

**Digital Communication.**  
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**Lecture-8.**

**Spectral Properties Of line Codes: On-Off/Polar/Bipolar Signalling.**

Professor: We have been now looking at the spectral properties of line codes, line coded signals used in PCM systems. And yesterday we derived an expression for the power spectrum of such line coded signals. The general expression was, just to recapitulate what we accomplished yesterday, that is line coded signal which uses pulse shape  $P(t)$  with the fourier transform  $P(\omega)$  has a power spectrum given by this expression. Right, where  $R_n$  denote the autocorrelation coefficients of the impulse sequence and in turn a dependent on the data sequence  $a_k$ , the nature of the data sequence  $a_k$ .

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$$S_p(\omega) = \frac{|P(\omega)|^2}{T_b} \left[ R_0 + 2 \sum_{n=1}^{\infty} R_n \cos(n\omega T_b) \right]$$

On-off signalling

$$S_p(\omega) = \frac{T_b}{16} \text{sinc}^2\left(\frac{\omega T_b}{4\pi}\right) \left[ 1 + \frac{2\pi}{T} \sum_{n=1}^{\infty} S\left(\omega - \frac{n\pi}{T_b}\right) \right]$$

So the important conclusion that we can obtain from this expression that we obtain is that the spectral, power spectrum or the spectral properties of line coded signals depend as much on the pulse shape as they do on the data sequence or the other way round actually. Not only they depend on the pulse shapes but they are also dependent on the properties of these  $R_n$ , right. Depending on what kind of autocorrelation sequence the data sequence has, we may have different features, different kind of power spectrum for the line coded signal.

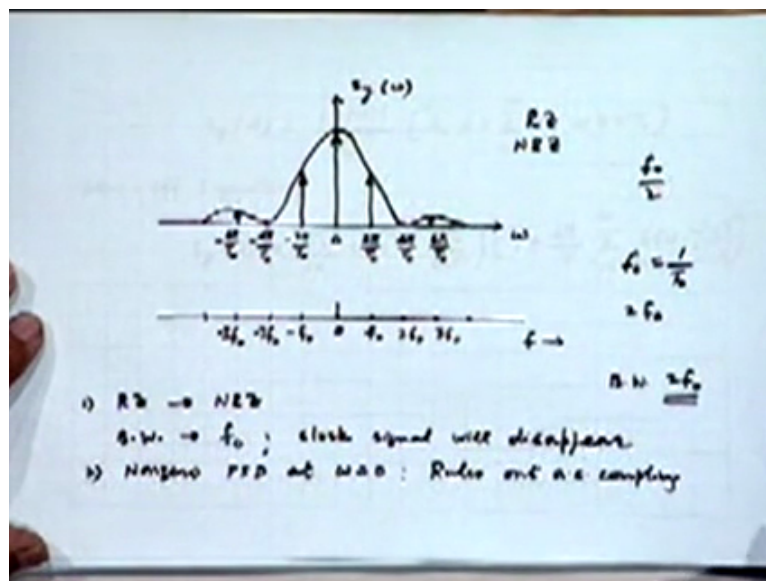
It is in fact this possibility of manipulation of the power spectrum through tuning the properties of the autocorrelation sequence and data spectrum, data sequence that we get

different line coding schemes, right. Different line coding schemes by virtue of the manner they exploit the relationships in ones and zeros and which ones are to be inverted and which ones are to be not inverted will determine what the values of this RI are going to be and in turn will control the power spectrum of the resulting signals that we finally transmit on the line, right.

And we will see that it has a very important bearing and it gives us a lot of flexibility in the design of line coding schemes. For a special case of on-off signalling, we went through this exercise a bit further, now we are taking specific line coded, line coding schemes. And the 1<sup>st</sup> one we have picked up for discussion is on-off signalling. And by appreciation of what these amplitudes  $R_0$  and  $R_n$  is should be in this case, we could obtain an expression for SY Omega, which you must be having in your notes now, is given by Sinc square function like this, multiplied by this impulse train, all right.

This is what you could do yesterday, we finished the maths of this, let us appreciate the significance of this mathematical results. And I will do so by means of this picture that we have here. If you plot the spectrum, well, from the mathematical expression itself you can notice that this spectrum contains 2 parts, this is a continuous function of Omega, this part, right, the 1<sup>st</sup> factor of the spectrum is a continuous function of Omega, it is a well-known sinc square function, right. It also has a discrete part which consists of impulsive components.

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And it is a product of these 2 continuous and discrete components that we get the overall spectrum which therefore looks like this. Each of these impulses by themselves is of constant

strength and constant height, but after multiplication with a sinc square function, obviously the strength changes. And therefore we get invert power spectrum which looks like this. This is a continuous part in this and these are the discrete impulses, right. So the power spectrum of an on-off signal, if we remember also, this is a power spectrum of a return to 0 on-off signal, right in which we are considering a half pulse, half width pulse rather than a full width pulse, right, so keep that in mind.

So what we find is that the spectrum contains these discrete components at  $2\pi/T_0$ , the 1<sup>st</sup>  $T_0$  crossing of this sinc square function cars at  $4\pi/T_0$ ,  $2\pi/T_0$  corresponds to  $f_0$ ,  $f_0$  being the reciprocal of  $T_0$  and therefore  $f_0$  is the data rate, right. And  $4\pi/T_0$ , you have the 1<sup>st</sup> 0 crossing and therefore the main lobe width, the main lobe width is how much?  $2f_0$ , right. But the interesting thing is...

Student:  $4f_0$ .

Professor: Well, I am only including the positive part of it, right, because if we are talking of the baseband signal, real bandwidth will of course correspond to only the positive frequencies. If it is a bandpass signal, it would have been twice as much but as the moment we are talking of simple pulses, baseband pulses which are not modulating any carrier or anything, right. So this is 0 frequency over here. So the bandwidth of the signal is  $2f_0$  and I will return to this point in a few minutes. More importantly we find that we have discrete components at  $f_0$  which is a nice thing, right, why is it nice?

Because that means I have a specific finite amount of energy available at the clock frequency, right. The clock signal is being represented in the received signal and I could use some kind of PLL of, phase locked loop to extract this clock component for synchronisation purposes, right. This is a nice feature of on-off signalling but other than this it hardly has any other nice feature as we already discussed. For example, this bandwidth is  $2f_0$  which is very high, right. Why is it very high? We will see later that the theoretical minimal bandwidth with which we can manage to transmit data  $f_0$ , at the rate of  $f_0$ , is  $f_0$  upon 2, okay.

We have not yet discussed this fact but we will be discussing that soon enough later on. So with respect to the theoretical minimum possible, we are consuming 4 times as much bandwidth in the signal, right. Not very efficient from that point of view of bandwidth utilisation and that is a very important resource to worry about all the time for us. Similarly we have already seen, not only the continuous power itself has a nonzero value at 0, right, the

continuous part of the spectrum but we also have a specific discrete components, very large amount of energy specifically present at DC, which we know is not a good thing to do.

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$$S_p(\omega) = \frac{|P(\omega)|}{T_b} \left[ R_0 + 2 \sum_{n=1}^{\infty} R_n \cos(n\omega T_b) \right]$$

On-Off signalling

$$S_p(\omega) = \frac{T_b}{10} \text{sinc}^2\left(\frac{\omega T_b}{4\pi}\right) \left[ 1 + \frac{2\pi}{T} \sum_{n=1}^{\infty} \delta\left(\omega - \frac{n}{T_b}\right) \right]$$

Less immune to noise than polar scheme

We have already seen why this is not a desirable thing to have on a cable or any kind of line. We could reduce this bandwidth somewhat by using not return to 0 signal but nonreturn to 0 signal, right. Then we will be essentially using a pulse of width  $T_0$  rather than  $T_0/2$ , right. And what will happen to the main lobe of this? So if you go from RZ to nRZ, the bandwidth will be reduced by a factor of 2, it becomes  $f_0$  which is still twice the minimum what we can work with, but at the same time only good, only nice feature that this signal would have will also go, right.

That is same pulses at this particular point, the clock rate, will disappear, right. And also the clock signal or, I should not really call it a clock signal, a clock recovery signal will also go, will disappear. That is as far as the spectrum is concerned, okay. These are brought conclusions that we can draw regarding the spectrum, okay, one there was one point, we already noted that we have a nonzero PSD at  $\Omega = 0$ , right. And we feel that is not good because that rules out AC coupling, right, we discussed that earlier.

AC coupling is something that we just cannot avoid typical lines in repeaters, at the places very put repeaters. Why? We need to use transformers, we need to use AC coupling to remove bias that maybe getting introduced by other circuits, right. So this kind of circuits are bound to be there in your line, particularly at the point of locations of repeaters. In which case we will have the DC wander and all those problems, right. Why do we require transformers?

We require them for let us say for impedance matching, right. We require coupling capacitors to remove any kind of biases that some circuits might be introducing. Therefore AC coupling a something that is just not unavoidable.

Student: What does pst stand for?

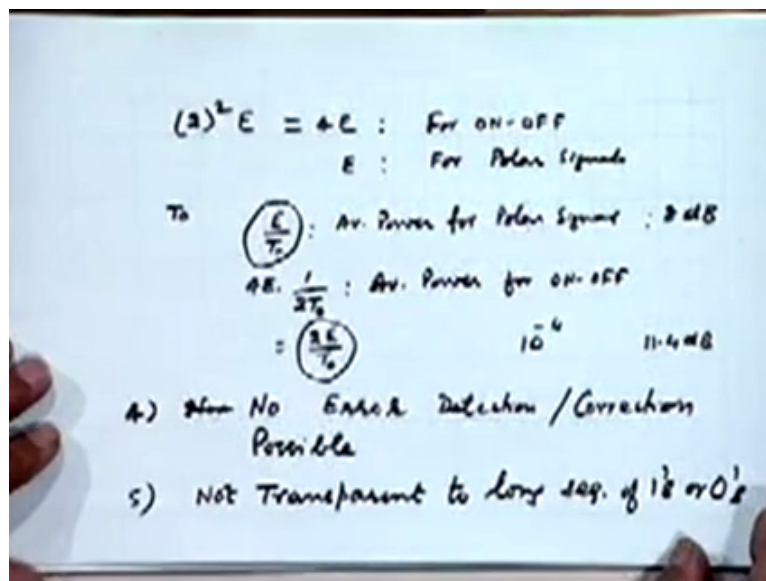
Professor: Power spectral density  $(\text{W/Hz})$  of as per omega. So we also see that the bandwidth is excessive, we have already seen that. Not only all this, in terms of its performance against noise, noise immunity, it is less immune to noise, let us say Delta Polar scheme that we will discuss soon again. The Polar scheme of line coding. Yes please?

Student: Larger  $(\text{W/Hz})$ .

Professor: Okay. It is less immune to noise than polar scheme, right. Let us see how. A typical on-off signal will have 2 levels, between let us say 0 and, I will just call it 2. Why I am calling it 2 is I am comparing it with, let us say polar signalling in which we have one level represented by let say +1 volt and 0 level by a -1 volt, right. In order for both the signals to have same kind of performance against noise, it is clear that it should have the same kind of amplitude swing, right. So that is 0 is not confused for 1 and vice versa due to occurrence of a large noise spikes for a certain duration.

Therefore we should have signal swing, if this is going between +1 and -1, for getting the same kind of noise immunity as unipolar, a polar, sorry an on-off signal that were discussing here or unipolar signal should have a swing from 0 to 2 volts, it is going from -1 volt to +1 volt. Is that okay? That is intuitive sense and theoretically also one can show that under this situation they will have identical performance against noise as measured by probability of error, right. The probability of making the wrong decision from 1 to 0 or 0 to 1.

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But under these conditions, this contains twice as much power than this contains, this contains 3 db more power than this, let us see how. In on-off signal, because of the amplitude considerations, what is the energy of each pulse? It is proportional to 2 square, let us call it some 2 square e where e is some constant. So that is the energy that is equal to 4e for on-off signals. Everytime a pulse is transmitted, we transmit an energy proportional to this factor 4 am I right, because the amplitude is 2. For the polar signal on the other hand, what will be the energy transmitted everytime a pulse is transmitted?

Only e because amplitude is +1 or -1, right. So the value is e for Polar signal. And suppose pulse duration is  $T_0$ , whatever the pulse, let us say, let us assume maximum pulse duration, this can be  $T_0$ , then your power, average power in polar signal transmission is, that is the average power for Polar signal. Because every interval of  $T_0$  seconds, you are either transmitting a +1 or -1, right and therefore you are transmitting energy e, the average value is e by  $T_0$ , right, where  $T_0$  is the relation for free the energy is targeted.

On the other hand for the on-off case we are transmitting pulses only for the half the because for the other half time when we have zeros, we are not transmitting and energy, right. So therefore the average power calculations here becomes this, because only half the time you are transmitting 1, the other half you are transmitting 0 which is represented by 0 amplitude. So that is the average power for on-off signals or unipolar signals which is twice e by  $T_0$ . So what we find is that we end up transmitting twice as much power here, average power here

than with, then we do for the case of polar signal while getting the same noise performance, same noise immunity, right.

So suppose we are interested in error rate of 1 in 10,000, right, if, if for example in this case, I think the corresponding figure is 8 db, required signal-to-noise ratio is of 8 db or 8.4 db, for the case of unipolar signals, we end up requiring 11.4 db or something. 3 db more power, which is not very good because every db of power adds up to cost, error of cost right. So these are the various properties associated with on-off signal, both from the spectrum point of view as well as the noise immunity and power efficiency point of view, right.

These are instant, in fact 2 most important features to worry about when we are talking of line coding when the power required for a given performance against noise and bandwidth used and also whether it has susceptibility to DC and all those things and all. There are some other points to mention here, it also has low error detection or correction capability.

Student: Okay. It is not possible easily to correct or detect errors and of course there is nothing in it which will help me to become what we call transparent to long sequences of ones and zeros, right.

Professor: We will continue to have that disadvantage, that is possible loss of synchronisation when there is a long sequence of one then zeros, so not, this is called a transparency property, okay. We would like line codes to be transparent to long sequence of ones and zeros, right, in the same way our timing circuit, timing extraction circuit should not get affected if such a situation arises, there is nothing in it which will help us to do that. So therefore for all these reasons on-off signalling is not a preferred mode of signalling, right.

That is what we can appreciate, except for one that it gives you, if you use half width pulses in the on-off signalling scheme, we get easy clock recovery, but that feature we can incorporate in other schemes also we will soon see.

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2) Polar Signalling:

$$1 \rightarrow b(t) : a_k^2 = 1$$

$$0 \rightarrow -b(t)$$

$$R_0 = \lim_{T \rightarrow \infty} \frac{T_0}{T} \sum a_k^2 = \frac{T_0}{T} \cdot \frac{T}{T_0} \cdot 1 = 1$$

$$\sum_{k+n} a_k a_{k+n} = 1 \text{ or } -1 : \frac{T}{2T_0} \text{ times each}$$

$$R_n = 0 \quad n \neq 0$$

$$S_p(\omega) = \frac{|P(\omega)|^2}{T_0} \cdot R_0 = \frac{|P(\omega)|^2}{T_0}$$

Let us then come to the 2<sup>nd</sup> kind of signalling that we talked about, go from unipolar to polar, in which 1 is represented by some pulse  $p(t)$  and a 0 is represented by a negative of that, right. That means you are a  $K$  square, remember we have to go to  $R_n$  now to compute the, in every case that we consider your, we have to go back to the basic expressions for  $S_Y(\omega)$  and look at the amplitudes are 0,  $R_1$  and so on, so as to calculate the nature of the power spectrum. Now  $P(\omega)$  square part of it remains same but the factor which depends on the amplitudes  $R_0, R_1$  etc. Will have to be calculated afresh depending on the kind of line coding scheme we have.

So in this case your  $a_k$ s are going to be equally likely to be  $+1$ , right. I will call the, the amplitude will be, hierarchy will be  $+1$  or  $-1$ , the strength of those impulses, right for a polar signalling 3. So  $a_k$  signalling will always be equal to 1. No matter in your transmitting 1 or 0, therefore you  $R_0$ , you may recollect what the definition of  $R_0$  was, limit as  $t$  tends to infinity,  $T_0$  upon  $t$   $\sum a_k^2$ , right. This will be into one which is always equal to 1. Similarly what are the other quantities of interest?

These kind of factors  $a_k$  into  $a_{k+n}$ , right, I think I am in the habit of writing a bit small, let me try to write big again, same thing,  $a_k a_{k+n}$ , this will be equal to...

Student: How do we get that  $t$  by  $T_0$ ?

Professor: For the same reason...

Student: Summation  $(\cdot)$ (23:18).



Professor: Total number of pulses in the interval  $t$  is  $t/T_0$  for each of which  $a_k^2$  is 1, so after summing this up we will get a total value of  $t/T_0$  and that is how we get 1, right. Same argument that we used for on-off signalling, right. And this argument is going to be repeated every time we consider a new line coding scheme, only the values will turn out to be different, okay. Alright, what is the product of these 2,  $a_k a_{k+n}$ ?

Student: Can be -1...

Professor: That is right. We can have, because just ones and zeros at random, which now become one than -1s at random, right. Therefore it is equally likely to be 1 or -1, each will take  $t/T_0$  terms, right,  $t/T_0$  times each. All right. So when you sum this up over  $K$ , what is going to happen, this is going to become 0. That means for any value of  $n$  we can come to that later  $R_n$  is going to be equal to 0,  $n$  not equal to 0 because for  $n$  equal to 0 we have already seen it is equal to 1. Which means that the 2<sup>nd</sup> term, summation term appearing in the power spectrum expression can be ignored.

Remember this expression, let me see if I can get the factor of  $A$ .

Student: So why do you ( ) (25:05).

Professor: This term  $R_n$  can be ignored, right. Yes, what is the question please?

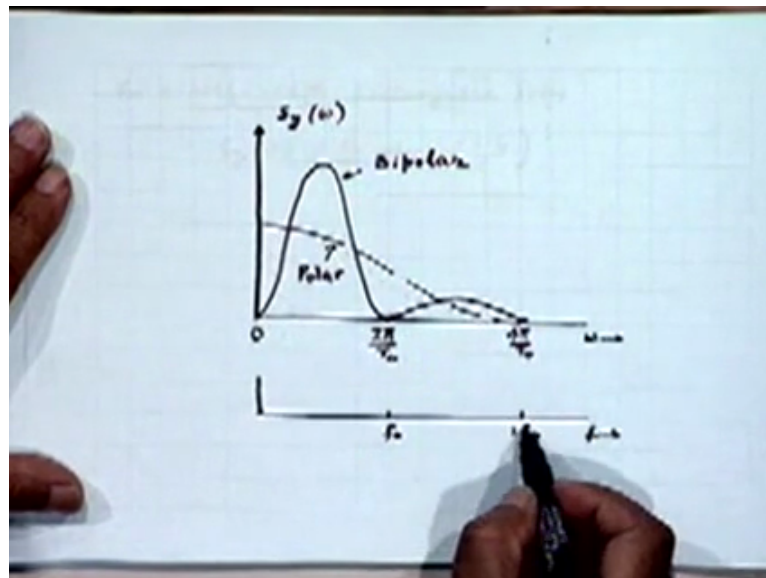
Student: Why do you assume that there are equal number of ones and zeros, ones and -1?

Professor: For one thing it is convenient for the maths that we have, that we are doing here, but more than that in reality we would like to them to be like this. So even if they are not like this, we will be converting our data to the form which makes ones and zeros to be equally likely because if they are not doing that, that means a lot of time you are transmitting within that information. We will talk about that later, we will talk about information theoretic aspects of information what we want to transmit, messages that we want to transmit.

It will always be preferred that we transmit digital data in the form in which one then zeros are equally likely, right. If you are not doing that, you are not making the efficient use of your available bandwidth. So we shall talk about that separately, it is a reasonable assumption. Of course you may have a situation where inside of our efforts, they do not become equally likely, right. Such situations are likely to be very infrequent and not to worry about too much and we can obviously modify any of these results for their sake if required.

Okay, let us return to polar signalling, so what will be your SY omega? It will be simply P Omega square upon T0 into R0 which is 1, right. P omega mod square upon T0. So we have discrete components here, right. Unlike the on-off signalling scheme we have no discrete components, it is only a continuous spectrum corresponding to the sinc square function if we use a rectangular pulse.

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For a half-width rectangular pulse

$$S_g(\omega) = \frac{T_0}{4} \text{sinc}^2\left(\frac{\omega T_0}{4}\right)$$

(i) Polar signalling, more efficient  
: most efficient

- suffers from all other disadvantages
  - No spectral null at d.c.
  - Excessive B.W.
  - Not transparent to long 1 & 0 sequences
  - clock comp. recoverable

So if we use a half width, a half width rectangular pulse, when we say half width, the implication obviously is half the duration that you can make, you can have in that particular interval, that is T0, right, T0 is the maximum duration, so half of that. So for half width rectangular pulse if you choose Pt to be half width rectangular pulse, your SY omega is going

to be  $T_0/4$  sinc square  $\omega T_0/4$ , okay. Which is identical to the continuous portion of the spectrum of the on-off signal, right, this portion is same and this is what it looks like.

Ignore the solid curve, we will only look at this curve, right. This is the main lobe of the sinc square function which have plotted here, only for positive sequences, right. And it has its 1<sup>st</sup> 0 crossing at  $4\pi/T_0$ , again corresponding to  $2f_0$ , right. Bandwidth required is again the same as that for unipolar signal, it has a main lobe which has the same width, all right. I can reduce the bandwidth by a factor of half again by going from half width rectangular pulses to full width rectangular pulses, right. That is the maximum I can do, account from  $2f_0$  to  $f_0$ , right and we will talk about that in a few minutes.

Now let us discuss the properties of the polar signals in relation to the properties of the on-off signal that we have just discussed. The 1<sup>st</sup> point to note is that polar signalling we have already noted is more efficient than on-off signal. In fact it can be shown that as far as binary signal transmission is concerned, polar signal transmission, the polar transmission is the more is the most efficient in terms of power, right. There I am referring to power efficiency, you can talk of power efficiency, you can talk of bandwidth efficiency in terms of digital signals.

Power efficiency refers to how much power is required for a given level of performance as measured by error rate, by fixed value of the error rate, right. And if you specify the error rate, you calculate the power and different schemes will have different power requirements for achieving that performance. More power efficient scheme will require less power and less efficient scheme will obviously require more power. So polar signalling is more efficient than on-off signalling, in fact it is the most efficient.

We will see is a line coding schemes also are less efficient than polar signalling, power efficiency, as far as power efficiency is concerned. But other than the fact that it is more power efficient, it suffers from all the other disadvantages of on-off signalling, right. You will see that yourself, it suffers from every disadvantage that polar signalling has. For example, we have component of DC, even though it is not an impulsive form, the spectrum is, there is no spectrum null at DC, right. There is no spectrum null at DC which is preferable to have because the AC coupling requirements we have discussed, right.

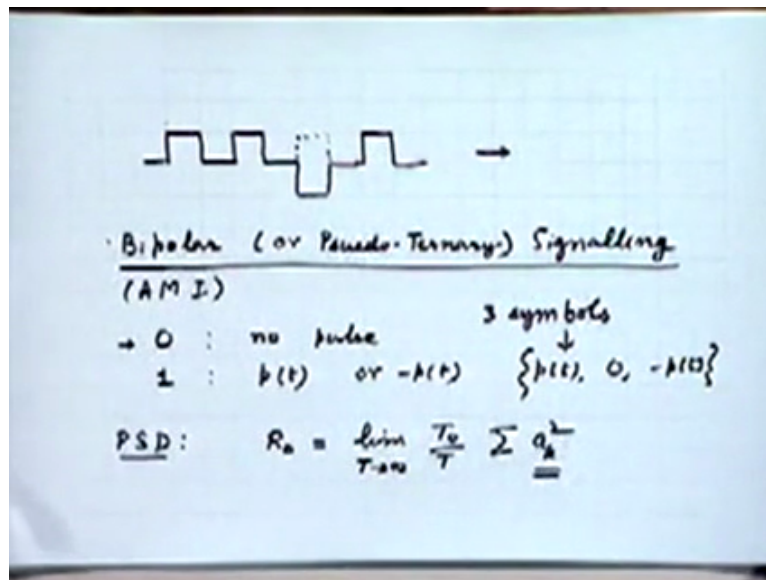
Also we continue to use excessive bandwidth, right, agreed. Also we are not transparent to long string of ones and zeros, right. And in fact we seem to have more disadvantages, that is we do not have clock recovery components present because that is no discrete spectral

component at  $f_0$ . But that is easily taken care of because if we rectify the received pulses we produce that, right, because we will convert the polar signal into a unipolar signal. We can easily convert the polar signal for the purpose of clock extraction clock extraction into a unipolar signal by simply rectifying the received signal, right.

And then generating the component at  $f_0$ . Right so clock component still recoverable through rectification, okay. Let me now write it because it will become too small. Any questions? Let me invite questions if you have any.

Student: ( ) (33:12).

(Refer Slide Time: 33:27)



Professor: Well, no problem, let us say you are using half width pulses, right. That is why, that is your polar signal, now if you want to convert polar signal, all you have to do is rectify it, right, this will come here and you will get a clock component, right. That is the basic idea is, in fact you can only get a clock component like this provided we use, provided we use half width pulses. If we full width pulses, there will be problems. That is why we are looking at all these schemes with half width pulses because it is an important function to perform at the receiver and at the repeaters to be able to recover the clock.

Also it is obvious that you are half width pulses required twice as much bandwidth as using full width pulses, right. We can always reduce the bandwidth by a factor of 2 by simply going from half width pulses to full width pulses in each scheme. But if we do that, we lose the flexibility of recovering clock through either directly or through a rectification process. Okay. So you keep that in mind.

Student: Sir excuse me.

Professor: What?

Student: It is clear that why it is polar is more power efficient than on-off signalling. You said that intuitively that the noise level seems to be the same because the amplitude seems to be the same in both the cases, right. But before that you commented that the on-off signalling is less immune than RZ and polar signalling.

Professor: Yes, the 2 statements are equivalent. Yes, if you require 3 db more power to get the same noise immunity, we say you are less immune to noise, okay, there is no contradiction in that. Any other questions?

Student: (( ))(35:28).

Professor: Okay. You may ask the question here, you may direct your question here if there is a problem. Am I going too fast? You can slow me down by asking questions, that is the standard thing all of us can use, right and that is equally true in a class, right. We next go to the 3<sup>rd</sup> scheme that we discussed, what do you do from polar, how do you improve polar? We went to buy polar, right, by using a scheme like AMI, right. This bipolar is actually if you remember a 3 level scheme, right. And therefore we sometimes call it a ternary or pseudo-ternary signalling.

For example, alternate mark inversion is an example of this situation. In fact that is the (( )) (36:42) we discussed here. This is more or less has become a de facto standard for PCM system, till recently that was... It was, these days this is used quite often. And in this a 0 is represented by a low pulse or a 0 level and 1 is represented by either a pulse  $p_t$  or  $-p_t$  depending on some logic, right. For example we may have alternate pulses to be positive, alternate marks to be positive and negative, right. Which is what we do in a.m. I. So really speaking of 3 levels, we use 3 symbols, namely  $p_t$ , 0 and  $-p_t$ .

Although we are doing binary transmission, we make use of 3 symbols, right. That is why we call it a pseudoternary signalling scheme because it is a 3 level scheme rather than 2 level schemes. Alright 1<sup>st</sup> thing we like to do is compute the PSD again, power spectral density function again. And we again start by looking at  $R_0$  which is given by the same expression as before and we have to look at what will be the signal  $a_k$  square in this case. Let us come to

ak, it is clear that in this scheme of things, half the aks will be 0, if you are again talking of equally likely zeros and ones, original zeros and ones.

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$a_k$ :  $\frac{1}{2}$  of cases 0  
 $\frac{1}{2}$  of cases 1 or -1 with  $a_k^2 = 1$   
 $R_0 = \frac{1}{2}$   
 $R_1 = \lim_{T \rightarrow \infty} \frac{T_0}{T} \sum_{k=0}^{n-1} a_k a_{k+1} = -\frac{1}{4}$   
 $n > 1$   
 $\sum_{k=0}^{n-1} a_k a_{k+n} = \begin{cases} 0 & \frac{3}{4} \\ 1 & \frac{1}{4} \\ -1 & \frac{1}{4} \end{cases}$   
 $R_n = 0$  for  $n > 1$   
 11  $\rightarrow$  -1  
 10  $\rightarrow$  0  
 01  $\rightarrow$  0  
 00  $\rightarrow$  0

Then half will be 0, very good, so  $a_k$  is half of them 0 and half of them 1 or -1, each contributing  $a_k$  square equal to 1, for each of them  $a_k$  square will be equal to 1. The value they will be contributing in the summation will be unity. Therefore we need to compute  $R_0$  equal to half, right, let us come to  $R_1$ , a case in which  $t$  tends to 0. You see you see the flexibility that you have now. By changing the rules of the game, that is how  $a_k$ s are related to each other, successive  $a_k$ s are related to each other in terms of positive and negative levels, you can do wonderful things to the spectrum as we will keep on seeing, right.

So this is  $T_0$  by  $t$ ,  $T_0$  by  $t$   $a_k a_{k+1}$ . Let me consider 1<sup>st</sup> the case of  $n$  equal to 1, we will discuss, in this case we need to consider  $n$  equal to 1 differently from general way, all right. All right, is it clear to all of you, do I need to explain? Somebody had suggested that it should be equal to -1 by 4 and answer happens to be correct, that means you can work it out. Let us assume that you can work it out, unless you want me to do it. Okay, very easy to work it out because  $a_k$  and  $a_{k+1}$  will have opposite signs, right.

And when you have, you know we can have 4 possible symbols as per symbols are concerned 11, 10, 00 and 01, right. In terms of levels, voltage levels, this product will be -1 here because if one of them is 1, the other will be -1 in terms of voltage level. And all the others will be zeros, so only one 4<sup>th</sup> of the time you are getting a contribution from these terms and that

contribution is equal to -1, right, and therefore the results. All right. And for  $R_n$ , for  $n$  greater than 1, let us consider a situation  $R_n$ .

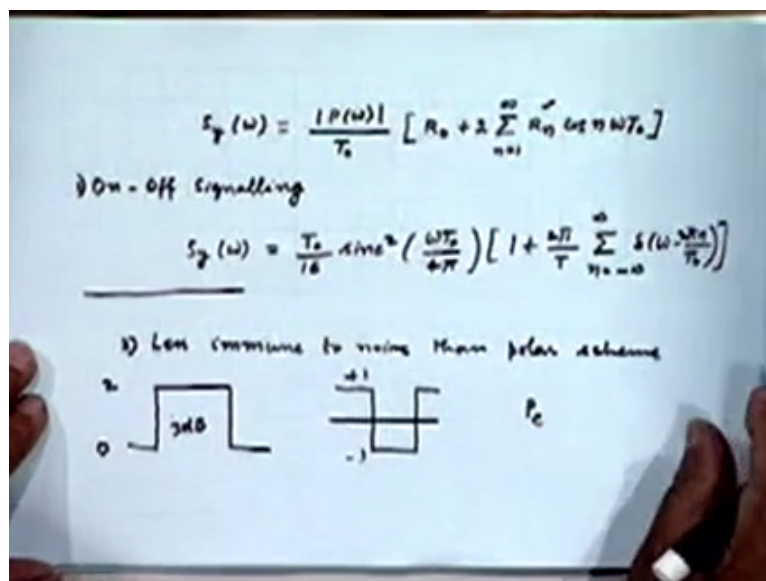
The situation for  $n$  greater than 1 and the value of  $R_n$ , okay. Let us therefore start by discussing  $a_k a_{k+n}$ . Now what can we have here?

Student: (0)(41:47).

Professor: This could be 0 or 1 or -1, right, depending on whether one of them is 0 or whether both, now both of them can be ones or both of them can be -1 or one of them can be one and one -1, you can have all possible combinations. And the situation is like this, 11 will give rise to either -1 or 1, right. Depending on whether they both have the same sign or opposite signs. Then all the 3 other combinations again will lead to 0. That means the probability of a 0 is going to be 3 by 4 and probability of these 2 together is half and each of, sorry, 1 by 4 and each of them individually is 1 by 8, right.

So therefore what is  $R_n$ ? Half the time, one 8<sup>th</sup> of the time they are 1, one 8<sup>th</sup> of the time they are -1 and for rest out of time they are 0, right. So again ones and -1s in this summation unequally balanced, right, for  $R_n$  equal to 0, for  $n$  greater than 1. That means the term which will matter now in the power spectral density expression are  $R_0$  term and the  $R_1$  term, right. Is it clear to everyone? If necessary I can go over it again if there is a problem. Fine.

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$$S_y(\omega) = \frac{|P(\omega)|^2}{2T_0} (1 - \cos(\omega T_0))$$

$$= \frac{|P(\omega)|^2}{T_0} \sin^2\left(\frac{\omega T_0}{2}\right)$$

(i)  $S_y(\omega) = 0$  for  $\omega = 0$  (d.c.) regardless of  $P(\omega)$   
: D.C. null guaranteed

(ii) Half-width Rect. Pulses  

$$S_y(\omega) = \frac{T_0}{4} \text{sinc}^2\left(\frac{\omega T_0}{4\pi}\right) \sin^2\left(\frac{\omega T_0}{2}\right)$$

So what happens to SY omega? We have got P Omega square magnitude upon 2T0 into 1 -, what is the value of this summation that, -1 by 4, right. So that is 2 Sigma something, right, so that 1 by 4 becomes 1 by 2 which have taken out, right. So you are only left with cosine Omega T0. I hope all of you know which expression I am talking about, the 3<sup>rd</sup>, let me recapitulate again for you. We are talking of this expression here, right, substituting for R1, all the rest of the terms being 0, right.

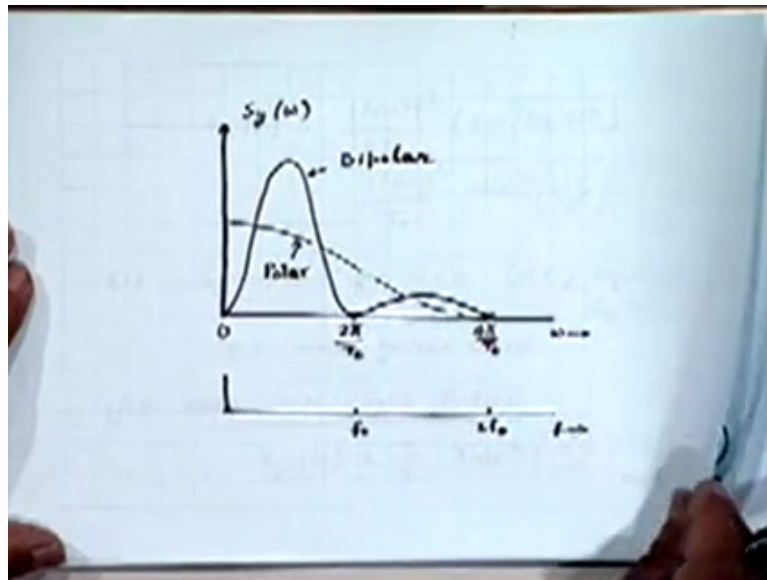
Have got -1 by 4 here, 2 by -1 by 4 is -1 by 2 which you are taking out, R0 is also half, okay, which you can write as P Omega square by T0, using trigonometric identity, sin square Omega T0 upon tau, right. Now this is very very interesting, let us see in what way we have interesting visuals here. 1<sup>st</sup>, but before that there should be mathematical problem, if anybody has any problem please speak out. 1<sup>st</sup> important thing is irrespective of what pulse shape I have and therefore what fourier transform is associated with that pulse shape at omega equal to 0, SY equal to be 0, right.

So we have a spectral null guaranteed to exist at DC, which of course is intuitively obvious because half the time we are transmitting 1 and half -1 and rest of the times they are all balance, 1 and -1s are balanced and rest of the time we are transmitting zeros. So that is DC, regardless of P Omega, right, that is DC null is guaranteed which is very desirable as we already know from the AC coupling point of view. Now let us consider the case of half width, we considered a specific P omega now, half width rectangular pulses. For this case you are SY omega will be equal to, again the same sinc function of width symbol 4 pie by T0, this becomes T0 by 4 Sinc square omega T0 by 4 pie into sin square omega T0 by 2. Okay.



Now what do you see? Therefore 2<sup>nd</sup> interesting property that we get from here, the 1<sup>st</sup> one was irrespective of P omega you get a PC null, right. What kind of P omega you might use? Now we are considering a specific P omega, namely half width rectangular pulses which we have already seen in the polar and the unipolar case lead to a bandwidth of 2 f0, right. Let us see what we get here. We get a bandwidth of, okay, you see it is this factor which is really making the difference.

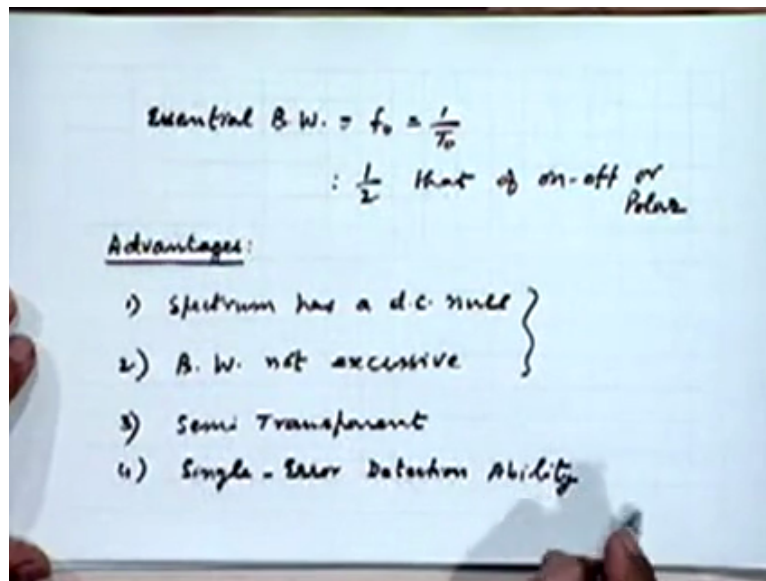
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The Sinc square function is going to have its 1<sup>st</sup> 0 at same point 2f0 but this function has a 0 at half that point, it will be between 0 and that point. I have drawn this somewhere earlier, yes. This is a spectral after bipolar signal, right. You get DC, you get a 0 at DC, right and you get your 1<sup>st</sup> null not at 4 pie by T0 but at 2 pie by T0 because in square omega T0 by 2 is 0 at this point. Substitute, what will you get, you will get sin square pie, sin pie is 0, right. So the product of these 2 functions goes through 0, so the main lobe width is reduced by half, right.

For the bandwidth requirement is now, for the same half width rectangular pulses is not 2 f0 but f0, right. Okay. So let me make this point very clear. The essential bandwidth, of course you do, when I say essential bandwidth other reason is that is still some energy in this lobe, right.

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Essential bandwidth is equal to  $f_0$  equal to  $1/T_0$  which is half that of on-off or polar, right for the same pulse width situation. However we are still off from theoretical bandwidth by factor of 2, remember that, theoretical minimum that we can go to. So let us look at in a nutshell what are the advantages we are getting here, bipolar signal. We have discussed that earlier an intuitive point of view, now we are taking from a very definite knowledge through analysis. 1<sup>st</sup>, spectrum has DC null, 2 bandwidth is now, I do not think we should call it excessive, we have twice the theoretical minimum but certainly it is not excessive, it is not 4 times that. Now what is more, these are desirable things, we are also somewhat, I think they are still not transparent to long sequence of ones and zeros.

Student: (())(51:15).

Professor: So we can say semitransparent or whatever. Now but we do have some other advantages, we have some error detection abilities, right. You can in fact up to single errors, so we have single error detection capabilities. Can you see how?

Student: Because of alternate inversion.

Professor: That is right. If at any time you find that alternate ones are of the same polarity, that means you have made some error in the decoding process, right.

Student: But how do you know that the thing which is coming, the next one that is the alternate part, that is 1, that could be a 0 (())(52:06) on account of noise.

Professor: That is by we are saying detection, we are not saying that we are able to correct. There only was to say yes there is an error, I think more than that. Right, that can help you in some situations, all right. There is a difference between error detection and error correction. And what about clock recovery? Okay, that is an important point. Do you have the possibility of clock recovery or not? Well in the same way it is Polar, right. Because I can always rectify it again and do it.

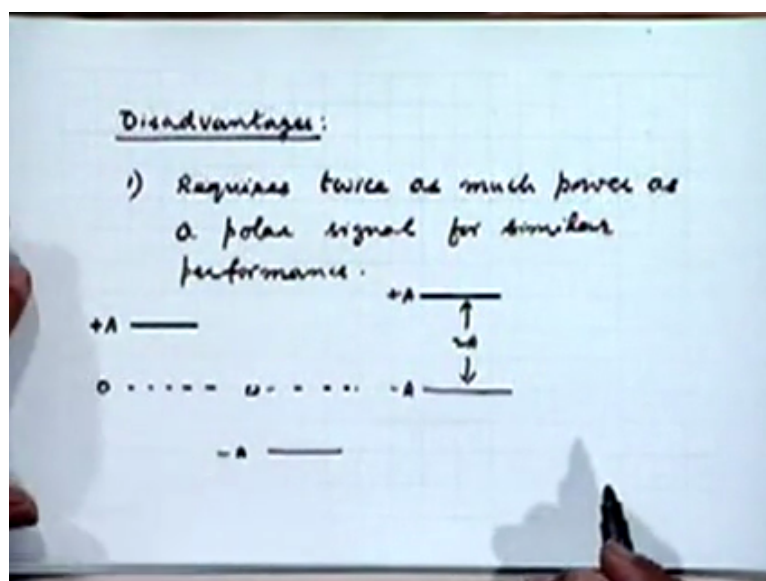
Student: (())(52:51).

Professor: As long as...?

Student: Zeros are...

Professor: As well as, well that is taken care of by this property, that is common disadvantage in all the cases. Even the discrete components that they have talked about in the spectrum makes sense if you do not have long string of ones and zeros, right because that is assuming that your signal is statistically behave, well-behaved, you do not have long string of ones and zeros, right, it is reasonably random. If it is no longer that random, that is why the transparency question comes up. Is the signal is always truly random, ones and zeros are always changing, right, then we always get the discrete components in the unipolar case. But even there you will not always get a discrete components since get long string of ones and zeros.

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That is if the signal is not truly random which is implied by this kind of analysis. What are the disadvantages of bipolar schemes. Let us talk of power efficiency, do you think it is as power efficient as Polar or less power efficient than Polar? Can you make any judgement?

Student: ( ) (54:17). Same as on-off signal.

Professor: Okay, the answer is correct, it requires again twice as much power as that, as the Polar signal does for similar performance. The reason is, in order to distinguish, let say between  $+A$  if  $A$  is amplitude of the pulse and  $0$  or  $-A$  and  $0$ , right, separately. As against Polar where I am comparing  $+A$  with  $-A$ , right, here I have a suppression of  $2A$ . And I am making this decision again in the presence of noise, if I want same kind of performance, I better have the same kind of swing between  $0$  and this point, right and not between these 2 points because the choice is not between this and this, the choice is between these 2.

And similarly the choice is between these 2, right, so that the average power becomes twice. Okay. Well that is the main disadvantage, other than the fact that it is not fully transparent.

Student: Full width pulse will become 4 times. Power for full....

Professor: No, we are we are doing a relative comparison, if they are full width here, they are full width there, if they are half width here, they are half width there, okay. It does not make sense to compare 2 things which are not similar.

Student: Noise performance will be changing.

Professor: Right. It does not have same amount of energy being transmitted in every bit interval, right in whatever you like to do it. Do we have time for...? Okay, so we have discussed today 3 line coding schemes Polar, bipolar and of course on-off unipolar, right. Out of which we are, we have really gradually gone from the unipolar to the bipolar and we have seen that we have considerably improved in our performance. And we will do a few more things, we will see that, we can come up with some still better and nicer properties in terms of line coding properties and we will continue to do that tomorrow or next time. Thank you very much.