Digital Communication
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Module 1
Lecture 36
The concept of channel capacity

So we will continue our discussion on this topic that we have started yesterday error free communication over noisy channels essentially the concept of channel capacity how it comes into the picture and how as Shannon told us it is possible to think of transmitting information at a finite non-zero rate but within certain limits and still guarantee error free communication, right? At least in theory in practice of course things maybe different but in theory how it is possible to think of a result like and let us go over the justification and if you remember I have started about the suspect by considering the role of redundancy in error free communication yesterday, right?

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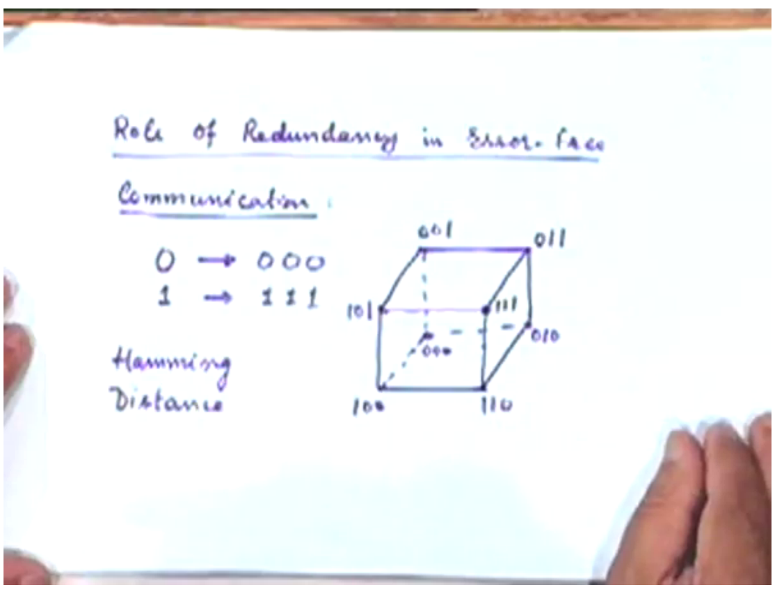

The crucial thing that we had tried to say wise that it is by the use of by the transmission of redundant information along with information that we actually want to transmit that it is possible to think of error free communication at all, how exactly we will still talk about that but construct of channel capacity in fact as we will see today and as I discussed yesterday is entice us that if you transmitting symbols which contain a certain amount of information how much of that information can be information of interest to the user and how much effect must be redundant information, right? Which is of no direct interest to the user, for example if every symbol of a if your channel capacity is 0.4 that for example we are taking if your channel capacity has a value of 0.4, right? That means whatever amount of information is being conveyed by every symbol of your transmission whether it is 1 bit or more bits only 0.4 reflection of 0.4 of that can be useful information rest of it has to be redundant information.

If you transmitting 1 bit for symbol then 0.4 bit has to be useful and 0.6 bits has to be redundant that is the implicational meaning of channel capacity when it is taken as a number less (())(3:31), right? Now to see how such a result comes about if you remember we were discussing this example of, excuse me repeated transmission of a bit for introducing redundancy, right? For example 0 followed by is transmitted as repeated transmission of three 0's 1 as repeated transmission of three 1's and we could (())(4:08) of picture for the operation here by thinking that these two code words are these two what is it in this hamming cube, right?

And we could do a similar extension with this idea nearly 5 repetitions or seven repetitions and then correspondingly we are thinking of a 7 dimensional hamming space or a 5 dimensional hamming space, right? And the cube is the hyper cubic here you consider is in that particular space but the important thing to note is that because we have chosen code words a bit carefully it is because we have chosen code words carefully we are able to get features of error correction in this schemes of things, right? The philosophy of choice is that for every code word that you select for every vertex that you select is a code word basically I have chosen this vertex and this vertex as code words for every vertex that you select is a code word you have to leave out all neighboring vertices or a few neighboring vertices unassigned as code words we should not resigned as code words they should not be chosen as code words.

In this case of course the maximum we can do is leave these vertices unassigned if you want to use at least two. In the case of five repetitions we can leave out more vertices unassigned and correct up to two errors, for example we could leave out all vertices which lie on a hamming sphere of radius 2 unassigned in a 5 dimensional situation, right? And therefore we could get a double error correction capability in that situation, okay. Now as you keep on increasing the number of repetitions it is quite obvious that the distance between these chosen code words will keep on increasing and in the limit we will get a situation where there is very small probability of confusing one for the other, right? But do you think that justifies channel capacity argument? No, because simultaneously what is a effective information rate which you are going to get, again 0, right?

So although we have introduced redundancy and we are hoping to get the channel capacity kind of situation that is finite information transmission without errors we do not see that kind of thing happening here, right? The reason why it is not happening is because our manner of introducing redundancy is not really very nice, you appreciate that? Excuse me for a second, okay because what we are doing is we are introducing a lot of redundant bits just to check on one bit whether it is going in error or not, right?

And that is not a good thing to do the better thing to do would be take a group of bits and have a common set of redundant bits which we will check on that complete group of bits rather than just designing system to guard against errors which maybe on single bits basis, right? That is not a good thing to do it is nice to give this argument which is given in the book also suppose you have a, basically what are these bits doing? Guarding against errors that might take place on the channel error take place in the channel you will be able to connect, okay is some kind of a guarding phenomenon guarding mechanism.

Suppose there is a street or a neighborhood in which there are lot of cases of burglaries and dacoities there are two ways of tackling that problem, one way is each one of you or each one of the residence hire his own guard not a very practical thing to do, the better thing to do would be to get together and hire one or more common guards to take care of the problem, right? It is the same kind of situation here.

So instead of trying to introduce redundancy to correctly transmit a single bit or a single digit we would like to introduce a set of check digits or redundant bits to guard against errors in a group of bits of information together that will be more efficient way of doing things rather than this way of doing things, okay. So now next therefore try to do a things in a slightly different way that is add redundancy in a slightly different way, we will take a groups of bits and I will introduce some notation for that, let alpha be the binary information rate that is the rate at which input information bits are coming along, okay and let me accumulate these incoming information bits over some period of time to get a group of information bits, right? Let say this time is t seconds, so in t seconds how many bits I will accumulate? Alpha t, right?

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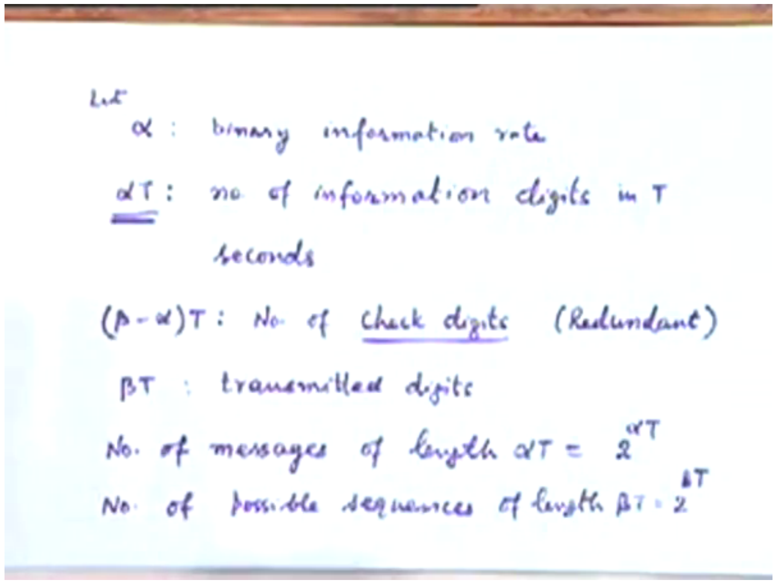

And these are the number of information digits in t seconds, okay. And what I will do is for every group of alpha t information digits that are coming along I will transmit a group of beta t digits actually on the channel that is my code words will have length beta t not beta not alpha t. So basically what I am going to do is add a few extra bits equal to beta minus alpha times t which will serve as the redundant information or it is also called checked digits, okay. So these are the number of what are called check digits and these check digits constitute the redundant information that we are transmitting, yes normally these are bits but one can also have a more generous scenario where one can talk of more abstract symbols you can have non-binary value digits also but for our discussion let us assume that they are all bits, okay. So they are beta t transmitted digits for every alpha t information for every alpha t and information digits.

Now let us look at things in that (())(11:50) framework that we were talking about. What is a number of messages that we have to convey when if you are conveying alpha t groups of bits together, how many different messages are of interest to us? We have a source which is emitting a group of alpha t information bits, right? What is a number of different messages source can generate for us? Two digit power alpha t, right? So number of messages which are of interest which we want to convey one time or the other which are of length alpha t is 2 to the power alpha t and what is our code word size what is a final length of the code word that we are transmitting beta t.

So how many different sequences of that size we can have? 2 to the power beta t so I will say number of possible sequences what is a physical sequence you can say of this two digit power beta t insert a number of possible received sequences because although you maybe transmitting only this 2 to the power alpha t sequences out of this 2 to the power beta t sequences after corruption with noise after degradation we can receive tactically any sequence theoretically, right?

So number of possible received sequences of length beta t is 2 to the power beta t or we can also look up on this in a different way our final code word is of size beta t, right? So in that hamming space of you are talking about what kind of hyper cube is defined by this length of code? It is a beta t dimensional hamming space and the number of vertices in this hamming cube will be 2 to the power beta t you can think of this in that way also. There are 2 to the power beta t vertices in the hamming cube corresponding to the hamming hyper cube in the beta t dimensional space, okay.

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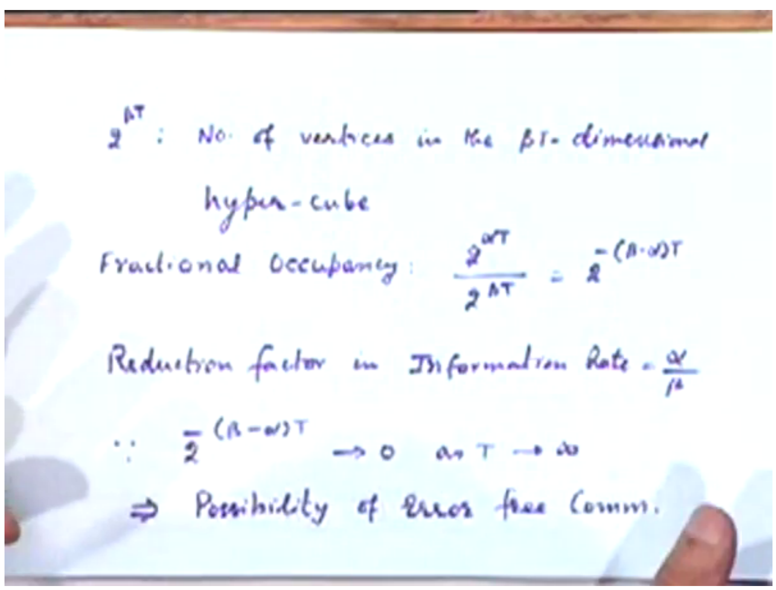

So let me write that you can also think of this as the number of vertices in the hamming cube in the beta t dimensional hyper cube, fine? Agreed? Out of this 2 to the power beta t vertices in this hyper cube how many of them will you finally select as code words? 2 to the power alpha t, right? If and of course how you will chose us code words is a matter of designing the code we are not at the moment going to that, right? We will go a bit into that to take our argument to completion but the important thing to appreciate here is that the friction of vertices which will be used as code words I will call that frictional occupancy, okay.

So this term frictional occupancy essentially tells me what is the friction of code words what is the friction of vertices which are being used as code words, no 2 to the power beta t is a total number of vertices in that cube, right? Out of this only 2 to the power alpha t are assigned as code words, right? If for the frictional occupancy of code word vertices with respect to total number of vertices will be 2 to the power alpha t upon 2 to the power beta t which you can write as 2 to the power minus beta minus alpha t, okay because beta minus alpha is positive so I have written it like that.

And what is the rate of information reduction? What is the reduction factor in the transmission rate of information? Alpha by beta, for every group of beta bits that you are actually transmitting or beta t bits which you are actually transmitting only alpha t of them is information bits, right? So is a reduction factor in the information rate involved I should not say transmission rate, I should say information rate, yes the alpha upon beta we have which is going to be less than 1, right? You can think of this beta t bits as being one symbol of its source, right?

Out of this beta t bits that is being conveyed by with this one symbol only the friction alpha t only the portion alpha t is really information number of bits the rest is a redundant bits. So this is linked with our reduction that we talked about in correction channel capacity, right? You know but as far as this argument is concerned again it looks as if it is only enough to have beta slightly greater than alpha, right? And I can make this reduction factor very very small it may look like that and get 0 (())(17:47) probability, why? Because as frictional occupancy becomes smaller and smaller that means around each assigned code vertex I have a lot of unassigned vertices, right? The number of unassigned vertices is very large and particularly as these may go to infinity, right? But there is something wrong here.

No, time getting at the moment let us not worry about time that (())(18:18) for, right? But in principle it seems that with almost no with almost very little reduction information rate alpha by beta nearly equal to (())(18:30) just make beta slightly greater than alpha this should become a positive quantity and if it is a positive quantity for a sufficiently large value of t it should be possible to make this sufficiently small, right? And it may look as if there is no concept of a channel capacity in that case that is one should be able to transmit one should eventually be able to have error free communication (())(18:57 to 20:25).

Student: And is really not going to happen and also the fraction is very small.

Professor: Let me complete the argument maybe you will understand, okay. Anyway this thus indicate because 2 to the power minus beta minus alpha t tends to 0 as t tends to infinity, right? It does imply that there is a possibility of error free communication this much this conclusion is right, okay but we cannot say from this that any n value of beta slightly greater than alpha is adequate, right? That is the channel capacity cannot be 1 that we cannot that argument the argument for that is not complete we cannot say what is the value of the question therefore that we ask next is, yes please.

Student: (())(21:38).

Professor: This comes from this fact that since a frictional occupancy can be made very small by simply increasing t.

Student: (())(21:52)

Professor: At least in fluid suppose I am waiting I am ready to wait for any (())(22:00) sufficiently large amount of time in theory it seems possible at the moment you only thought of theoretic possibility not a practical solution this is not a practical solution to doing case we not really do things in this particular way when we want to introduce redundancy we will not wait for infinite time.

Student: (())(22:18) previous case t point introducing a lot of redundancies.

Professor: Yes.

Student: Then the probability of (())(22:23)

Professor: But simultaneous our information it was also coming down to 0 because only you understand because only 1 bit information was being conveyed by infinite number of bits, what is effective information bit? 0, right? Now we are not having that situation we have a finite number of alpha t information bits beta t code bits, alright? So our information right is reduced only by a factor of alpha by beta and I can chose beta just redundant alpha to make this positive so that this happens.

Student: Sir then some of the value (())(23:00) should be if we take three four long time we can end up (())(23:04).

Professor: That was source coding now we are talking of channel capacity, do not mix up with (())(23:08) we are discussion source coding here now we are discussing the transmission of information over a channel whether if a context is different the arguments are totally different, yes in both cases infinite waiting of was required but in different context.

Student: The value should be defined (())(23:28).

Professor: That was just by way of telling you what CS means, at that stage we had not introduced any argument of t, right? That was just trying to tell you the concept of the meaning of the channel capacity, right? How much redundancy one has to introduce at that stage I just want to give you an example but now I want to give a justification for where does this concept of channel capacity coming from, okay yeah this implies that rather than the other (())(24:01), right? Anyone have questions? Sachin? Fine?

So this is agreed that as t tends to infinity we at least have a possibility for error free communication coming from the fact the reflection occupancy over this is becoming very very small, okay. The question really now is can we choose is there some limit or alpha by beta or can I chose alpha arbitrary close to beta or beta arbitrarily closed to alpha except it distributes like is greater alpha, right? The answer is no, there has to be a limit on minimum value of beta that you must have for a given value of alpha, right?

And therefore it implies that you must sacrifice a minimum amount of information rate in order to achieve this, okay and that brings in the concept of channel capacity, channel capacity is precisely this concept that alpha by beta has to be less than a number a which is the fraction alpha by beta has to be equal to or less than a given fraction which is not unit friction of course is not (())(25:18) how does that come?

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Any limit on $\frac{\alpha}{\beta}$? Yes

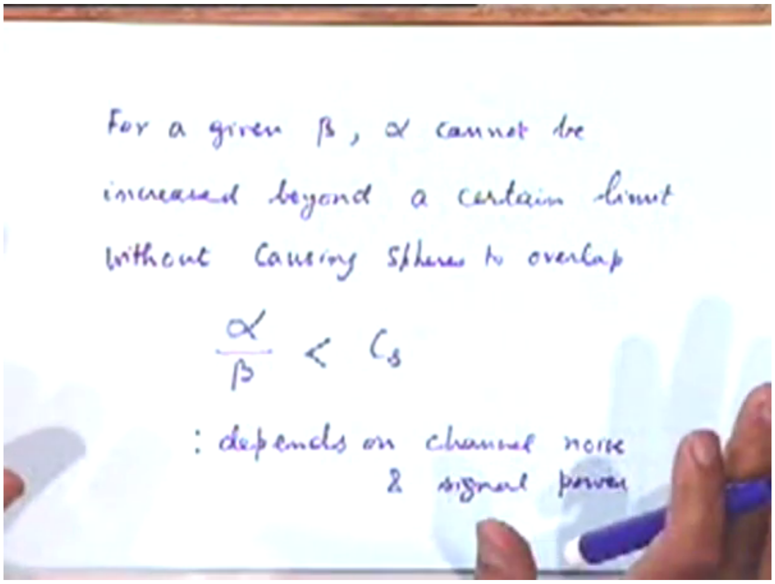
Let P_e : digit error probability

No of digits in error in a seq. of length βT = $\beta T P_e$ as $T \to \infty$

$\overset{\beta T}{\therefore}$ Must leave all vertices unoccupied within spheres of radius $\beta T P_e$

around each of the $2^{\alpha T}$ occupied vertices

$\therefore$ We must be able to pack $2^{\alpha T}$ non-overlapping spheres, each of rad. $\beta T P_e$ into Hamming space of βT dimensions.



So it is a question next is there any limit on alpha by beta and answer is yes there is a limit to understand that limit let us return to the our example of binary symmetric channel in the context of which we have discussed things earlier, right? Let us say that binary symmetric channel, remember what was a binary symmetric channel? Let us say it is associated with a parameter P sub e which is error probability, right?

So let P sub e be the digit error probability in a binary respective channel, now we are transmitting a code word of length beta t I am assuming t sufficiently large, alright? So we have very large number of bits which we transmit out of this transmitting number of bits how many of them on an average can be in a if P is large and therefore beta t is large beta t times P sub e, right? So you transmit if when you are transmitting a code word of size beta t the number of digits which will be in error on an average in a sequence of length beta t will be equal to beta t times P sub e particularly as t tends to infinity, agreed?

And this puts a restriction, that means what does this mean? That the received sequence you are transmitting a sequence of 1's and 0's of length beta t a received sequence will be different from the transmitting sequence in how many places on an average beta t P this is it will be at a hamming distance of beta t times P from the transmitted sequence. Now if I want to use the nearest neighbor rule which was implied by the (())(27:44) that we discussed earlier I must make sure that from this received sequence there is no other code word other than the one which I transmitted which is at hamming distance of beta t P from this received sequence there should be only one code word which is a transmitted code word, right? At this hamming distance from the received, what does that mean in term?

That means for every code word for every vertex that we chose as a code word you must leave all those vertices unassigned which are at a hamming distance of beta t P e (())(28:24) because all those are candidate received sequences corresponding to that transmitting sequence, is it clear? Let me write this down for those who have not appreciated my argument this implies that I must leave, let me complete this then we will come back to your question all vertices unoccupied within spheres of radius, what is a sphere radius of the sphere? Beta t P e P sub e around I will have to the next sheet have you copied this around each of the 2 to the power alpha t occupied vertices, is that clear?

Student: Sir from the received signal it should not be like there should be only one coded vertex within the (())(29:54).

Professor: But what is the implication of that? The implication is see you are transmitting a particular code word which is a vertex the received one could any neighboring vertex within this neighborhood, right?

Student: There would be decoding as a vertex.

Professor: And if you are within that you will come back to your vertex code vertex if not you will go to some other vertex, it will go to the nearest vertex, right? So you must put a assigned vertices as code words in this manner you must essentially pack spheres in this hamming cube, right? How many spheres you have to pack? 2 to the power alpha t spheres, right? If sphere must be of radius beta t and all this (())(30:39) for alpha t spheres must not overlap with each other, is it clear? Can I request no talking please, that is we must be able to pack 2 to the power alpha t non-overlapping spheres each of radius beta t P e P sub e into the hamming cube hamming space of beta t dimensions, fine?

Now obviously if you want to do this kind of packing there has to be a certain relationship between alpha and beta we cannot just chose beta slightly greater than alpha and hope to do this, right? That is if you have a particular value of beta alpha cannot be more than the number of spheres that you can pack will be limited of that radius, right? And therefore it has to be a limit of and the radius will depend on the value of P sub e, right?

So how many spheres you are able to pack and therefore what is the value of alpha that you have to have, what is the reduction in alpha beta that you can have will depend on the kind of channel we have, have a good channel your radius is smaller you can pack more spheres your channel capacity is larger, right? If you have very poor channel in which your error probability is large, right? Your spectating spheres have to be of larger radius, right? In which a channel capacity comes down.

Student: Sir this beta dp has to be integer value (())(32:42) two integers for.

Professor: Well that is secondary to the argument we will assume that we take the nearest integer as t tends to infinity how does it matter, right? (())(32:55) t coming to infinity kind of argument so one does not have to worry about that small p.

Student: Sir beta dp is also very (())(33:01), or is it generally well?

Professor: Which are t?

Student: P e that we are saying.

Professor: See, after all if beta t is sending to infinity beta t p e also will be a sufficient in large number.

Student: No, is it very poor when t put as infinity or it is valid for only a value of t.

Professor: This argument is all asymptotic argument, right? Because again we are using the..

Student: That is what (())(33:25).

Professor: Direct frequency is definition of probability which is again valid only as a number of trials becomes infinite, right? So you solve a asymptotic argument that is why I am saying this is only theoretical argument really it therefore you may well ask what is a significance of this but let us come to I will come to that in a minute important thing for you to appreciate right now is that this brings us to a limit on alpha by beta and that limit is precisely the notion of channel frequency, right? You all understand that now the concept of channel capacity where it comes from and but now we know that if our information (())(34:07) is less than channel capacity we can think of the possibility of error free communication , right? Which is the new result for us it was a real major breakthrough when it came, is this possibility was never thought of right communication engineers before.

So for a given beta alpha cannot be increased beyond a certain limit without causing spheres to overlap and then the scheme will fail, therefore it implies that alpha by beta has to be less than some fraction which is what we call C of s and that depends on the channel parameters.

Student: Sir we have derived a formula for Cs.

Professor: Depends on channel noise and signal transmitted signal power and things like that.

Student: Sir we have given a expression for Cs on based of P e can we relate the (())(35:33).

Professor: (())(35:34) now this two arguments are slightly different things this is just to justify the existence of a concept like general capacity and what it means for us the next job is how do you calculate this channel capacity for various types of channels so that is what I gave you that formula was an example of the calculation of Cs, right? That comes next so at the moment we do not mix with it. What are the practical difficulties with this approach and what is a significance of this result practical difficulties is obvious that it is an asymptotic argument it cannot be really done in real life situation.

So therefore and it is very complex this we are doing, in that case why it is important to talk? I think I have given you enough reason for that we all appreciate that it is still important this idea in spite the fact that this does not give us a practical method of achieving this possibility because we now have a benchmark we know that we can achieve this much but not beyond this, right? It is a very major breakthrough for communication engineers from that point of view, okay it does is there is an upper limit on the rate of error free communication of information, right? And let upper limit is given by the channel it is really benchmark for.

As per as practicality is concerned since this result came coding theories and modulation theories and communication theories and basically working on trying to realize this dream that was shown at that time this possibility and we are in a very lucky age that we are now living in a situation where we are very close to this stream that is we have schemes of modulations, we have error correction and modulation schemes which allow us to achieve our very near to channel capacity if not exactly channel capacity, right?

And this is the result of the continuous research of more than last 3 decades, right? Last 3 to 4 decades and number of breakthroughs have come within this decade also. Now, yes any question before I proceed further? No questions? Now I do not have time to go through what I had plan to do but I plan to do after this for you was to take up channel capacity calculations for both discrete channels, discrete model channels as well as for continuous channels, right? If some of you are interested and I hope you will be will take that up in one some of the special lectures which we will take up after your majors, right?

Otherwise for the moment what I will do for you is just give you formulas for channel capacity for these two classes of the channels, one formula I have already given for the discrete channel two classes of channels what channels which are modeled as discrete channels or channels which will rather model a continuous channel, by continuous I mean they are not using a binary symmetric model you are working with analog channel in which you have a signal power and noise power and you want to express channel capacity directly in number of sequence fall in noise power rather in terms of error (())(39:19), right?

Because there are two ways you can (())(39:22) things, one can evolve various kinds of working with channels it is kind of models for the channels and have a channel capacity formula for that kind of situation that you want to work with, if you for example work with a binary symmetric model we already know what is the result for Cs, right?

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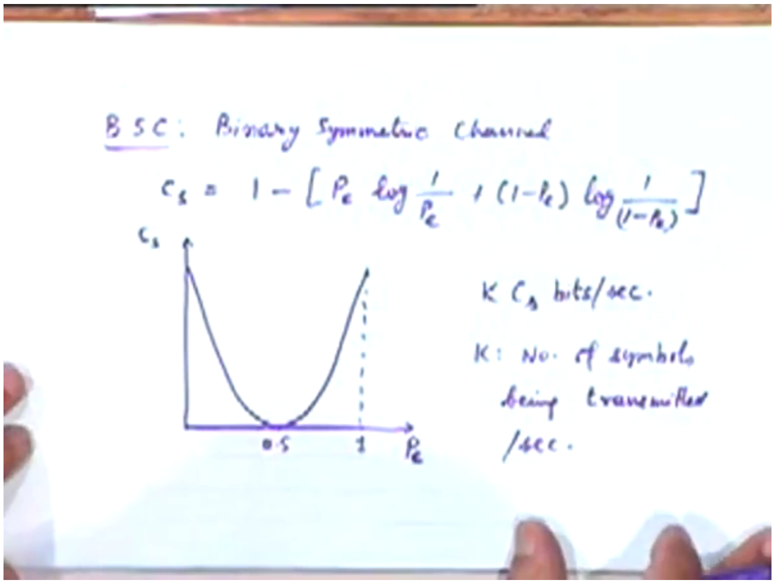

I gave you the result last time Cs is equal to maybe I can just repeat it, okay I have it somewhere, maybe I will just give you that result and discuss that briefly, yes it is 1 minus P sub e log of 1 by P sub e plus 1 minus P sub e log 1 minus P e, okay.

I think I gave you this formula yesterday, right? P sub e that is the error probability associated with binary symmetric channel model, yes, okay. It will be now this is I am not going to derivation I think I just gave you this formula because derivation requires at least one complete lecture on their which I have to skip. Now it will be useful at this say just plot this formula as a function of P sub e, right? It will be instructive to do that if I just plot C sub s versus P sub e what kind of curve you will like to get can you make a guess?

Student: At a half (())(41:21)

Professor: At P equal to 0 well, it is 1 and becomes 1 again it is very interesting and you might be surprised to start with, is it?

Student: If it is already error is 1 then also to be noted bits are to (())(41:44).

Professor: Correct, okay. So really speaking the worst channel is 1 corresponding to an error probability of 0.5, right? Of course this is just an academic argument because we do not normally work we do not normally communicate over channels with such large error probabilities, right?

Our typical error probabilities is the order of 1 in ten thousand or one in one million, right? Not 0.5 but this is just an purely an academic height. The second point to remember which I pointed out last time is how do you relate this conceptual channel capacity which is a fraction with channel capacity which is usually given in bits per second, right? Remember the argument that is going to be dependent on what is the rate at which you can transmit symbols on a channel, right? Without from other considerations from other considerations we know that for a band limited channel the number of symbols you can transmit is a certain basic value whatever it is maybe 2b or it may be something else from some other considerations then your absolute value of channel capacity will be that factor K multiplied by C sub s bits per second, right? K can be 2b for band limited channels or it could be something else from other considerations, right?

Because this 2b is also after all a upper limit you remember you may be able to transmit only less than there because we are using excessive bandwidth we are using (())(43:19), K depends on other considerations, right? For example bandwidth for example the kind pulse shape that you are going to use to transmit your things. So K is basically the number of symbols you end up using from other consideration being transmitted per second.

Now you maybe also interested to know whether one can express the channel capacity of a channel without regard to a particular digital implementation because this way of looking at things implies that you have a digital communication system for which an error probability is defined and then you are going to use that error probability in this calculation, right? This is the implication of this use of the binary symmetric channel model, right? But suppose after all what is P sub e depend on? (())(44:22) to noise reduction and natural question is can we express the channel capacity directly in terms of that signal to noise reduction, right? It is a very reasonable question and the answer is of course again yes I have gonna done it again if we had time it was on my agenda but I will just give you the result.

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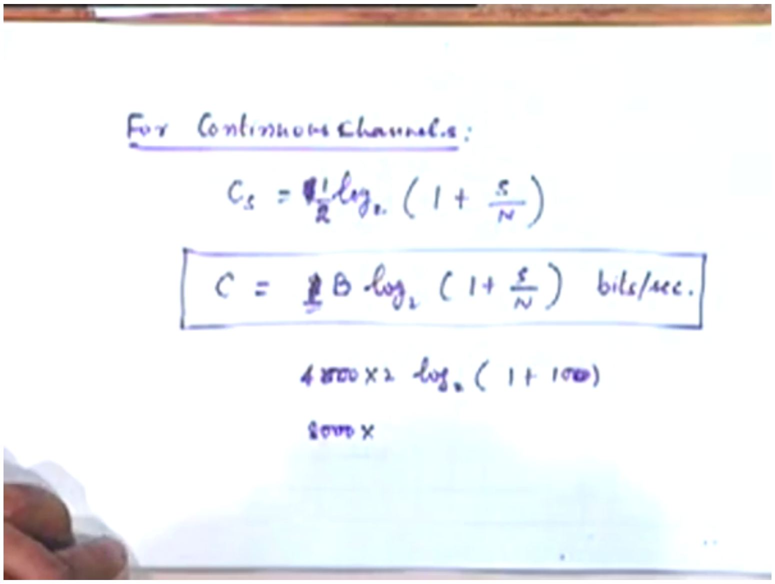

The result for continuous channels is as follows but both these results can particularly this one which I am going giving you now are very fundamental results and you must know them and that is C sub s will be log to the base 2 of 1 plus S by N of course everywhere it is assumed that we are working with (())(45:15) channels nothing there is a fraction of 2 also, no this 2 is not there, okay and of course the corresponding band limited channel will imply that you also put a 2b there so in terms of this is as a fraction and this is in terms of bits per second absolute value, this is a very famous Shannon formula for channel capacity of continuous channels.

Let us take a typical example, over telephone lines a typical simulation noise that you may have maybe of the order of 32 to 40 dbs, right? And then it was the order of let us say 3 to 4 kilo hertz, right? What will be the channel capacity like? Can you make it quick sample calculation? Let us have 4 kilo hertz, so this is 4 into 4000 into 2 into log 2 of let us say 30 to 40 db means about let us say 30 db, right? 30 bd will be how much ratio of 1000 power ratio of 1000 will be 30 bd, right? So this is about 1000, 1000 log 2 of (())(46:53) about 10.

So that will be 8000 into there is something wrong, will this (())(47:20) that is about 10 but this is too large where is the mistake this figure is bit on a higher side.

Student: (())(47:38) is 3.1.

Professor: Maybe 30 to db is a bit too larger value if a telephone channel we have to check what is a synchronization, it may be more of the order of 20 db I will have to check now I think I am forgetting what is synchronization that we typically use in telephone channels, suppose it is 20 bd it will be 100, right? And 100 will be corresponding to 7 and so I think I am also getting confused about this too we will just check this I will have to check, 2b has to be there but maybe there is a half here which I am missing out, yes that is the problem, I am sorry there is a correction this is half of this so this becomes b times this let me rewrite this so there is no confusion.

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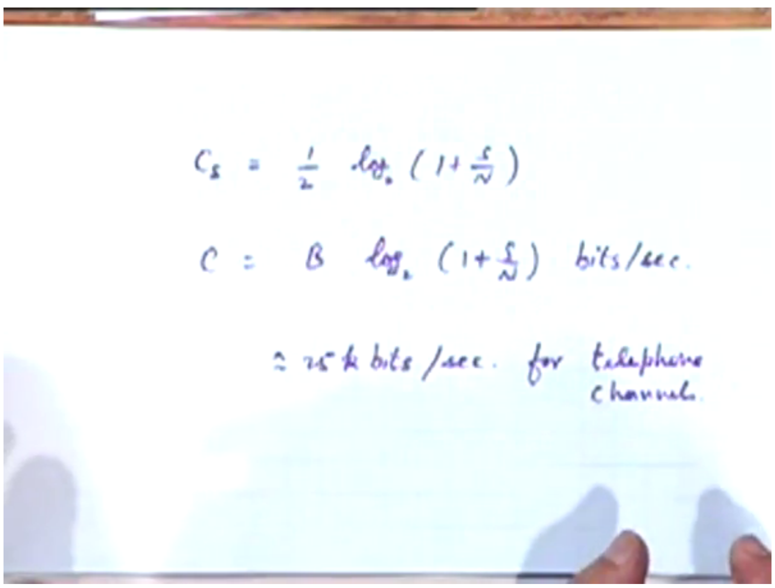

For continuous channels Cs is half of log 2 1 plus S by N and C is B times log 2 1 plus S by N in bits per second. And now our result will for telephone channels it turn out to be 4000 and 2 about what is more like what is the log of 10 100 to the base 2? 7, so about 28000 25 kilobits per second, right? So this is the typical figure that I am familiar with and I was trying to cook to that figure, right? 25 kilobits per second for normal telephone channels.

So there is what this most modern sophisticated modems telephone modems try to achieve, right? With some unit tells you that he has designed a modem which will give you channel capacity data rate of information rate of 3000 bits per second or 40000 bits per second you must know that it is making a fool of you, right? It is not possible but the current state of the art is that there modems which can go up to about a 19.6 kilobits per second, right? And the other which are in a fairly high set of development which take you very close to this limit they are in the lab prototypes (())(50:20), okay.

I think this is where we finished today, in the next two classes that is on Friday and Monday.