Digital Communication.
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Lecture-7.
Spectral Properties Of line Codes: General Relations.

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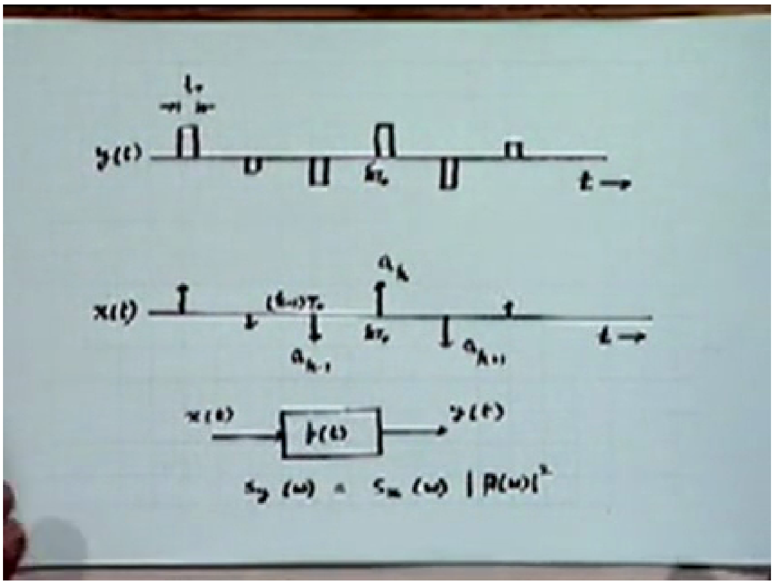

Professor: We shall talk about spectral properties of line codes. You may recall we were interested in looking at the power spectral properties of line codes which may typically like this, line coded signal will typically look like this, consisting of positive and negative pulses in general of arbitrary amplitudes with pulse widths equal to let say some value t sub 0 and with pair of t sub 0 which corresponds to the data rate, the symbol rate or the pulse rate that you are using in your digital system, digital transmission system.

As we discussed that day, instead of computing the power spectrum of directly this pulse train, it will be more convenient to 1st look at the power spectrum of the corresponding impulse train whose strengths depend on these amplitudes or amplitudes of these pulse trains and impulse trains and this impulse trains are related to each other. Because once we have the power spectrum of this kind of impulse train we can use that to compute the power spectrum of not only this kind of pulse, pulse rate but any arbitrarily kind of pulse train in the sense that the pulse shape could be different from the rectangular one we are using here.

All we have to do to do that is to regard this pulse train as having been obtained from passing of this impulse train through a filter whose impulse response is pt where pt is the shape of this pulse, right. For example you want a rectangle pulse of width t0, all you need a filter of impulse response with, with a width of t sub 0, right. Because passing of this impulse train through a filter of that kind will obviously produce a person of this kind, this hardly needs any explanation.

And in that case the power spectrum of this pulse train yt we obtained from, in terms of the power spectrum xt by simply the well-known result sy omega here would be the sx omega which is the power spectral how the input multiplied by p omega magnitude square, all right, this is what we were talking about last time. So we will now be interested in obtaining the power spectrum of this impulse train xt, all right, which occurs here periodically, it is not a periodic impulse train but the consecutive impulses occur every t0 seconds, capital t sub 0 seconds.

And we have the impulse trains in general, the kth impulse has a strength of a sub k occurring at the time instant k t0. Now how do we go about obtaining the power spectrum of such an impulse train? Any suggestions on that? How do you go about obtaining the power spectrum for the impulse train of this kind?

Student: Spectral integration (())(4:39).

Professor: Any specific definite approaches?

Student: Autocorrelation, auto (())(4:49), autocorrelation and take fourier transform.

Professor: Very good, what we have to really do is compute, you have to realise 1st that these amplitudes are random in nature, right. And therefore we have to obtain the autocorrelation function of this impulse train, at least the time autocorrelation function and then what is the relationship between power spectral density in autocorrelation function? They are fourier transform pair, fourier transform pair by the virtue (())(5:20) theorem that you must have heard of. The autocorrelation function and power spectral density function are fourier transform pair, that is the result we shall use.

And to do that is what we 1st need to do is to compute the autocorrelation function of this impulse train. In order to do that we shall make another transformation from this impulse train back to rectangular pulses. What will be like to do is, all of you are familiar with this fact, you can regard an impulse as a limiting form of a specific kind of pulse shape that you may choose. For example we could choose a rectangular pulse shape whose width and height are such that the area under the curve should be equal to the strength and the limit as the width is decreased to 0, the amplitude of that pulse will go to infinity becoming an impulse with the strength a sub k, right.

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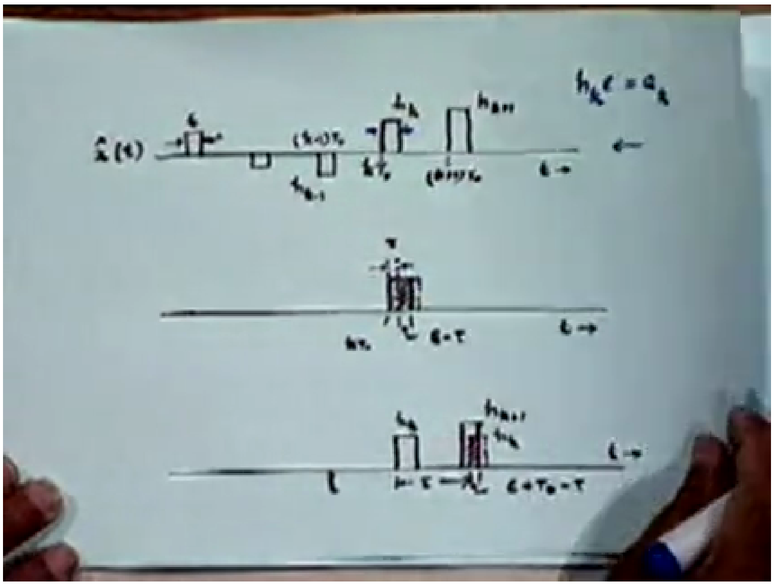

So we shall do that, we shall therefore approximate to start with this impulse train once again with the rectangular pulse train but with a different purpose now. And if the limit as the width of those pulses is made 0, we shall obtain our impulse train back. So that is the situation depicted here, each of these pulses is supposed approximate the impulse corresponding to it in the impulse train sequence that we discussed a few minutes ago. The height x of k of the kth pulse and the width epsilon k, epsilon, epsilon will be constant for all pulses are chosen such that hk epsilon would be equal to a sub k, right.

So that in the limits like this, epsilon is made 0, area is kept constant, by increasing the height we will again get back our impulse train, all right. So therefore to start you will not compute the autocorrelation function of the original impulse train, what we will like to do is compute the autocorrelation function of this kind of a pulse train and then take the limit as epsilon tends to 0 keeping the area constant. That is the approach we are going to follow. Is the approach clear?

Student: Sir.

Professor: Yes?

Student: Why go from a pulse train to an impulse train and then back to the impulse train, why not…?

It is convenient, you will see that it is very convenient, there are certain kinds of things we can understand nicely if we follow this approach, we will see that as we follow. Now let us look at the autocorrelation function calculation which we do by, i will come back to this diagram in a minute but let recalls the definition of an autocorrelation function that you are all familiar with. Let me also, let me just denote this approximation by x hat t, right, this member that xt was the original impulse train and x hat the t is approximating it by rectangular pulses of relatively small amplitudes, relatively small duration epsilon. All right.

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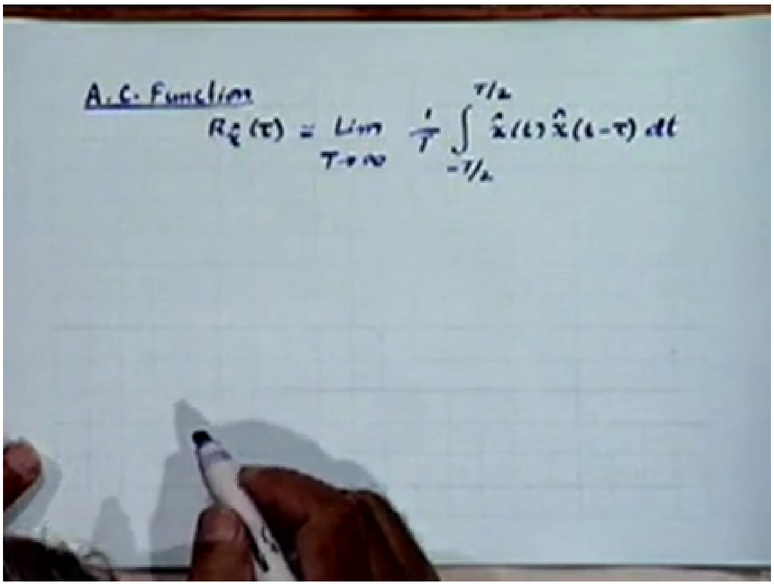

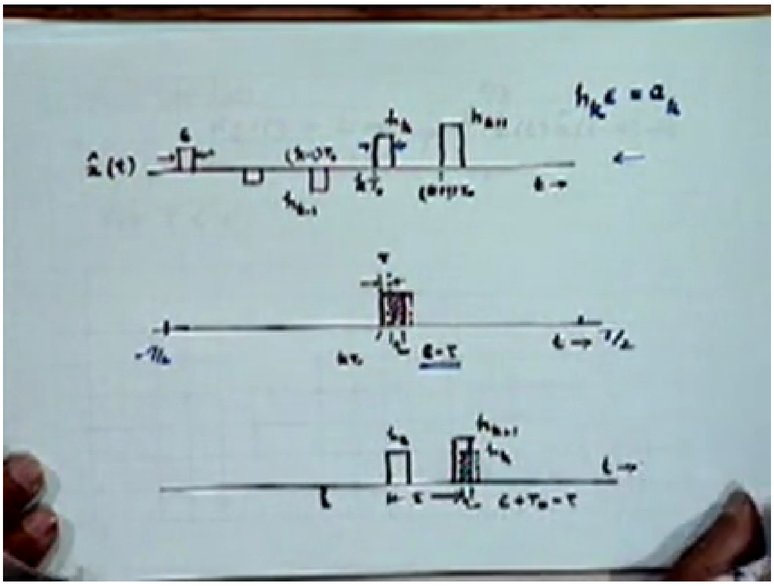

So you may recollect the definition of a time autocorrelation function of r x hat tao, how should you define it, we will define it as limits as some t, capital t tends to infinity, 1 by dark t is into integral from - t by 2 to + t by 2 x hat t into x hat t - tao dt. That is the autocorrelation function definition. Now recollect this definition, we basically take an observation interval which is sufficiently large, take original waveform x hat t, take its shifted version with the delay of tao, corresponding to which you obtain, you want to obtain the correlation between the 2 waveforms.

You are basically obtaining correlation with the lag tao of the waveform with itself, right. So just we take an observation of length t, compute these integrals over these limits and that take the limit as t tends to infinity, that is the definition of time autocorrelation function. As the tao is 0, the 2 waveforms, the undelayed and the delayed versions will perfectly coincide and you will get the peak of the autocorrelation functions. You will get the maximum value of autocorrelation function for tao equal to 0 which is a well-known property of an autocorrelation function. And also we may remember that autocorrelation function is an even function of tao for real signals, right.

That is whether you delay one way or the other, the value of the autocorrelation function is the same for equal values of tao on both sides, right. So whether you delay or advance it by tao, you will get the same kind of average behavior of this product, this integral. Let us consider to start with the values of delay which are small. Let us consider taos which are small, specifically smaller than the pulse duration epsilon that we have been using. Let me show it here. We are considering this waveform and its delayed versions and i have picked it here only one of these pulses instead of the whole of it because it is not really required.

So what you can say is that this dotted waveform is shifted from the original waveform, original pulse by an amount which is equal to tao which is shown over here, very small amount which is less than epsilon the pulse duration, right. And obviously the contribution to the integral, what we are interested to know is what is the value of this product because that product is what is going to contribute to the value of this integral. This product will be contributed now by a number of such pulses, right, all of which lie in the interval between - t by2 + t by2 which we are integrating. And where looking at the contribution of the kth pulse, kth pulse here is, between x hat t and x hat t - tao which is depicted like this.

Student: (())(12:58) gap between 2 pulses is greater than epsilon.

Professor: That is right because epsilon the anyway assume it to be vanishingly small eventually, that is understood. Okay. So what will the value of the autocorrelation function for this value of tao? Well it depends on this overlapping area as we know and that overlapping area will depend on, will actually be proportional to epsilon - tao, more specifically it will be hk into epsilon - tao, right. And every pulse in the region which we have chosen, let say between - t by2 to + t by2 will contribute a similar value but the only difference will be the amplitude hk is different from pulse to pulse.

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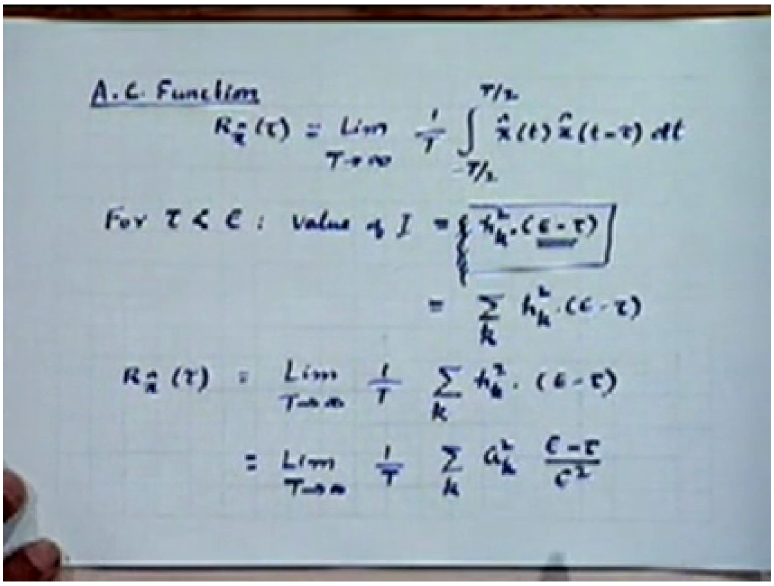

So we can say therefore for tao less than epsilon, the value of the integral, i will just call it i, value of the integral will be equal to hk square because you are multiplying hk with hk in the kth pulse pair situation into epsilon - tao which is the duration of the overlapping interval, right. Or if this is confusing you can write epsilon - tao into hk square. So as not to confuse this with the argument of hk, this is a product. And okay, this is, this is the contribution to the integral from only the kth pulse pair.

Really speaking i must modify this to be this equal to sigma hk square into epsilon - tao for all those values of k, all those values of the index k which keep the corresponding to the pulses in the interval between - t by2 + t by2, right. So we can therefore write that rx hat tao equal to limit as t tends to infinity of 1 by t, i can replace this integral now with this summation, right. Is that fine? Further, i note that hk square hk is ak upon epsilon because hk was elected to be such that hk epsilon is equal to a sub k, right.

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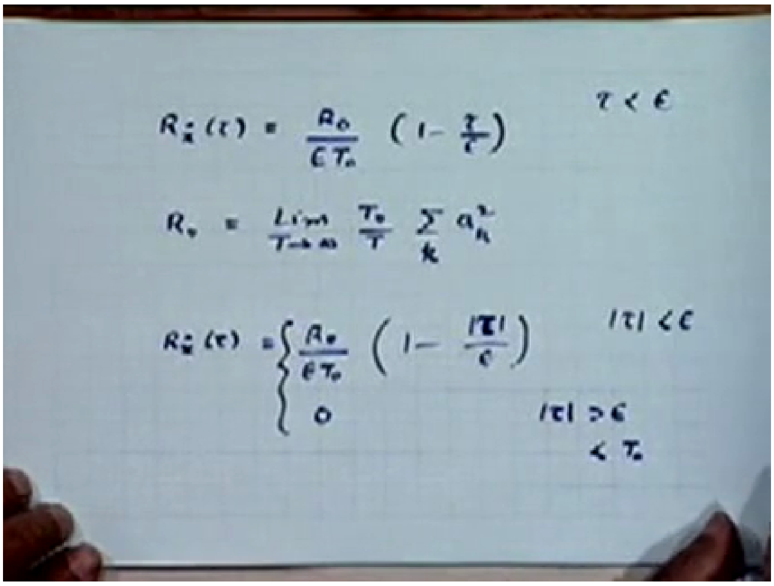

So let us substitute for that and we will get ak square into epsilon - tao upon epsilon square, all right. Any doubts? Please speak out if you have any problem. We will go proceed further then. Or further we can write r x had tao, i will define a quantity r subzero in a minute, upon epsilon t0 into one - tao by epsilon, where let me just quickly look back it is difficult to see, or we see like this. This epsilon square i am taking outside, so that gives you this expression 1 - tao by epsilon, actually we are taking epsilon outside because this one epsilon cancel.

And r 0 is defined by limit t tends to infinity t0 upon t, just convince yourself that is okay, all right.

Student: (())(18:07) is arbitrarily chosen?

Professor: No, t0 is the, just, just for some convenience i have introduced this t0, it was not required to be introduced, right. All that we really need to define was limits 1 by t ak square, right. We have deliberately done it because it is convenient to do so as we will proceed further, right. T0 is the period of the… it is the repetition period of the pulse train. It is not exactly the repetition period, it is the time interval at which the impulses are going or the pulses are undergoing, corresponds to the data rate, this is the interval corresponding to the data rate, right. This expression is fine, we can proceed further then.

Now to proceed further, the 1st thing that you would like to notice is this expression has been obtained under the assumption that tao is positive in less than epsilon. And we have already discussed, it is an even function of tao, autocorrelation function is an even function of tao. So the same expression should be valid or similar expressions should be valid for negative tao, except that this will become + or better it is more convenient to write like this, 1 - mod epsilon by, sorry mod tao by epsilon, right. Because (())(19:45) shift on one side, r on other side, you expect the same value as autocorrelation, right, for the same value of tao, provided mod tao is less than epsilon.

What will happen now as, as tao is increased? It is

Student: (())(20:09)

Professor: Good, very good, it will become 0 at what point, at tao equal to…?

Student: Epsilon.

Professor: You have a triangular shape up to tao equal to epsilon and 0 beyond, not entirely beyond but after sometime.

Student: Then next one will start.

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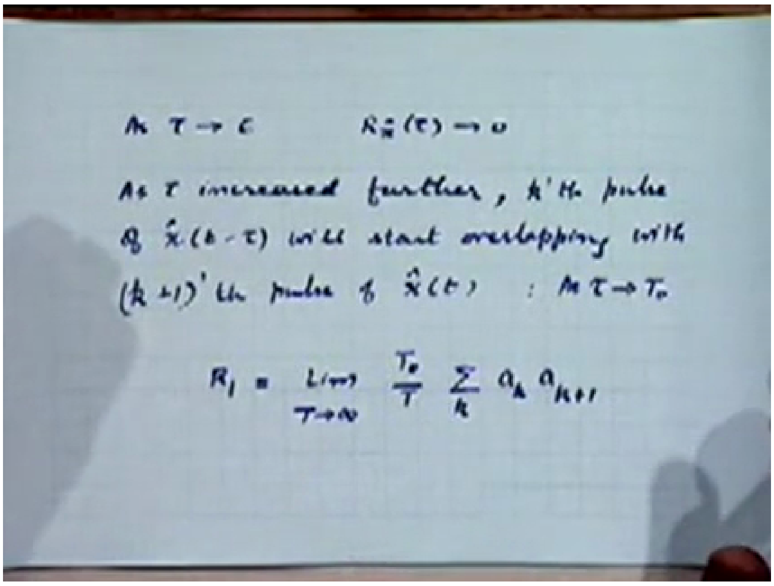

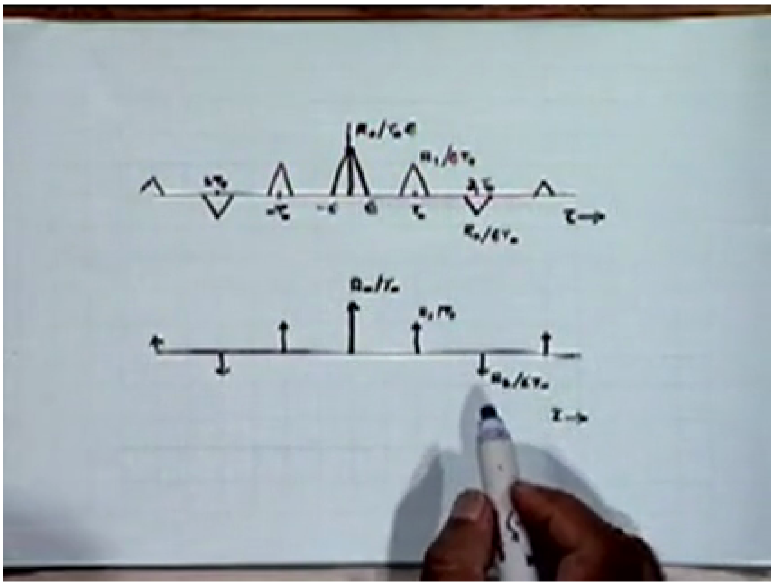

Professor: That is right. So we will say that this is, this is not really a complete expression, this is for magnitude of tao greater than epsilon but less than t0 and so on. We will try to appreciate it more and more as we go along. So is this fine? So as we just noticed as tao tends to epsilon, r x had tao tends to 0 because of the 0 overlap between the kth pulse of x hat t and the kth pulse of x hat t - tao. But as tao is increased further…

Student: Excuse me sir.

Professor: Yes please?

Student: In the previous expression it will be 0 for tao greater than epsilon but for tao less than t0 - epsilon.

Professor: Yes, that is right.

Student: Because right half of it will…(())(21:43)

Professor: Yes, you are absolutely right, this should be t0 - epsilon because as soon as tao becomes t0 - epsilon and overlap between 2, 1 will be a displaced version of each other, crossing of each other will start to take place. Very good. Now as tao is increased further, this is what just as been pointed out, the kth pulse of the tth pulse of x hat t - tao will overlap with, will start overlapping with k + 1th pulse of x hat t, right, very clearly as tao approaches t0, more specifically at the point where it becomes equal to t0 - epsilon has just been pointed out, right.

So what will happen then? We will get another triangle, right, around tao equal to t0 with the difference that the amplitude of this triangle would be slightly different because now it will involve product of ak and ak +1, right. So i think all of you now appreciate this fact that r1 that will come into picture will be limit as t tends to infinity of t0 by t summation over k a sub k into a sub k +1, right. Again we are assuming that all those terms will matter for which, which lying between the interval -t by 2 and + t by 2 that you are considering. Similar things will happen for tao equal to 2 t0, 3 t0 and so on, right. As you…

Student: (())(24:15) expression will also change in the…

Professor: The corresponding expression in r1, r2, etc. Will also change.

Student: (())(24:20) 1 -…

Professor: That will stay say as so because basically you will get a triangular shape over 1, over 2 epsilon period around each t0, around n t0, right. Because as you move along the overlap 1st increases, as you approach the next period the overlap 1st increases gradually, becomes maximum and then starts to decrease.

Student: (())(24:46) it will be tao - k t0 only, like you put the…

Professor: I am only talking about the shape. Obviously each of these triangles is centred around n t0 whatever, tao equal to n t0, you are quite right, let me show that pictorially. The thing that all of you recently mentioning is precisely what i also mean. That is at the centre around dot equal to 0 we have a triangle like this of duration 2 epsilon going from - epsilon + epsilon, amplitude which is r 0 by t0 epsilon, around t0 you have the amplitude which is r1 by t0 and so on. Okay. This r0 by t0 epsilon if you notice, comes from this here, do not worry about that, it is okay.

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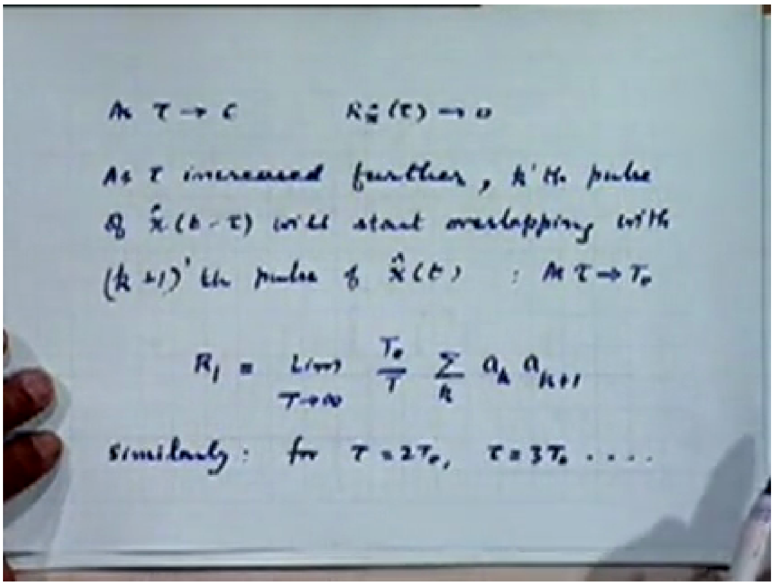

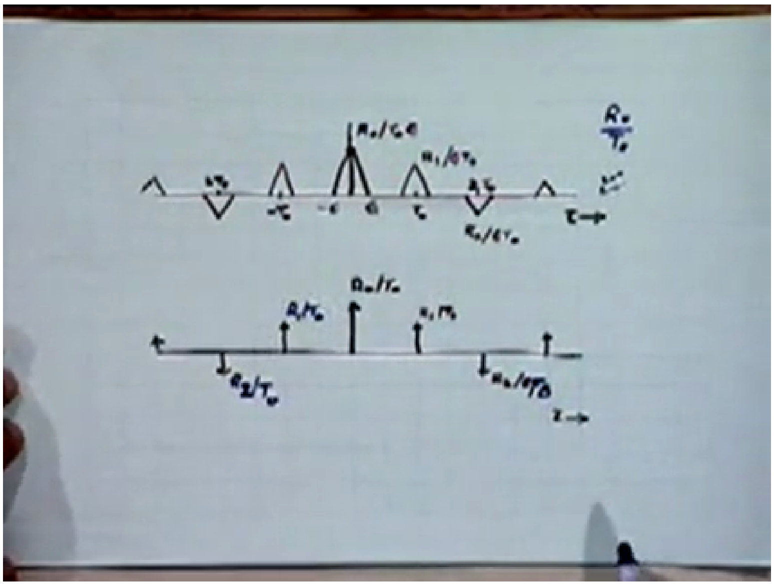

This way, this amplitude here, arbitrary angle. So this is the autocorrelation function of that approximating pulse train, right. And obviously what you would like to do is get back to autocorrelation function of the original impulse train. Let me before i do that, we must note the generalisation of this expression would be, this will become r sub i and everything else is same, this will be a sub k into a k +1, right. That is the value that will govern the amplitude of the triangle located at tao equal to m t0, right. So r sub m will be governed by the summation a sub k into a k +, a sub k + n.

Now to get back to the original situation what we would like to do is make epsilon vanishingly small, make epsilon smaller and smaller, keeping hk into ak constant or keeping the area under this triangle constant, right. So what is the area of this triangle, it is twice of this, sorry half of this into this, right. So r 0 by t0, right, that is the area, so as we make epsilon smaller and smaller, this amplitude obviously becomes infinity, once again to get back and impulse located at each of these pulse. The impulse trains are r0 by t0, right.

Student: Should not we decrease epsilon keeping the condition epsilon into ak equal to hk?

Professor: No, we are now talking about what will happen to this limit, what will be the limiting form of this impulse, this triangular pulse train.

Student: That is true but to get the corresponding autocorrelation function for the pulses from where we have started by making…

Professor: No, we have obtained for that pulse train, this autocorrelation function, right. As epsilon tends to 0 and that area is constant, we get that pulse train. We would like to see what happens to this autocorrelation function which has an amplitude which also tends to infinity, right. So we get a different kind of impulse train whose amplitudes are these, okay. So you get a limiting form of this as the autocorrelation function of the limiting form of the, also an impulse train whose autocorrelation function we really desired.

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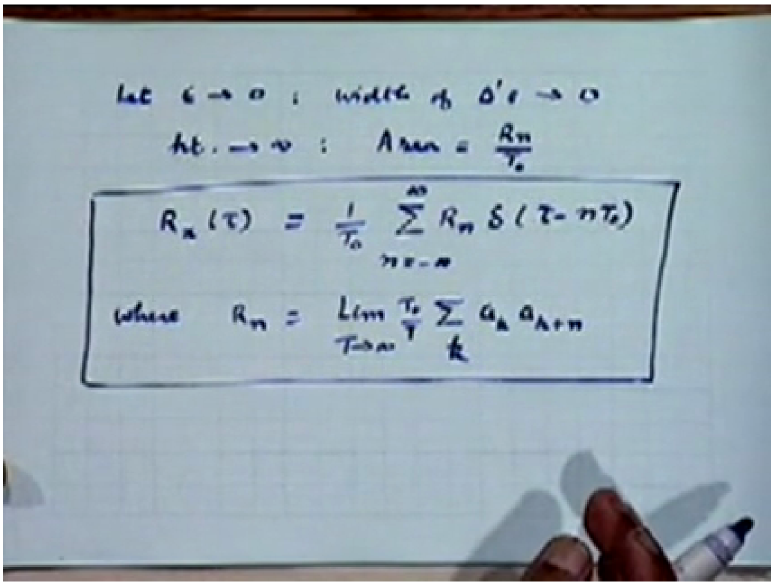

So as you can see this will be either very symmetrical, this is r1 by t0, this will also be r1 by t0, this should be r2 by t0, not epsilon t0, similarly this will be r2 by t0 and so on. So mathematically we let epsilon tends to 0, this will make the width of each of these triangles tend to 0, also the height will tend to infinity because the amplitude is r sub m by epsilon t0, there is an epsilon in the denominator of the amplitude which will make the height of the impulses of the triangles tend to infinity. But the area will remain finite, which is equal to r sub m by t0 for each of these.

And therefore in the limit as shown in here, you get r sub x now equal to1 by t0 sigma rn delta tao - n t0, n equal to - infinity to + infinity, where we keep in mind that rn is limit t tends to infinity t0 upon t a sub k a sub k +1 whole k, right. So this is our result for the autocorrelation function. This is what we would like to study in detail in connection with various kinds of line codes which may be of interest to us, which will give us a better insight about what kind of spectral properties the various line codes possess and therefore what conclusions can we draw regarding their behavior.

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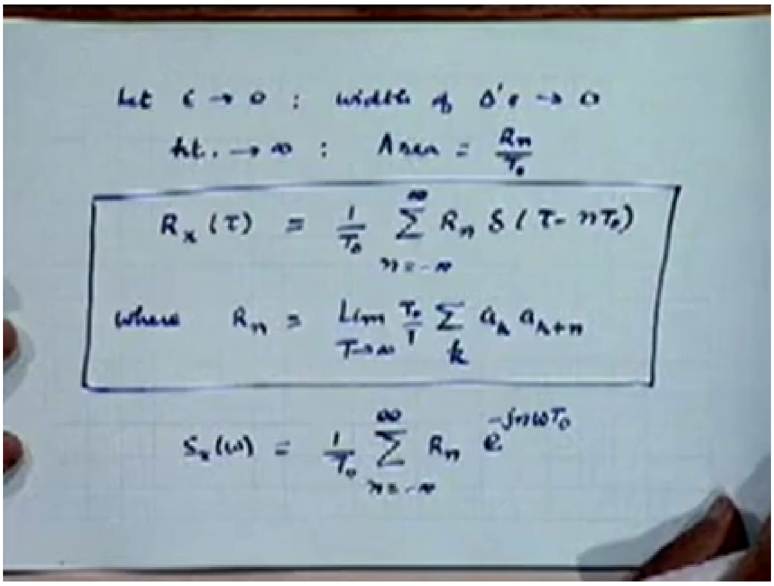

Before doing that, is it okay, to take it away? Before doing that we should go from the autocorrelation functions to the spectral domain and that we do simply by taking the fourier transform of this function. I think i can do that here itself so that you can appreciate, sx omega will be equal to… very easy, is not it? The fourier transform of an impulse function located at m t0 which spent r sub n will be given by e to the power - j omega n omega t0, n going from - infinity to + infinity.

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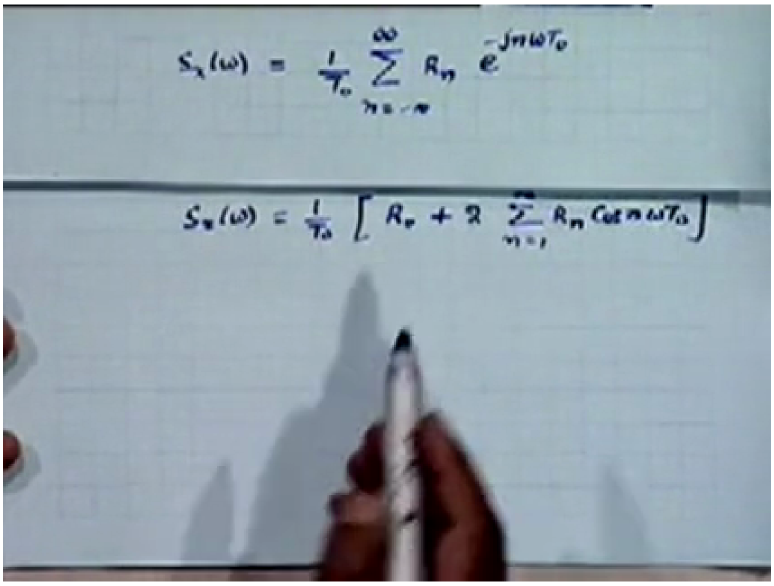

Now we can also use the fact that r sub - n is the same as r sub n, right. And we can obtain alternative expression which is dependent only on cosine functions, 1 by t0, i have taken out the r0 term here from this + 2 sigma n equal to1 to infinity r sub n cosine n omega t0, right. We can combine the positive frequencies and the negative frequencies to the cosine terms for nonzero values of n. So this is an alternative equation for sx. This is the power spectrum of the impulse train and if you remember, we started by saying that we can obtain the power spectrum of a pulse train with an arbitrary pulse shape pt by passing xt through a filter with impulse response pt.

And let say we are now interested in that, power spectrum of a pulse train yt, pulse train yt which uses pulse shapes, basic pulse shapes p of t. And obviously we discussed that earlier, sy omega would be p omega square into this expression over here, sx omega and substitute for that upon t0 r0, right, so that this general result. Any questions? As earlier, this pulse shape pt could be totally arbitrary, for example i could have, in fact i probably have something here. I could have a pt like that, right.

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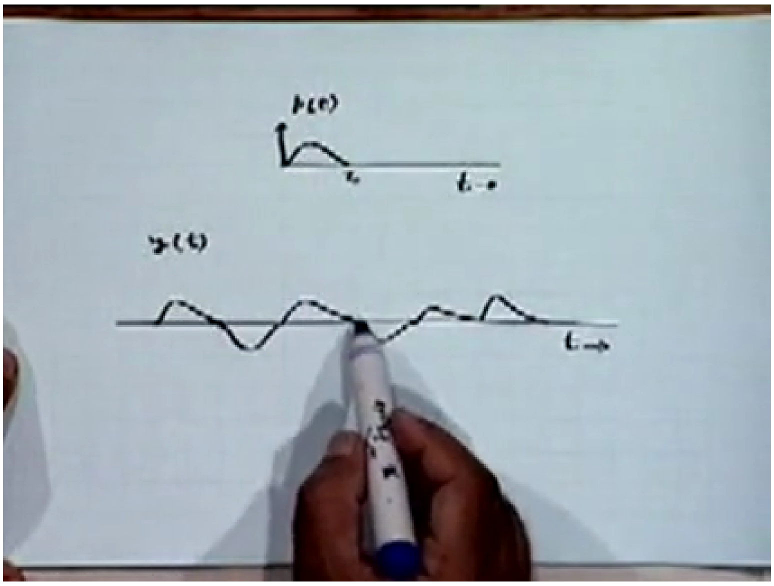

This is the basic pulse shape in the interval of 0 to t0 and then we get a pulse sequence like that, need not necessarily be rectangular in shape, right. But what may seem desirable from what we have been talking about is that pt should be limited to the interval t0, 0 to t0, right. But as we will see later, in reality we do use pulses in data communication which may not necessarily be limited to the data rate, basic data rate interval, right, for reasons which will become clear soon enough. But problem remains, we shall impose that condition and consider line codes with this property, right. Any questions we have? I will stop for a few questions as you may have?

What we will do next is take-up various line coding schemes which when we use and see what kind of spectral properties have and derive important and useful information about them. Any questions? Okay. Let us 1st take the class of unipolar signals which we, we have argued earlier not to be a very good thing to do, right. We will never see that from a number of points of view that is so, we have already seen some, we look at them again but we will start by looking at a description of a class of unipolar pulse trains.

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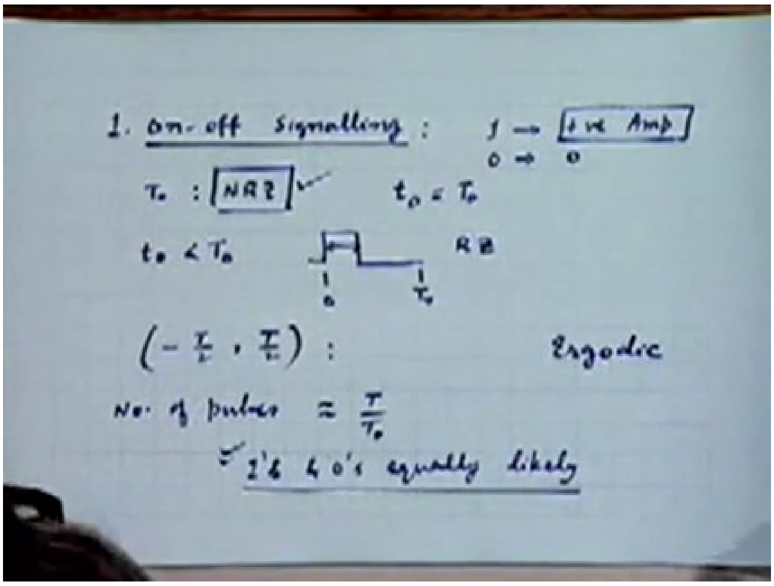

And this class of pulse trains is typically known by the name of on-off signalling, right. Because when you are transmitting 1, you are transmitting a finite energy, a finite amplitude level in a pulse which may have some arbitrary duration. It will not necessarily be equal to t0, the pulse duration need not essentially be equal to t0, it may occupy one of the intervals. If it occupies whole of the interval, we are one kind of condition, if it does not we have another kind of condition but 1st basically what we are talking about is the fact that 1 is represented by a positive amplitude, a positive, maybe negative but usually a positive amplitude and a 0 by 0.

This positive amplitude pulse may have a duration equal to t0 or less, right. When it has a duration equal to t0, we get a situation which is similar to nrz, in fact this is the most general name of nonreturn to 0, right. That is if your pulse duration t0 is taken to be equal to t0, during a 1 we are not returning to the 0 level and that is what we call a nonreturn to 0 signalling scheme, right. If however we take t0, which is less than t0, right because we are transmitting a pulse, this is your t0 let us say, what pulse width is less than that, even during a 1 you are at the 0 level for part of the interval, right.

Let us call it return to 0 signalling unipolar signalling scheme, right. This is the general definition of nrz, we will discussed only special case of that last time when we discuss bipolar schemes, right. But this is a general case of nonreturn to 0 and this is what is called return to 0. We will consider both of these another class of on-off signalling first. Now remember we have taken for our analysis interval equal to, interval from - t by 2 to + t by 2, right. And then of course you make t tends to infinity.

Let us 1st try to understand what happens, what kind of properties you will like to get, we will get for a sequence of unipolar pulses in this interval. 1st of all how many pulses you are likely to get in this interval, if t is sufficiently large? Number of pulses in this interval that you will see is of the order of t upon t0, right, a new pulse is coming every t0 seconds at the rate of at the data rate. Approximately we will get t0 pulses and if t is sufficiently large and if we assume that ones and zeros are equally likely, that is some of the assumptions we make.

Then we can go and calculate our autocorrelation function of power spectrum. But the important thing to note here is that your power spectrum depends not only on the pulse shape but also on the specific sequence of aks that are being used to transmit via those pulse shapes. Aks, the strength of these pulses also have a bearing on the spectrum, remember that expression, in fact i forgot to point this out earlier but this is a very important fact that one should remember.

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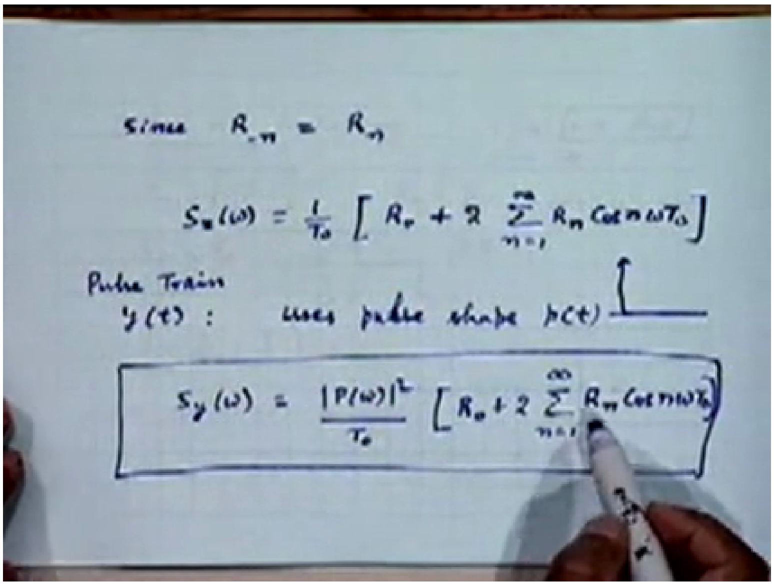

The power spectrum of s sub x or sub y depends on these ri or rns and rns in turn depend on the summation ak into ak + n. So the specific data sequence that you are transmitting is going to have a important bearing on the power spectrum that finally results. Not only just the pulse shape, which is just a multiplying factor, this of course has a very important role to play, the shape of p omega but equally important is the role of these functions, ri, rns, okay. That is why you have to consider what kind of aks we are talking about. That is why i have got a model for aks that we shall use in our calculation of power spectrum and that is one of the assumptions we shall make when ones and zeros are equally likely.

Student: Why number of pulses are approximately equal to t by t0 (())(43:00).

Professor: Well, I have got said here that t is a multiple of t0 and all those things, one can do that but it since (())(43:10).

Student: When you say ones and zeros are equally likely, that means we tend to set of particular kind of sequence strength pattern ak, ak?

Professor: No, data sequence is random yes but in general if you take, if you observe the data sequence over a very large period of time, you are likely to see as many ones having occurred as number of zeros, right.

Student: Within this there may be different sequences of…

Professor: Within this you can have infinite set of sequences, right. So it is hardly imposing any restriction, it is only talking about the fact that your source of data is balanced in ones and zeros.

Student: Each combination would have a difference spectrum, that is what is reflected power…

Professor: But since we are talking of everything ak into ak + n over k, right, it is going to become sequence is independent more or less, provided the average interval is sufficiently large, under certain conditions. Strictly speaking what you are saying is true but under certain assumptions like ergodicity of sequence and things like that, it will become, you know the time averages will become equal to ensemble averages, right.

So we are assuming that the source, we can assume that the source is ergodic in which case even if we compute specific sequence, if the averaging interval is sufficiently large, we can assume it to be more or less representative of the structure for any sequence. As we saw, this assumption was satisfied. So i did not want to go into those questions but since you have raised it, we can assume that the force is also ergodic , okay, you can replace time averages with you can replace ensemble averages with time averages. There is a whole motivation for taking of time autocorrelation function.

Right, otherwise one has to do a slightly different kind of treatment. We can also do an alternative treatment in which we can do a statistical averaging rather than time averaging, by and large we will get similar results, this is more convenient so i did this.

Student: If ones and zeros were not to occur with equal probabilities… (())(45:18) then only this would have mattered.

Professor: No, it does not affect the expression that we have derived, it will affect what we are going to derive. Right, what we are going to derive from this because we went to compute the value of rns and that computation we are going to do under that assumption, okay. It does not affect this computation, it does not affect the result that we already derived. But it is not going to affect for a specific case of on-off signalling, the computations that we are going to reach, right.

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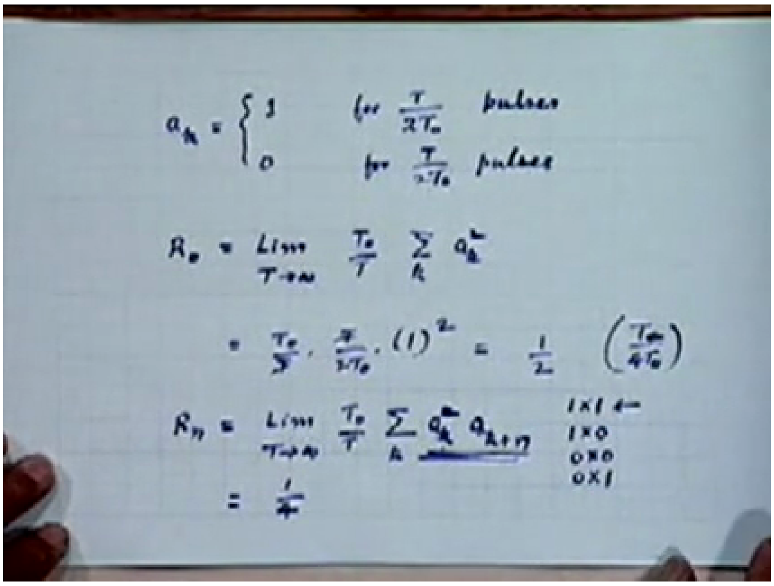

Okay so if we assume ones and zeros equally likely, what we are saying is the value of ak is going to be 1 for approximately half the number of pulses that we are observing in the double t, right, which was t by t0. So it is 1 for t upon 2 t 0 pulses and equal to 0 for equal number of pulses. Consider then, is it all right, any questions in that? Consider then r0, that is given by limit as t tends to infinity t0 upon t summation a sub k square, right. Now what can we say about this summation?

This in the observation interval t we are considering which is large, ak square equal to1 for half the time, right, and zero half the time, only those pulses which are 1 will contribute to the solution, those which are 0 will not contribute to this. So this will be equal to t0 by t, this is going to be t by 2 t0 into 1 square, right because each has an amplitude 1. I have forgotten the limit here because this t and this t will cancel, so it will become independent of t as far as t is sufficiently large.

So we will get precisely equal to half, similarly consider now r sub n. The case of r sub 0 is okay? R0 for this kind of situation will become equal to half and we shall now consider what is r sub n.

Student: 0.

Professor: No, it is not going to be 0, let us see that. This is equal to limit t tends to infinity, yes, that is right, i am sorry, this should be ak into ak + n. And once you got ak into ak + n, we can have this pair, consider the pair ak ak + n, and we can have the values 1 into 1, 1 into 0, 0 into 0 and 0 into 1, right. Again under the equally likely assumption that we have made, only one 4^{th} of these terms will really be contributing to the, assuming that t is sufficiently large, only one 4^{th} time you will have this situation, we get this answer to be equal to 1 by 4, right, just like we have half over here.

Student: T by2. One by 4. 1 by 4.

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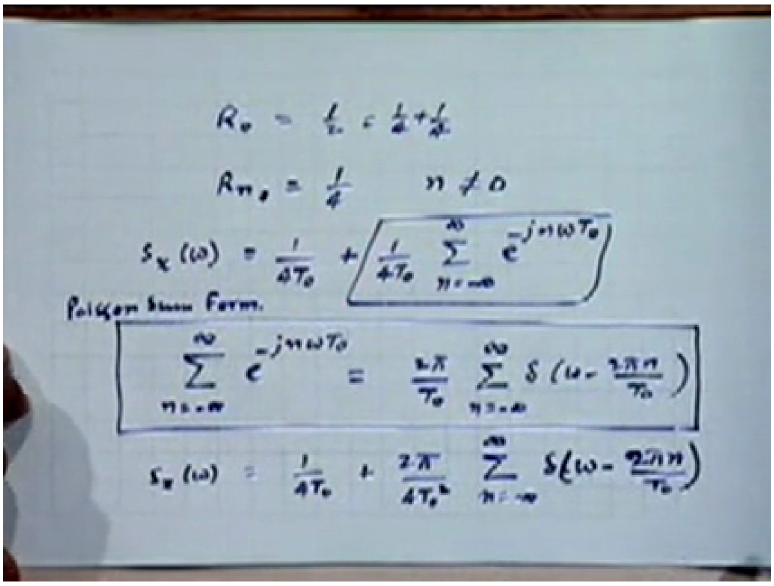

Professor: This is 1 into 1 when they are both 1 and that occurs one 4^{th} of the time, so this will become, this is going to be t0 by 4 t0, t by 4 t0, right. So the conclusion is that r 0 equal to half, rn equal to 1 by 4 for n not equal to 0, substitute that in your expression for s x omega, we have 1 by 4 t0 + 1 by 4 t0, now this is a complex exponential form that i have taken. I have deliberately skipped the r equal to r0 term into 1 by 4 + 1 by 4, where i can write this, right. Now let me give you a small exercise, if you do not know this result already, try to prove this result.

This is known by the name of poisson's sum formula, it can be obtained from simple diagram in single in signals and systems that we already have. We have a result e to the power - jn omega t0 summation over - infinity to + infinity, can be written as 2 pie by t0, that is right, delta omega - 2 pie n by t0. Please try to prove this result, okay. This is one of class of formula known by the name of poisson's sum formulae, right. Very easy to prove, by just looking at the fourier transform properties and we shall use that here to get sx omega equal to 1 by 4 t0, i will substitute this by this over here to give you 2 pie by 4 t0, delta omega -2 pie n by t0. Yes, you have a question?

Student: (())(52:20) 1 by 2, rn is from - j n omega t0, so we will not get that constant term.

Professor: This term?

Student: Yes.

Professor: I just told you are sitting with life that must have all the ri equal but my r0 is half, otherwise i have 1 by 4, so i am splitting this into 1 by 4 + 1 by 4, taking one of the 1 by 4 out and writing like this, right. Any other doubts? Today we have a lot of maths but this is more or less the final point of maths. Let us see a little, just a few more equations before we start discussing the results.

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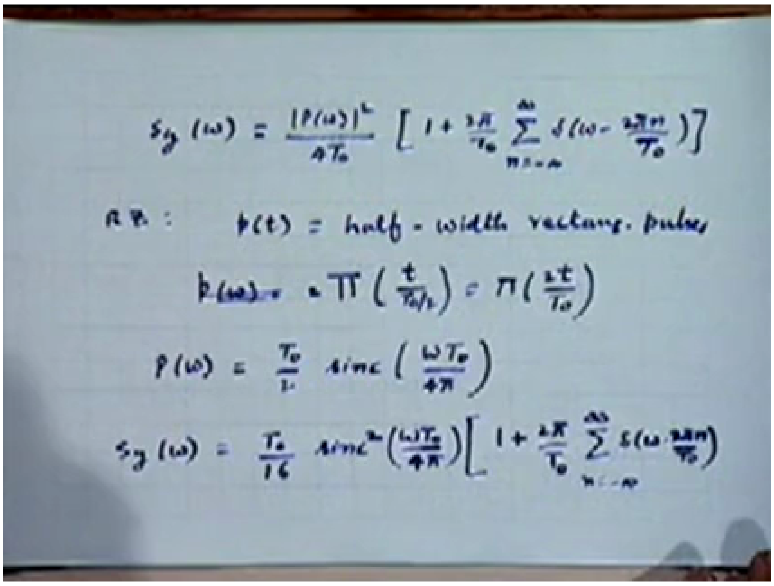

Coming back to sy omega for an arbitrary pulse shape, this will become p omega square up on 4 t0 into one +2 pie by t0 delta omega -2 pie n by t0, n going from - infinity to infinity, where p omega is the fourier transform of pt. Now let us consider the return to 0 situation assuming that your pulse pt has a duration equal to t0 by 2, we call it half width rectangular pulses, okay. So your p omega is going to be, let me specify pt more precisely, we will denote it by pie, this is just a notation, this is a notation for rectangular pulse of width t0 by 2, right, generally we write t by now, sometimes we can also write equal 2 t upon t0.

This is a notation for this kind of rectangular pulse. What is the fourier transform of such a rectangular pulse?

Student: Sinc function.

Professor: Sinc function, more specifically we will not really go into that, i assume that all of you can verify this, all of you know this, that is this function. And we get our final result which we shall look at in detail now which is this, right. We are just at about 11, so i think we will have to stop here, we look at this expression again next time and try to understand the nature of the power spectrum of the rz signal and nrz signal with respect to on-off signalling and go on to other forms of line codes signals.

Student: (())(56:22).

Professor: That is something which we will discuss separately if you do not mind.