Digital Communication.

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Lecture-4.

Digital Representation of Analog Signals A: Nonuniform Quantisation in PCM.
B: Quantisation Noise.

Professor: So last time we talked about pulse code modulation, right. And we saw how to digitise messages using quantisation followed by encoding and also looked at the kind of quantisation noise that we have to cope up with when we do pulse code modulation, right. Under the simplifying assumption that we use the uniform quantisation, we also obtained an expression, explicit expression for a signal-to-noise ratio, signal to quantisation noise ratio that we will be having to work with, having to cope with any PCM system.

And under the other option that the signal also have the uniform distribution over the same interval for which your quantiser is designed, we found that a nice rule of thumb for predicting the performance of a PCM system is that for every additional bit that you introduce in your representations, you get a 6 db advantage in signal-to-noise ratio, right. Because your SNR in db is given by 6N, right. So that is a good rule of thumb to remember even when your underlying signal that you are trying to quantise may not be uniform, right. It gives a good idea of what kind of performance you can expect from a given PCM system of a given number of levels.

Now towards the end of the last class we talked about the possibility of using a nonuniform quantiser, right. That is in which the levels are not equally spaced but have variable step size and the motivation for that is that many real life signals that we actually deal with are not really uniformly distributed in the finite interval. They may be having a finite range all right but they may not be uniformly distributed over that range, typical situation is that lower amplitudes are more likely than the higher amplitudes, right. So for example Gaussian distribution is a very commonly encountered distribution of the signals that we deal with.

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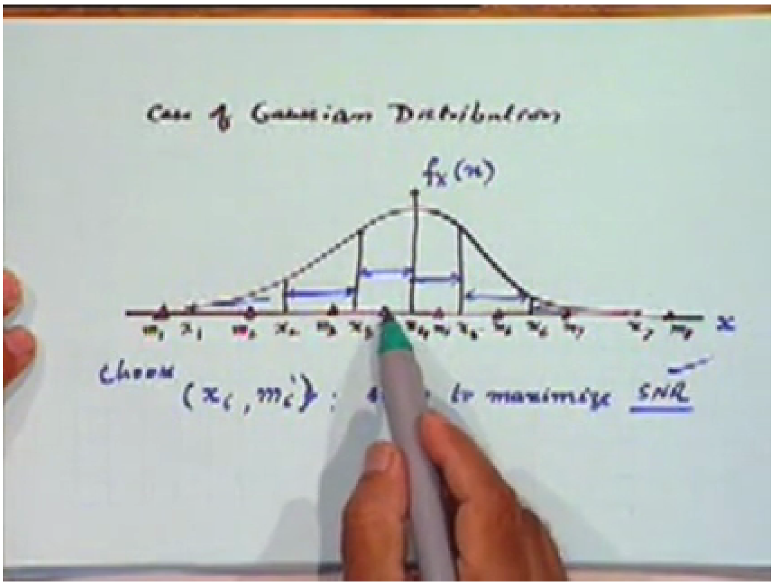

The other distributions are there too but Gaussian is one of the most commonly used models for many signals. And in such a situation it makes sense to have quantiser levels distributed non-uniformly over the range of values of X, right, we discussed that last time. In particular we would like the levels to be, the various quantisation levels to be more crowded in that region where there is a higher probability of the signal value lying, right. Whereas we like to space them out more sparsely towards the tail of the distribution whereas the probabilities of signal levels associated with those amplitude are much smaller.

One can actually obtain an optimum set of values for these boundaries X1, X2, X3 and the actual quantisation levels m1, m2, m3 by suitably optimising with respect to this distribution so as to maximise the signal to quantisation noise ratio. However we will not go into those details here, perhaps we can take up one or 2 such problems in our extra class lectures or tutorials, whatever you like to call them. And at the moment it is sufficient to take note of this fact that nonuniform quantisation can give us significant advantage and one can optimise it in this particular way.

Now since there can be a large variety of signals to deal with in practice and one may not know a Priory in particular communication system what kind of signals you are going to deal with, what kind of probability distribution function you may have to work with, it also makes sense to design a quantiser which may not be optimised for a specific distribution. It should be kind of robust towards the variations in the distribution, right. Because in reality, in reality it will never be the case that you know a Priory that you are always going to deal with Gaussian signals.

Maybe it is not exactly Gaussian distribution but some other distribution, right. So therefore even though one can optimise these kind of things, you can choose the, you can design an optimum quantiser by appropriately selecting these X I's and m i's, it actually makes more sense not to optimise too much for a specific distribution, because your distribution is likely to change from time to time. So what is more important is to have some kind of a robust quantisation which is going to give a reasonably good performance over a fairly large variety of distributions, right. So we will talk about these 2 issues in particular.

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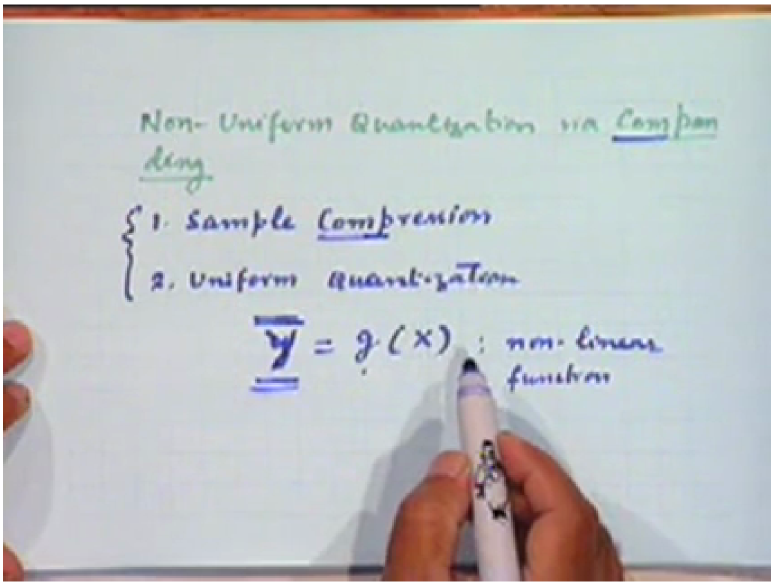

1st the manner in which this nonuniform quantisation is usually done. Instead of doing things in the way as depicted in this diagram, what is typically done is that you go through a process of Companding, so nonuniform quantisation via companding. Let me try to tell you what we mean by companding. Companding is a two-step process, is the 1st step and that is done at the transmitter you carry out what is called sample compression. And the 1st 3 letters of this word com actually come from the 1st 3 letters of this word compression or perhaps the 1st, 1st 4 perhaps, I think it is the 1st 3, all right. And this is at the transmitter followed by a uniform quantiser, uniform quantisation.

So what we are saying is that instead of doing this a nonuniform quantisation with levels which are separated by variable steps from each other, you do any equivalent thing, somewhat equivalent thing by 1st compressing the range of the amplitude of the input signal, right and then follow it up by a uniform quantisation. Now this compression, I will just illustrate this idea by a diagram, this compression is usually achieved by a transformation Y is equal to G of X, right. So it is a memory less transformation of a kind which compresses the amplitude range of the signal into a finite region, right.

The higher amplitudes are also compressed to lie within a fixed dynamic range. So this is a typical non-linear distribution that is used, non-linear function. And this function should be chosen in such a way that Y, what kind of random variable should Y be?

Student: Uniform distribution.

Professor: It should be uniformly distributed, right. Because if you are going to follow it up with a uniform distribution, the motivation is that we expect this Y to be very closely, very close to uniform distribution, right. So this G is chosen so as to make Y nearly uniformly distributed if not exactly, right. In fact I am sure you are aware of transformations which will render a given random variable X into another random variable Y which will be uniformly distributed, you must have done that in your probability theory course, if you have done a course in probability theory.

And we can take that up separately as an exercise to remind you what kind of transformation is that, right. There are transformations if you know what kind of distribution X has from that knowledge we can obtain a transformation G such that Y has uniform distribution. And since Y has uniform distribution, you can now use a uniform quantiser, it makes sense to use a uniform quantiser. And as far as X is concerned, effectively you are carrying out a nonuniform quantisation, right. This is best illustrated by means of a picture which I have here.

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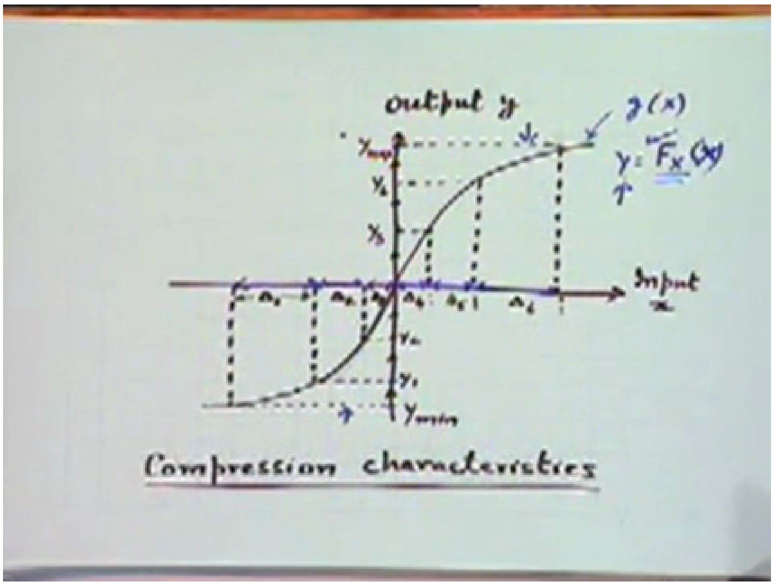

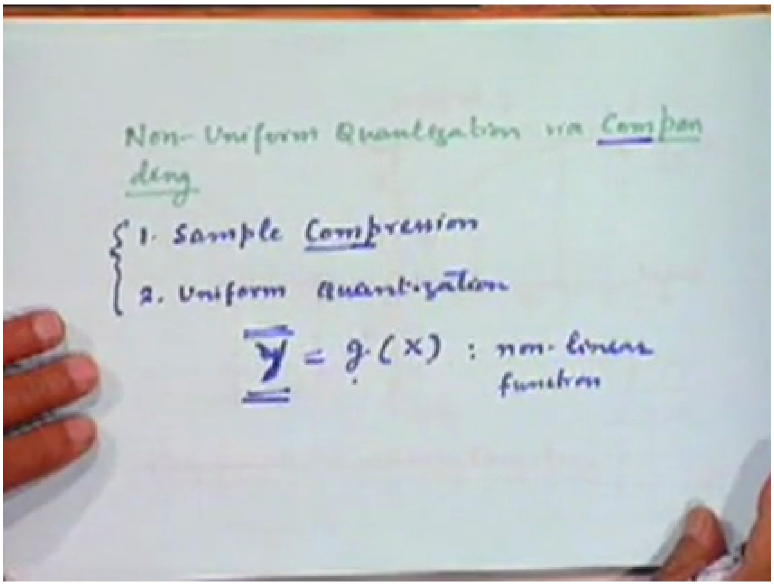

This is your, this function that I have got here represents your output Y from the nonlinearity and therefore this corresponds to G of X, right. As you can see a large dynamic range of X is being compressed into a finite dynamic range of Y going from Y sub min to Y sub max, so this is one end of the boundary and this is the other end. And if this G is appropriately selected, they can expect this Y to be uniformly distributed, right. Do you remember what that, what kind of G would give rise to?

Student: G should be (())(11:28).

Professor: No, it actually depends on the distribution function of X, right. The most general form of G is if you, if X has a cumulative distribution function fx X, then you can use this function as a memory less transformation square F, capital Fx is the cumulative distribution function of the random variable X. This is just to recapitulate for you what you might have done, if you have not done, do not worry about this, okay.

Student: But how can we realise (())(12:03) this fx X.

Professor: Well, there are many ways of realising this practice these days. If nothing else, you can, you can always realise any kind of a function by a read-only memory, right. Any kind of memory less nonlinearity can be a Priory programmed into a rom and depending on the value of X which can serve as the address of the rom you can obtain the value of Y, it is very easy to do these things these days. So one does not have to worry about its analog implementation, even that is also feasible, but the digital implementation is extremely simple and convenient to work with, right.

Student: Sir could you give an example of an analog device which in general can manipulate or…

Professor: Well there are log, as somebody mentioned you can use a logarithmic amplifier and many of these amplifier would be very close to this kind of distribution, this log characteristic. In general actually that is of academic interest because we will very rarely use such a precise distribution, for reasons that have already just mentioned to you, because we do not want to optimise the quantiser for a specific distribution. We would rather have a nonlinearity G which is more robust and can work across a larger class of distribution, right. So that is really of academic interest.

Student: (())(13:23) how do you say, how do you choose G?

Professor: G is fx, right. Because that is very easy to show that if Y is equal to fx X like that, then this Y would always be, independent of what fx is, Y would always be uniformly distributed. This is a very standard result in probability theory which you can check up or we can discuss separately. Now after having obtained Y in this manner, you can do a uniform quantisation over Y, have uniform steps, this picture does not exactly sure you uniform steps but they should be across Y. And effectively as per X is concerned you are obtaining nonuniform quantisation as you can see, right.

These are the steps corresponding for the X, right. If you do uniform condemnation across Y, you are effectively at any nonuniform quantisation across X, right. Of course this process will introduce distortion for the signal, right. Because Y equal to G X is non-linear function, so you are distorting the signal. There is a nonlinearity being introduced in the signal. In fact process of nonuniform quantisation itself is equivalent to, introduces some kind of distortion. But this is very specifically obvious from here that if you do the nonuniform quantisation in this particular manner, we are going to introduce deliberate distortion at the transmitter which is not good.

So what you have to do therefore is make sure that this effect is compensated for at the receiver, right, so as to remove the distortion. And therefore at the receiver you carry out an inverse transformation, G inverse Y, all right, so as to obtain X, to undo the effect of this distortion. And that obviously would be, this inverse function would obviously expand the dynamic range of the signal, expand the range of the signal. So therefore what we have to have at the receiver is the opposite of this compressor as obviously we can call it an expander the relatives from this combination of compression and expansion that this name companding is derived, right.

Student: (())(15:55).

Professor: The expander is the inverse functions which you must have, which you must implement at the receiver so as to undo the non-linear operation that you did at the transmitter, which otherwise would introduce distortion for you, right. Any kind of nonlinearity will introduce, will distort your signal. For example what are the kind of things that you are familiar with non-linear operations me to do the signal. Can you name a few?

Student: Harmonics.

Professor: It will introduce intermodulation terms, right. Right for example a signal has only single frequency components, it will introduce multiple frequency components, it will generate additional frequency components which were not originally not there in the signal, right. If we will have more than one frequency components, then you will get all kind of sum and difference stance of those frequency components also reduced after the nonlinearity which you do not want, right. Therefore you must undo the effects of these distortions, that you are deliberately calling out from the point of view of a good quantisation at the receiver by introducing their inverse function which obviously would be some kind of an expander, it will be inverse of this function, right, mathematical inverse of this function, right, which would be an expander.

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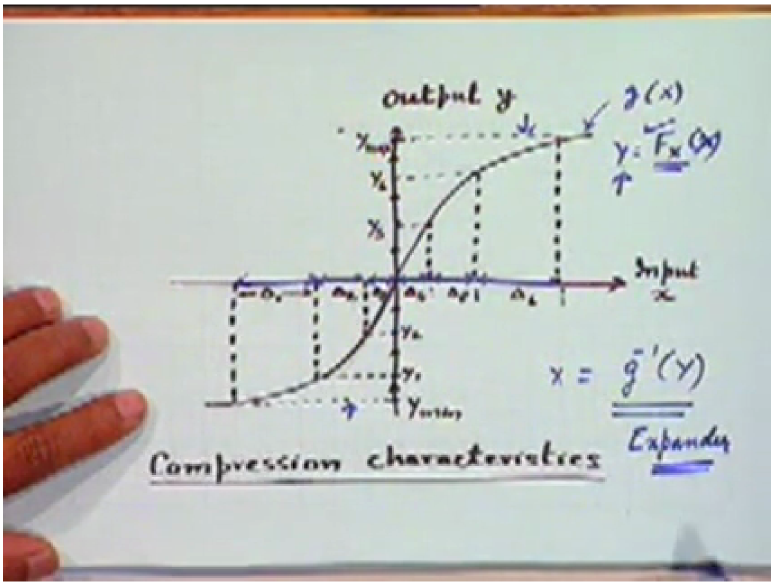

And using these terms we get the term companding, right. So nonuniform quantisation in general is done through the process of compression at the transmitter, followed by uniform quantisation and at the receiver we have to compensate for the compressor characteristic by having an expander, appropriate expander, okay. I think that is sufficiently clear.

Student: You talked of dynamic compression in the transmitter, like the thing is just (()) (17:54) with respect to the internal signals.

Professor: One can do that but it is usually more convenient to have a robust quantiser than optimum dynamic quantiser. For one thing it is simpler to implement, one can standardise it and for another thing it gives you nearly as good results, okay.

Student: What do you mean by robust this thing?

Professor: I will just talk about that. Basically by robust quantiser I only mean the fact that it should be distribution free, it is not depend that much, its characteristics, its performance should not depend that much on the specific distribution with which it is faced in practice, okay.

Student: It means to have a constant function (())(18:39).

Professor: I am just coming to that, I will give you some specific functions which are used typically in such robust quantisers. Now typically the compression characteristics and expansion characteristics have been the subject of widespread in the PCM literature during the early stages of development of PCM and people tried to look at various kinds of compression and expansion characteristics. And out of a lot of research, most of it, a lot of it was theoretical but finally a set of laws have been given which have more or less adopted as standard was used in PCM systems.

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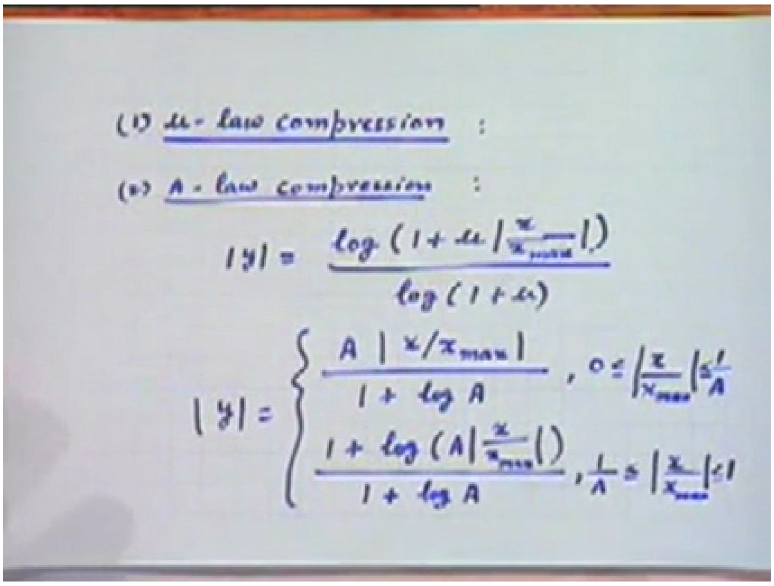

One of these is called mu law compression. Mostly these use logarithmic function as somebody pointed out, which makes sense, it makes a lot of sense. And the other is called the A law compression. These specific laws are the culmination of a lot of studies in this area about theoretical as well as experimental and empirical. So I will not be able to give you a justification for it here because of lack of time as well as context, it will take us, digress us quite a bit. But just mention what kind of laws these are. This mu law compression is what is normally used in the European continent. So most of the European standards, PCM standards have the mu law compressor…

Student: Mu law is in America, A Law is in Europe.

Professor: Maybe I have got the thing other way round, I will have to check up on that. If you are very confident, maybe it is that way, my impression was that mu law was Continental and A law was American, North American. But I may have got that is other way round, I do not remember these things very well sometimes, we will have to check on that. But let me give you what these are. Good, somebody at least has heard of it and knows better than me. What is that? Speak out, no problem.

These names actually have no significance, they only come from the fact that somehow this parameter mu as used in this expression and it is just known as mu law from that point of view. They both logarithmic functions, log functions, okay, that is the mu law compression. And A law compression is slightly more complicated. It looks atrocious, okay. It is basically some complicated looking mathematical expression, which as I said have come out of a lot of study, both empirical as well as theoretical and the main reason why they are, they have been adopted as standard is that both these laws yield an average quantisation noise which more or less is independent of what kind of signals you are feeding in it, right.

So that is what I mean by being robust, right. The quantisation noise that we introduced to some extent does depend on the signal, that is being quantised, but the variation is very very small, okay. And therefore in that case they are robust. It is better to use such laws or such nonlinearities than specifically design optimum nonlinearities, even if they can be made dynamically adaptive. Right, because they are more complicated to deal with.

And this, in fact these standards are now absolutely taken for granted, many of the codecs, so-called codecs, coder decoders which do the PCM transformation of a voice signal that are commercially available would be available with these options, either mu law companding or A law companding, they are available in IC forms.

Student: Even they can be varied?

Professor: In fact they have also been optimised more or less. For example mu is typically taken to be 100 and so is A, right. So that is about nonuniform quantisation, any questions? Anything about nonuniform quantisation?

Student: (())(24:51).

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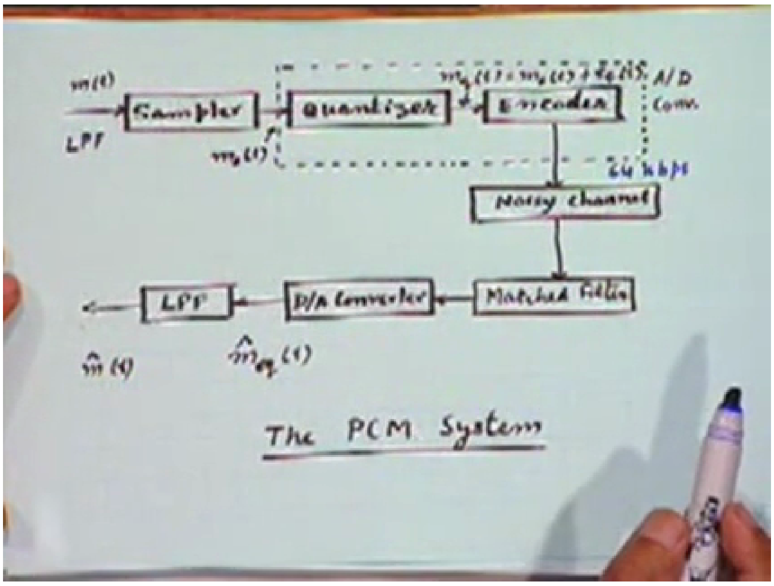

Professor: Yes. That will be quite complicated and I have not (())(24:58). Okay, let me now get into the system aspects of PCM system, typical PCM communication systems. The transmitter you are now quite familiar with, as a transmitter, the things we need for PCM system or to start with, we have message mt which must be lowpass filter, I have not shown the lowpass filter here but that has to be there because this is immediately going to be followed by a sampling operation. And to make sure that there is no aliasing distortion, you must make sure that your sampling rate and the lowpass filter bandwidth are properly matched, all right.

Typically for voice signal this lowpass filter will have a bandwidth of less than 4 kilo hertz and the sampling will be done at the rate of 8 kilo hertz. This is followed by the quantiser and the encoder and typically these 2 are realised as one block, typically known by the name of analog to digital conversion, right. This quantisation process and encoding process are really, slightly indistinguishable at the implementation level, they go more or less hand by hand. And as I said in speech signals, we used typically 7 to 8 bits of quantisation, which brings us to the data rate of, if we use 8 bits, 64 kilobits per second, right.

Because your sampling rate is 8 kilo hertz, each sample is being represented by 8 bits, so the final rate at which you have to transmit this data, this data is 64K low bits per second. And that is the standard PCM data rates, 64 kilobits per seconds. If you are going to do voice transmission over a digital channel, you require a 64 kilobits per second line, right, or bandwidth, bandwidth corresponding to that. Now this is transmitted onto a channel which is usually noisy, it introduces noise, introduces noise and to take care of the noise at the receiver or to the other repeater, this diagram could be that of the receiver and the final destination or of the receiver which is used at the repeater for requantising the signal and retransmission.

So this receiver diagram will be more or less the same. Now to take care of noise you use at the front-end what is called as matched filter. We will talk about the need for a matched filter and its characteristics at a slightly later time in our course, but broadly at this time it is sufficient to appreciate that the main purpose is to be able to make correct decisions regarding whether in a particular bit interval a 1 or a 0 has been transmitted or received. Well, because this decision is a nontrivial decision to make because this bus that is coming along if coming along with a lot of noise, it may not be obvious from the noisy signal, which is very weak to immediately see whether it contains a 1 or a 0, right.

To be able to make a good decision to a certain kind of processing on the received signal, which processing is known as by the name of matched filtering.

Student: Is it sort of noise based, I mean filtering of the noise?

Professor: Yes, the purpose is to remove the noise and to get a sample value based on which you can make the decision more reliably, right, depending on the output of this matched filter. But we will go into the details of that problem slightly later. At the moment we just have to know that such processing is required to take care of noise, all right. Once you have got that done, then you can take a decision whether one or a 0 was transmitted and then follow it up with D to A converter, these bits are collected as such and put into a D to A converter and that output is usually quantised version of the input signal and therefore you lowpass filter and finally get your estimate of the transmitted signal.

Student: Sir what is the (())(29:26) A to D converter.

Professor: D to A converter?

Student: A to D converter, what is the relation…? Mathematical formula what is that?

Professor: Just a second, one question at a time, what is your question?

Student: In A to D converter what is the relation of mt?

Professor: Okay, I have just got some notation is your, this mt is the original signal, m sub S if the sample signal, m sub q is the quantised signal, right, which is the sum of the sample signal + error term, the quantisation error term E sub q, right. The output of the D to A converter converter, you have an estimate of the quantised signal and finally after lowpass filtering again you get an estimate of the original signal mt.

Student: Why do we need to lowpass…?

Professor: Because 1st of all there could be still some noise that has trickled through and more importantly your quantiser simply introduces steps, staircase kind of thing which will be again removed by, smoothen it out, whatever the error the quantisation is introduced, the quantisation noise has also broad spectrum, you remove as much of its energy as possible from the band of interest.

Student: (())(30:45).

Professor: If it is a repeater, this will be again followed by quantiser and encoder, sampler, quantiser and encoder, right. But usually at the repeater level one can just clean up the pulses and retransmit rather than calling out the process of D to A convergence and lowpass filtering. Basically you have to still just retransmit. Let us go into these things are little more in detail, although I think it is probably not necessary, I am sure you know how A to D converters are typical implemented. This is a block diagram of an A to D converter which I am sure you can recognise.

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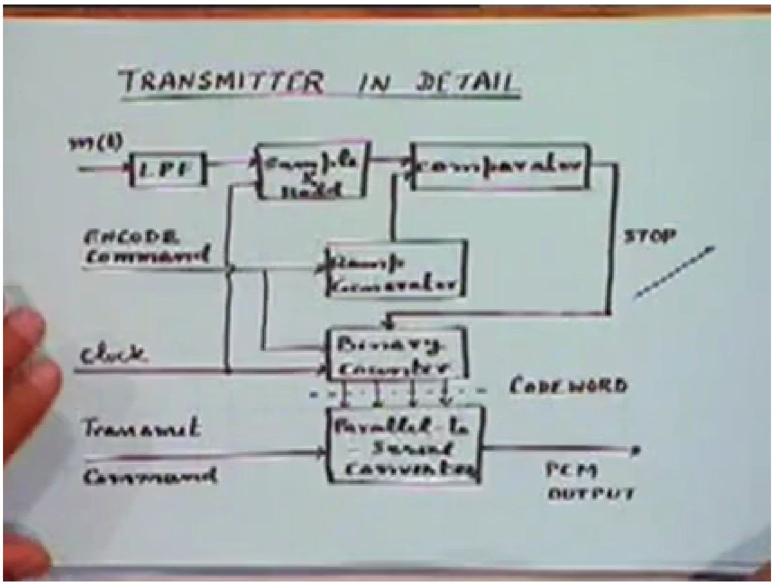

What kind of A to D converter is this?

Student: Ramp type.

Professor: Ramp type. And what does it do? You have this sample signal which is held and this is compared just as this, just as this sample is being generated, you also start, you also give what is called an encode, to the converter which initiates the generation of a ramp signal. This incoming sample is generated, as compared with this ramp output and when this exceeds this, the comparator output generates a signal which becomes a stop signal for the binary counter. And as long as this ramp is operational, this binary counter the, this encode command also comes to this binary counter.

This binary counter is counting from 0 onwards, right. And the moment this level is exceeded, the counter is made to stop counting and the output of the counter now represents this level to which this ramp has reached, which also represents the level of the input signal. And therefore this becomes a binary representation of a digital signal, which is converted into a serial form and transmitted as PCM. That is the essential difference between a normal a A to D converter and a PCM system. Normal A to D converter output is in parallel form because typically it is going to be used in I machine with all the bits available, where all bits are required to be available simultaneously.

But on a line you have to transmit these bits in a serial mode, right. So the essential difference is that you need to have a parallel to serial converter for transmission onto a line.

Student: What is the connection between sample and hold and the binary counter?

Professor: This clock is being counted, and the same clock is used for, well you may divide this clock somewhat because this counter will be operating at a higher rate than the sampling rate here, but those relationships are not very clearly given over here, I am sure you can work them out yourself. In any case this is something I am sure is not new to you. You want me to keep it there for sometimes?

Student: Yes.

Professor: All right.

Student: Sir but ramp type A to D converter is slow…?

Professor: Well the specific type of A to D converter that you shall use will depend on the rate at which you want to do things. If it is a voice signal, this will be good enough but if it is a picture signal which you want to quantise at a very fast rate, maybe you require a faster A to D converter and a different kind of A to D converter. So this is just an example, this is not to say that all the time you are going to use this kind of A to D converter. Technology has made available, made available many different kinds of A to D converters and we will choose the most appropriate ones, both from the point of view of cost as well as from the point of view of meeting your needs, right.

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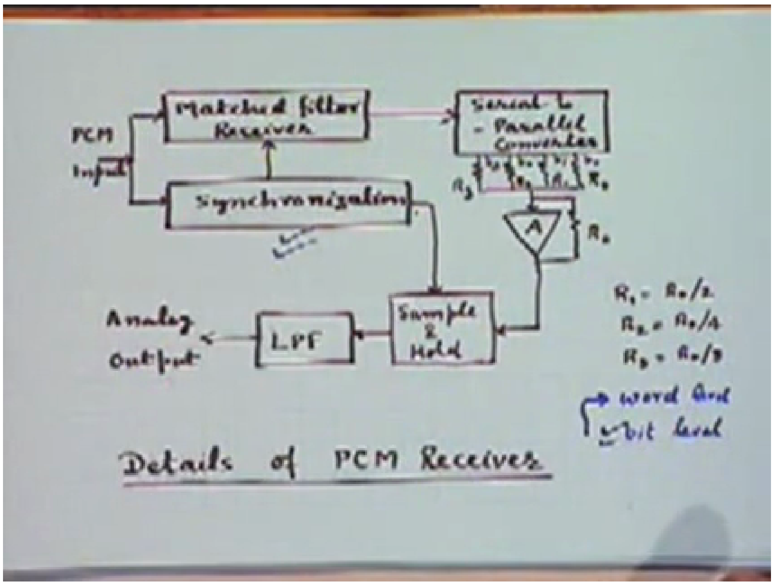

Always it is a tradeoff between cost and what you can live by. Okay. Can I remove it now? At the receiver, this slightly blown up diagram of the receiver. You have the matched filter followed by serial to parallel converter and D to A converter which is essentially a resistor network, weighting, resistor weighting network followed by an operational amplifier. Now the important different thing here is this thing here, this block, which I have not talked about so far but which is very crucial to any digital communication system.

And this is the complexity, this in fact makes the digital communication receivers much more complex than corresponding analog communication receivers, the need for synchronisation.

Student: Synchronisation is there an analog also, the (())(35:59).

Professor: That is all, that kind of synchronisation is also needed here, we will see that later. Well this synchronisation is slightly different, that, the kind of synchronisation you are referring to in analog communication is basically that of the carrier, right. We need carrier synchronisation you are also when we use a carrier, right. But remember what we were doing in the process of encoding, we were taking a sample, putting them into a group of bits and these bits were transmitted, right.

Now at the receiver you do not know where bits, where a particular word starts and where it ends, right. So you require synchronisation at the word level, right, so as to demarcate different groups of bits from each other, clear. And you also require synchronisation at the bit level, you need to know precisely where each bit starts and where each bit ends, so as to enable the matched filter to do its job properly, right. Because the matched filter has to remove noise for a particular pulse, so as to be enable you to make a correct decision whether in that bit interval a 1 or a 0 was transmitted, right.

To be able to make the decision correctly for that interval, it must know where datable starts and where it ends, right. So the matched filter requires such information. Of course usually if you have good bit level synchronisation, it is very easy to derive the corresponding word level synchronisation because you know how many bits constitute a word, right. But you may still require to know precisely where the boundaries are, right. So for which one has to do additional things, which are, which are slightly irrelevant to the basic job of communicating information.

For example you may like to transmit some kind of a preamble, rights, for the synchroniser to get ready. Some protocols have to be involved, so that your receiver is setup to receive the information when it, when it started, when it is transmitted.

Student: Start stop bits…

Professor: There are various kinds of protocols, we will talk about them at an appropriate time.

Student: You said preamble?

Professor: A preamble word, some kind of a protocol.

Student: So that is done only at the beginning of the transmission or…?

Professor: That is right, something at the beginning of the transmission. Or something which may be done periodically if your channel is bad and you may like to set up the receiver again and again, right. So that is a slightly toned up picture of the receiver. Any questions on this?

Student: What is the synchroniser?

Professor: I just…

Student: How is it implemented?

Professor: Okay, at the moment we are only looking at the block level implementation, needs. We will be going to details of many of these things later, right. Because they will be required in many other contexts rather than just the PCM context. Now we come to an important question which was raised by somebody last time. We have looked at 2 different methods of converting or representing analog information by, into a digital form, right. Namely the delta modulator and the PCM and it makes sense to ask the question how do they compare with each other, how do I know which one I should choose, right.

I do know that Delta modulators is a simple thing to implement but and in fact it does not have anything like word synchronisation necessary for it to be necessary at the receiver because you just can, the demodulator is extremely simple, much more than the modulator the delta demodulator is extremely simple, all you require is an integrator and a lowpass filter, no synchronisation problems, right. Except that you have to take care of noise somewhere but that is something that can be done. So on the one hand it looks that delta modulator may be a better option, from the point of view of system complexity but before we jump into confusion, we must also see what kind of performance we can get from delta modulator for similar situations.

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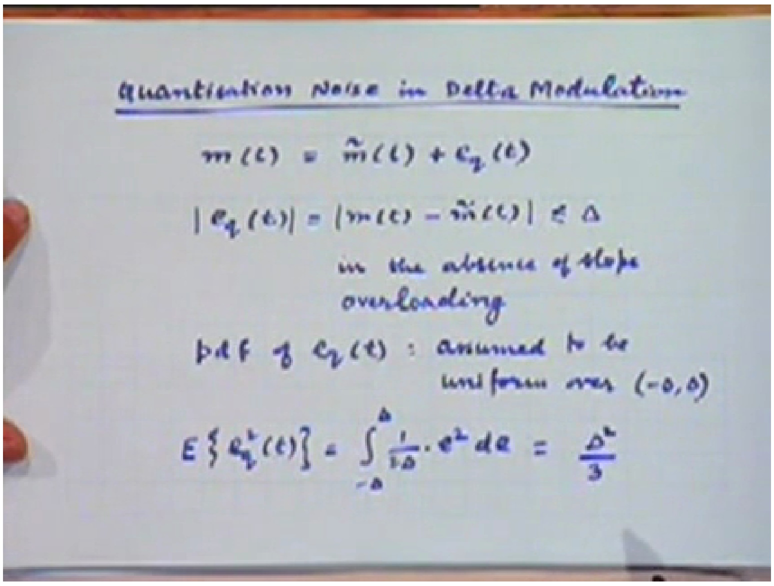


And let us therefore look at the quantisation noise in delta modulation. So now we are returning again to delta modulation having understood PCM. Let us do a bit of analysis here. Let me represent my, the signal mt which I am now quantising by delta modulator as the sum of 2 components m tilde t + E sub q t by this represents the staircase approximation that the delta modulator generates of mt, right. Remember what does Delta model it do? Effectively it gives us a sequence of energy pulses from which we can generate a staircase approximation of mt which I am representing by this notation, okay.

And therefore this represents the error of representation at time t. So E sub q t is nothing but mt - m tilde t. And of course magnitude of this will be always less than or equal to Delta by delta is the step size, in the absence of slope overloading. If there is slope overlooking mother this error can become large, right. So this has to be qualified, this will be less than this, only in the absence of slope overloading. Okay. Now to be able to obtain the quantisation noise for this case, you will have to make some assumptions and one assumption to be like to make is that the PDF of E sub q t, e sub qt is a symmetric, its magnitude is less than delta, that means its value lies between - Delta to + Delta, right.

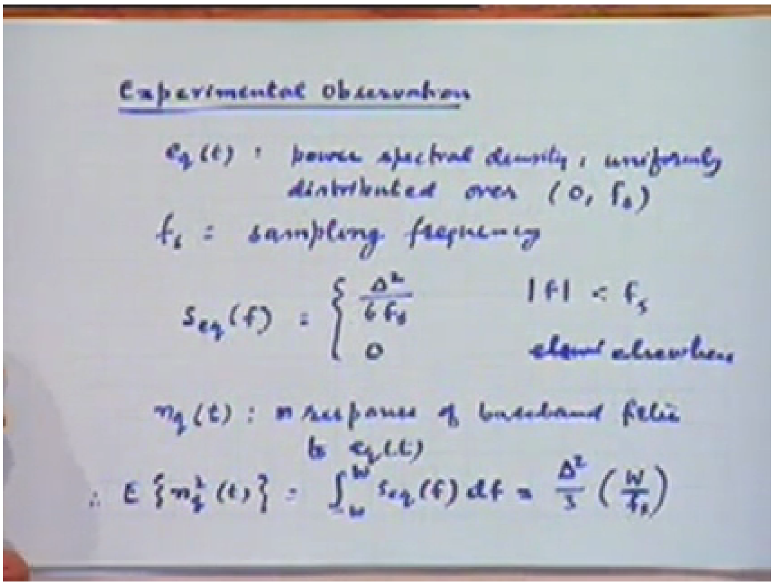
You can have - Delta or you can, maximum you can have on one side - Delta and on the other side + Delta. Is that okay? So we will assume that the PDF of e qt is uniformly distributed over this range, right. And this is supported by observation in practice for many kind of signals, so assume to be uniform. Therefore to make this assumption I can write expected value of eq square t as 1 by 2 delta which is the distribution of eq, distribution function, density function of Eq, right, multiplied by e square, eq square or e square de.

Which should be simply delta square by 3, right. If you evaluate integral, the value will turn out to be delta square by 3. Now fortunately for us not all of this noise is what you will finally hear after demodulation. Why?

Student: LPF.

Professor: Because that is an LPF whose bandwidth is going to be governed by the signal bandwidth. A signal has a bandwidth W, you are going to put a lowpass filter with that bandwidth, whereas this noise will be some kind of a broadband noise because this is having very fast fluctuation, all right. This error signal is undergoing very fast fluctuations, it is essentially some kind of a broadband noise. In fact experimentally it is observed , can I remove this sheet if you have written all all these things down?

(Refer Slide Time: 45:45)



Experimentally, it is an experimental observation of the, regarding the power spectrum of this noise, that e sub qt has a fairly constant kind of power spectral density function between 0, that is DC, 0 frequency and FS, F sub S, where F sub S is the sampling rate or the clock rate used in the delta modulator, right. So it has a power spectral density function which is nearly uniformly distributed over this range, Fs is your sampling frequency. That means if I write the power spectral density function of S e sub q, what will it look like? Its total power we know, delta square by 3, right, we just computed that.

This power is uniformly distributed over the span of frequencies, right.

Student: (())(47:08).

Professor: That is from - Fs to + FS, so it will be delta square by 6 FS for this region and 0 elsewhere. Okay. How much time we have? Okay, just, I will derive this expression, we will do the discussion of this result next time. Therefore, now if you consider the noise n sub qt which is coming out of the lowpass filter in the demodulator, that means. So this I am distinguishing between the noise that is generated by the process of delta modulation and the noise which is finally observed after demodulation, after lowpass filtering, right.

So N sub qt is the, let us say the response of, response of the lowpass filter or the baseband filter to this e q t. Right. The input is this noise along with the signal, output is bigger presented by this noise. Essentially the 2 are going to be different in the frequency domain, in the fact that this has a bandwidth equal to W whereas this has the bandwidth given by FS, right, this is the essential difference between these 2 noises.

Student: In this case the bandwidth will be FS by 2.

Professor: Now, FS.

Student: No, the 2nd one.

Professor: No, no, you remember our sampling rate in delta modulation is much larger than the Nyquist rate, so there is nothing like FS by 2, right. It is going to be much less in fact, right. It will depend on the signal bandwidth W because the lowpass filter must pass the signal, that is the main requirement. Baseband signal must have a bandwidth corresponding to mt, right and usually FS now is going to be much larger than 2W, right, in delta modulation, much larger as I mentioned last time when we discussed delta modulation.

Therefore your expected value of m sub q square t is essentially going to be the energy of this noise or the power contained in the noise in the bandwidth from - W to + W which is very easy to evaluate, is delta square by 3 into W by FS, okay. I think we will have to stop here and we will start from this point next time.