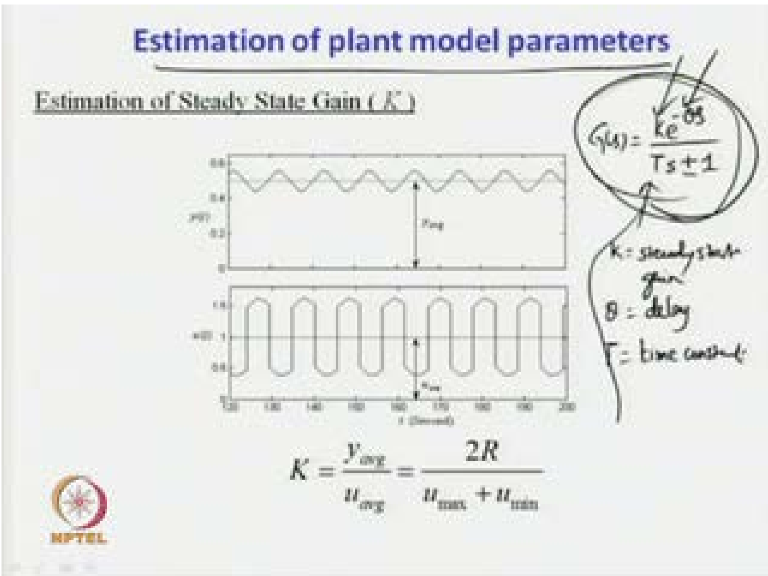

Now, how can we estimate the plant model parameters? Assuming some transfer function model of the form G(s) is equal to K e to the power minus theta s upon T s plus minus 1 for the dynamics of the plant. We find that, the transfer function model for the dynamics of the plant is having 3 unknowns. So, K, theta and T. Now, the K is known as the steady state gain steady state gain of the plant; theta is the time delay delay and T is the time constant of the dynamics of the system. Now, the dynamics of the system is going to be mimicked by or represented by that of a first order plus dead time transfer function model of this form, which has got 3 unknowns.

So, estimation of plant model parameters means, it is all about finding either explicit expressions for K, theta and T in terms of the measurements or measured quantities or getting indirect expressions or analytical expressions. Those can be used to estimate the values for K, theta and T. Now, estimation of plant model parameters will begin with the most important parameter associated with the transfer function model that is known as the steady state gain. How to find or estimate the steady state gain of the transfer function model?

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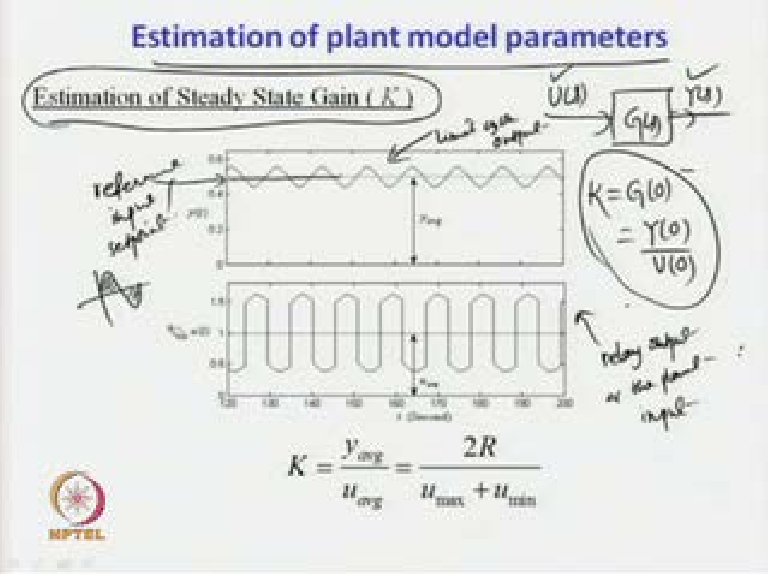

$$K = \frac{y_{avg}}{u_{avg}} = \frac{2R}{u_{max} + u_{min}}$$

To find the steady state gain, what we mean by steady state gain? Given the dynamics or given the plant dynamics of this form G(s), when the output is Y(s) and input is U(s), the steady state gain K can be given as, G(0) is equal to Y(0) upon U(0). This is what, we mean mean by the steady state gain. How to estimate the steady state gain? As you have seen from the typical output, you get during the relay test using this information and and using the input to the system. At the time of relay test, using both informations it is possible to estimate the steady state gain.

Now, this is the limit cycle output, limit cycle output when the system is subjected to a relay and this is the relay output or the plant input the plant input. As you know, Y is the output of the plant; U is the input to the plant. Therefore, I shall make use of both informations to estimate the steady state gain. Now, I have skipped the other part of the typical output. We concentrate on the result or the output, we get during relay experiment; that means, we confine our discussion for the time period from time from 120 seconds to 160 seconds. So, when that is shown over here, in some convenient form. I get the limit cycle output in this form.

Now, what is the dotted line shown over here? The dotted line shows the reference input or the set point. So, we have got the oscillatory output superimposed over the reference input. Now, if I find the average value average value of the limit cycle output, that will be equal to the set point value or reference input. Why that is so? Because the limit cycle output is symmetrical and over a period of time, total area will be 0. Therefore, the contribution or the area contribution by the limit cycle output, during 1 cycle will be 0; rendering the average value of the output, during the relay experiment to be equal to the set point or reference value.

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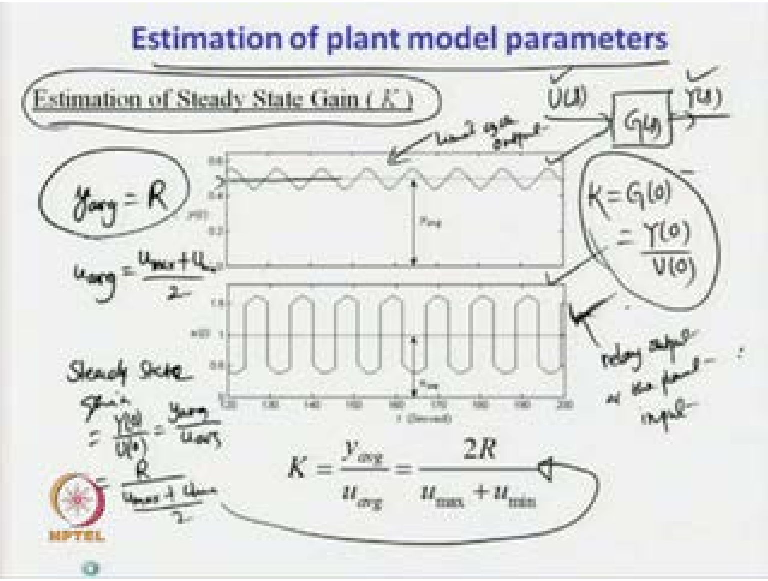

So, let let us write the average value of the output has the set set point value or the reference. Now, we obtain a typical relay output of this form or the plant input of this form. So, how to find the average value of that signal? I can use the expression U average is equal to Umax plus Umin divided by 2. So, this is how, you find the average of any signal easily. Now using this 2, now the steady state gain will be steady state gain is equal to y 0 divided by u 0 Y(0) divided by U(0) is equal to Y average divided by U average, which will be equal to R divided by Umax plus Umin by 2.

So, which will ultimately give you an expression of the form, K is equal to ratio of average values of output to input is equal to 2 R times Umax plus Umin. So, using this simple expression, it is possible to find the steady state gain or it is possible to estimate one dynamic parameter model. That is the steady state gain from measurements made on the limit cycle output and the relay output signal. Now, we will go to the analysis of the closed loop relay control system now, to find the remaining 2 parameter. So, what remaining parameters we have to be identified or estimated or we have to find explicit expressions for those.

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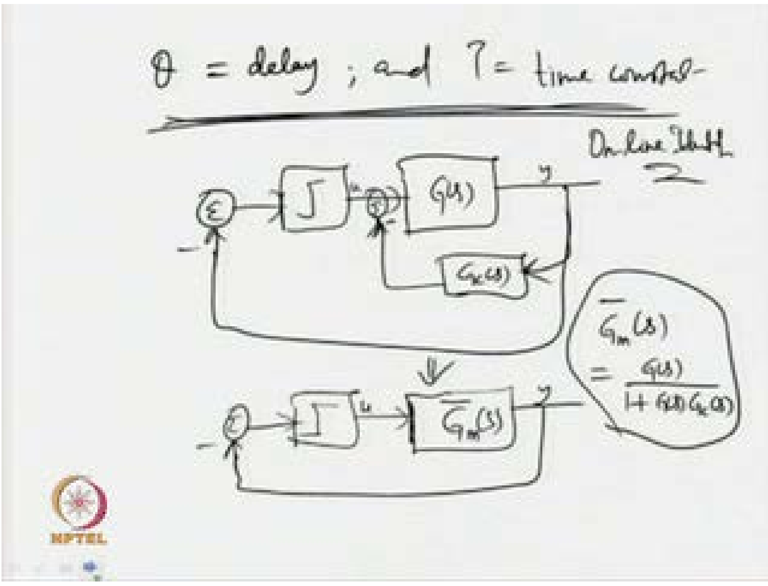

Those are the 2 remaining parameters known as theta, time delay and T, time constant. So, how to estimate the 2 parameters now? For that, we have to have analysis of the closed loop relay control system. So, the relay is connected to the plant and now, the plant is having some inner feedback controller. This is the controller, because we are considering online identification scheme. online identification. So, this is the equivalent representation of the online identification scheme.

From this equivalent representation, the block diagram can be reduced further to give us a block diagram of the form relay and I will get some transfer function of the form Gm bar s. This is the output and this is the relay output u relay output u. So, this is how, you get a reduced block diagram. What is Gm bar s now? Gm bar s can be obtained as, when the controller dynamics is given by a transfer function Gc(s). Gm bar s will be equal to G(s) divided by 1 plus G(s) Gc(s). So, these things are very simple and you can easily obtain expression for G m Gm bar s. So, the modified dynamics of the process that the relay sees or experiences is given by Gm bar s.

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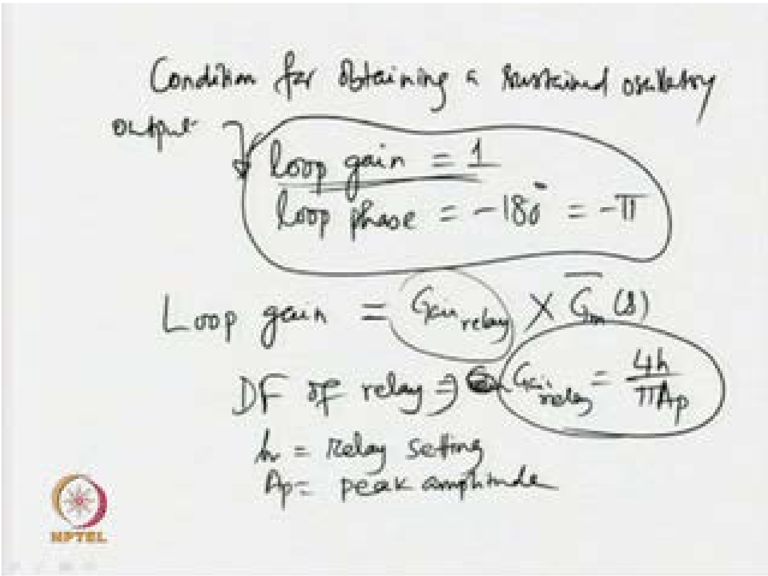

Now, when limit cycle occurs or when we obtain a sustained oscillatory output, the condition for obtaining condition for obtaining a sustained oscillatory output or limit cycle output is given by the conditions; that the loop gain at that time must be equal to 1 and loop phase must be equal to minus 180 degree or minus pi. So, these are the 2 conditions must be made to obtain sustained oscillatory output from the relay control system. Now, what is loop gain for our case? Loop gain is now given by the gain of the relay or gain of the relay times Gm bar s. This is the loop gain and similarly sorry loop phase. This is the loop phase angle.

Loop phase is minus 180 degree and loop gain is 1 to obtain a limit cycle for the relay control system. Now, the loop gain is given by the expression; loop gain is equal to gain of the relay times Gm bar s. What is the gain of a relay? Using the describing function describing function analysis of a relay, where can enable us to get the gain for the relay; which is given by DF of a relay gives us the gain of a relay gain of a relay as 4 h by pi Ap. So, this has been already discussed at length, in our previous lecture now. What is that h in the gain of the relay? h is the relay setting and Ap is the peak amplitude of the limit cycle output or the sustained oscillatory output. So, peak amplitude. So, recall all these things. Now, I will go to the expression for loop gain now.

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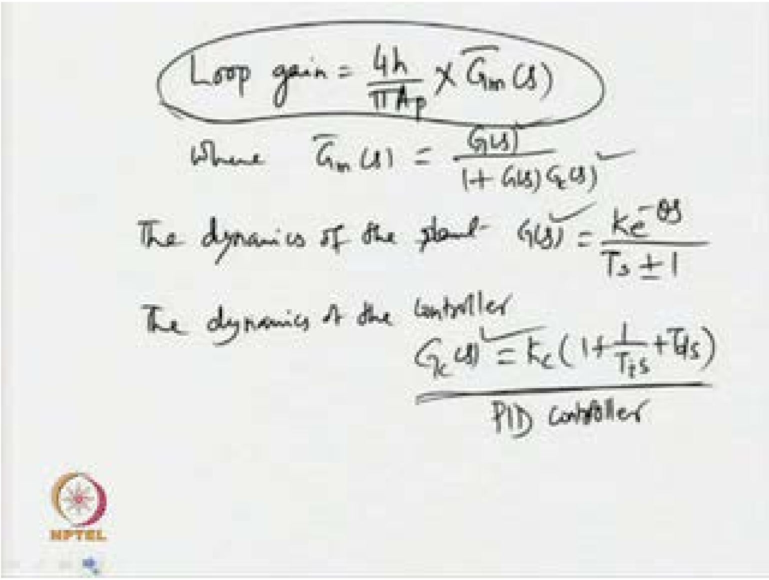

So, thus we have found that, the loop gain is equal to gain of the relay which is 4 h by pi Ap times Gm bar s, where Gm bar s is found to be G(s) upon 1 plus G(s) and Gc(s).

So, to carry on with the analysis, we required to consider some explicit transfer function models or transfer functions for G(s) and Gc(s). So, please allow me to have for our analysis. The dynamics of the dynamics of the plant dynamics of the plant given by G(s) is equal to K e to the power minus theta s T s plus minus 1. and the dynamics of the controller Now, the dynamics of the controller is assumed to be of the form Gc(s) is equal to Kc 1 plus 1 upon Ti s plus Td s. So, we are using a parallel form of PID controller for the analysis; a PID controller of this form Kc times 1 plus 1 upon T i s plus T d s. ok now. So, I will substitute all these expressions for G(s) Gc(s) in the loop gain expression for the loop gain.

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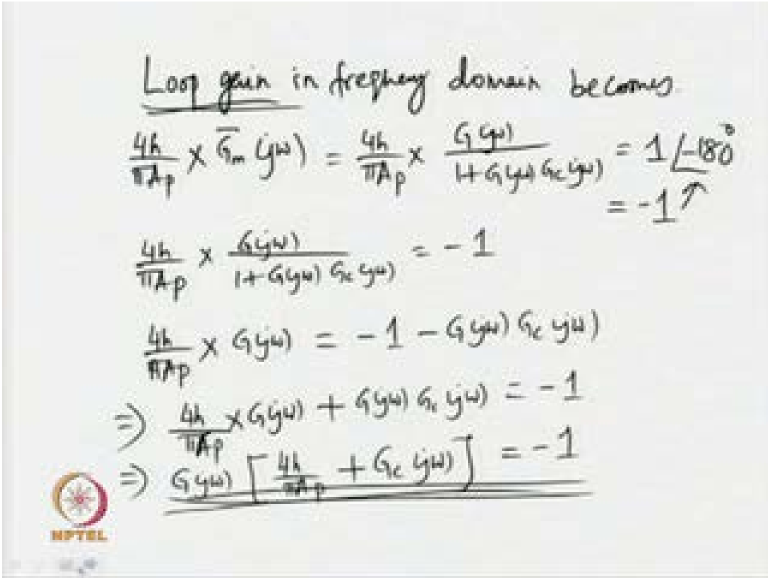

So, we have got the loop gain in frequency domain now. Loop gain in frequency domain becomes 4 h by pi Ap times Gm bar j omega, where omega is the frequency of the output signal; is equal to 4 h by pi Ap times G j omega divided by 1 plus G j omega times G c Gc j omega. We know that for limit cycle to occur; for inducing limit cycle, the loop gain has to be equal to 1. And for limit cycle to occur, the loop phase has to be the phase angle has to be minus 180 degree; which can ultimately be written in the form of minus 1 now. So, 1 with phase angle of minus pi or 180 degree gives us a real value of minus 1 or minus 1 in phasor form can be written as 1 with angle minus 180 degree. Is it ok?

Then, I get an expression of the form 4 h divided by pi Ap times G j omega divided by 1 plus G j omega Gc j omega is equal to minus 1. So, now cross multiply and obtain the terms, 4 h by pi Ap times G j omega is equal to minus 1 minus G j omega Gc j omega. Then, I can further write this in the form of 4 h divided by pi Ap times G j omega plus G j omega Gc j omega is equal to minus 1; which again becomes collecting the common term, G j omega 4 h by pi Ap plus Gc j omega is equal to minus 1. So finally, we have got from the analysis of the loop gain in frequency domain an expression of this form G j omega times 4 h upon pi Ap plus Gc j omega is equal to minus 1. Now, you substitute the expression for G j omega and Gc j omega.

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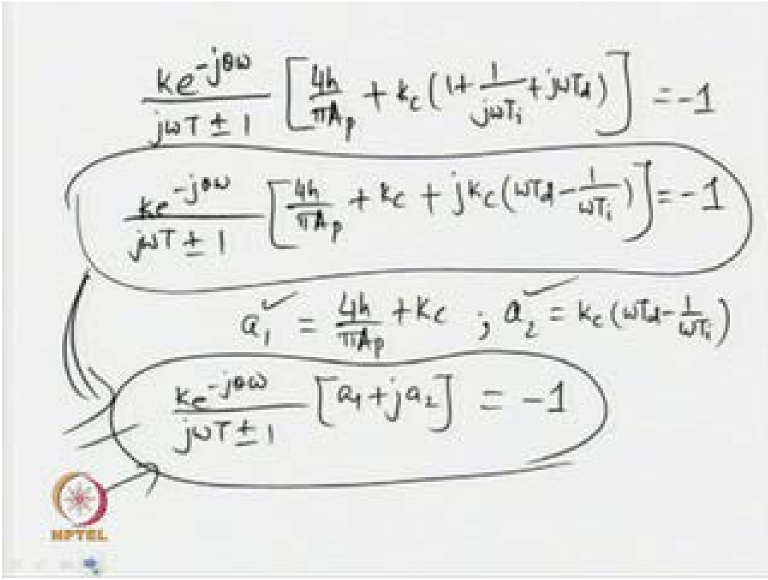

Then, you will get G j omega in frequency domain, will be K e to the power minus j theta omega by j omega T plus minus 1 4 h by pi Ap plus the dynamics of the controller now, Kc 1 plus 1 by j omega Ti plus j omega Td. So, this is equal to minus 1. So then, I can further simplify this in the form of K e to the power minus j theta omega divided by j omega T plus minus 1 times 4 h divided by pi Ap plus Kc plus I will bring j as common. So, we will get j Kc omega Td minus 1 upon omega Ti. So, this is how, you get the same expression finally expressed in this form. Now, allow me to introduce some constant a1 as 4 h divided by pi Ap plus Kc and either a a2 or b1, it does not matter.

So, a2 is equal to Kc times omega Td minus 1 upon omega Ti. Why I am doing so? So that, I will get finally this expression, in the form of K e to the power minus j theta omega divided by j omega T plus minus 1 times a1 plus j a2 is equal to minus 1. So, this expression becomes a simpler one, when the two constants are considered in this form. Now, I can consider the magnitude and phase angle of this final expression. So, let me rewrite first, equating the magnitudes of both side of this equation will result in further simpler expression, that can be used to find explicit expression for one unknown of the transfer function model. So, when I consider, next you please consider the magnitude of both sides of this analytical expression.

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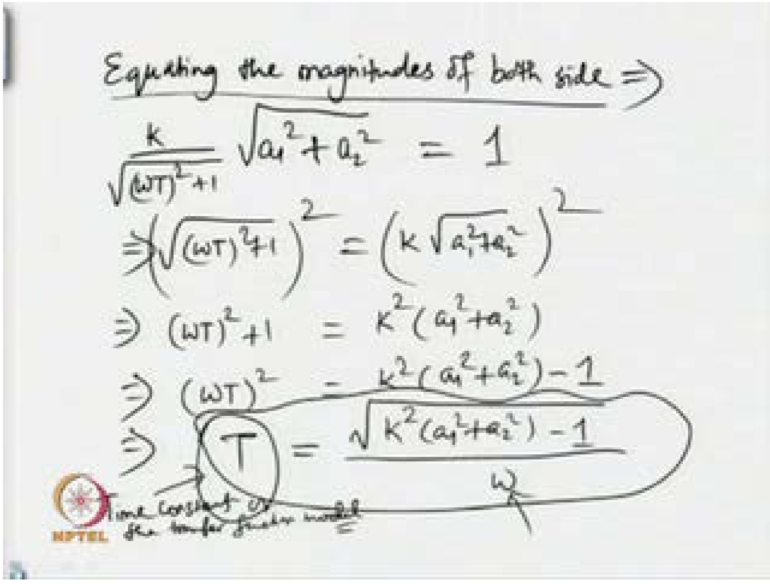

So, magnitude when I consider magnitude in the numerator, we will get; let me write the text first. Consider the or I would rather say, equating equating the magnitudes magnitudes of both sides of the analytical expression results in the numerator K in the denominator, you will have omega T square plus 1 root. The magnitude of this will be omega T whole square plus 1 root and for this one, a1 square plus a2 square. The magnitude of the complex number a1 plus j a2 will be a1 square plus a2 square root is equal to 1.

So, this is what, you get from the equating of the magnitudes of both side of this expression. Now, I will simplify this one. It implies omega T square plus 1 root is equal to K times root of a1 square plus a2 square. Take the square of both sides; that gives us now, omega T square plus 1 is equal to K square time a1 square plus a2 square. Then, omega T square is equal to K square time a1 square plus a2 square minus 1 implying T is equal to finally, directly I will write now; root of K square a1 square plus a2 square minus 1 square root of this divided by omega.

So, the beauty of doing this describing function based analysis is that, we have been able to get an explicit expression for one unknown associated with the transfer function model, in terms of the measurements. What is the measurement? That is, their omega and there are also measured values used in a1 and a2. So, this is how, I got an explicit expression for the time constant of the transfer function model. So, this is the time constant of the transfer function model for the dynamics of the original system. Similarly, when I equate the phase angles of both sides of the equation; now, this is the equation. I will equate the phase angles now.

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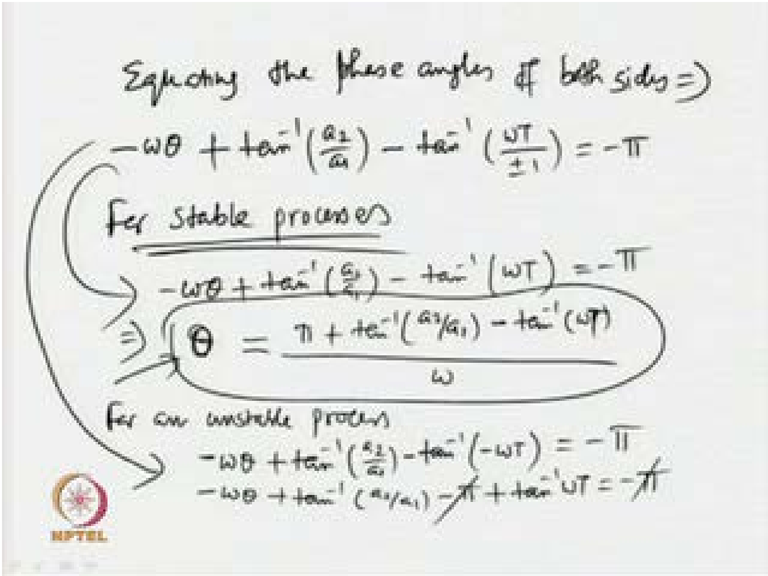

Equating the phase angles of both sides will give us minus omega minus omega theta minus omega theta from this term. The phase angle of this exponential term is minus theta omega. The angle given by this will be tan inverse a2 by a1 and then, we will have angle given by this term. So, the phase angle will be plus tan inverse a2 by a1 minus… why minus? because this term is there in the denominator. So, phase angle will be given by minus tan inverse minus tan inverse omega T divided by plus minus 1. So, that will be the phase angle contributed by the pole. So, it will be minus tan inverse omega T divided by plus minus 1 and that angle is nothing but, minus pi.

As we have seen earlier, the net loop phase is minus pi; then only, some limit cycle can occur. So, how to find the expression for theta now? So, with little modification for stable processes, when you are the system or plant is a stable process; for stable processes, this equation can be written as minus omega theta plus tan inverse of a2 upon a1 minus tan inverse of omega T now. Because here it will be plus 1 in the denominator is equal to minus pi giving us an expression of the form theta is equal to… I am directly writing now. So, pi will come here; pi plus tan inverse a2 by a1 minus tan inverse omega T divided by omega.

So, the explicit expression for the time delay or theta associated with the with the transfer function model is obtained in this form. Now, again I am telling the measurements; we make we make use of to obtain or estimate theta or omega and the measurements you have for a1 and a2. And theta has T has been already estimated earlier. So, I can use T in this expression. So, thus for unstable system, the explicit expression can be written as for an unstable process for an unstable process the this expression or the original expression sorry for an unstable process the expression can be written as minus omega theta plus tan inverse a2 by a1 minus tan inverse here.

Please keep in mind; you will have minus 1 in the denominator. So, I will remove this minus 1; rather write here the angle as minus omega T is equal to minus pi. Then we know that, tan inverse minus theta can be written as pi minus tan inverse theta use that one. So, tan inverse minus x; do not write theta; theta will confuse. So, tan inverse minus x is equal to pi minus tan inverse x. So, using that identity, it is possible to write this further as minus omega theta plus tan inverse a2 by a1. Now, it will be minus pi plus tan inverse omega T is equal to minus pi.

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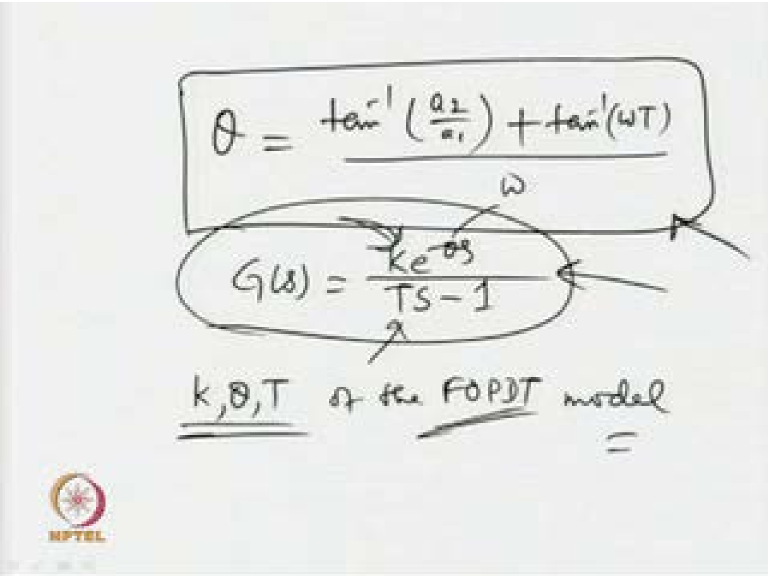

So, like the earlier case; now finally, I can have this minus pi minus pi of both sides cancelled out; giving me theta is equal to theta is equal to tan inverse a2 by a1 plus tan inverse omega T divided by omega. So, when the dynamics is having unstable pole, then at that time please make use of the two formula; we have derived the T this one and the theta given by this expression to correctly to estimate correctly parameters of the transfer function model; that will be for an unstable process as, K e to the power minus theta s T S minus 1.

Please keep in mind, when the plant is having unstable dynamics that time, the model its transfer function model is having a first order plus dead time model including 3 unknowns; K, theta and T. And this expression for T and theta we have found and K has been obtained using the ratio of average values of output and input. So, this is the way or procedure that can be that that will enable you to estimate all the unknowns associated with the transfer function model. So, K, theta, T of the first order plus dead time transfer function model can be estimated in this way. Now, I will extend. Now, you will see the constants. What are the a1 and a2?

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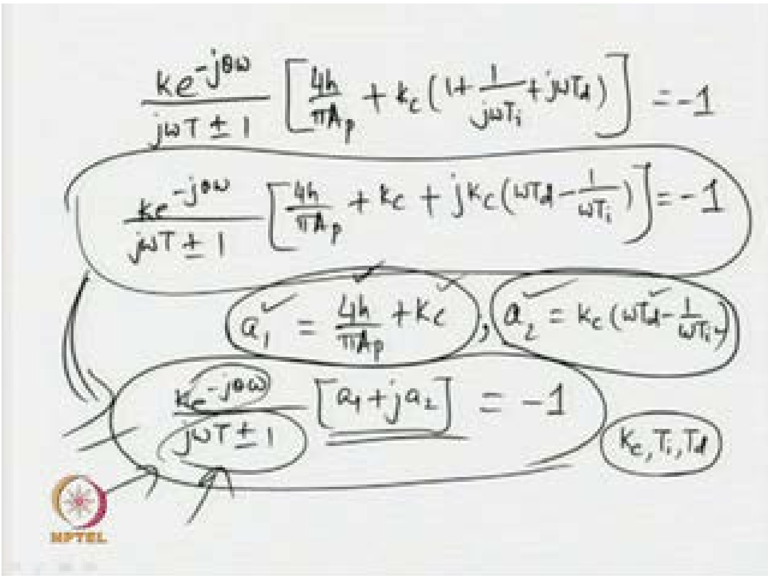

If you carefully look at the two constants a1, and a2; you see it contains not only the gain of the relay, it contains parameters of the controllers as well. So, Kc, Ti and Td, omega is measured; omega is obtained from the limit cycle output, but how to how to choose Kc, Ti, and Td during the relay experiment or test. That possesses a challenge.

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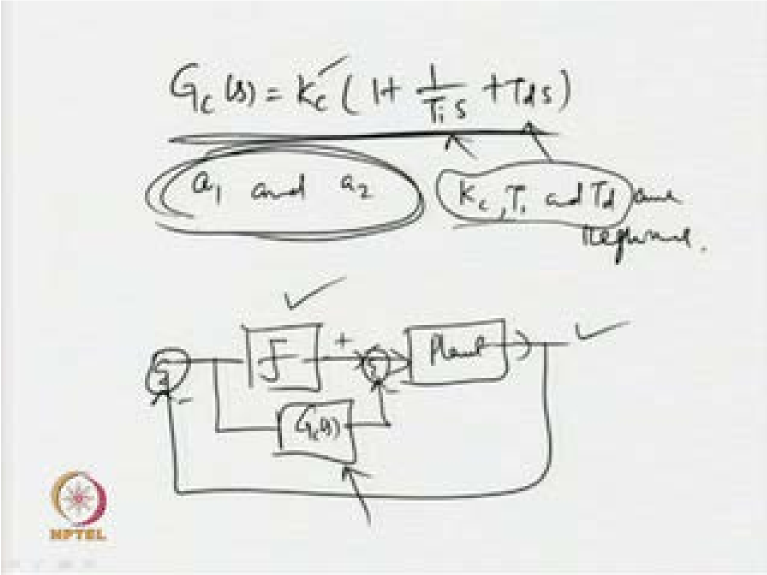

Now, I mean to say at the time of relay experiment, what you have? You have a controller in the loop, and you must have some controller parameters Gc(s) given by Kc 1 plus 1 upon Ti s plus Td s. So, those initial values are going to determine the values for a1 and a2. Please keep in mind; a1 and a2 are functions of Kc, Ti and Td. And when you are conducting a relay test, when you have no information about the process dynamics, you have to be very careful; judicious judicious choice of Kc, Ti, and Td are required. So, what what I mean by that? When you are going to commission a plant, when you are going to design a controller for a plant that has not been in use so far; then for that case, what happens? You do not know any anything about the plant. So, that way, how to design a controller for the plant unless you conduct the relay test?

So, for that you have to make use of judicious values or proper values of Kc, Ti and Td, while conducting relay experiment. So, when you are conducting a relay experiment, these PID controller parameters are of immense effect. So, they will result in either they will lead to some sustained oscillatory output or they may fail. So, it is very important to see, what values of Gc(s) are used at the time of relay test? Because the a1, a2 parameters are there and indirectly speaking theta and T. The two explicit expressions are functions of a1 and a2. So, in practical values for a1 and a2 are not going to give us correct estimated values for T and theta. Now, I will try to extend the analysis for some higher order plant dynamics.

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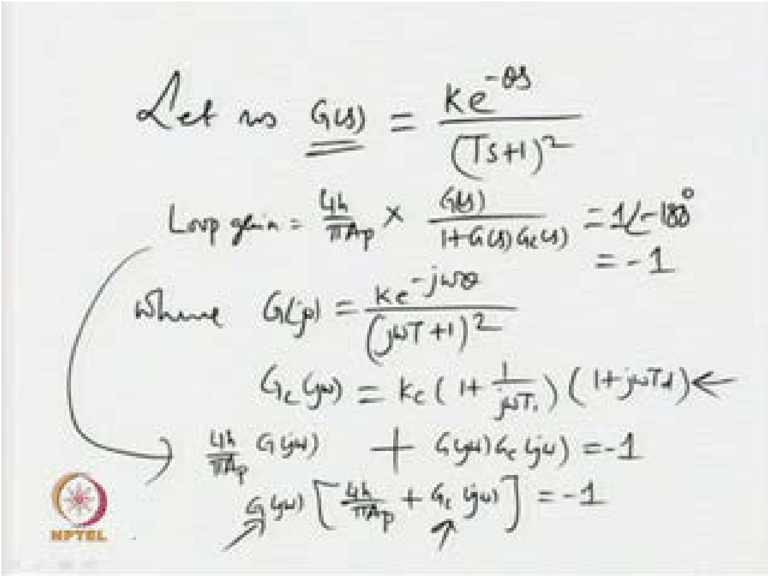

So, let us assume for this case. Let us assume that, the plant is a second order system where the dynamics of the plant is given by G(s) is equal to K e to the power minus theta s upon T s plus 1 square. So, I am not using unstable plant in this case. So, for this case again using the same analysis, what you will get? You know that, you have got the loop gain given by loop gain given by 4 h by pi Ap and further you have got G(s) upon 1 plus G(s) Gc(s) is equal to minus 1. Finally, I am writing loop gain is equal to 1 and loop phase equal to minus 180 degree.

At that time, you get loop gain in the form of 1 with phase of minus 180 degree, which can be written as minus 1. So, directly I am writing. Now, I will substitute now these values for or or sorry expressions for G(s) and Gc(s), where G(s) will be sorry G(s) in frequency domain will be G j omega is equal to K e to the power minus j omega theta theta j omega T plus 1 square. And Gc j omega, please allow me to use a controller of different form; I can use a series controller now; given by Kc 1 plus 1 upon j omega Ti and 1 plus j omega Td. So, what we have done? We have used a series PID controller in place of the parallel one; then that will result in finally, an expression of the form.

Again, please do the analysis of this expression, which will give us 4 h upon pi Ap; keep G(s) in one side. So, G(s) or I will directly write G j G j omega in frequency domain. So, G j omega is equal to minus 1 minus G j omega Gc j omega. So, when I take this to the left side, I will get this as plus is equal to 1 or I get this again in the form of, yes minus 1 plus 1. So, we have got again this given by G j omega will be common 4 h by pi Ap plus Gc j omega is equal to… no this will be minus 1 sorry this will be minus 1. When you cross multiply, you get that in this form; minus 1 will remain in the right side.

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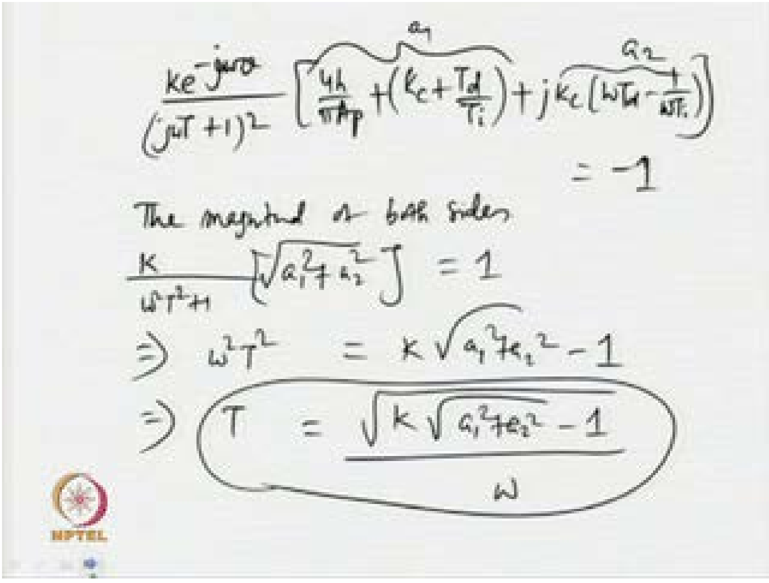

So, substitute for G j omega and Gc j omega, that will give us now; the G j omega is K e to the power minus j omega theta divided by T j omega T plus 1 square 4 h by pi Ap plus the dynamics of this… when you get in frequency domain, if I multiply I will get Kc plus j omega Td by Ti. So, Kc plus Td by Ti as one term, then plus if I take common; So, j will be the common j Kc omega Td minus. So, j Kc omega Td minus 1 upon omega Ti. So, this is equal to minus 1. Now, again find the magnitude of both sides giving us K; the magnitudes let me write few texts also.

The magnitude of both sides will be K divided by omega square T square plus 1 and I will have 4 h by pi Ap plus. Why not to use some a1 and a2? So, let allow me to choose this as a1 and this as a2, then we will get the magnitude as a1 square plus a2 square root is equal to 1, which will give us again omega square T square plus 1 is equal to K root of a1 square plus a2 square. Then, take this to the right side giving us minus 1 here implying T is equal to K root of a1 square plus a2 square. If I take root root of this one and square, yes it is ok then minus 1 root by omega. This is how; you obtain an expression for T now.

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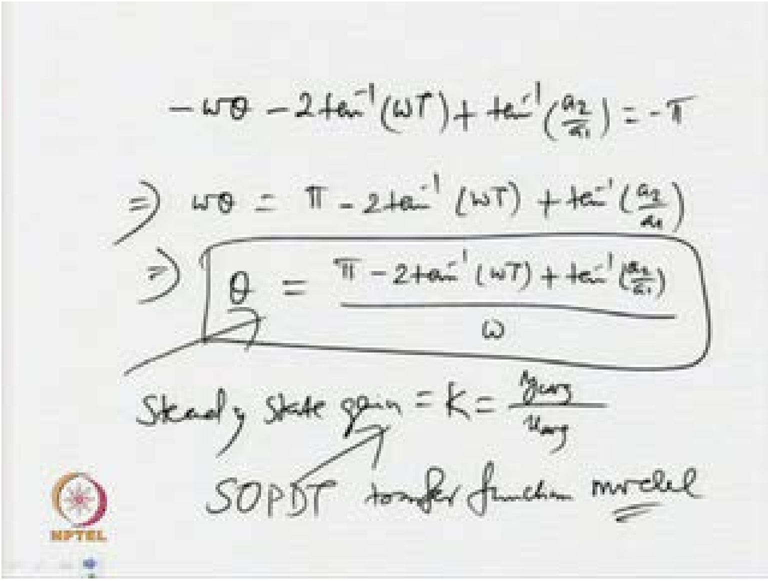

So, again equating the phase angles of both sides of this expression will give you, for the phase angle it will be minus omega theta and phase angles 2 times of minus 2 times of tan inverse omega T and I will get plus tan inverse a2 by a1 is equal to minus pi implies; So, this will be equal to omega theta is equal to pi minus 2 tan inverse omega T plus tan inverse a2 by a1 implies omega sorry theta is equal to pi minus 2 tan inverse omega T plus tan inverse a2 divided by a1 whole divided by omega. So, this is how, for the second order plus dead time dynamics, we have been able to find explicit expressions for the unknowns; time constant T and the second unknown, the time delay theta.

So, this procedure can be applied for any transfer function model. Ofcourse, the transfer function model should have limited number of unknowns. ok now. So, basically for this transfer function model, we have got also 3 unknowns theta, T and K. The K, the steady state gain the steady state gain can be obtained again from the ratio of average value of the output signal, when you have got relay in the loop and the average value of the input signal. Thus, all the parameters of the second order plus dead time transfer function models, stable transfer function model can be estimated.

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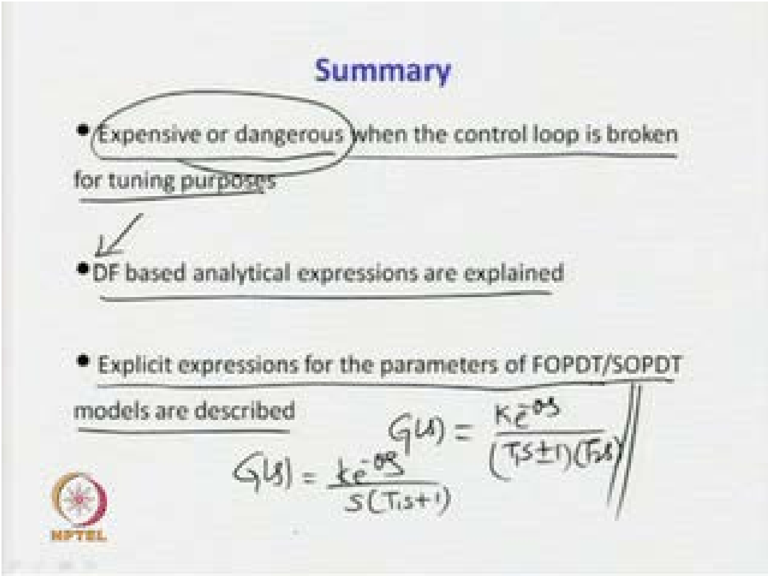

I'll go to the summary now. Expensive or dangerous when the closed loop is broken for tuning purposes. What we mean by this? That, when you are doing offline identification, there is no controller in the loop and to operate a system or process or plant with no controller in the loop is often dangerous. So that, I mean by this expensive not only expensive, it is also hazardous to perform relay test in the absence of a controller. We have done describing function based analysis of the relay control system giving us two explicit expressions for the unknowns associated with the transfer function model.

The explicit expressions for the parameters of first order plus dead time and second order plus dead time models are explained. Now, I can extend the analysis for a plant with a dynamics of the form, G(s) given by K e to the power minus theta s T1 s plus 1 plus minus 1 T2 s or it can be extended for integrating processes as well. G(s) is equal to K e to the power minus theta s s times T1 s plus 1.

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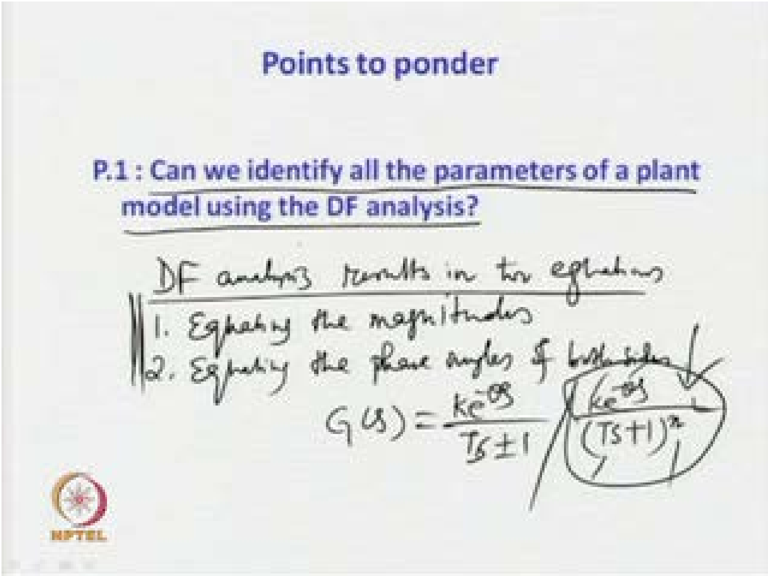

Can we identify all the parameters of a plant model using the describing function analysis? Generally, no. The reason for that is that, the describing function analysis describing function based analysis results in basically two equations. What are the two equations? The one equation, you get from equating the magnitudes of both sides of the analytical expression. And secondly, equating the phase angles of phase angles of both sides of the analytical expression obtained with D describing function. So, basically you will get limited number of analytical expressions.

Therefore, it is it may not be possible to identify all the parameters of a plant model using the describing function analysis. So, basically the type of models, those should be considered are of the form K e to the power minus theta s T1 s or T T s plus minus 1 T s plus minus 1 or of the form K e to the power minus theta s T s plus 1 to the power n. Again, when you put n is more than hm some value means; if n is known, then it is possible otherwise not. So, you can use higher values of n, suppose I wish to find a transfer function model of this form.

It is possible, because of the numbers of unknowns are three, but when n is not known, then it will be very difficult to find the number of parameters, more number of parameters associated with this type of transfer function model. Because the number of unknowns here are K, theta, T, and n; n is also one unknown associated with the transfer function model. So, describing function analysis has got this major limitation that we have to use simple plant or process transfer function models for identification of system dynamics or plant dynamics, using online identification. Thanks.