Information Theory, Coding and Cryptography
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Module - 35
Space Time Codes
Lecture - 35

Hello, and welcome to our next module on Space Time Codes. Let us start with a brief outline of today's talk.

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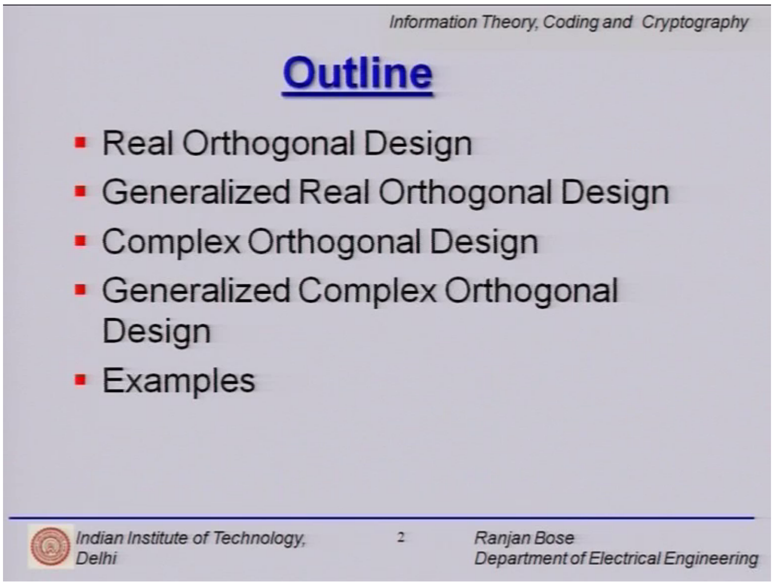

We would look at a real orthogonal design. We will look at a generalized real orthogonal design will revisit that. And then we have studied complex orthogonal design what we will do is then move on to quasi orthogonal design which will help us move from single symbol decoding to decoding in pairs. And finally, we would look at some examples.

So, let us see how we can proceed we have done some of these earlier and they are important enough to be revisiting them again. So, let us go with that. So, let us recollect what we learnt about real orthogonal design in the context of space time block codes.

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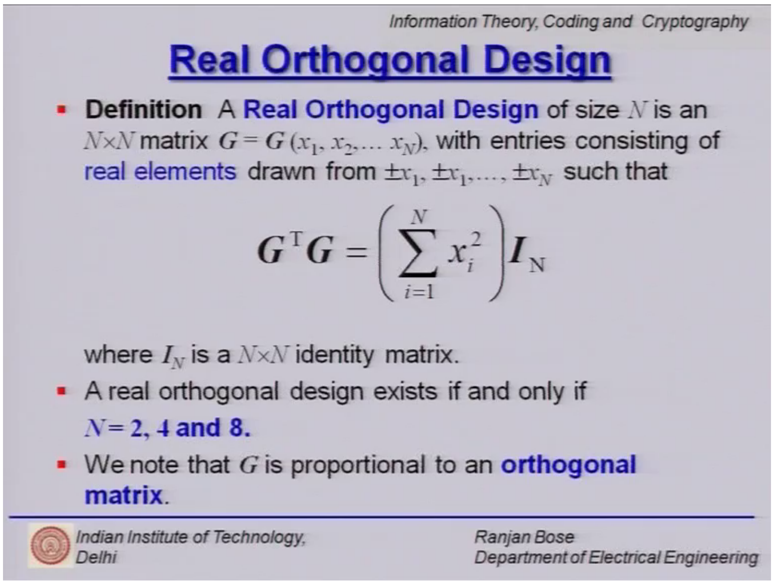

We defined a real orthogonal design of size N as an N cross N matrix. So, it is a square matrix G with the increase coming from x 1, x 2, up to x N such that G transpose G is summation i equal to 1 through N, x i squared I N, where I N is an identity matrix. And we have also seen that real orthogonal designs exist for limited number of cases namely for N is equal to 2, 4 and 8 and we also know that G is proportional to an orthogonal matrix.

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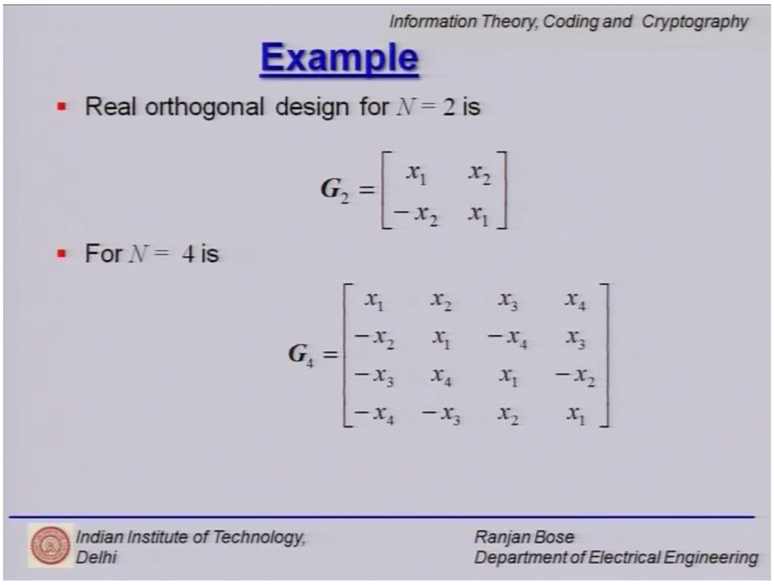

We look at some examples well that three examples possible G 2 for N is equal to 2, G 4 for N is equal to 4. So, both of these cases if you do G 2 transpose into G 2 you will get elements only along the diagonal and on diagonal will be 0.

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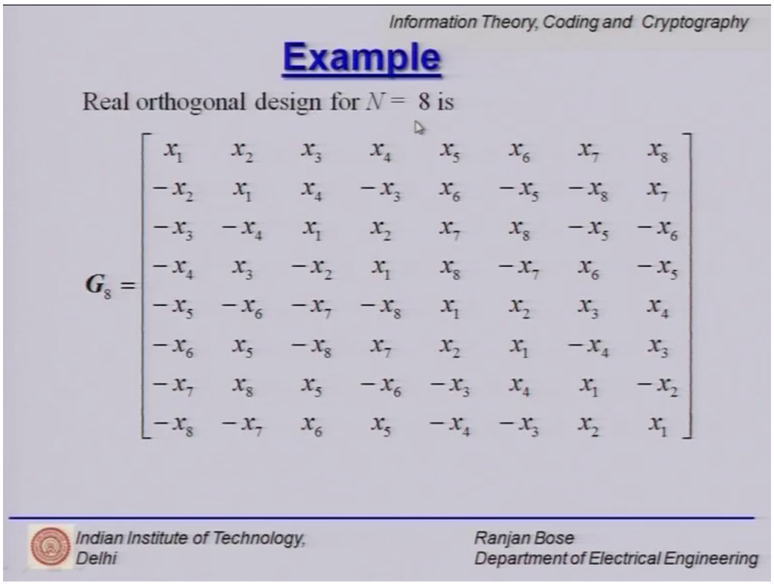

And finally, the example for N is equal to 8 which will allow you to transmit 8 symbols through 8 antenna elements in eight time slots. So, please note the columns correspond to the antenna elements. So, clearly this is the symbol that I will be transmitting from antenna element in time slot one the first row corresponds to what we are going to do N time slot 1. So, if I were to implement this in real life time slot one I send out x 1 from antenna element one, x 2 from antenna element two so and so forth to x 8 till antenna element 8.

Having done that, we go to our next time slot where we load minus x 2 from antenna element one x 1 from antenna element two, x 4 from antenna element three so and so forth till x 7 from antenna element eight and again I fire them and then I go to the next time slot which will transmitted row three and we repeat this till we are at the last row which is the time slot 8 where we send out these 8 signals.

So, what we realize is at the end of eight time slots we have been able to transmit x 1 through x 8. Therefore, the rate is 1 and we have seen that this orthoganality leads us to single symbol decoding.

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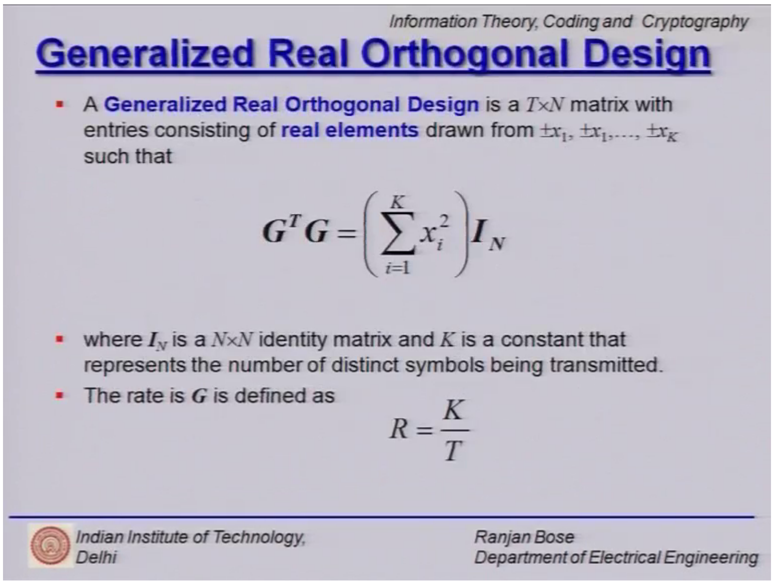

Then we said that we need to go beyond N is equal to 8 and therefore, we learn to tweak the real orthogonal design and we went on to study generalized real orthogonal design, where the matrix generative matrix is T cross N, where we go from plus minus x 1 plus minus x 2 up to plus minus x K, again the constraint is G T G is nothing but summation I equal to 1 through K x i squared into this identity matrix.

But, please note that this time we use the T time slots and therefore, since we have been able to send K symbols effectively the rate is K over T should we compromise on the rate.

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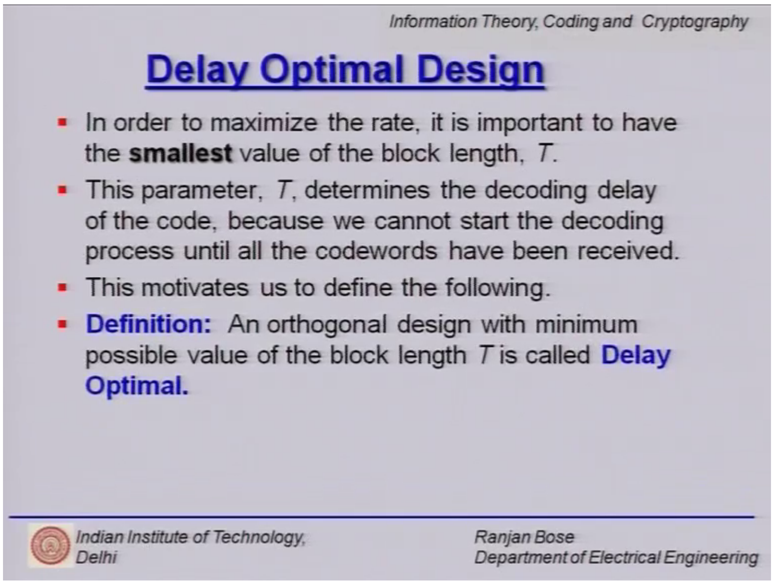

Then we would be interested in finding out the smallest value of T. So, we would like to send out the symbols in minimum number of time slots, because T determines the decoding delay, because it is very obvious that we cannot start the decoding process unless until all the symbols of all the T time slots are received.

So, we define the delay optimal as an orthogonal design with minimum possible value of the block length T.

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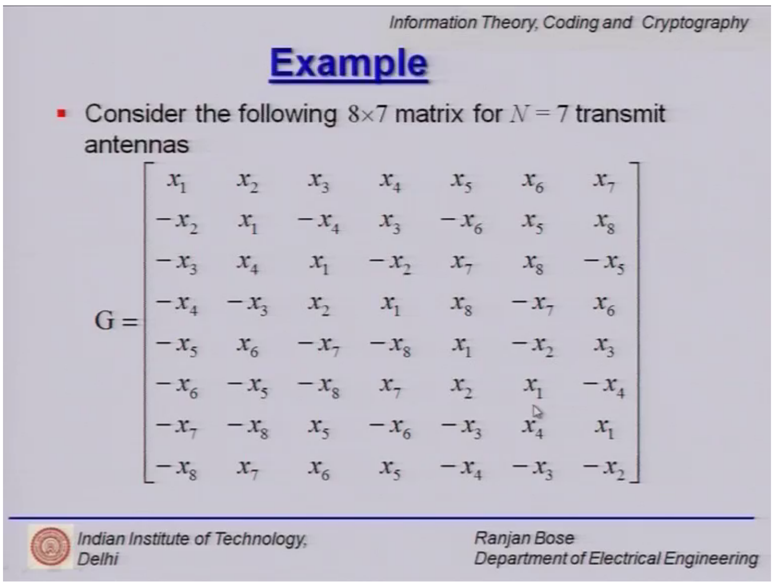

We did this definition earlier. Here is an example. This is an 8 cross 7 generator matrix, clearly it will be using 7 antenna elements it is. So, this x 1, x 2, up to x 7 goes through the first time slot and then the next and next one, but we are using eight slots to send out seven.

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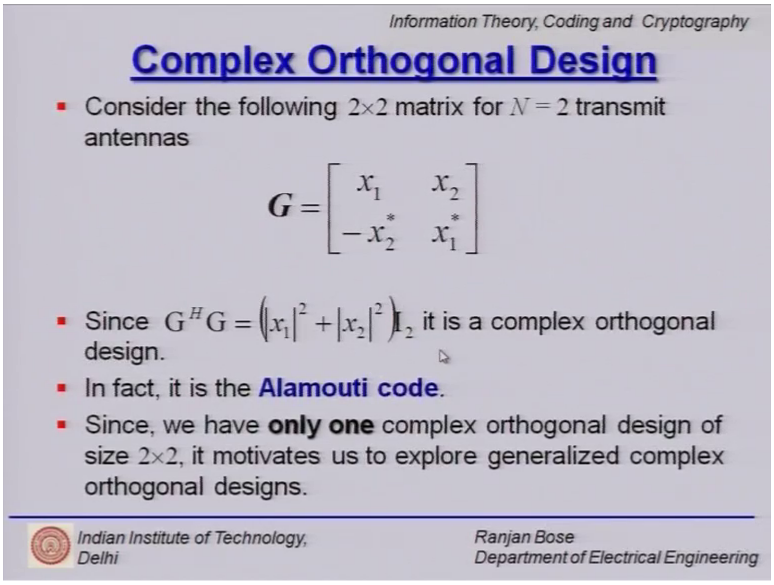

Then we moved on to complex orthogonal design when we remove this condition of real symbols. So, our constellation could be QPSK, QAM, but here we would like to use complex design and again we can verify that this is the only possible solution where G Hermitian G gives this x 1 squared plus x 2 squared into I 2. This is the Alamouti code we have studied and this is the only possible design forcing us to look at generalized complex orthogonal designs.

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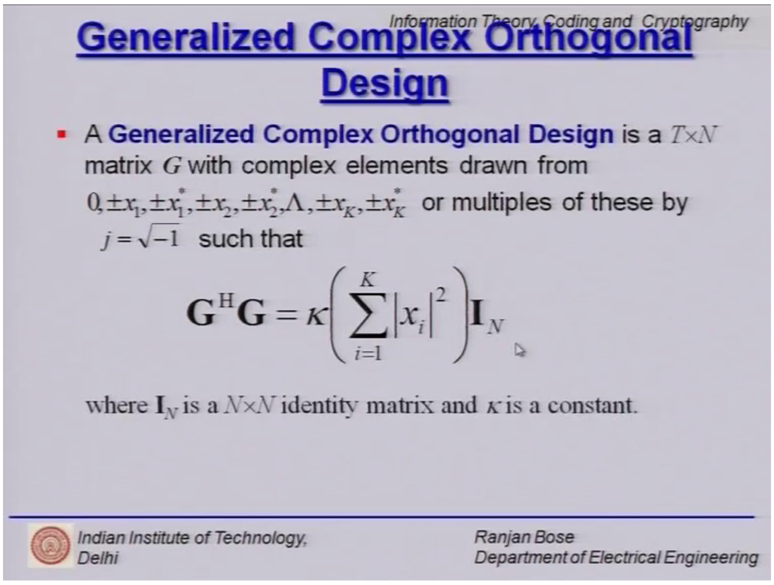

Here we start with the possible symbols coming from 0 plus minus x 1 plus minus x 1 star which is the complex conjugate plus minus x 2 plus minus x 2 star and so on and so forth up to x K star. And we again put this condition G Hermitian G is equal to some kappa times this standard I is equal to 1 through K summation x i absolute value squared into this identity matrix cross N cross N.

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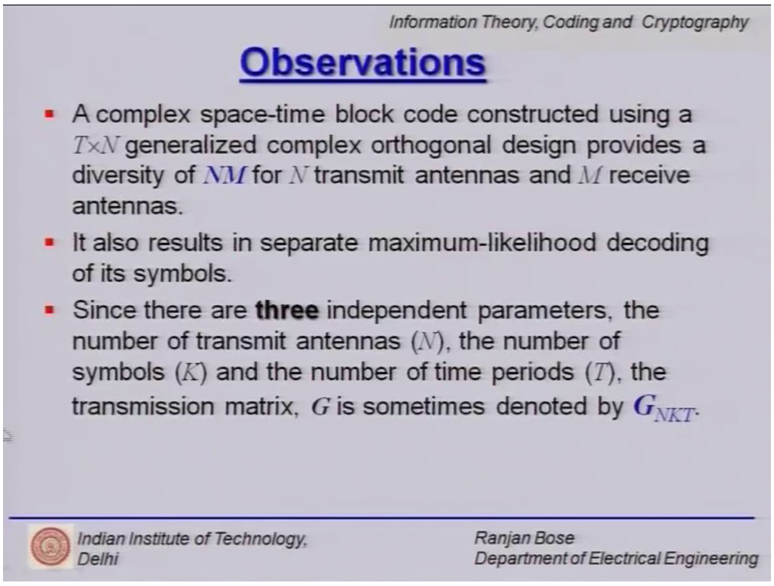

Then we make some observations that the generalized complex orthogonal design provides a diversity of N cross M. So, this is one of the very important things, we get tremendous amount of diversity gain using space time block codes and N is the number of transport antennas and M is the number of receive antennas. So, it is directly the product. The other good part is the decoding complexity. The maximum likelihood decoding the symbol by symbol because they have been able to decoupled the decoding operations at the receiver by using the information about the channel gains.

So, what we see is that we have three independent parameters, N, K and T. N is the number of trans mate antennas I can fix that will, I can have the number of time slots K, sorry the number of time periods T and the number of symbols K and I can play with them and come up with a generator matrix which can be denoted by G sub NKT.

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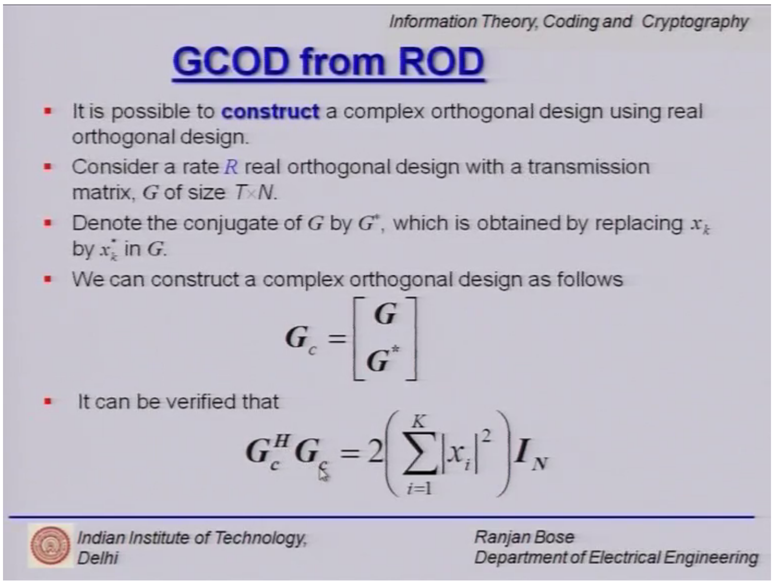

Now, we move forward and we see that we have already available some real orthogonal designs. Can we construct generalized complex orthogonal designs from this real orthogonal design? If we have that then we have a recipe. So, answer is yes, it is possible to construct a complex orthogonal design using real orthogonal design. When we say real orthogonal design you just mean that we have the generator matrix for that.

So, considering rate are real orthogonal design with transmission matrix G of size T cross N. So, this could be a generalized real orthogonal design. So, we denote the conjugate of G by G star and this is nothing but your placing all the elements x k with x k star x 1 by x 1 star and so on and so forth. And therefore, we construct a complex orthogonal design as follows. G c right is nothing but G the generator matrix of the real orthogonal design and G star.

We can easily verify that if you take this and come or compute G c Hermitian G c then it will you can check with the properties of G which is itself generalized real orthogonal design to give you the following. So, thus by construction we have generalized complex orthogonal design and we can go on and on with this.

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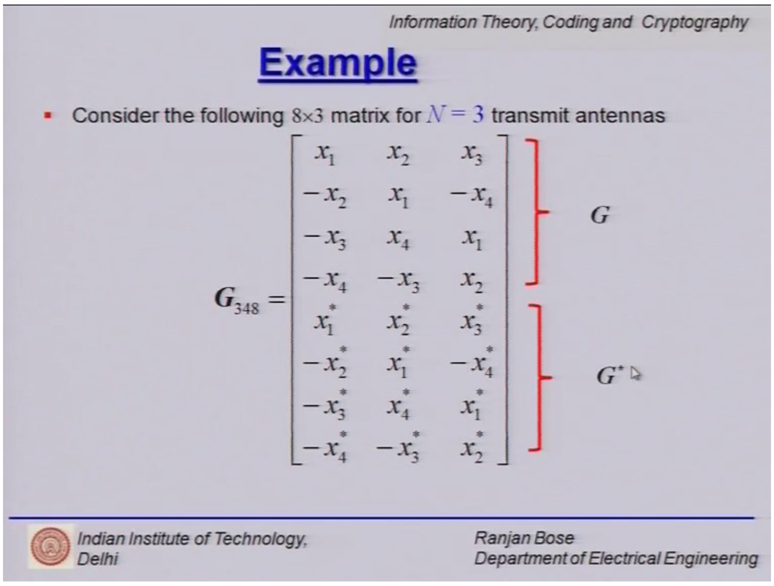

Let us take a very quick example. So, we would like to consider a three antenna system there are three transmit antennas. So, N is equal to 3 and we would like to construct generalized complex orthogonal design using a real orthogonal design.

. So, what we need to do is, we will put the first G here and G star here. So, if we substitute this G by its elements here I have this design that we studied earlier again this is a three antenna system and we have four time slots. So, it is kind of a generalized real orthogonal design, but please note in four time slots it is able to send x 1, x 2, x 3, x 4. So, it is a full rate generalize real orthogonal design.

All we have to do is substitute each x i by x i star and we fill up the lower half of this matrix. So, we have just mechanically substituted the complex conjugate of each of this. So, the top portion is my G and the lower portion is my G star and low and behold we have are new constructed complex orthogonal design a generalized complex orthogonal design available and one can verify that this will indeed give me G Hermitian G will give me elements only along the diagonal.

Now, please note here N is equal to 3, so, if you see the subscript N is equal to 3, 3 comes in here and we have used four symbols to be transmitted as you can see x 1, x 2, x 3, x 4. So, x so, K is 4. So, the next number is 4 and if you see this total design will require eight slots, right. The number of rows represent the number of slots, time slot. So, 8 is the third number here. So, we can denote this generator matrix as G 348. Later on we will look at the performance of this as we go along the lecture.

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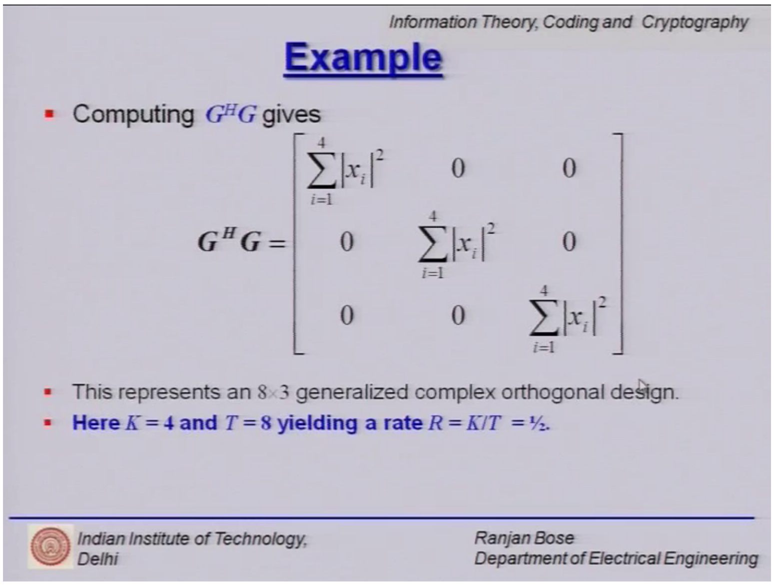

So, if you do the computation of this matrix G Hermitian G you will get the following along the diagonal you have summation i is equal to 1 through 4 x i squared and rest of the terms are all 0. So, this is the generalized complex orthogonal design.

So, K is 4, T is 8. So, consequently we have rate 1 by 2. Please, note here the top half was a full rate, this guy is also full rate, but will not increase the number of symbols they remain from x 1, x 2, x 3 and x 4 consequently doubling the number of time slots had led to halving of the rate. So, we pay in terms of rate. So, we were able to easily design it where the rate has gone down.

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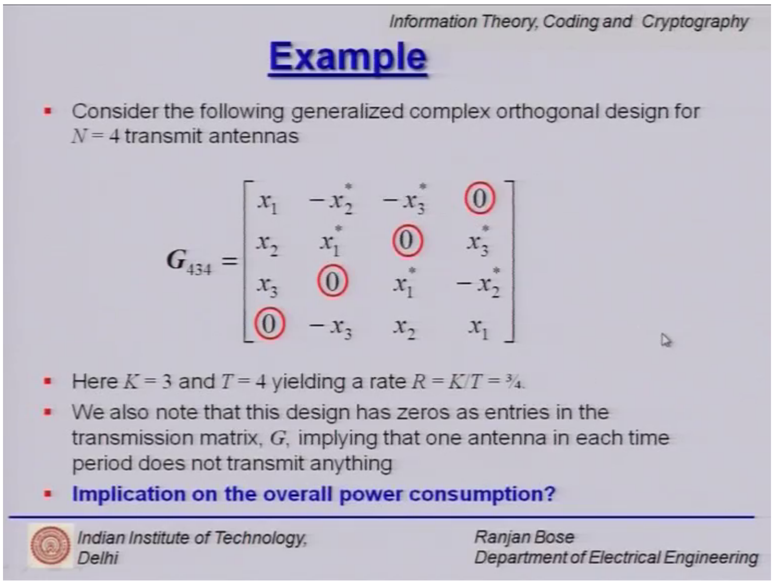

Let us consider another example. This time we have N is equal to 4, four transmit antennas. So, clearly there are four columns and then this a 434 design. So, we are trying to basically try to see that four transmit antennas N is equal to 4, but the number of symbols that we are sending is only three. So, x 1, x 2 and x 3 and their complex conjugate. So, at the second number is three and the number of time slots is four, that gives you the last 4. So, this is a G 434 design K is equal to 3, T is equal to 4.

Now, the definition of rate is K by T. So, 3 by 4 it is also clear from this that I have taken four time slots to send out three symbols. So, the rate is actually 3 by 4. But, what is very interesting is that even though four antennas and the rate is not too bad in the first time slot three antenna elements are firing and the fourth one is silent, there is a 0, I do not transmit anything.

In the next time slot antenna one, antenna two and antenna four of firing with antenna number three silent. So, where antenna number two does not emit anything and in the last time slot the first antenna element does not radiate anything. So, every time slot one of the antenna elements is silent, right. This is very interesting this has an implication on the average power that we are transmitting through the antennas. The rate is not bad, right and this is a complex orthogonal design of course, generalized complex orthogonal design.

So, these are very interesting variations that give you some additional benefits without compromising much on the rate. So, this of this thing shows you that each and every time slot make sure that one antenna element is silent. So, it has a very interesting implication on the power consumption that is also an important parameter here.

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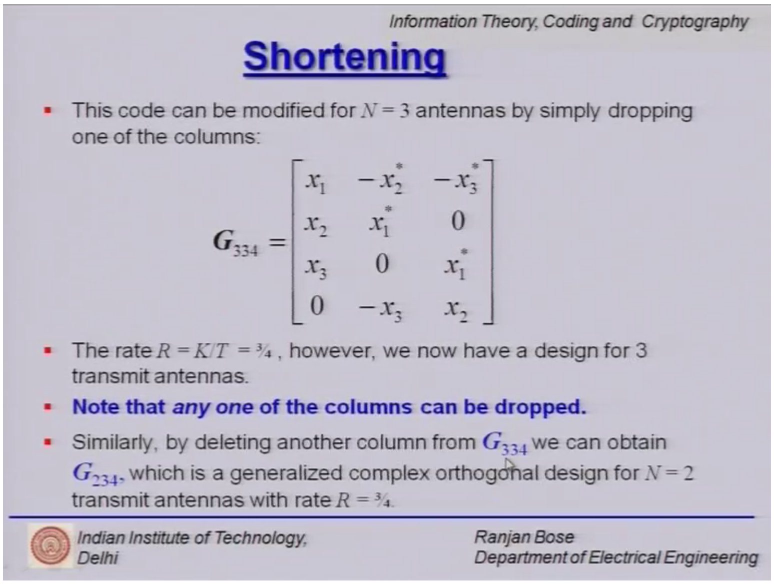

Now, let us talk about shortening, where we had earlier observed that it is possible to obtain code designs with fewer number of antenna elements simply by dropping columns. Why? Columns represent the what you transmit from certain number antenna number three. So, this column represent what you send from antenna element one, this column represents what you sent antenna element two along the four time slot from antenna element three across the four time slots.

So, what we do is we look at the earlier example here and we are curious to see what happens to this code when you simply drop the last column, ok. We would like to say that we are now instead of using four transmit antennas we would only use three transport antennas. So, we just keep the last antenna element turned off across all the time slots. So, if you do so, you will get this following code matrix.

Now, again what have we gained? We are again transmitting x 1, x 2 and x 3 using four slots. So, the rate has not changed, but magically now we were design for three transmit antennas. Is it orthogonal? sure.

Now, we dropped the fourth column, but we could have as well dropped the second of the first any one of them because no antenna is holier than the other antenna. So, we could have obtained deleting other columns another parallel G 334 design and then we if you are not satisfying we want to go to a two antenna system I would again like to drop off any one of further to get a G 234 design that will be a design for N is equal to 2.

So, the shortening method is again giving me a mechanism to use fewer number of antenna elements right, but still giving you this design. So, if I draw this third column again some left with this matrix which corresponds only two, N is equal to 2 antenna elements, but what is very interesting is we have x 1, x 2 and x 3 all 3 being sent over tack over the four time slots. So, we have not really compromised on the rate; rate still remains 3 by 4, but the number of antenna element has gone down. So, it will have implications on the diversity gain.

So, yes there is no free lunch, we are cutting back on the number of antenna elements and leading to the lowering of the diversity gain, which shortening is a very simple way to make my life simple from a larger design I can go down to a design with fewer number of antenna elements.

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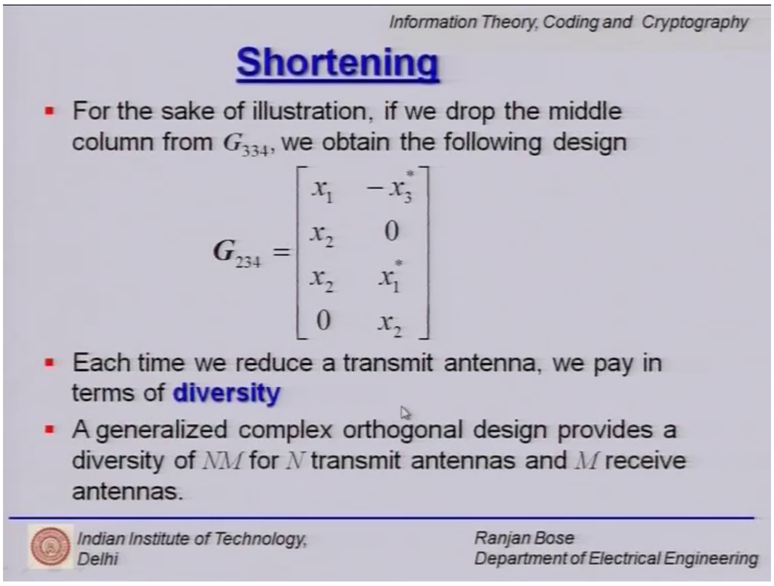

So, here I have dropped the third column as well. So, effectively from the original design of 434 I have been able to drop the last two columns and retaining the first two columns and I still have this design, but each time we do so, we are paying in terms of the diversity. Because diversity is N into M, where N is the number of transmit antennas which we have halved and so, the diversity gain will also drop down by half. We have observed this that the diversity gain is N cross N.

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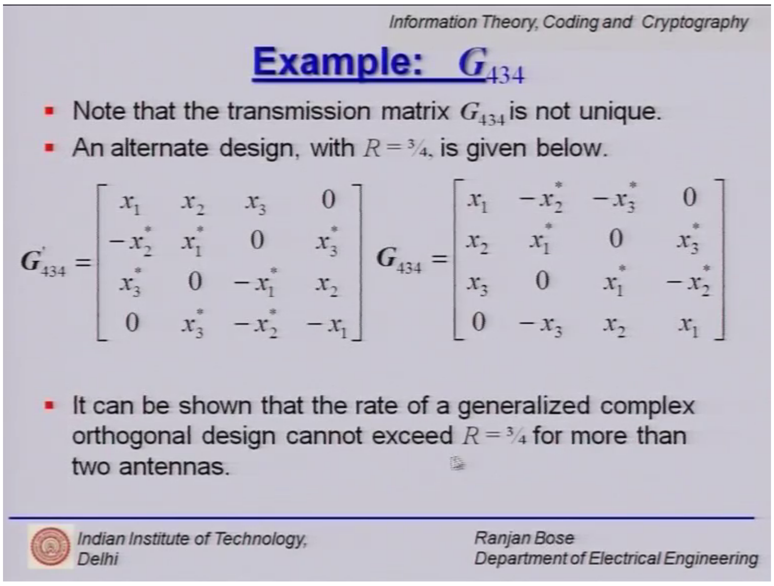

Please note that the transmission matrix of G 434 is not unique. So, again we have here another parallel design, an alternate design with rate 3 by 4, it is given here let us talk about this as G prime 434 to distinguish it from the original design that we looked at G 434. So, this is different if you do not remember the earlier one will bring it in. So, this is the first design you saw, this is the second design. Here is if you see it is a more simple way. So, we have x 1, x 2, x 3 from the first three elements nothing from the fourth element whereas, in this first design we had the complex negative earlier and so on so forth, but again zeros one 0 in every row one 0 and every row.

Now, you can check the both of these designs are orthogonal. Now, it can be shown that the rate of a generalized complex or eternal design cannot exceed 3 by 4 from more than two antennas. So, this has been shown earlier. So, this is the best we can do for a generalized complex orthogonal design for more than two antennas.

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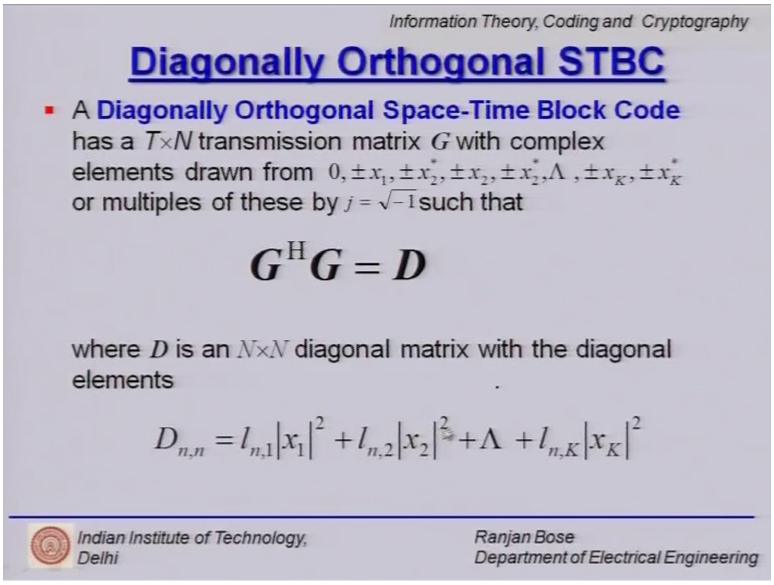

Now, let us move a little bit ahead and talk about something called is diagonally orthogonal space time block code. So, we are not looking a different variations and what we can buy, what we can compromise and sacrifice. So, diagonally orthogonal space time block code is a T cross N transmission matrix G. Again, clearly it is using complex elements from x 1, x 2, x 3 and so and so forth up to x K all the real and complex and the condition is that G Hermitian G should be a diagonal matrix.

Earlier, it was something multiplied with an identity matrix, but this time I am happy if it is just purely a diagonal matrix. So, D is an N cross N diagonal matrix and what will be the diagonal elements? Well, diagonal elements can be of the type some coefficient absolute value of x 1 squared there is some coefficient absolute value of x 2 square and so and so forth up to some coefficient absolute value of x K squared. So, these are the diagonal elements of this diagonal matrix.

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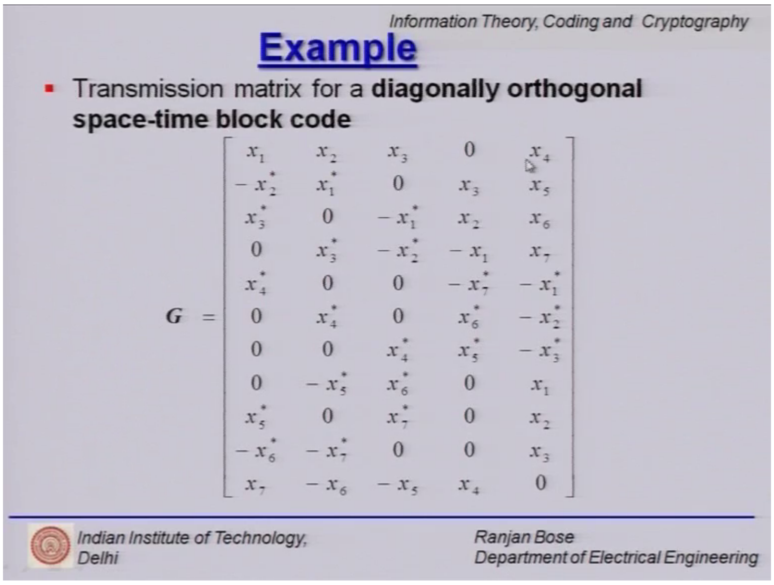

Let us look at a quick example of our transmission matrix for a diagonally orthogonal space time block code. So, here what we have is if we count the columns 1, 2, 3, 4, 5 we have N is equal to 5. So, we are going to use five antenna elements to transmit and if you count the number of rows 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 so, we have eleven time slots with which they are going to send this out.

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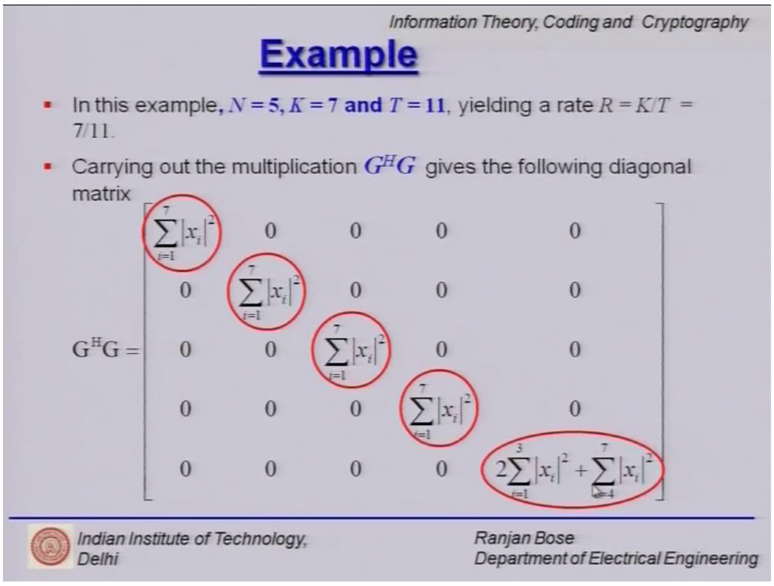

So, T is equal to 11, N is equal to 5 and if you observe carefully we have x 1 through x 7 only. So, we have K is equal to 7. So, seven symbols being transmitted using your eleven time slots. So, rate is obviously, 7 by 11, we can carry out for are sanity check G Hermitian G and what we get is as follows.

So, up to this point it looks pretty much like your regular generalized complex orthogonal design, but here if you see the last diagonal element we have some coefficient x i squared is equal to 1 to 3 and then from 4 to 7 this one. So therefore, we could not represent it as something multiplied by an identity matrix. So, thus we have this design. This is clearly had a similar expression come in the last diagonal then we would have said that, no, it is really a generalized complex orthogonal design.

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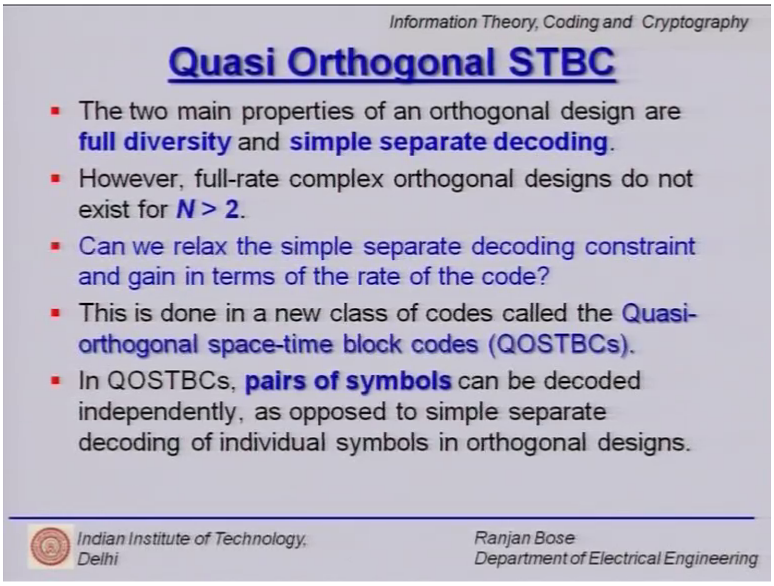

Now, what we have studied so far is this single symbol decoding. This has really improved are decoding complexity and we do not have to worry too much about doing exhaustive search over a pair. So, the two main properties of this orthogonal design are full diversity, ok. We have to looked at so many examples that is giving us full diversity and simple separate decoding. These are the two trump cards that we have, but we have seen also at full rate complex orthogonal design do not exist for N is equal to 2, we made that observation.

So, we have to relax something to gain something more. Can we realize the simple separate decoding constraint and gain in terms of the rate of the code? Which is more important separate decoding constraint which has a direct implication on decoding complexity or the rate of the code which has a direct relationship to bandwidth? So, on one side we have to weigh whether we would like to gain in terms of bandwidth or we would like to reduce the receiver complexity; these are the two requirements and orthogonal code so far were giving us the single symbol decoding thereby reducing to the extent possible, the receiver complexity. And at the same time giving us full rate or wherever rate was not possible for N is equal to 2 we do not have full rate codes. So, question is now can we maybe go for double symbol good decoding or pairwise? So, let us explore that possibility.

This is done in a new class of codes called quasi orthogonal space time block codes or QOSTBC for short, quasi orthogonal. So, we are going to relax some properties of the orthogonal design. In quasi orthogonal space time block codes pairs of symbols can be decoded independently. So, we have compromised in terms of single symbol we are now going back to pairs it is not too bad if the number of symbols are not too large, right.

So, we are now going to look at pairs of symbol as opposed to single symbol decoding.

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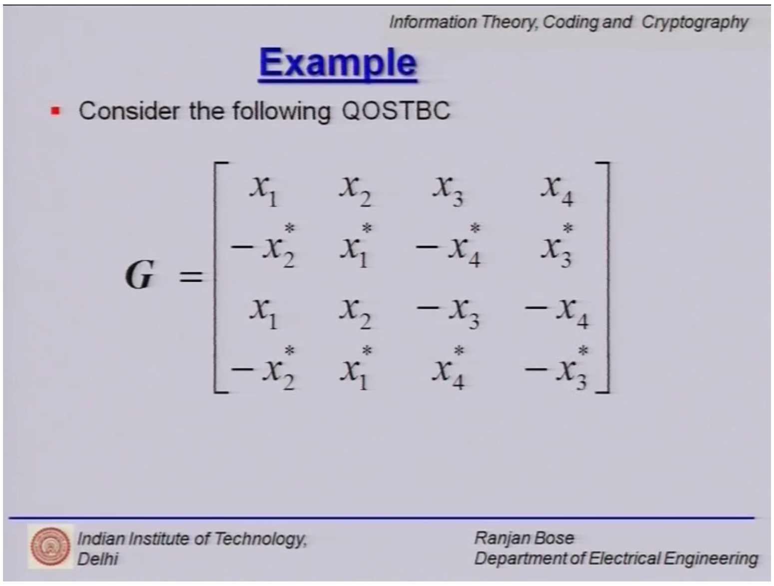

Let us start with an example. Consider the following quasi orthogonal space time block code. So, here if you see there are four antenna elements so, N is equal to 4 and here if you see we are using four time slots. So, we are back in business earlier to transmit four symbols we are started to take more number of timeslots thereby reducing the rate, but here again we are back in business we are transmitting four symbols over four time slots and thereby where looking at the full rate course, but then what happens to the orthoganality? Clearly, this is not an orthogonal design.

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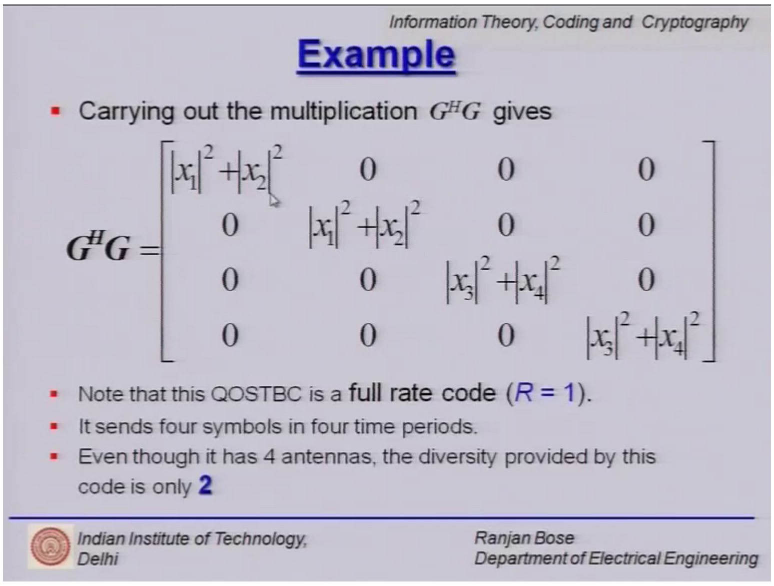

So, let us check it out we do this multiplication G Hermitian G, what does it give us? Well, if you do that you get as a first diagonal element x 1 absolute value squared plus x 2 absolute value squared same for the second and then this one is x 2 absolute value squared plus x for absolute value squared and similarly, here.

So, what has happened is we have kind of coupled these, but what have we gained well the QOSTBC is a full rate because it sends four symbols in four time periods. So, even though it has four antenna elements the diversity is only 2, that is the question we have to ask ourselves. Yes, diversity to why? Why are we saying that the diversity is do can we observe this and say the diversity is 2 that is whenever we do coding at one time only two symbols if only one of them fades the other one can be used. So, the diversity gain of 2 is there, similarly a diversity gain of 2 is there. Had there been 4 elements are together then we would have looked at a diversity gain of 4 as required because N is equal to 4, but the gain is 2.

Please note, intuitively we have explained that diversity gain comes from the fact that each of the symbols coming from different antenna elements goes through a different path and a path gain. Now, if that is fading then if we assume independent fading if one of the symbols is fading the other one is not. So, the diversity gain comes from that.

Here, if you look at this carefully you can see immediately that the diversity gain is 2 and not more.

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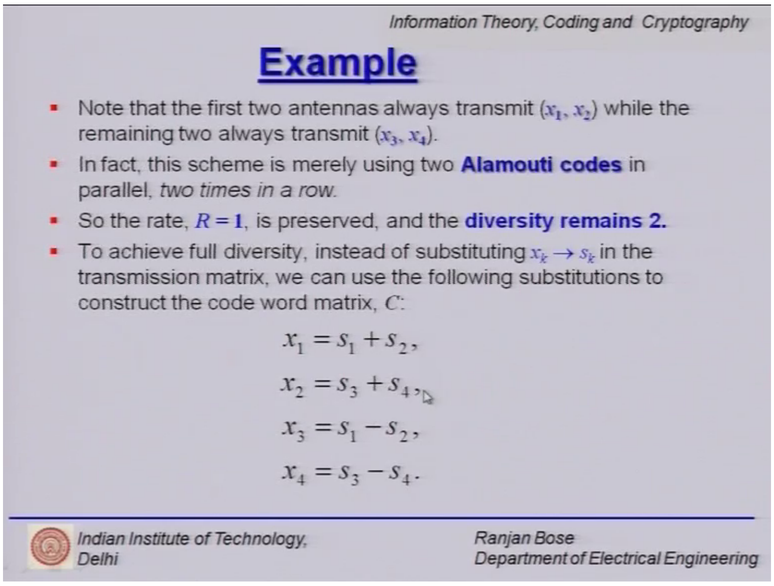

So, what does it mean? Well, the first two antenna elements always transmit x 1 and x 2, while the remaining two always transmitting x 3 and as x 4 and some combination thereof, but that is it. So, effectively what we have done is we have just concatenated to Alamouti codes and we are trying to say that we are sending x 1 and x 2 and x 3 and x 4, but we are just using Alamouti code two times in a row, but Alamouti will only give me a diversity gain of 2, rate is preserved because 2 times in a row we are using a full rate code. So, the effective rate is one, but diversity there is no improvement.

So, to achieve full diversity instead of substitute x k by s k in the transmission matrix we can use the following substitution to construct the codeword matrix. So, now we started mixing. Earlier instead of x 1 being dependent only on x s 1 I made x 1 dependent on s 1, s 2 now the first transferred antenna will be dealing with s 1 and s 2 the second one is now dealing with s 2 s and s 3 and s 4. So, brought in back this other two symbols I have started mixing them up and thereby hoping to get diversity.

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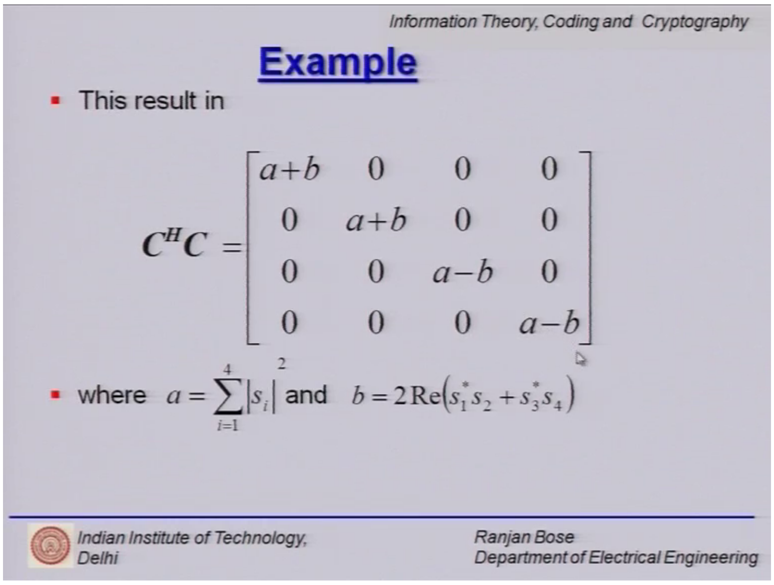

So, what does it lead us to? Now, if you look at this C Hermitian C you will get this kind of a solution, right and what is a? a is given by this. So, you again immediately get back all the four terms in summation, immediately a diversity jumps to 4, because see we assume all the signals are fading independently. So, if you are taking the sum of the all the four symbols effectively even if one fades other three will not fade giving you that diversity gain now there independently fading. So, if you carefully observe this C Hermitian C is immediately leading you to a diversity of 4.

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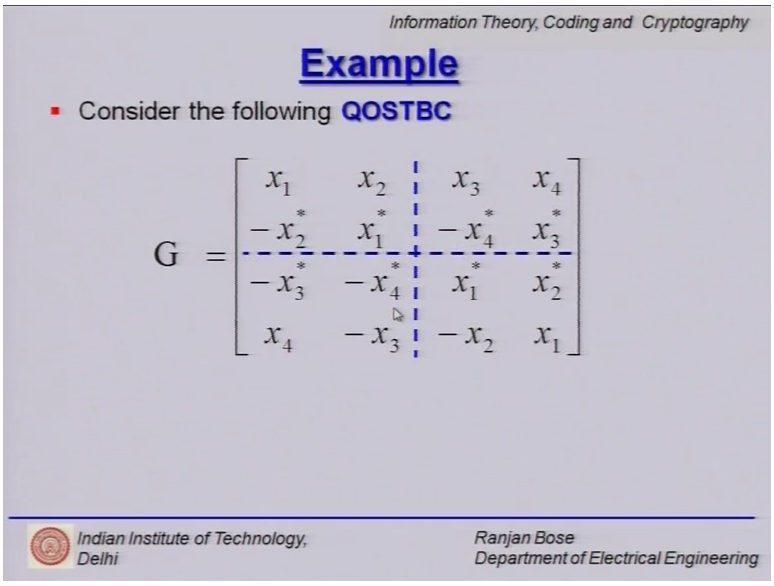

So, now, let us consider another quasi orthogonal space time block code, these are best understood using examples. Now, if you logically partition it as follows. So, we divide into four quadrants.

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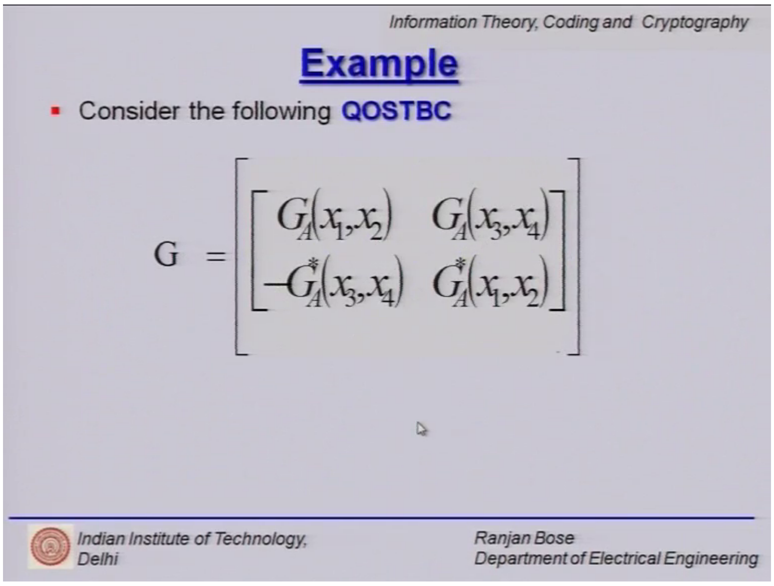

And, we see that the first quadrant is G A x 1, x 2; first please note, it only depends on x 1 and x 2, this depends on x 3 and x 4, this is depends on x 3 and x 4, this one depends on x 1 and x 2. So, I do that x 1 and x 2, x 1 and x 2, but if you look at carefully this is these two our complex conjugate. So, this is G 1 x 1 comma x 2 and this is G 1 complex conjugate. Similarly, this is G 3 and this is negative sign complex conjugate. So, I get x 3, x 4, G A and minus G A complex conjugate.

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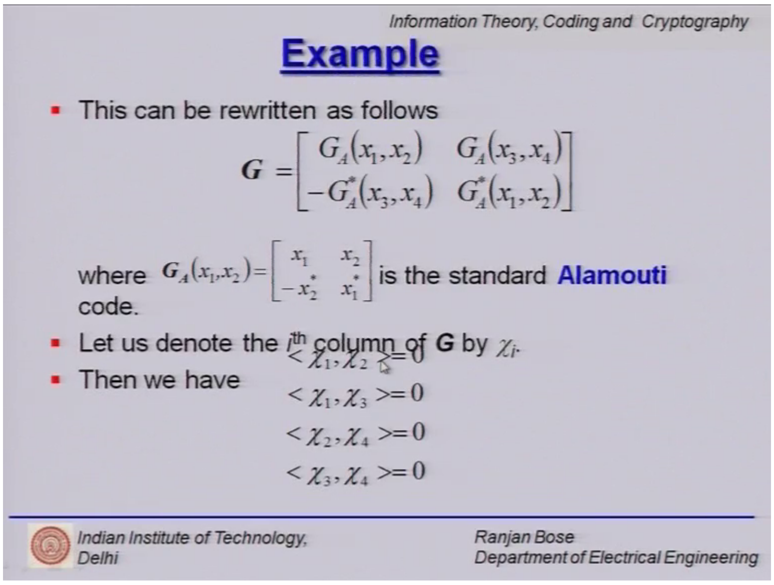

So, we right G as follows we just now observed, but what is G A? Well, this is the standard Alamouti code. So, we denote the i-th column of G by chi i and we look at chi 1, chi 2 in a product that is 0 chi 1, chi 3 – 0, chi 2, chi 4 is 0, chi 3, chi 4 is 0.

So, we are looking at this pairwise orthoganality and therefore, the name quasi orthogonal.

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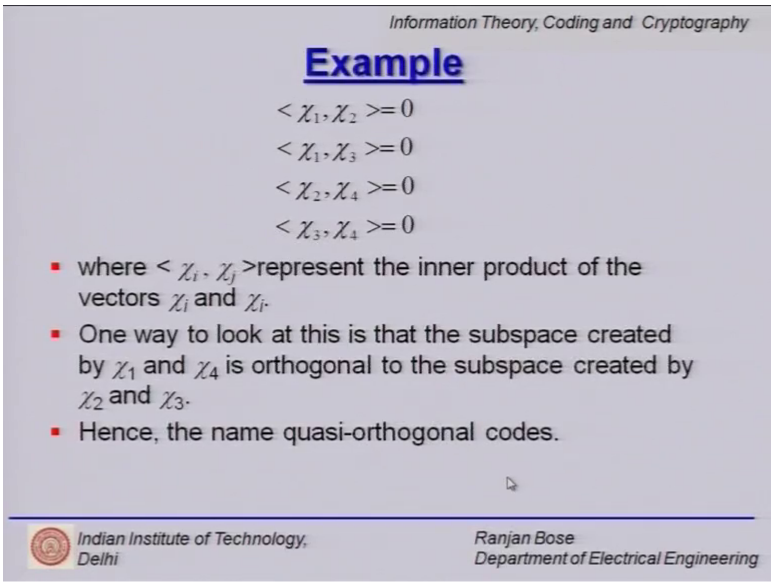

So, here this notation is the inner product of the vectors. So, one way to look at it is that the sub space created by chi 1 and chi 4 is orthogonal to the substrate created by chi 2 and x chi 3. So therefore, we have these quasi orthogonal codes, right, this justifies the name.

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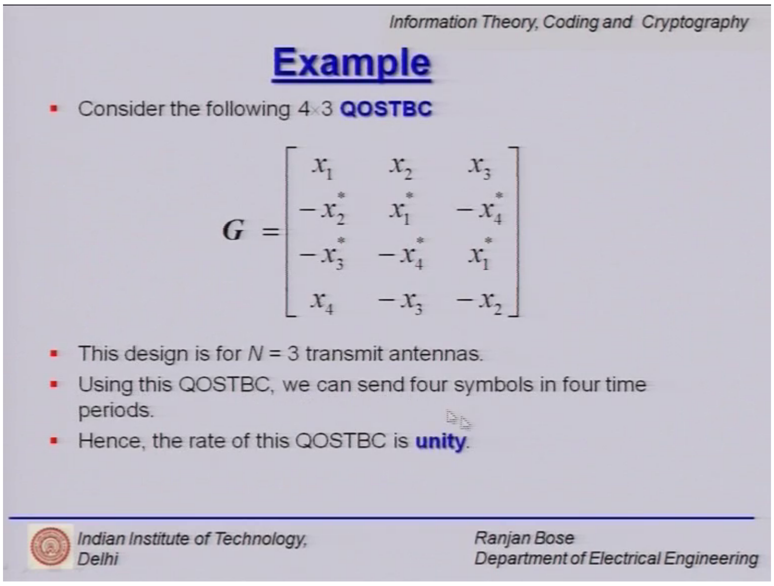

Let us look at yet another example of a 4 cross 3 quasi orthogonal space time block code, ok. Four antenna elements as shown by the four columns, well we do not want them to disappear all of them and we have got four time slots and these are quasi orthogonal space time block code that we have seen, all right. So, if you make this observation this is only a function of x 1 and x 2, this guy this quadrant is G A star which is the complex conjugate this is G A x 3 coma x 4 is starts stands for the Alamouti and this is minus G a x 3 coma x 4. So, this is what we have done this.

And, now let us drop one so, we erase one of the columns. So, suddenly we are left with three antenna elements, but please note here the beauty we still have x 1, x 2, x 3 and x 4 all 4 symbols being transmitted in four time slots. So, just by dropping this column we reduced are diversity gain, but of course, few number of transmit antennas has other practical benefits and. So, we replace that four columns to three columns, ok. So, we go this one and we write it properly and we express it in terms of the three columns.

So, we have now obtained a simple design for three transmit antennas. So, this using this quasi orthogonal space time block code we are now going to send three symbols x 1, x 2, x 3, x 4 which will represent if I map s 1, s 2, s 3 and s 4 in four time periods. So, clearly the rate is unity.

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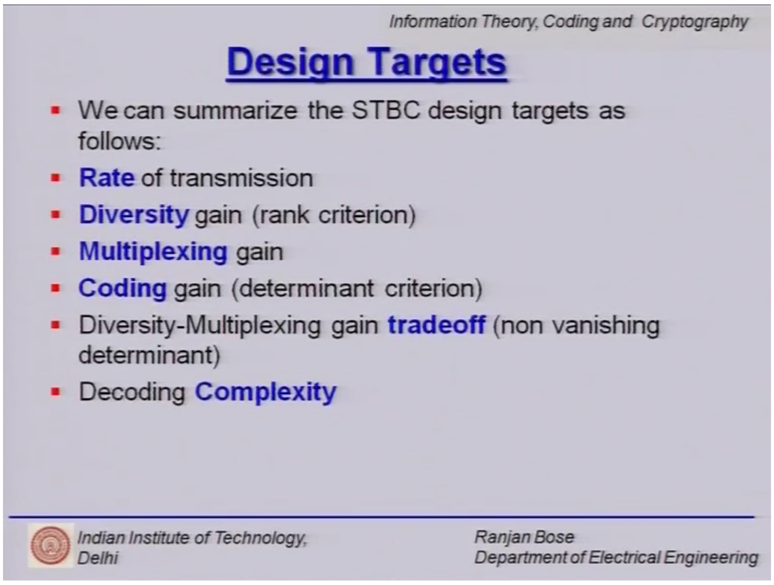

Now, just let us spend a couple of minutes on our design targets. What are the design targets, what we gain, what we can compromise is there a free lunch, let us look at that. So, the design targets for space time block codes are as follows. First is the rate of transmission, direct implication on the bandwidth. Today, all practical designs require us to transfer data at higher and higher rates. So, one of the greatest benefits is the rate improvement, we are can get full rate codes and therefore, we would be very happy to have rates as high as possible as one of the design criteria.

The number 2 important thing is a diversity gain, we have been talking about it all along. Space time block codes give you humongous diversity gain specially at high SNR conditions and this comes from I rank criteria that we discussed earlier. Then we have also got multiplexing gain because simply because of this mimo system you have different channels and different channels independently give you multiplexing gain.

Then we had looked earlier into the determinant criterion were coding gain was also coming from the design of good space time block codes. So, that is an add on and finally, there is a diversity multiplexing gain trade off, that comes from a non vanishing determined condition we have not talked about it in detail, but intuitively you can look at either from the independent channels that we get from different antenna elements we have diversity or we can utilize it as a multiplexing gain, but not both. So, we have a tradeoff of sorts.

And, finally, we have been talking about the single symbol decoding and at most a pairwise decoding. So, decoding complexity is important specially if you are looking at larger and larger constellation size. So, you have this platter of constraints or requirements or design targets if you will that we have to keep in mind while designing or choosing good space time block codes.

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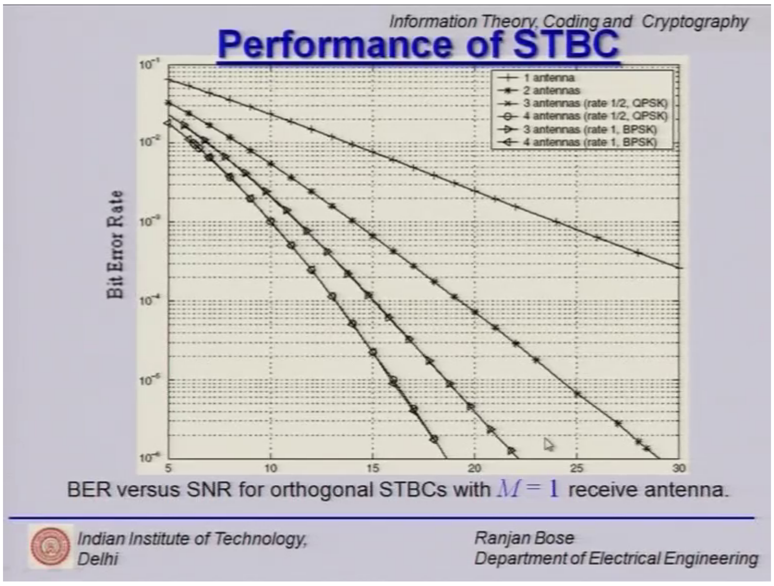

Let us now move over to the performance of some good space time block codes. So, if you see on the x axis we have the SNR in dB, on the y axis we have the bit error rate,. We have lots of curves here let us look at this top one which is the worst performing curve, it is for a single antenna system. So, you do not have space time block codes. For the sake of discussion only let us make sure that we understand that m is equal to one that is it is a single receive antenna.

So, when we talk about multiple antennas we are putting them at the transmitter. So, the first case is SISO. So, single input single output system, no diversity gain whatsoever and we have a happy go lucky curve going out here, not to go not too bad, but can we improve it. So, we immediately put in two antenna elements and we have the second curve a quick dip here and the slope increases and the slope is an indicator of the diversity gain. So, just by observing these two curves we can say that look putting two antenna elements of the transmitter has immediately given me a diversity gain of 2.

The other important thing is to note that the slope increase means that this paths diverge. These two curve diverge as we go along which means that at higher and higher SNR the diversity gain is more and more. So, the diversity gain the real benefit comes at higher SNRs then why should we stop we go ahead and look at 3 antenna systems, but this is a case we have looked at many N is equal to 3 cases, but this is the rate half and modulation scheme is QPSK and then we have this curve which is giving you the 4 antenna systems and then again we have some overlapping curves with 3 antenna and 4 antenna systems. So, that the symbols are plotted on top of each other shown that there are co incidental with rate full rate and BPSK.

So, the point is that as we go from one an antenna element at the transmitter to 2 antenna to 3 to 4 this clearly diversity gain slope improves, improves and improves, but the improvement is decreasing. So, we are a humungous improvement from N is equal to 1 to N is equal to 2, yes improvement from N is equal to 2 to 3, but not so much as we saw in the first case and we go from 3 to 4 improvement is there, but even less. So, we see some sort of a law of diminishing return that as we go along.

So, we just cannot say that look give me as many antennas and I will pack them in my transmitter after a while it really is not worth it.

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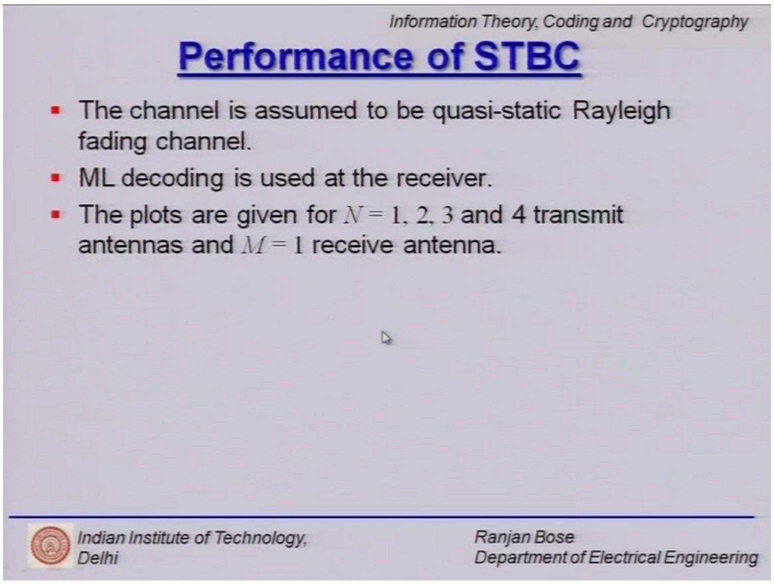

So, this like basically summarizes that plot that we have seen just now. We have assumed a quasi static related fading channel. So, over the symbol that we have transferred in the channel really does not change of course, we are using ML decoding at the receiver and we have looked at the plots and compared for N is equal to 1, 2, 3 and 4 provided there only one receive antenna at the receiving end.

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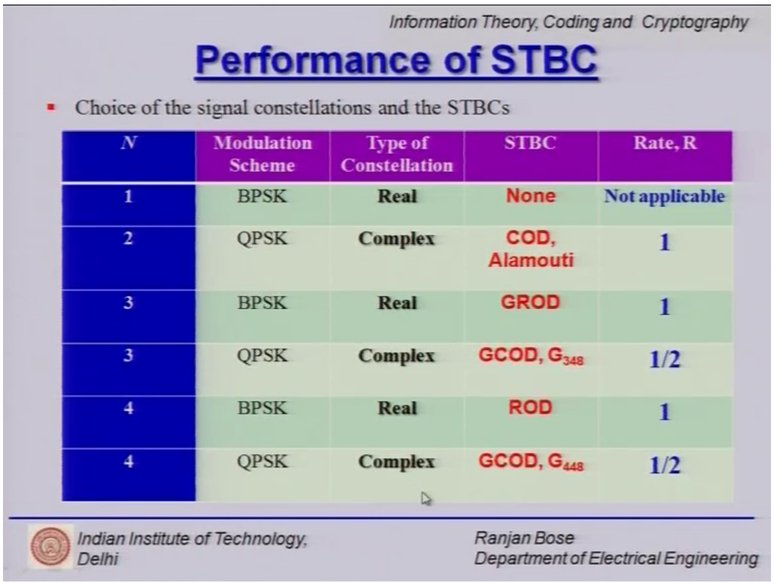

And, this table summarizes which space time block code actually did we use what relation scheme did we use.

So, if you see then they were six curves they were four distinct curves there were some overlapping curves and so there is six rows here. So, this first case was the standard BPSK which is a real constellation obviously, we cannot use paste and block codes so, rate is not applicable. Here in the second case when 2 antenna elements are used we used QPSK. So, we had four symbols to choose from which is obviously, a complex constellation and we used complex orthogonal design the only known Alamouti scheme be used full rate.

The third one when we moved on to 3 antenna elements we used a real orthogonal design using this generalized orthogonal design we studied full rate and then again for N is equal to 3 we just do not have to go with a real design, I can go with a complex design and complex design QPSK and this G 348 we have already studied in today's lecture and it is rate is 1 by 2 right and then we if we look at 4 antenna elements again we have an option for real design on a complex design.

So, what it illustrates is that you have a choice, you can go with a real design, we have looked at so many examples or a complex design, generalized complex orthogonal designs, you can have a choice for modulation schemes as well and so, for this stuck with simple QPSK, BPSK types it does not stop you from going to slightly higher modulation schemes, we have to see what do you sacrifice in terms of the rate.

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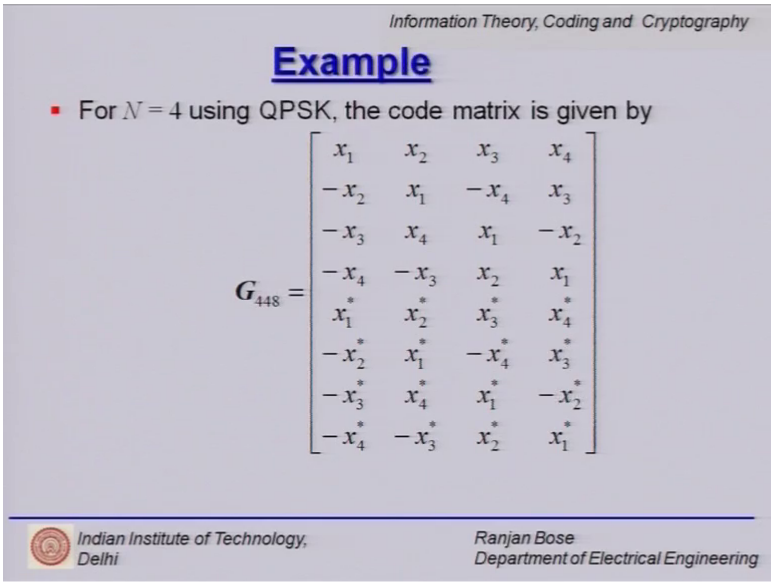

So, just to bring that to a conclusion for the N is equal to 4 QPSK condition. So, if you look at this last row right, if you have to just take this last row into consideration what exactly did we do well we have a complex generalized complex orthogonal design given by this thing G 448, 4 antenna elements, using 8 time slots sending only 4 symbols. So, we have a rate of 1 by 2. This is the case we just now studied, ok. So, this is the last curve that is coming out from the design we just talked about M is equal to 1.

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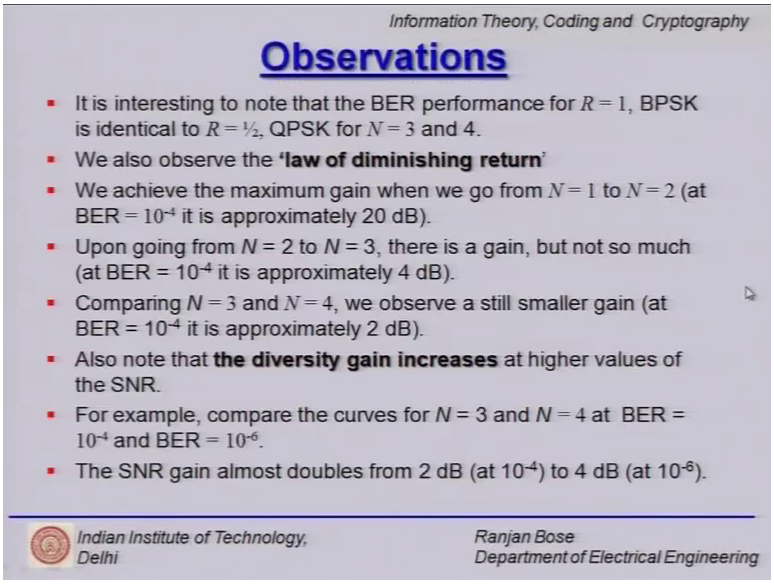

Now, a couple of observations we have made some of them are earlier. It is interesting to know that the performance for rate R is equal to 1 BPSK identical to read half QPSK, for N is equal to 3 N 4; that is why they were overlapping, ok. This is interesting to note. Also that the law of diminishing return was observed, right and as we go to high and higher SNR then the diversity gain increases this improvement we have seen earlier also.

So, we can look at some numbers which are given here which tells you that exactly you get a better diversity gain at higher SNRs.

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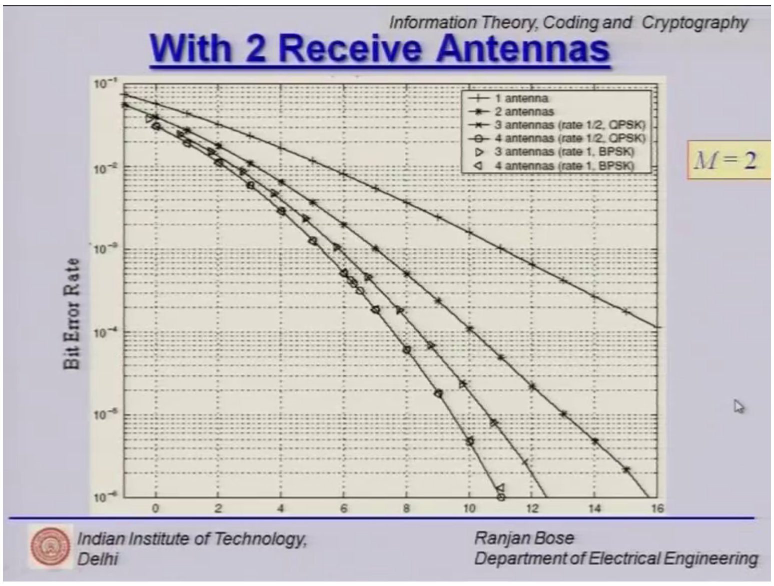

Just try to conclude this taught by looking at now what happens if we put in 2 receive antennas. So, N is equal to 2. So, my variable now is N, where I vary my N from 1, 2, 3, 4. So, for up to 4 transmit antennas and 2 receive antennas. Clearly, we have seen that the diversity gain is M into N. So, if you compare the first case of single antenna at the transmitter, but two antennas at the receiver. So, you get some gain, but again if you look at a 2 antennas at the transmitter and 2 antenna at the receiver will get a following curve and again at 3 antennas at the transmitter and 4 antennas at the transmitter you again get this law of diminishing return.

Once again for 4 antenna and 3 antenna curves this either you use rate one BPSK a rate half QPSK you have the curves coinciding. So, this cements the fact that we have a big potential for performance gain if we use space time block codes and no wonder for newer wireless communication standards they are becoming more and more popular and we are going to use space time block codes to improve the performance of wireless communication systems.

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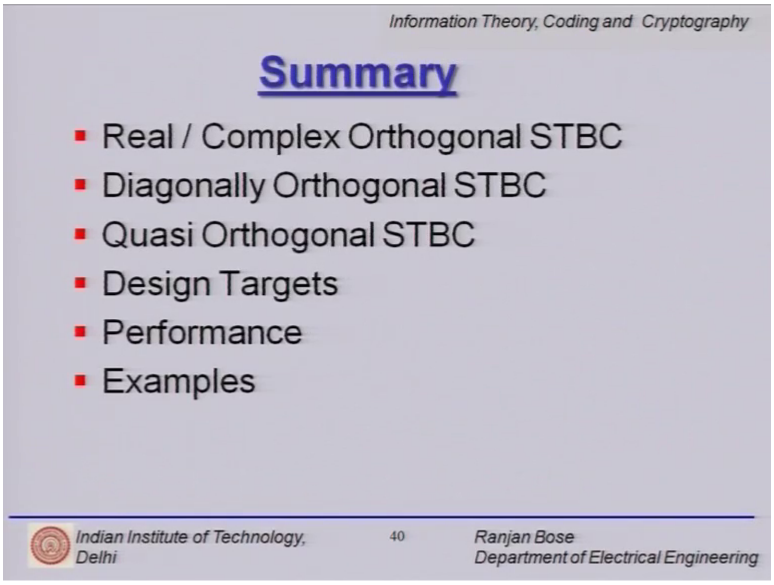

Let us look at our lecture what we have studied today. So, we revisited real and complex orthogonal space time block codes, we talked about generalized real orthogonal designs and generalized complex orthogonal design. Then we talked about diagonally orthogonal space time block codes followed by relaxing the condition of single symbol decoding and we went on to quasi orthogonal space time block codes. We then talk about what are the design criteria, what are the design targets. And finally, we looked at the performance curves for N equal to 1, 2, 3 and 4 for different number of receive antennas.

With that we conclude our lecture today.