Advanced Control Systems
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Module No. # 01
Model Based Controller Design
Lecture No. # 02
Time and frequency domain performance measures

Are we satisfied with the controller in a closed loop control system, if we can switch off, controller can be ascertained quantitatively with the help of performance measures. We shall discuss certain performance measures in today's lecture.

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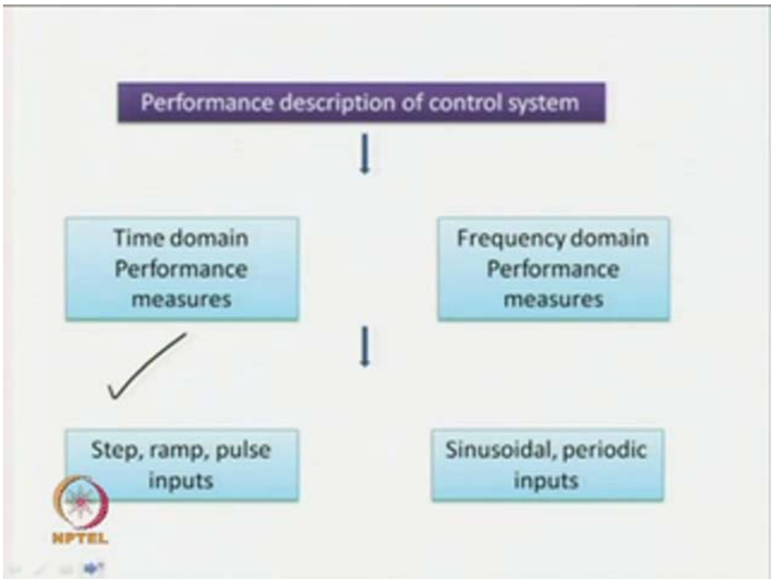

What is Performance description of control system? We can have performance description in time domain and in frequency domain. To obtain the time domain performance measure usually step, ramp or pulse type of inputs are applied to the closed loop system whereas, to obtain frequency domain performance measure one apply sinusoidal periodic inputs to a closed loop system to know in detail how to find the performance description of a control system.

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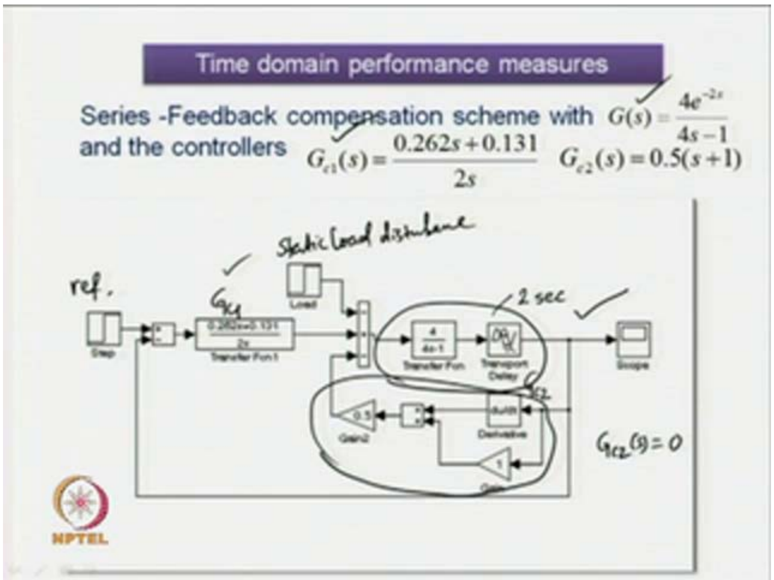

Let us consider the block diagram shown over here. This block diagram shows us a Series Feedback compensation scheme where we have got a controller GC 1 in the feed forward path and a controller GC 2 in the Feedback path, thus we bring us a series feedback compensation scheme.

Let us assume that the process has the transfer function given as 4 e to the power minus 2 s upon 4 s minus 1, the transfer function shows that there is 1 right half plain pole of the transfer function. Thus the process each open loop unstable also we have got transportation delay in the process thus the process is having a time delay of 2 seconds. Let the feed forward controller be having the transfer function GC 1 s given as 0.262 s plus 0.131 upon 2 s and the feedback path controller GC 2 s having gain of 0.5 with a derivative controller given as s plus 1 for the derivative time constant t d equal to 1 second.

Now, this scheme helps us to find out the closed loop response of the series feedback compensation scheme when it is subjected to some reference input and some static load disturbance.

What will happen when GC 2 equal to 0 when GC 2 s equal to 0; obviously, the unstable time delay process will be subjected to a P I controller in the feed forward path and we may or may not get stable output, that we shall see after analysis. Let us first assume that GC 2 s equal to 0 for the time being and do the analysis of the closed loop system.

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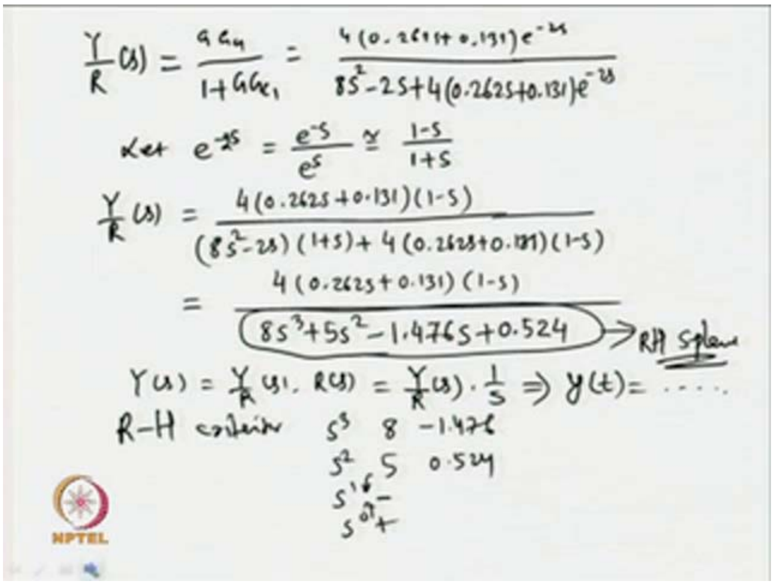

When the GC 2 s is assume to be 0 the closed loop transfer function with respect to the reference input can be obtained as y upon R s equal to GGC one upon 1 plus GGC 1 since GC 2 equal to 0, which can again be written as 0.262s plus 0.131 upon 8 s square minus 2 s plus 4 times 0.262 s plus 0.131 e to the power minus 2 s. Now there is 1 delay term as well in the numerator, let us assume that e to the power minus s be minus 2 s be represented as e to the power of minus s upon e to the power s, which can be approximated as 1 minus s upon 1 plus s, with this approximation for the time delay. It is possible to get the expression for y upon R s in the form of 4 0.262 s plus 0.131 1 minus s upon 8 s square minus 2 s times 1 plus s plus 4 0.262s plus 0.131 1 minus s, again which can be simplified and obtained in the form of 0.262 s plus 0.131 times 1 minus s upon 8 s cube plus 5 s square minus 1.476 s plus 0.524. Thus to find the response of the closed loop system to unity reference input, we have to either a find by expression y s equal to y upon R s time R s which is nothing but one upon s for unity step input y upon R s times one upon s. Then from there one can find its laplace inverse and ultimately find expression for the y t.

Instead of doing all those things if we concentrate on the denominator polynomial of the closed loop transfer function also, we can derive some conclusion. If I look at the polynomial and make use of Routh-Hurwitz criterion which can be given as s cube s square s 1 s 0 with the coefficients 8 minus 1.476, then 5 0.524 and the coefficient at this place will be definitely a negative value. And then I do not mind what would be here now. It shows that it may come a positive value suppose in spite of that what we will have there will be 2 sign changes here, thus giving us employing that there will be 2 roots of the polynomial located in the right half of the s plane. Thus the response will be unstable. Let us see the response actual response one obtained from the series feedback compensation scheme as expected when GC 2 s equal 2 s 0 the output of the closed loop system goes unbounded for bounded input the output has exploded in this session.

Because the closed loop system is not stable when GC 2 s equal to 0. So, this is what we get as the time response of a closed loop system for unstable process in the absence of some inner feedback controller.

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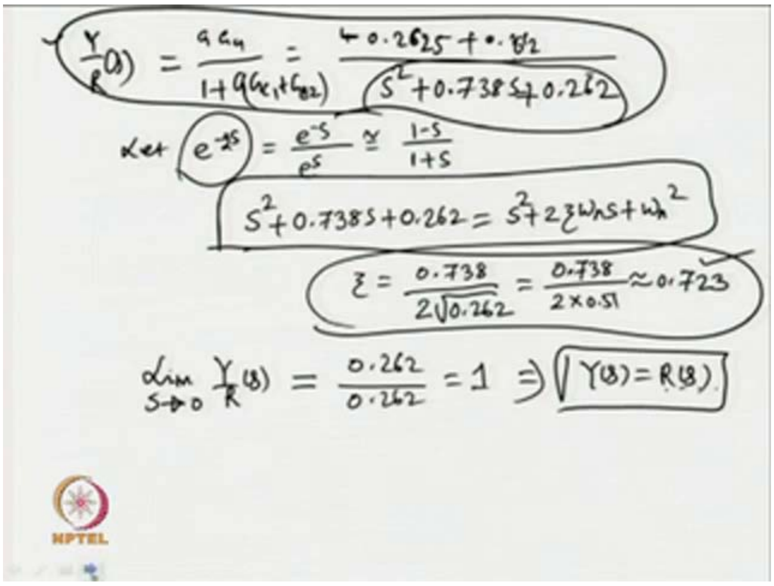

Now what happens when GC 2 s is not 0, that time one can find similar expressions. Now when GC 2 s is not equal to 0 y upon R s can be given as g GC one upon 1 plus g GC 1 plus GC 2 and using the approximation e to the power minus 2 s. One can find the expression as minus 0.262 s plus 0.262 in the numerator upon s square plus 0.783 s plus 0.262.

So, this is what we get when the inner feedback controller is present in the loop. So, when the inner feedback controller is present in the loop, we get a closed loop transfer function whose denominator is given by s square plus 0.783 s plus 0.262. So, if I compare this polynomial s square plus 0.783 s plus 0.262 with the standard second order denominator polynomial s square plus 2 zaye omega n s plus omega n s square. Then I get the zaye as saye which is known as the damping ratio as zaye equal to it will be equal to 0.783 upon 2 times root of 0.262 which can been written as 0.738 upon 2 times 0.51 approximately which can approximately be written as 0.723 ultimately.

So, this dumping ratio zaye gives us the information that we get an under damped response for the closed loop control system.

Now, when the input is unit step reference input at that time, the output of the system will be under damped response because the damping ratio is less than 1 that is 0.723.
What other information one can obtain from the closed loop transfer function of the system which has got. Now both the feed forward and feedback controller the steady state response that can be found with the help of the expression limiting s to 0 of y upon R s that will give us 0.262 upon 0.262 equal to 1 implying that y s equal to R s. So, the output becomes the desired output in steady state. This information can easily be obtained from the closed loop transfer function; that means, the steady state error of the closed loop system response is 0 whether those are true or not let us see from the simulated result.

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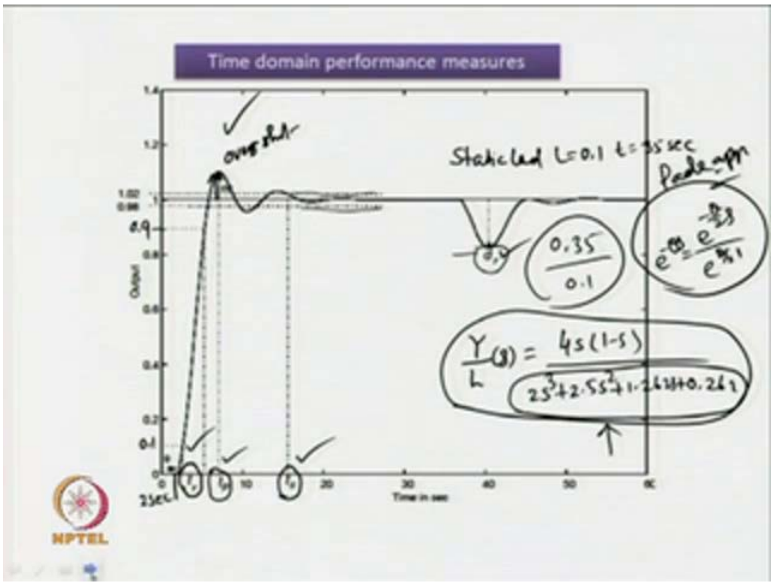

We can see that the closed loop response of the system is obtained as shown over here where we have got the overshoot undershoot before settling down. The output has got overshoot, undershoot, overshoot little bit of undershoot over here, before settling down to the reference value. Similarly, when a static load disturbance of magnitude load of magnitude l equal to 0.1 is applied at time T equal to 35 second, we have got the disturbance response in this session.

Now, what one can inform from this plot that the disturbance response has successfully been rejected with the set of controllers? What other information we get from this time response of the closed loop system, we say that the response starts after certain time that is of 2 second as expected because the process has a time delay of 2 second. Therefore, its time response starts after 2 seconds. So, before settling down it has got some overshoot, undershoot this magnitude is known as the overshoot and the time at which the overshoot takes place is known as the peak Time P T. Any other information we can obtained from this time response. As you know the time taken for the response to go from ten percent of the output to ninety percent of the output is known as rise time and when the output response enters the band of plus minus 2 percent of the reference value, that time is designated as the settling time. The time at which the output enters into the plus minus 2 percent band is known as the settling time. So, these are some critical parameters one obtained from time response of closed loop systems, what are those.

Let me again repeat, an overshoot rise time, peak time, settling time, disturbance response magnitude; in this case which is approximately of the value 0.35 for the load input of 0.1. So, these are certain critical parameters those are upon consider as far as time domain performance measures of a closed loop system are concerned.

Why the load response is of this form? To find that, again you need to find the closed loop transfer function with respect to the static load input which can be given as y upon l s in the form of 4 s 1 minus s upon 2 s to cube plus 2.5 s square plus 1.262 s plus 0.262. So, the closed loop transfer function of the system with respect to the static load disturbance input can be given in this form using the approximation for the time delay e to the power minus theta s as e to power of minus theta by 2 s upon e to the power theta by 2s. So this sort of approximation known is as your pade approximation, pade approximation. Now using that approximation, the closed loop transfer function denominator polynomial becomes this. As it appears that all the roots of this polynomial are located in the left half of the x plane. Therefore, we obviously, expect stable disturbance response or we say successful disturbance rejection by the closed loop system.

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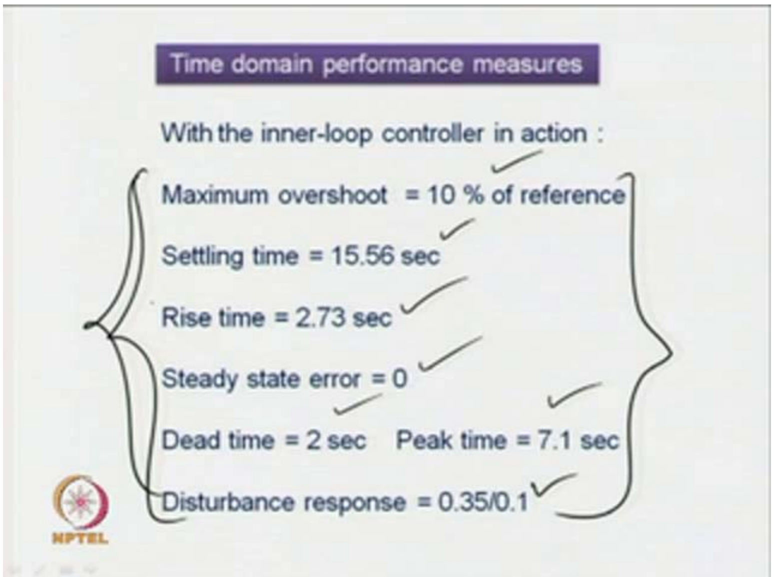

Now, let us see what are the exact measures of those time domain performance measures we have got from the simulation result. The simulation result with both the controllers in the loop give us a maximum overshoot of ten percent settling time of 15.56 second, rise time of 2.373 second, steady state error of 0, dead time of 2 second, peak time of 7.1 second and disturbance response of 0.35 upon 0.1. Ideally what should be the these values? These values should be as small as possible. The better the controller, the smaller these values 1 obtained from the closed loop arrangement.

Now, how minimum those values can be? There are some due to that controller may not be able to provide us minimum values for all these performance measures. But when it provides those values minimum values often we call the controllers to be optimal controllers.

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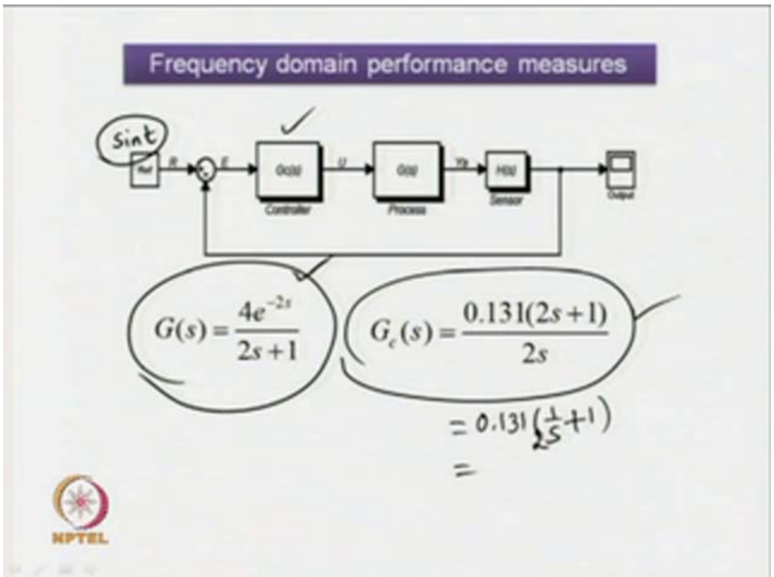

Now, let us go and see what we have in our Frequency domain performance measures. To study Frequency domain performance measures, let us assume the input to be a simple input given as sin t which has got a magnitude of 1 and molar frequency of 1 radian per second. When the input to the system is sin t and we have got a controller which dynamics is given by the transfer function GC s which is same as 0.131 1 upon s plus 1 which gives us again 1 upon 2 s plus 1.

Then which is the same controller we have in the previous scheme, but the process dynamics is now given by 4 e to the power minus 2 s upon 2 s plus 1. We do not have any internal controller GC 2 s in this scheme. So, this series compensation scheme will definitely give us satisfactory time response as we have seen earlier because we are using the same modified process for the series feedback scheme. Now when the input to this series compensation scheme is sin t, what sort of output we often expect from this arrangement?

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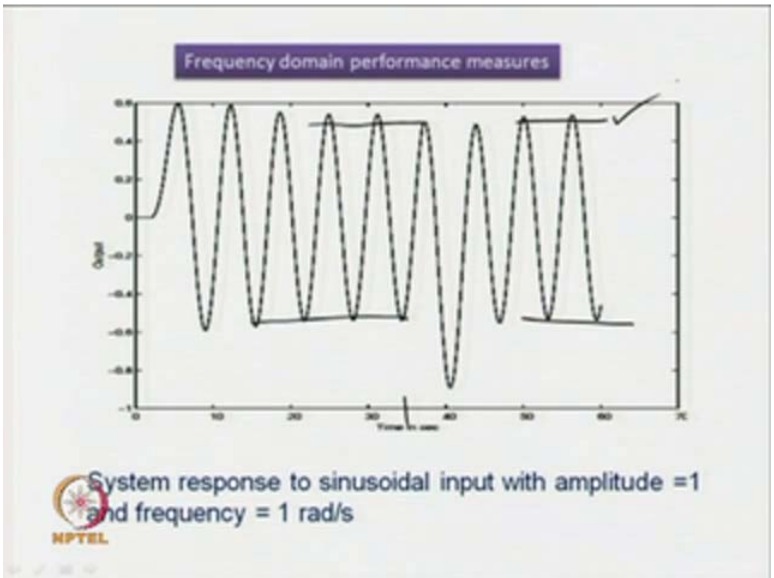

The output can be of this form. Now it is very difficult to make out what the system output is giving us. Apart from the stability of the system, as the input is sinusoidal, obviously, out the output the output is sinusoidal which is a stable one if we see over here before some disturbances occurs at Time t equal to 35 second, again the disturbance has been rejected successfully and we come back to the stable response.

Apart from this information that the system output is stable for sinusoidal input to the system, it is very difficult to make out any other quantitative performance measures from this output. It is difficult to visualise the effects or good effects of a properly designed controller from this output. Therefore, we need to consider the loop gain of the closed loop system, loop gain of the closed loop system for frequency response analysis of the system. What is that loop gain of this system?

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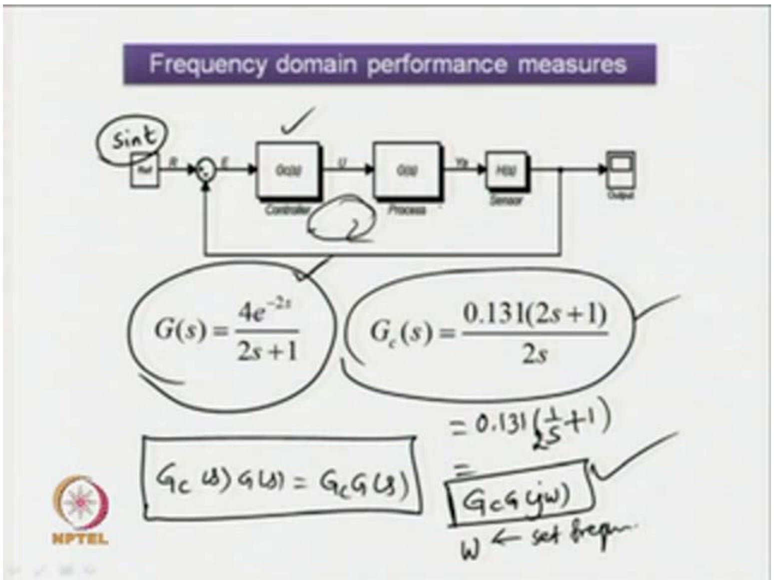

The loop gain of the system can be given as GcG s which can again be written as GcG s. So, this gives the loop gain of the closed loop system. When the loop gain is analysed by substituting j omega for s, we get the frequency response characteristics of the system.

So, now we shall concentrate on the loop gain which is available in the form of GCG j omega where omega is the set of frequencies applied to the system in the form of sinusoidal input with various input frequencies. How can we find some performance measure using the loop gain? That we shall see now. As we know that when the loop gain is one and the gauge loop gauge is also minus 180 degree, then the system will be under herds of instability. Using that at the will be possible now for us to find some frequency domain performance measures. How can we return our system stable in spite of or any external disturbances?

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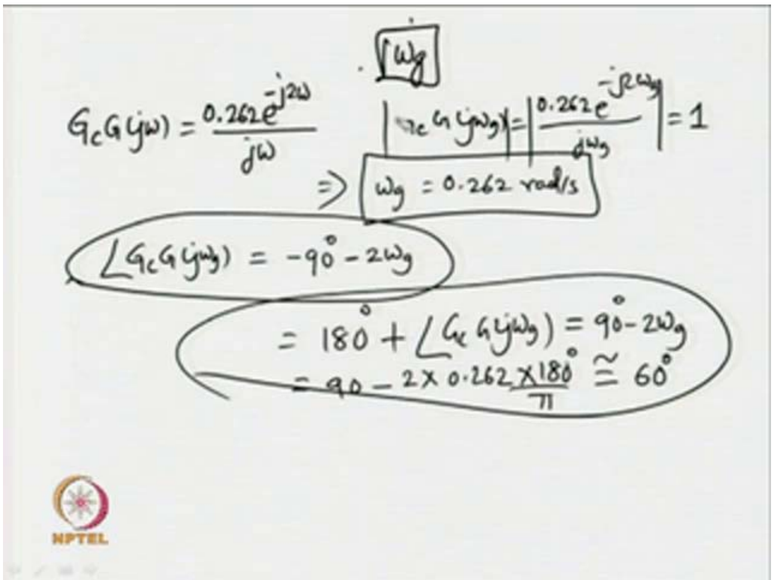

Now, we shall consider the loop gain in the frequency domain given as G c G j omega which can be, for our case as 0.262 e to the power minus j 2 omega upon j omega. So, let me write the loop gain once more clearly, GcG j omega will be equal to 0 .262 e to the power minus j 2 omega upon j omega. Now we shall define a few parameters those are associated with frequency domain analysis of a system. Those are the gain cross over Frequency omega g.

The gain at the frequency at which the gain of the loop is unity is known as the gain cross over frequency. To find that, what we have to do, set G c G j omega g equal to 0.262e to the power minus j 2 omega g upon j omega g is magnitude equal to 1.
So, this will give us obviously, omega g equal to 0.262 radian per second. So, this is known as the gain cross over frequency of the system.

What information we get from this gain cross over frequency? Let us find the phase of the system at this gain cross over frequency. So, the phase angle can be obtained at this frequency as minus 90 degree then minus 2 omega g equal to minus 180 degree which upon shall bring give us an equation of the form no the phase of this system will be given by this magnitude.

Now, how much additional phase can be added to this system before bringing the system to the words of instability? That is known as the phase margin of a system. So, we define the phase margin of a system as phase margin equal to 180 degree plus G c G j omega g which will be equal to ninety degree then minus 2 omega g and when this is put in the degree form ninety degree minus 2 into 0262 into 180 degree by pi upon solving this, one will obtain this nearly to be of 60 degree. So, thus the phase margin of the system can be obtained in this session, once we are able to find the gain cross over frequency of the loop.

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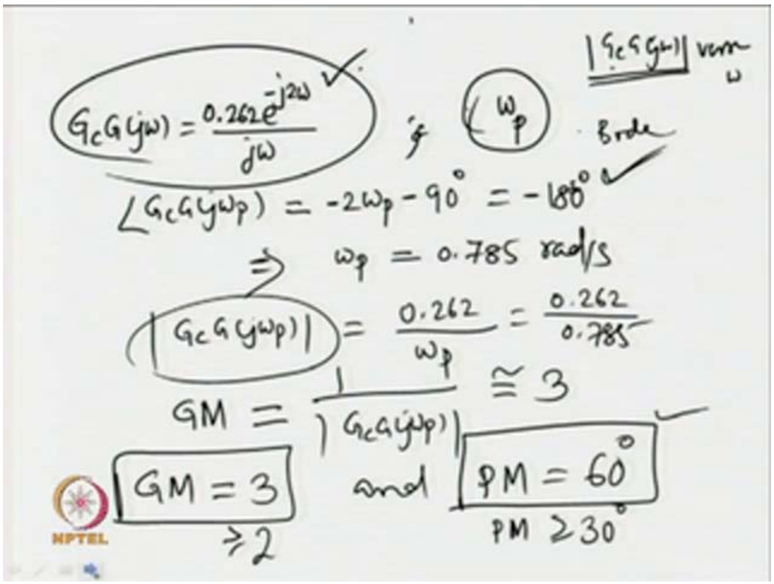

Similarly, using the loop gain, it is possible to find the phase cross over frequency at which the phase of the system is minus 180 degree. That can be obtained finding the phase. So, we will define one more parameter that is the phase cross over frequency omega p which is defined as the frequency at which the system phase will be minus 180 degree. So, G c G j omega p will give me an angle of minus 2 omega p minus ninety degree, which should be equal to minus 180 degree, which gives upon solving omega p h 0.785 radian per second.

Now, at this phase cross over frequency, what is the gain of the system. Let the gain of the system at this phase cross over frequency be G c G j omega p magnitude equal to 0.262 upon omega p equal to 0.262 upon 0.785. Now how much additional gain one can have or to have unity gain in the loop when the phase is minus 180 degree again which drives the system to the herds of instability, we have to find the inverse of this gain that will give us the gain margin of the system. Now the gain margin of the system becomes 1 upon G c G j omega p magnitude which will be approximately 3. So, what we have found from the analysis of the loop gain that the loop has got a gain margin of 3 in absolute value term and a phase margin of 60 degree. So, these things are possible from the analysis of the loop gain.

So, what information this gain and phase margin gives us? That when the gain margin is greater than equal to 2 or the phase margin is greater than 30 degree, we get a very good or performing closed loop system. Since in our case it is greater than 2 and phase margin is quite high 60 degree, therefore, as expected, its frequency response measures are quite satisfactory and therefore, its time response also will be satisfactory. The same closed loop system is found to have the time response of this form and let us see what are its frequency response curve when the loop gain G c G j omega is plotted. When the magnitude of the plot of the loop gain is plotted versus the frequency we get the bode magnitude plot.

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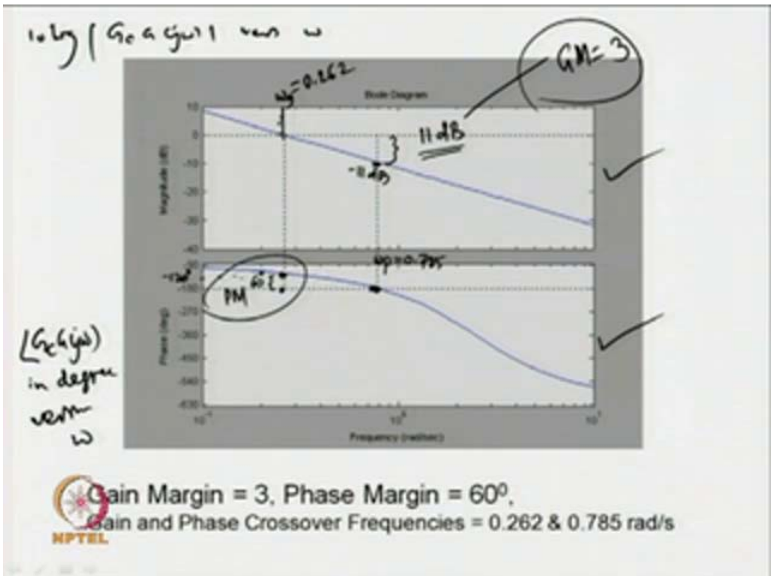

Let us see what sort of bode magnitude plot are series feeback system gives us. This is the bode magnitude plot we obtain. So, this bode magnitude plot is obtained from the plot of the magnitude G c G j omega 10 log versus omega. Similarly, angle G c G j omega in degree versus omega plot gives us this bode phase plot. So, the second one is the bode phase plot and the first one is the bode magnitude plot. This frequency plots also gives us the same information whatever we have obtained using analysis. What are those crossover frequencies here? When the gain of the system loop gain of the system is unity, the frequency at that time gives us the gain cross over frequencies omega g. When the phase of the loop is having minus 180 degree, that frequency is known as the phase cross over frequency.

So, we can get from here from the plot the gain cross over frequency to be of magnitude 0.262 because this is point 2, this is 0.262. Similarly, phase cross over frequency can be obtained as 0.785 if the values are read accurately from this plot. Now at this gain crossover frequency, what is the phase of the system? The phase is the phase is this much. The phase is minus 120 degree approximately. So, the system phase is loop phase is minus 120 degree when at the gain crossover frequency. Therefore, additional 60 degree of phase like can be introduced to the system before driving the closed loop system to the herds of instability and this additional 60 degree is known as the phase margin. Similarly, at the phase crossover frequency, when the phase angle is minus 180 degree, the gain approximately of minus 11 minus 11 dB. Therefore, additional gain of 11 dB can be added to the system before bringing the system to the herds of instability. This 11dB in absolute term will be of magnitude 3. So, the gain margin is 3 and the phase margin is 60 degree. So, same information can be obtained from the bode magnitude and phase plots.

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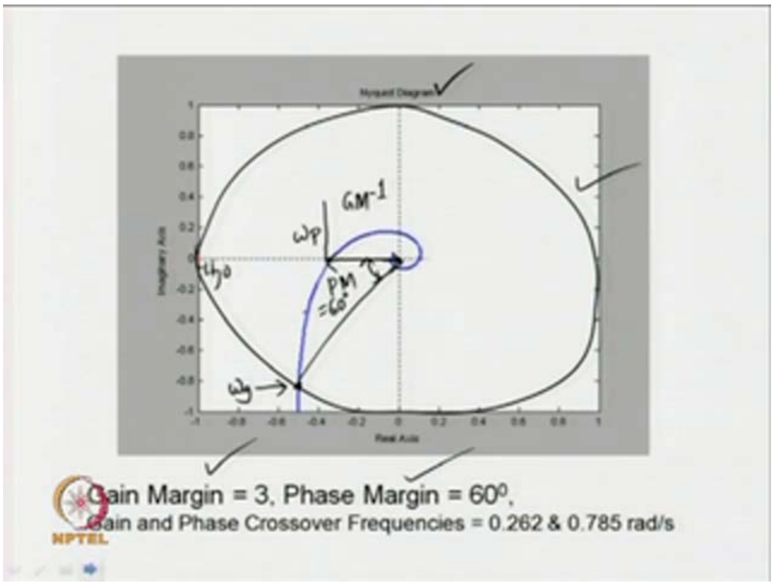

Next we shall see what is there for the frequency domain performance measure. Often we get the polar plot of the loop gain given in the form of Nyquist diagram. So, what a nyquist diagram is? It is nothing but the polar plot of the loop gain which has got real axis and the imaginary axis and the loop gain obtained in this typical form. From here also, it is not difficult to make out the crossover crossover points. So, this point, this point will give us the phase crossover point. So, at phase crossover frequency we are at this point. Similarly, when the unity circle is plotted, approximately of this form, this point gives us the gain crossover point omega g. So, at the gain crossover frequency, we are at this point where the gain equal to 1. So, similar information like the one we get from bode plot can be obtained from nyquist diagram. Basically we are able to locate the phase crossover point and the gain crossover point. How can we find the phase and gain margin from this plot conveniently.

Now, draw a line from the gain crossover point to the origin. Then whatever angle its obtains with respect to the negative real axis, that gives us the phase margin of the system. So, if one accurately draws the unit circle and identify the gain and phase crossover points and draw this line and measure this angle, then definitely he will find the phase margin to be of 60 degree. Similarly, draw a line from the origin of the plot till the phase crossover point. Measure the span. Inverse of that span inverse of this span will give us the gain margin. So, the gain margin can be given as gain margin inverse can be rather given as this span, can be given as this span. So, some convenient form of representation of nyquist diagram always enables one to easily obtain the phase and gain margin information. And the phase and gain crossover frequencies may not be a directly apparent from the nyquist diagram, but those can be calculated from the analysis.

Now, what benefit we get, what information we get from all these margins that how far our operating point is? How far our operating point is from the unity circle or from the minus one plus 0 point. That gives us information about the stability of a system. So, the better gain margin and phase margin one has, the better stable closed system one has. So, stability information. How stable is my controller, how robust is my controller; all those answers can be obtained from the information on phase and gain margins.

Now, we shall see, are there any other type of performance measures apart from the time domain performance measure and frequency domain performance measures. Yes, we have got measures like decay ratio which is given as the ratio of the heights of successive peaks during step input test. What are those successive peaks that can be explained with the help of the time response of a closed loop system.

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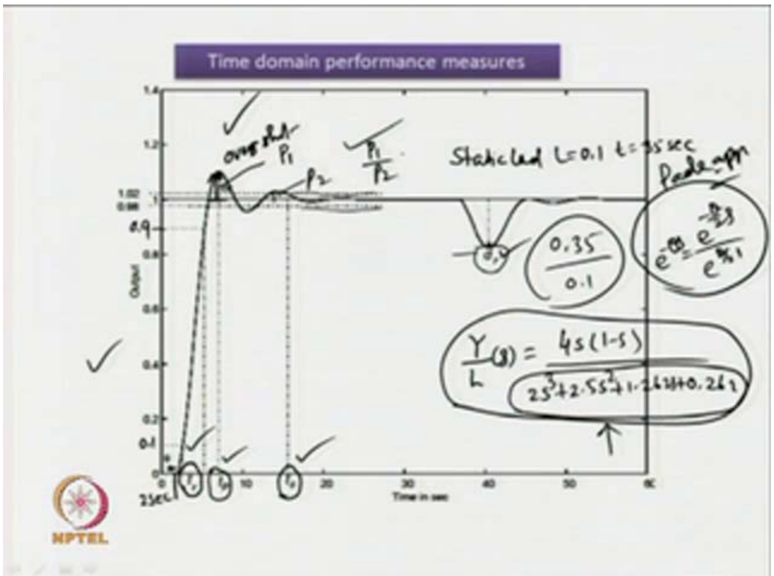

So, this is one peak and the magnitude of peak with reference to the reference input can be given as suppose peak 1 and we have got another peak over here, P 2. So, the ratio between P 1 and P 2 gives us the decay ratio. The higher the decay ratio the better the system response h, in the sense, it takes less time to settle down. The system takes less time to settle down whenever there are reference input changes or disturbance input to the system.

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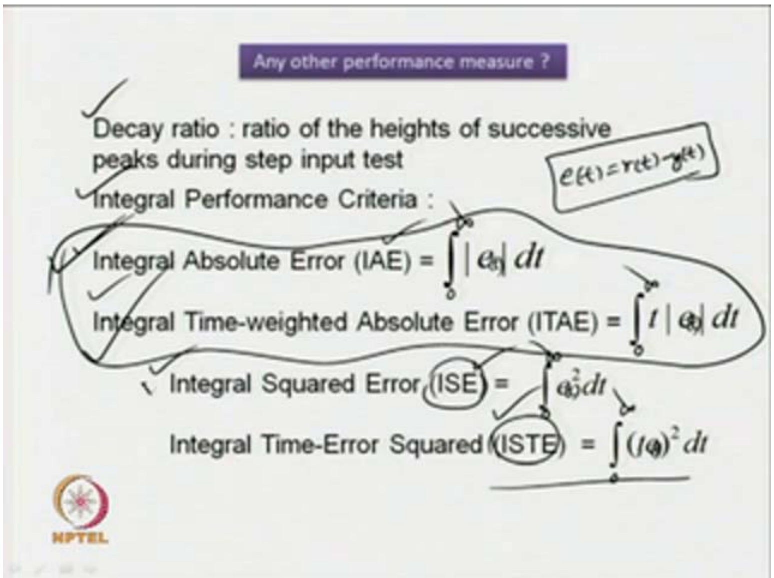

Therefore, high value of values of decay ratio are often desirable in a closed loop control system. Apart from the decay ratio, we have got various performance in this s upon known as integral performance criteria. What are those integral performance criteria? One such criteria needs integral absolute error, in short form it is known as IAE and expressed as integration from 0 to infinity mod of et dt. Another performance criterion is integral time weighted absolute error criterion given as integration from 0 to infinity t times mod of et dt. What are those e t? e t is nothing but difference between the reference input at the actual output of a system. Another performance criterion is integral squared error often abbreviated as ISE and given as integration from 0 to infinity E square t dt. Similarly integral time error squared criterion can be expressed as integration from 0 to infinity time et square dt. So, all these performance index indices give us quantitative measure of the performance of a closed loop system. So, controllers can be designed by minimizing all these performance criteria and those type of controllers often give us optimum parameters for the controller. Such controllers give optimum performance of a closed loop control system. Therefore, integral performance criteria are often used in designing controllers.

It is very difficult to estimate IAE and ITAE analytically where as it is not so difficult to compute ISE and ISTE using analysis. Using frequency domain analysis, it is easy to compute the integral square error and integral time error square performance criteria.

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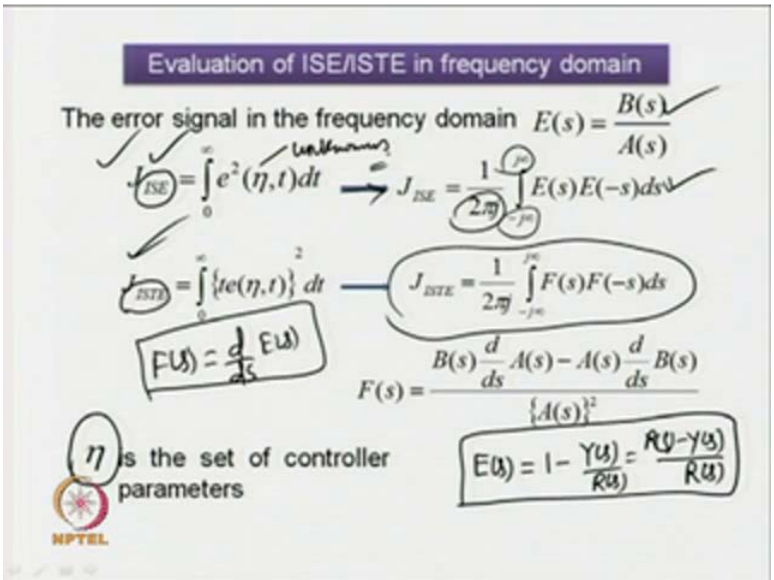

How can that be done? Using the relations that the ISE criterion expressed as integration from 0 to infinity E square eta t dt, in its equivalent representation in frequency domain in the form of one upon 2 pi j times integration from minus j infinity to j infinity E s E to the power minus s d s, where E s is available in the transfer function form. So, E s can always be available in the transfer function form because once we know the closed loop transfer function E s is nothing but 1 minus y s upon R s which is R s minus y s upon R s. So, E s can be obtained in the polynomial form. Suppose E s is expressed as B s upon a s, then employing B s upon a s and b minus s upon a minus s, it is easy to compute or evaluate the ISE performance index.

Similarly, IST performance criterion can be estimated using the formula which is 1upon 2 pi j integration from minus j infinity to j infinity F s times f minus s d s, where F s is the first order differentiation of E s with respect to s. Now eta is the set of controller parameters which are the unknowns in the expressions. So, these unknowns can be optimised by the minimization of the performance in this s jISE or j ISTE.

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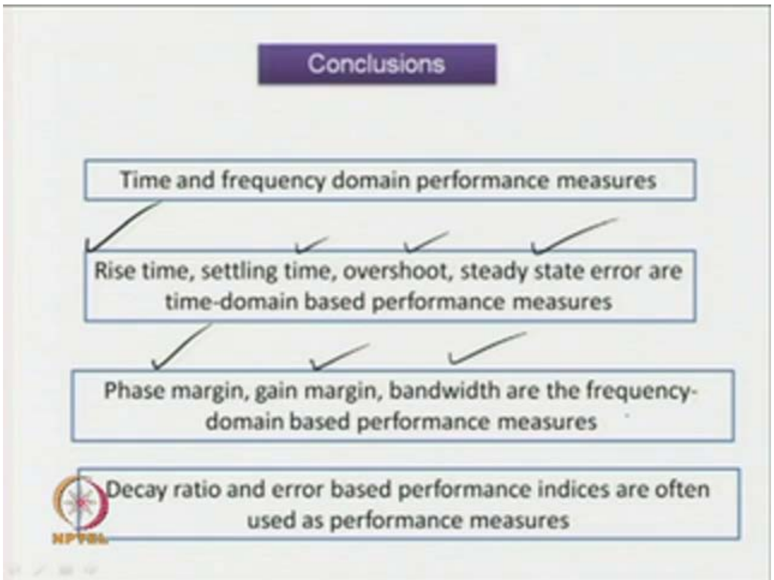

In summary, what we have learnt from this lecture that time and frequency domain performance measures are essential to evaluate the performance of controller, whether we are really happy with the performance of a controller or not can be ascertain from the fact that whether the closed loop system is giving us satisfactory time and frequency domain performances or not. Certain time response performance measures are often known as rise time, settling time, overshoot steady state error, decay ratio and so on. Also we have got frequency domain performance measures known as phase margin, gain margin, bandwidth to name a few. Often decay ratio and error based performance indices are also used as measure performance measures to ascertain the efficacy of controllers in a closed loop control system.

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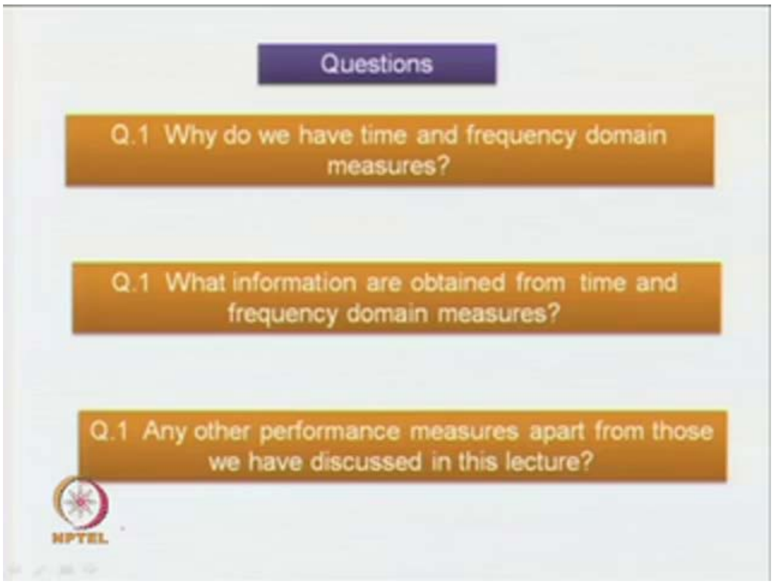

Next, we shall go to the questions. Suppose the questions are like this. Why do we have time and frequency domain measures? Time and frequency measures, time and frequency domain measures enable us to ascertain the efficacy of a controller. Generally controllers are design to meet certain time and frequency domain specifications and those are what we call as time and frequency domain measure. If the question is like this that what information are obtained from time and frequency domain measures, time domain measures give us satisfactory information about satisfactory performance of a closed loop system and frequency domain measure give us information about robustness, robust performances of a controller.

When the question is any other performance measures apart from those we have discussed in this lecture? Yes we have got certain other performance measures known as resonant peak frequency, resonant peak magnitude, bandwidth to name a few. That is all in this lecture.