Advanced Control Systems

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Module No. # 02

Frequency Domain Based Identification

Lecture No. # 02

Relay Control System for Identification

Welcome to the lecture titled, Relay Control System for Identification. In the earlier lecture, we have seen the benefits of relay control system. Relay ensures sustained oscillatory output in a closed loop system. Relay induces limit cycle in a closed loop system.

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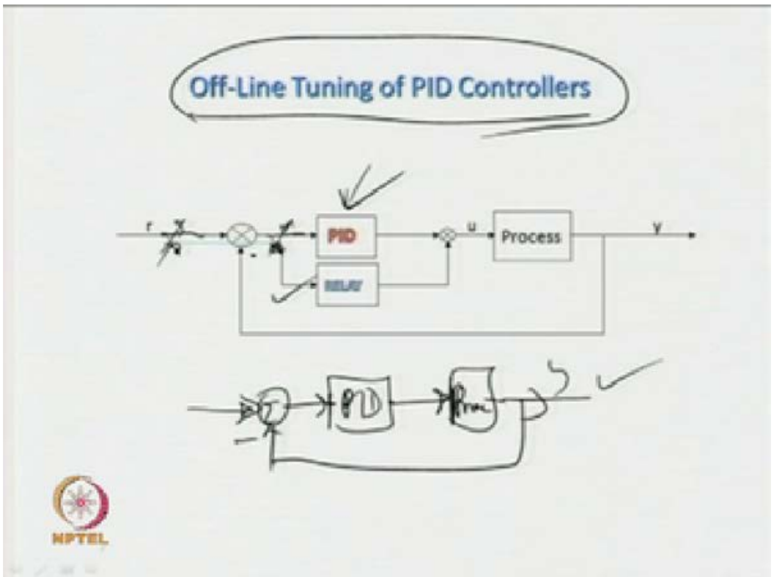

Let us consider, a relay control system of this form, where we have a PID controller, a controller in the PID forward path and a relay also there in the PID forward path; whereas, we have got two switches, one switch over here and another switch over here (Refer Slide Time: 01:03). When the switches are down, then the relay gets connected to the process and we have got relay and process in the closed loop system.

Now, the system can be represented in the form of relay, process and then we have got the output. Now, this is what we have got now. This type of system is known as autonomous relay control system. There is nothing beyond the process and the relay in the closed loop system. Now, using this, initially the parameters of models of a process are estimated.

So, using this relay control system autonomous relay control system, process dynamics, information on process dynamics are acquired. Then based on the parameters of the process model, PID parameters are set. So, this scheme is known as off-line tuning of PID controller. In the off-line tuning of PID controller, what is basically done? Initially, relay is switched ON to acquire the process information using the process information parameters of the PID controllers are set.

Then the switch is moved upward and it gets connected, here disconnected and the closed loop system becomes a PID and a process, output and negative feedback. So, this is the scheme we get. So, in the off-line tuning of PID controllers, a relay is connected in parallel with a controller, but a switch is provided, either the controller or the relay will be in operation at any instant of time, both cannot be present at any instant of time.

So, either the relay is to be connected or the controller is to be connected with the process. And this scheme of identification is known as off-line tuning or off-line this scheme of identification and control of a system is known as off-line tuning of PID controllers.

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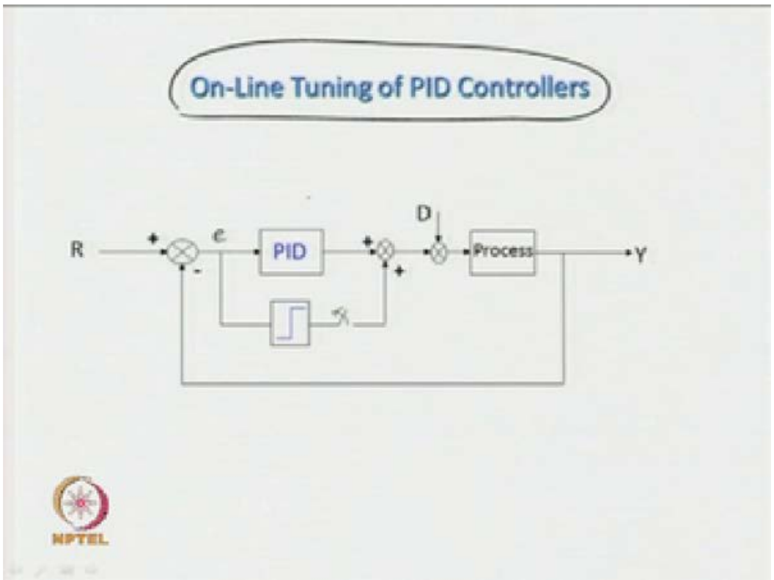

Let us see some other scheme, we have got also on-line tuning scheme, where the controller and the relay are connected in parallel. Now, the controller is always present in the closed loop operation. So, the process is subjected to the controller throughout its operation. This is the beauty of on-line tuning schemes, where the process information is acquired by switching on the relay momentarily. So, when I switch on the relay that time, the output of the process will be subjected to sustained oscillation (Refer Slide Time: 04:46).

So, when the output is at some steady state value, over that steady state value there will be oscillation. Suppose, this is the performance curve then, when the relay is switched at this instant, then we shall have sustained oscillatory output; and using this information, this signal model transfer function model or model working model either it is time domain or frequency domain model of a process is obtained. And based on the model information or the based on the parameters of a model, parameters of the controllers are set.

So, the controller parameters are tuned as and when required for that one can do to keep some checks; suppose, whenever this error signal goes beyond some specified values then in that case, the relay can be switched ON and process information can be acquired and based on the updated process information, the PID parameters are updated. This is what is done in on-line tuning of PID controller. So, design of controllers we have already studied in our previous lecture. Now, focus will be made on identification of the process dynamics using off-line and on-line tuning schemes.

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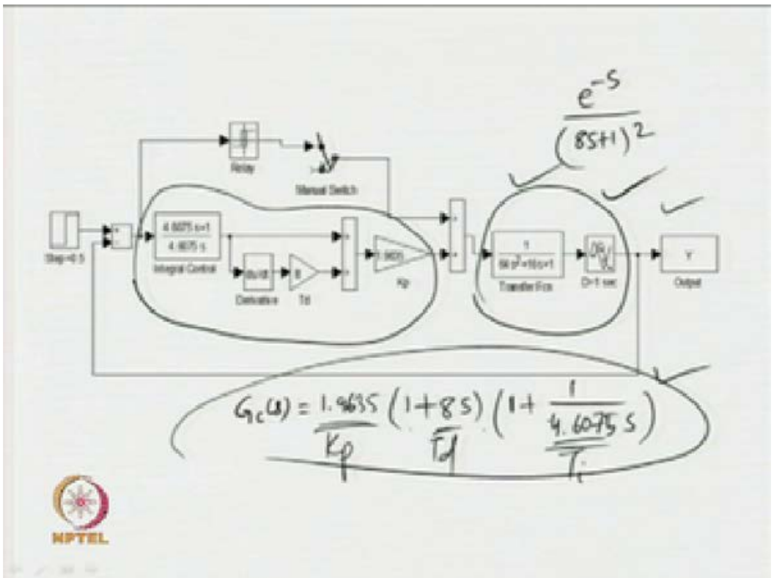

Now, let us come to this simulation diagram. So, in this simulation diagram we have considered a second order process dynamics which is given by e to the power minus s upon 8 s plus 1 square. Now, the process is connected to a series PID controller; the series PID controller is given as, G c s is equal to 1.9635 times (1 plus 8 s) times (1 plus 1 upon 4.6075s). So, the process has been subjected to a PID, series PID controller.

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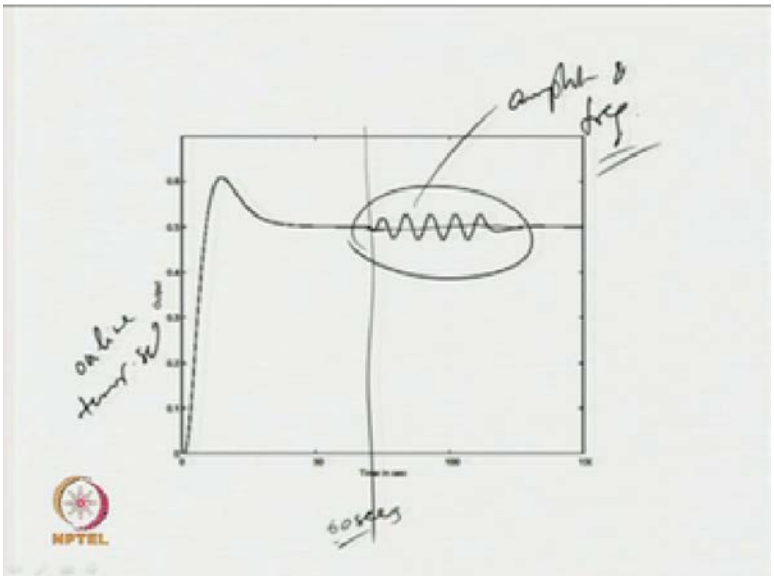

When a unit step input of magnitude 0.5 is applied then the output of the closed loop system becomes like this (Refer Slide Time: 07:40). So, till this point suppose we have got the output of the closed loop system, when the series controller is in action. Suppose, now we found that the controller is not performing, it is not giving desired or satisfactory time and frequency responses, then we need to acquire updated process information. So, it is necessary to obtain this process information, accurate process information based on which again the controller parameters can be updated to achieve desired time and frequency responses from the closed loop system.

Now, relay will be switched ON. So, when the relay is switched ON at time t equal to 60 seconds then, the output of the closed loop system becomes likes this. So, we have got this is the steady value. So, the output is oscillating around the steady state output of the closed loop system.

Now, measuring its amplitude and frequency and using the frequency it is possible to estimate dynamic model of the process. And from the parameters of the dynamic model, it is not difficult to find updated values for K p, T d and T i. So, this is the K p proportional gain. This is the T d derivative time constant, and this is the integral time constant of the series PID controller. This is how this simulation diagram can be used to induce limit cycle output in the case of on-line tuning scheme. So, this plot basically shows of the results of the simulation of some online tuning scheme.

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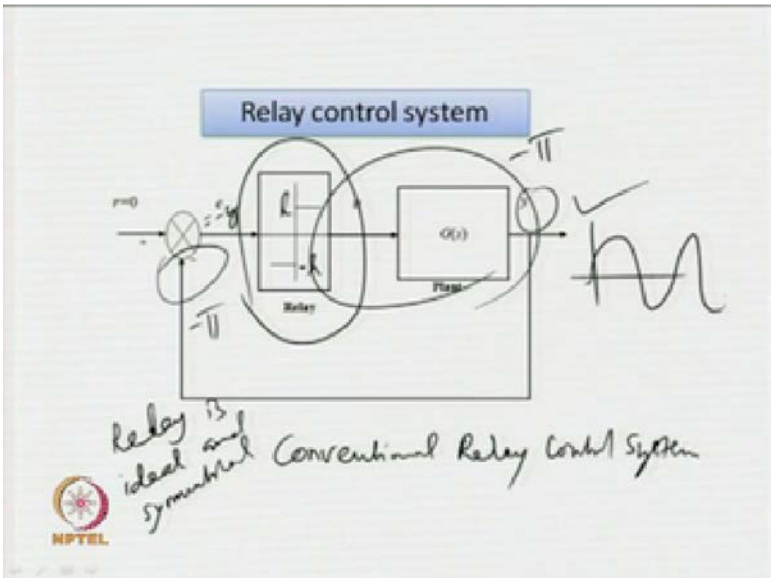

Let us go back to the original conventional relay control system. So, this is our conventional relay control system, why I say conventional? Because, the relay is assumed to be ideal we assume that, the relay is ideal and symmetrical. So, let the relay height be h and this be minus h. Then we call this as the relay setting. Now, these will certainly the relay with certain heights h will with the relay heights of h and minus h, a process will certainly be subjected to limit cycle output; and what sort of output we will get in this autonomous relay control system?

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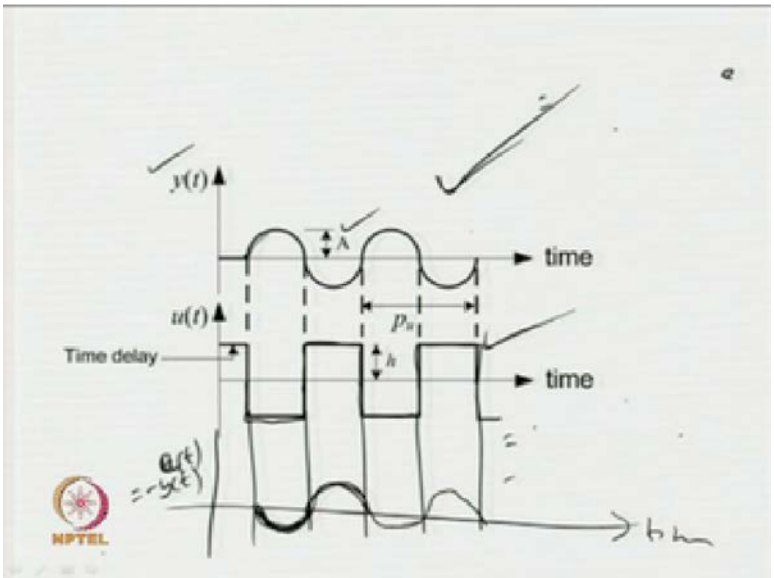

The output can assume this form (Refer Slide Time: 11:08). So, let the output of the autonomous relay control system be shown as like here. So, y t versus time t is given, where the oscillation starts from here and we have got sustained oscillatory output of this form.

Now, let the peak amplitude of the output be A, this is the peak amplitude. So, A is the peak amplitude of the limit cycle output signal; and let p u be the time period, p u is the time period of the limit cycle output signal, the subscript u stands for ultimate; it is also known as ultimate period. So, we can give two names to this variable p u, either it is known as time period or ultimate period. Why it is called ultimate? Going beyond that, there will be unbounded output. So, this is the ultimate period of the limit cycle output from the relay control system.

Now, when the output assumes this form then, the output of the relay will assume this form. Why the output of the relay will assume this form? Because, y t is like this, then definitely u t e t sorry e t versus time t can be plotted. So, this is y t, e t is equal to minus y t. If you carefully observe the block diagram here, r equal to 0, e is equal to minus y. So, e is equal to minus y. Therefore, we will plot the e first. So, when the e takes this shape then when as per the relay, relay is a device which gives us; if you look at, when the input is negative, the output will be output of the relay will be less to some minus h value.

So, when the input is negative for this span of the input relay input, the relay output will be like this (Refer Slide Time: 13:54). Similarly, for the positive span of the relay input, the relay output will be like this. Thus we get rectangular pulses as output of the relay. So, we get typical closed loop output, relay output from a conventional relay control system. So, please keep in mind, whenever there will be conventional closed loop relay control system, the output will definitely assume some form, which will be of the form of limit cycle and the output of the relay will be of rectangular or square shapes.

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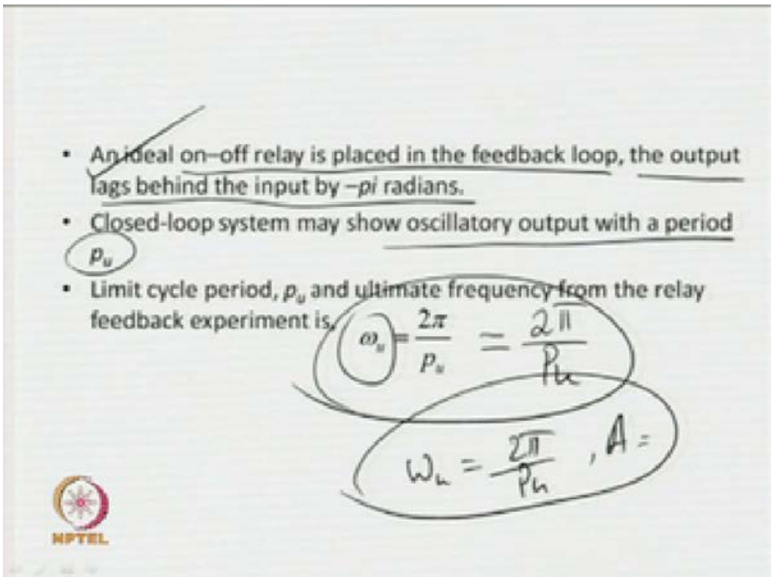

Next, some characteristics of relays will be studied. What we have done? An ideal on-off relay is placed in the feedback loop then, the output lags behind the input by pi radians. So, the relay is forcing, this relay is forcing (Refer Slide Time: 15:11), the planned output to be of sustained oscillatory and that relay is also introducing a phase lack of minus pi; and another phase lack of minus pi is there, due to the negative feedback. Therefore, we get sustained oscillatory output from the relay control systems.

Now, the oscillatory output is assumed to have a period of p u. And let the frequency of the output signal be denoted by omega u, which can be also obtained from the ultimate period or time period from the relationship 2 pi upon p u. So, omega u is equal to 2 pi upon p u; and A is the peak amplitude; these are the two information's, which can be obtained easily from the oscillatory output, and these two measurements also can be made easily.

How will be able to obtain these measurements so easily? Now, with the help of peak detectors it is not difficult to obtain the peak amplitude of a signal. Similarly, using the zero crossing detectors, it is possible to measure the periods or time period or half period of a rectangular pulse. So, using peak detectors and zero crossing detectors, we can easily estimate omega u and A from the limit cycle output.

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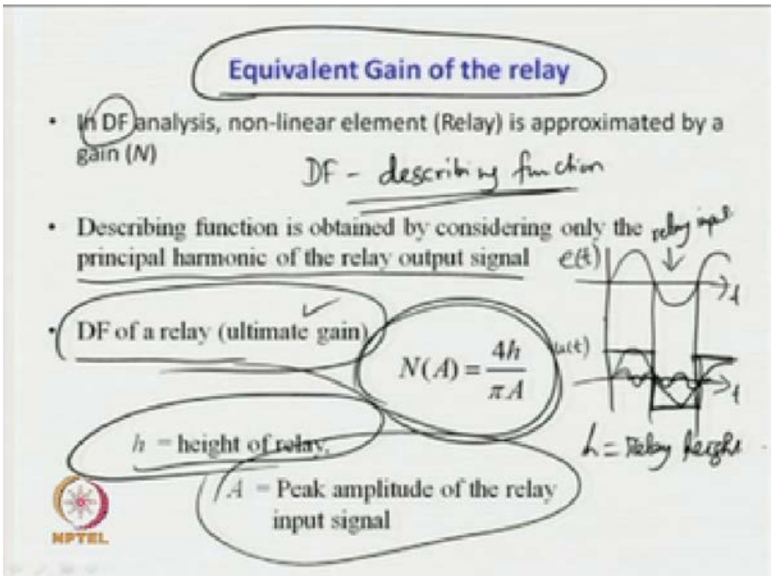

Now, we have interest to analyze to the closed loop relay control system. Now, if the relays as we know, let us go back to the original system (Refer Slide Time: 17:22), we started with the frequency domain identification technique with a closed loop, which had got a proportional controller initially. So, we had some proportional controller; and when the proportional controller gain was available to us, it was very easy for us to estimate two unknowns of transfer function model of a process dynamics.

So, if the relay also can be represented by some equivalent gain, if the relay dynamics or the relay characteristics can be represented by some equivalent gain like the gain of a proportional controller then, we can make use of the same loop phase and loop gain criterion to estimate unknown of the transfer function model of a process dynamics.

There is the reason why we shall go for calculating the equivalent gain of a relay. What we mean by equivalent gain of a relay, can we represent the relay, can we represent the dynamics of a relay by a simple gain which will be known as equivalent gain? Yes, that is possible using the Describing Function analysis, this D F stands for Describing Function.

So, using the describing function analysis, it is possible to find equivalent gain of relays. Please keep in mind, the gain will not be exact because, we shall make use of approximations in the analysis and that will enable us to get simple expression for the gain of a relay. Now, using describing function analysis and using the principal harmonic component of the relay output signal, it is possible to find equivalent gain of a relay.

What we mean by principal harmonic of the relay output signal? What we have seen, the relay input is found to be of the form like this (Refer Slide Time: 19:35), when the relay input is like this, this is suppose e t versus t; then we can have a plot for the relay output, this is the relay input, so relay input signal can be shown like this. Then the relay output signal will be of this form, so it is our u t versus t.

Now, if I carefully observe this relay output signal, basically these are rectangular or square pulses; therefore, it contains all harmonics infinite number of harmonics is present in this signal. Now, this signal can be analyzed using some techniques. Now, using the Fourier series technique definitely, the rectangular pulses can be represented by infinite number of sinusoidal term. So, if we concentrate on the principal harmonic or fundamental harmonic component of the output signal, then we can get the equivalent gain of a relay.

So, this signal u t can be made up of infinite number of terms I can try with a simple one first, then let us go to third harmonic components; then we will have fifth harmonic components and so on, infinite number of terms when added that will give us a rectangular pulse.

So, when we use only the principal harmonic component of the relay output and relay input, ratios of the two then we get the describing function of a relay. So, the describing function of a relay an A can be given as 4 h upon pi A, where h is the relay height h is the relay height. So, h is the relay height and A is the peak amplitude of the relay input signal.

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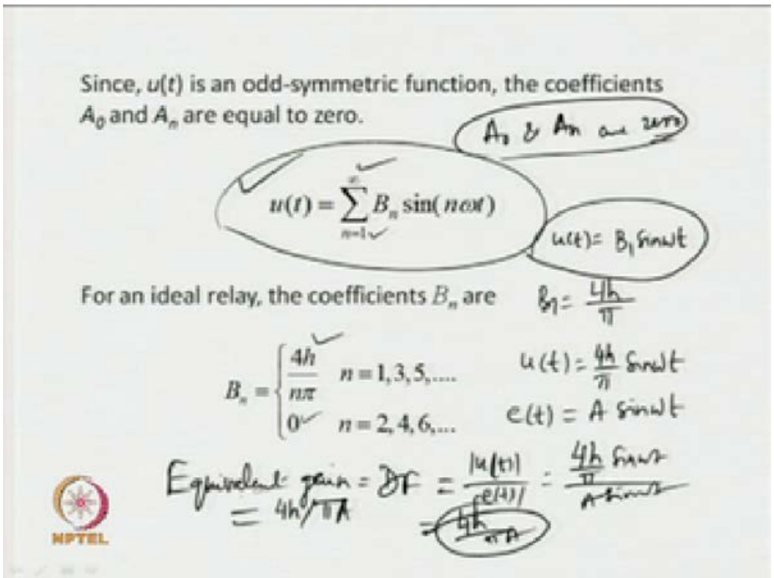

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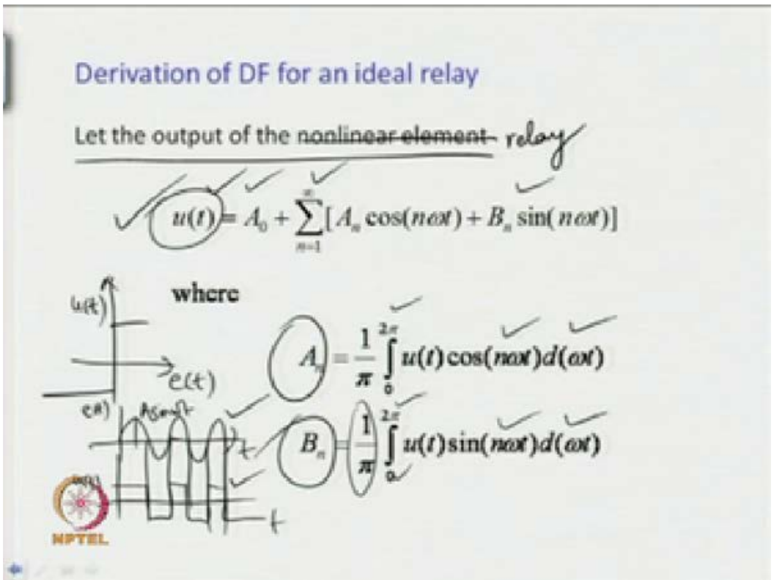

So, the equivalent gain of the relay is given by 4 h upon pi A. The equivalent gain of the relay is derived as 4 h upon pi A. How we have obtained that one we shall discuss in detail.

Let the output of the relay in this case, the non-linear element is a relay; for our case, we have employed a non-linear element named as relay. Therefore, let the output of the relay be expressed as u t is equal to A 0 plus sum of n from 1 to infinite A n cos n omega t plus B n sin n omega t; where A n is further defined as 1 upon pi integration from 0 to 2 pi u t cos n omega t d omega t; and B n defined as, 1 upon pi integration from 0 to 2 pi u t sin n omega t d t. So, this is the Fourier series expansion of the rectangular pulses u t.

Let us go back to the signal level again. What we are having basically is? We have a relay, the input to the relay is e t, and the output from the relay is u t. Now, e t is given as some sinusoidal signal like this; then let us assume this amplitude to be A therefore, the input to the relay can be represented by A sin omega t. The outputs of the relay are rectangular pulses now, which have got infinite number of terms like this (Refer Slide Time: 23:47).

So, this is u t versus t and here you have got e t versus t. So, we have got infinite number of transfer u t, when we concentrate on the principal harmonic component only, then u t becomes u t is equal to B 1 sin omega t. When we concentrate on all the terms which give us rectangular pulses then, u t expression for the u t becomes u t is equal to summation of n from 1 to infinite B n sin n omega t.

Now, keep in mind A 0 and A n are 0, because we have got symmetrical output; the input is also symmetrical, the output is also symmetrical, they are not only symmetrical they have odd symmetricity also. That is why both the coefficients A 0 and A n are 0. Now, the principal harmonic component of the output signal, relay output signal can be given as u t is equal to B 1 sin omega t for B 1 is equal to 4 h by pi.

Please keep in mind, the B co-efficient are computed and found to be B n is equal to 4 h upon n pi for n having odd values, odd numbers from 1 and B n is equal to 0, when n assumes even numbers starting from 2. So, when B 1 is equal to 4 h by pi I get u t as 4 h by pi sin omega t, but the input to the relay e t is A sin omega t. Now, describing function is nothing but, the ratio of the principal component of the output signal to the input signal.

Therefore, the equivalent gain of the relay equivalent gain of the relay is nothing but, the describing function which is given as now magnitude of u t to magnitude of e t. So, that gives us 4 h by pi sin omega t upon A sin omega t and ultimately we get 4 h by pi A. Therefore, the equivalent gain is found to be 4 h by pi A. So, that is how we have to obtained the equivalent gain or describing function of a relay, which is often known as ultimate gain as N A is equal to 4 h by pi A. So, I am introducing so many symbols, N A where N A appears in the closed loop system.

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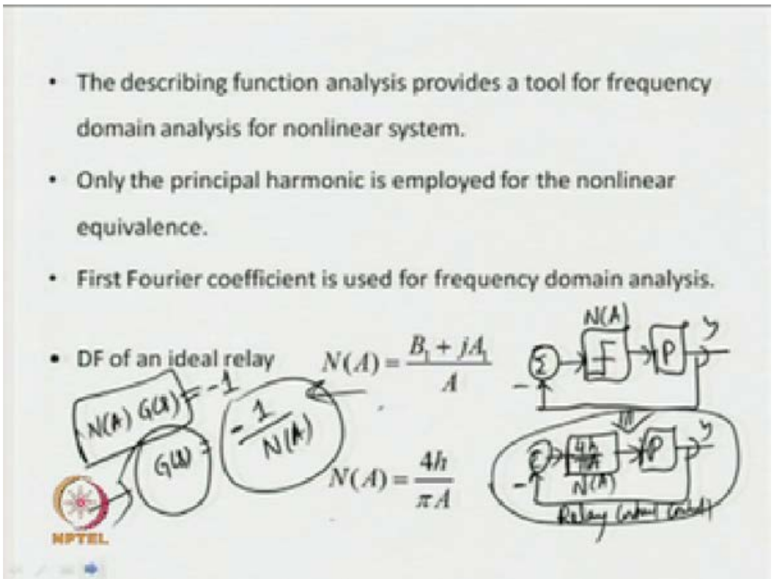

Let me go to this page and let me explain. So, we have got the autonomous relay control system given by a summer; then we have got the relay over here, then we have got the process, then output and negative feedback. These can equivalently be represented by an autonomous loop with the gain of 4 h by pi A, p, y. So, this equivalent closed loop transfer closed loop system very much looks like the closed loop system, we had with a proportional controller.

So, as if we have a proportional controller in the closed loop. So, imagine that we have got a proportional controller in the loop; then the frequency domain based identification technique can be made use of now to estimate unknown parameters of specific transfer function models of a process. Now, this is the N A, the relay is represented by the describing function N A.

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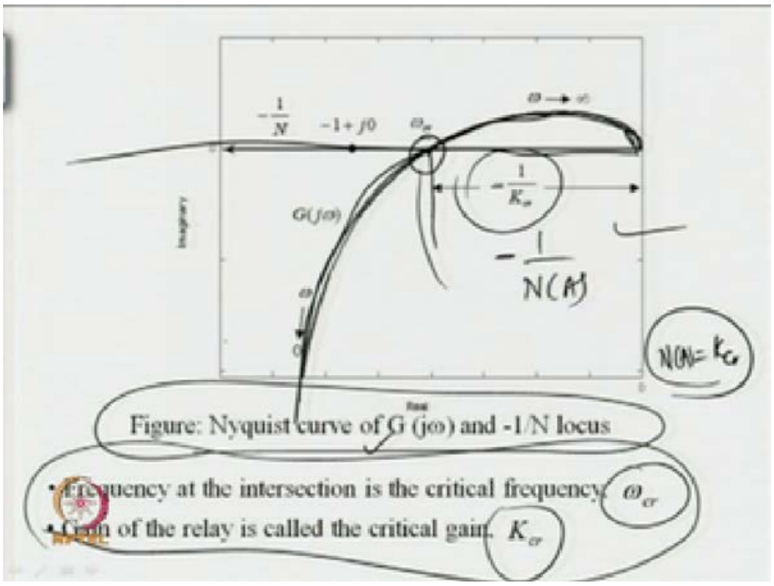

Now, let us have the Nyquist curve of the process dynamics and minus 1 upon N A. Go back to this, what we have in the relay control system now (Refer Slide Time: 28:59), this is our relay control system. The relay has been represented by a describing function N A. So, the loop gain is nothing but, N A times G s in frequency domain. So, this can be plotted, the loop can be plotted and the Nyquist curve can be obtained.

Now, individually we can plot the Nyquist curve of the process dynamics. Suppose, the Nyquist curve of the process is given by this one, then the describing function can be plotted also. So, when I take the plot of minus 1 upon N A, why we are doing like that? Because, we know that for the closed loop system N A G a 1 plus N A G a is equal to 0 or N A G a is equal to N A G s is equal to minus 1.

So, as we know the characteristic equation of the closed loop system is nothing but, 1 plus N A G s is equal to 0. So, that can be re-written as, N A G s is equal to minus 1.Further, I can write G s is equal to minus 1 upon N A. So, now if we plot the Nyquist plot of G s and Nyquist curve of minus 1 upon N A, the meeting point will give us the critical point. Now, this is the Nyquist curve for the process and the negative real axis is the Nyquist curve for the minus 1 upon N A inverse of the describing function.

So, when both are plotted, this is the meeting point and the meeting point definitely gives us the phase cross over point or the critical point. From this critical point, it is possible to estimate the critical frequency or phase cross over frequency and gain of the relay is now called gain of the relay is actually, if it is denoted by K c r. Suppose, the gain of the relay is N A is represented by some symbol K c r. Then we have got the gain over here as minus 1 upon K c r, definitely minus 1 upon K c r which is nothing but equal to minus 1 upon N A over here. So, this is how using the Nyquist plot one can estimate the critical gain and critical frequency of a relay control system.

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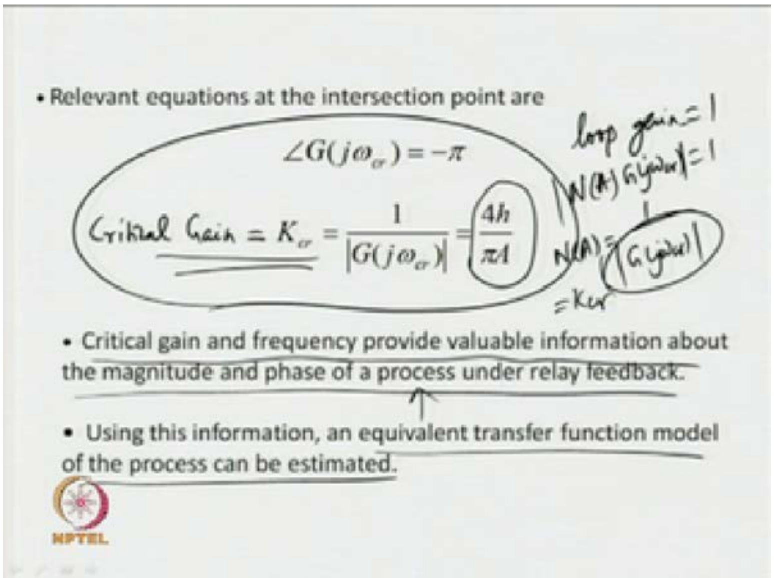

What more we can get using the Nyquist plot, we can get some simple expressions. Now, the phase of the process at the critical frequency will be equal to minus pi; why that is so? If we consider the loop gain, loop phase also we know that loop phase is equal to minus pi. In this case, what we have in the loop? N A G s as the loop gain, which in frequency domain gives us N A G j omega; at the critical frequency, the loop gain will be N A G j omega c r. So, this phase will be equal to minus pi.

So, the same information is obtained from the Nyquist plot. Again the gain of the, critical gain of the relay, so the critical gain, it is denoted by K c r is given as 1 upon G j omega c r magnitude which will be definitely as 4 h by pi A which is nothing but, the describing function of the relay.

Now, why that is so? If we go back to again the loop gain has to be 1. Again, I will write the same thing we have got N A G j omega c r magnitude is equal to 1, which will give us the N A is equal to 1 upon G j omega c r magnitude; but, N A is denoted by K c r.

Therefore, K c r is equal to N A is equal to 1 upon G omega c r magnitude, a magnitude of G j omega c r inverse.

Now, all these things can be made use of later on, to estimate unknown parameters of a transfer function model of a process dynamics. Some conclusions from the analysis, above analysis can be made like this. Critical gain and frequency provide us valuable information about the magnitude and phase of process under relay feedback.

So, the critical gain and frequency bears same information as the information we get from loop gains and loop phase. Using the above information, an equivalent transfer function model of the process can be estimated.

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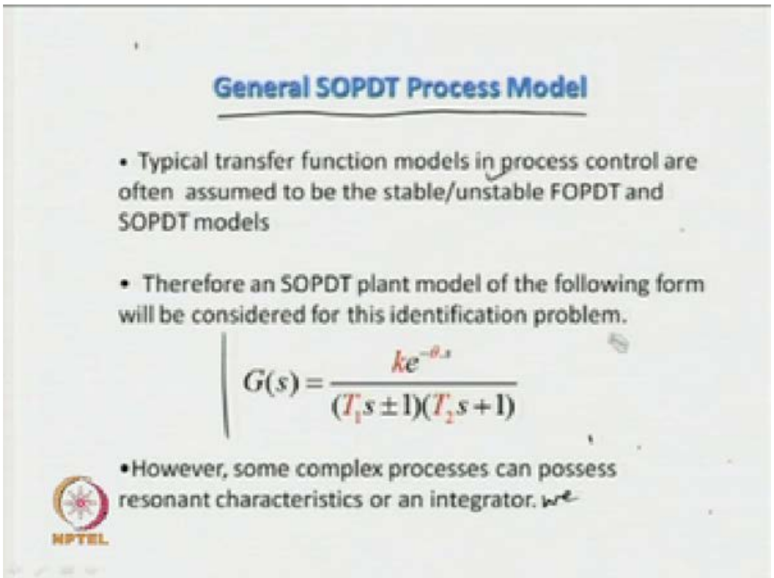

Now, we shall go to the varieties of process model; whenever, we talk about process model, what are process models for the case of identification of process dynamics? We have to imagine, we have a real time system or real time process, it has its own dynamics I do not know the dynamics of the real time process.

So, initiating some tests, either in frequency or in time domain, we can measure input and output parameters of the closed loop system or of the process or relay individually; and then we should be able to get enough information about the process, real time process. So, the real time processes can be represented by some assumed form of dynamics. What are those one? Those are known as transfer function models.

So, a process dynamics can be represented by transfer function models. In that case, we can have infinite number of models for a transfer function; then which one model to be chosen or how to go about that, how to choose specific type of transfer function model, lot of questions can arise.

Now, we shall see, what is the typical type of transfer function models used in literature for representing dynamics of real time processes. Let us consider a general second order plus dead time process model. Now, typical transfer function models in process control are often assumed to be the stable unstable First Order Plus Dead Time and Second Order Plus Dead Time types. What is the full form of F O P D T? This stands for First Order Plus Dead Time. Similarly, S O P D T stands for Second Order Plus Dead Time.

Apart from this, we have got stable first order plus dead time, stable second order plus dead time model, unstable first order plus dead time model, unstable second order plus dead time model, stable integrating plus dead time model, unstable integrating plus dead time model. So, varieties of transfer function models can be described. So, let us give the transfer function of all those models, because ultimately we are going to identify the parameters of those models in subsequent lectures.

Therefore, we need to know, varieties of transfer function models available in literature are helpful in representing the dynamics of real time processes. Now, the process model transfer function of a of a process dynamics in second order plus dead time form can be given as, G s is equal to K e to the power minus theta s upon T 1 s plus minus 1 times T 2 s plus 1. Now, what is K, K is the steady state gain of the process. What we mean by steady state gain? When some d c signal is passed through the process, the gain of the process will be K; suppose then the output of the process will be K times the d c signal.

Suppose, u 0 is the input to a process, it is a d c signal u 0 is d c in that case, the output of the process will be K time's u 0; then we call K as the steady state gain of the process. What is T 1, T 1 and T 2 are the time constants of the process model. What is theta? Theta is the time delay or dead time or time or transportation delay transportation delay associated with a process.

So, after introducing all those unknowns or all those variables in a second order plus dead time model; let us see, how we will be able to obtained varieties of process models from this general second order plus dead time process model. Why this is known as general second order plus dead time model? Because, by setting various values for K, theta, T 1 and T 2, it will be possible to obtain a variety of transfer function models from this general second order plus dead time process model.

So, when theta equal to 0, then G s gives us K upon (T 1 s plus minus 1) (T 2 s plus 1) an all pole transfer function model; when we have got T 1 plus 1 in their denominator, we have got an all pole second order stable transfer function model. So, we have got a stable second order transfer function model of all pole form; when we have got in the denominator, T 1 s minus 1 over here, T 1 s minus 1 then we have got an all pole unstable second order transfer function model.

So, by the setting of theta equal to 0, we have been able to get stable and unstable second order transfer function model, all pole transfer function model. Now, when theta is equal to 0, and T 2 is equal to 0, then G s becomes K upon T 1 s plus minus 1, which can give us transfer function models for all pole first order transfer function model. And the transfer function can be stable or unstable depending on the sign positive or negative in the denominator.

Now, limiting the value of T 1 to infinity; suppose, T 1 is very large such that K upon T 1 is finite. Then what G s will be? Now, G s can be represented as K e to the power minus theta s upon T 2 s plus 1. So, when T 1 is very large and T 1 inverse K is finite at that time, we get a transfer function which is often known as a stable first order plus dead time model.

Similarly, when T 2 is very large such that K upon T 2 is finite, and G s is obtained as K e to the power minus theta s upon T 1 s minus 1, then we get some unstable first order plus dead time model. So, thus it is possible to obtained stable and unstable first order plus dead time models with the suitable choice of T 2 and T 1. Next, when T 1 tends to infinity and again K upon T 1 is finite in that case, what we get? We get a transfer function model G s is equal to K e to the power minus theta s upon T 1 s (T 2 s plus 1).

So, we have got, when T 1 tends to infinity and K upon T 1 is finite; we have got a transfer function model which is often known as stable second order integrating transfer function model. So, we get stable second order integrating plus dead time transfer function.

Now, when T 1 and T 2 tends to infinity or becomes very large; T 1 tends to infinity, T 2 tends to infinity such that, K upon T 1 and K upon T 2 are finite; then, we get integrating process models. But, when T 1 is 0, then we get stable first order plus dead time model. So, when T 1 is 0, T 1 is equal to 0, then we get G s is equal to K e to the power minus theta s upon T 2 s plus 1. So, thus we get some stable first order plus dead time transfer function model.

When T 2 is equal to 0, we get a transfer function of the form K e to the power minus theta s upon T 1 s plus minus 1; therefore, that gives us stable or unstable first order plus dead time transfer function model. So, with the choice of proper choice of T 1, T 2, K, theta, it is always possible to get a varieties of transfer function models from this general second order plus dead time process model.

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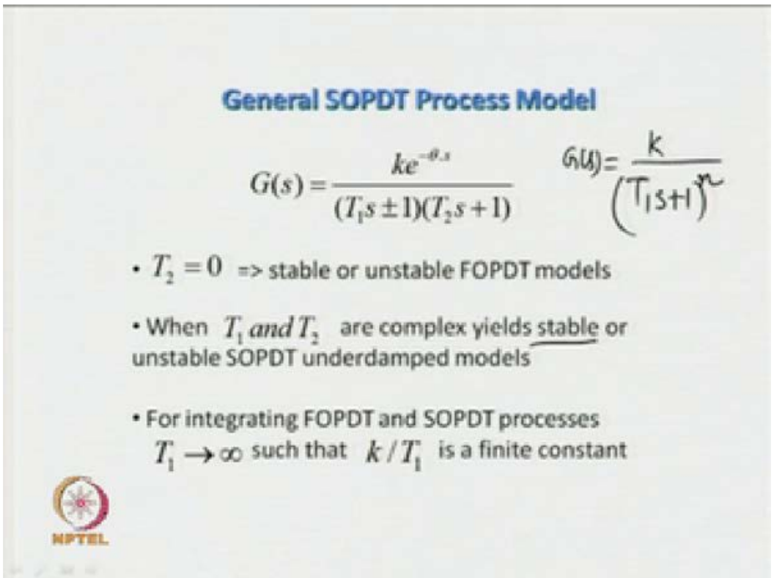

Now, this general second order plus dead time model, when T 2 becomes 0 gives us stable or unstable first order plus deadtime models. Similarly, when T 1 and T 2 is complex, this is very important; when T 1 and t 2 assume complex values or complex numbers, when T 1 and t 2 become complex numbers at that time, we get a stable or unstable underdamped second order plus dead time transfer function models.

To obtain under damped transfer function models, T 1 and T 2 must be complex numbers. Again, for integrating first order plus deadtime and second order plus dead time process models, T 1 should be a large value such that K upon T 1 is a finite constant.

So, using all these, it is possible to get a variety of transfer function models from the general second order plus dead time process. Are there any other types of representations? Yes, we have got a variety of representations, not only like this. For processes with time of significant time lags, the representations are given in the all pole form.

Now, for a process with sufficient time lags are having pole multiplicity, the representations are often given as in the form of T 1 s, K upon (T 1 s plus 1) to the power square for second order process model or it can be of power n for higher order process models.

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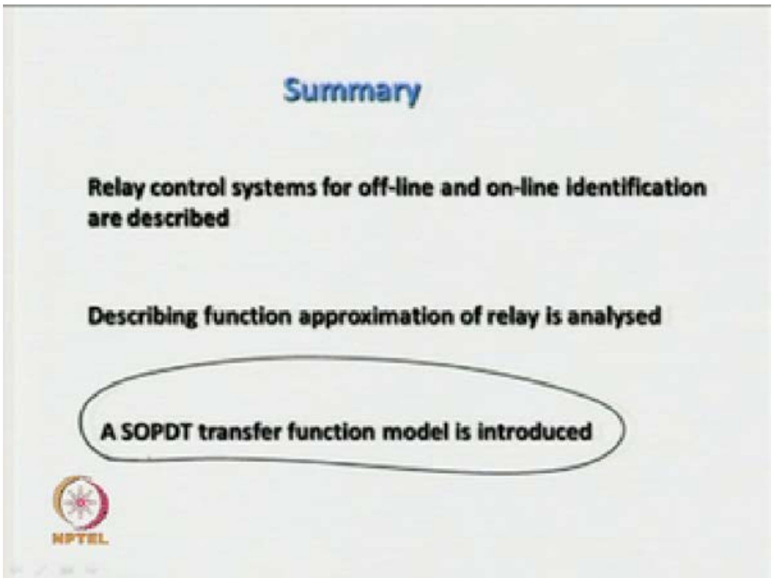

Now, in summary we have seen various types of relay control systems. We have discussed two types of relay control systems often known as off-line and on-line relay control systems, which are used for not only identification of process dynamics, but also for off-line and on-line tuning of controllers.

Again, we have discussed describing function approximation of relay. Describing function approximation of relay enables us to make use of the critical gain and critical phase in estimating unknown parameters of a process model. A second order plus dead time transfer function model is introduced which is general in nature. It is general in the sense, it has got generality in the sense that, the same second order plus dead time transfer function can be used for representing stable and unstable integrating underdamped processes.

Also, the same general second order plus dead time transfer function model can be used for representing a variety of transfer function models; those transfer functions can be stable, unstable, integrating, resonating and so on.

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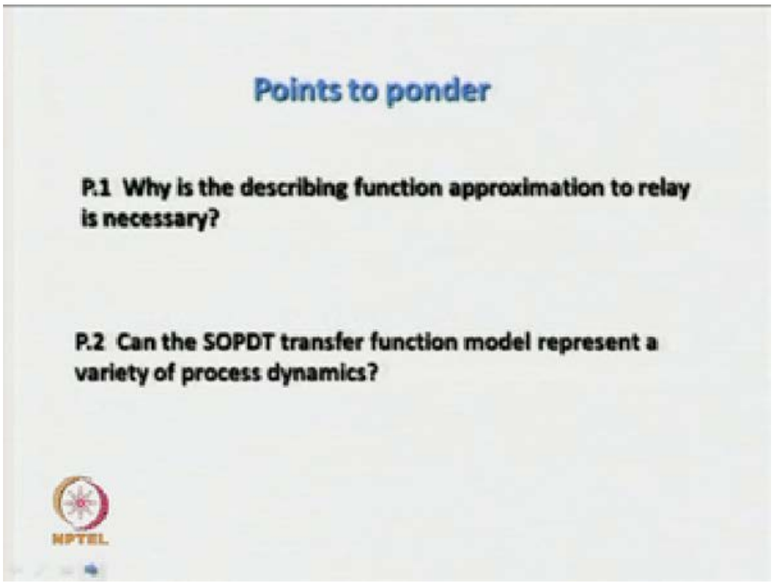

Points to ponder: first point could be like this, why is the describing function approximation to relay is necessary? It is easier to estimate the unknown parameters of an assume transfer function model with relative each, when the relay is described by an equivalent gain, when the relay dynamics is available in the form of equivalent gain.

Second point is, can the second order plus dead time transfer function model represent a variety of process dynamics? Yes, it can represent a whole variety of process dynamics, often encountered in process industries with a suitable choice of values for the variables of second order plus dead time process model. It is possible to obtain stable, unstable, integrating, resonating process models with or without time delays and time lags that is all in this lecture.