Digital Communication
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Module 01
Lecture 30
Demodulation of DPSK and M'ary Signals

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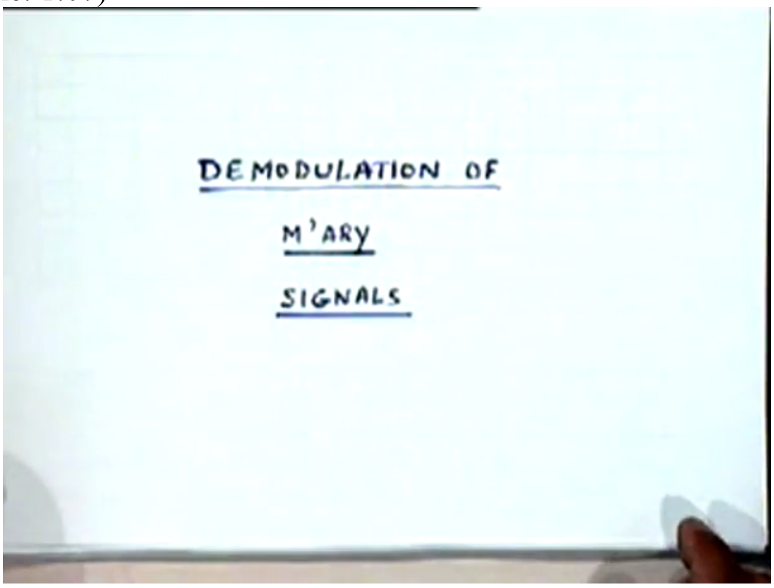

Today will discuss de-modulation of M'ary signals but before I get into that I like to complete our discussion that we started last time on de-modulation on differential (PSK)
BPSK differentially M'ary PSK. If you may recollect let me quickly go over a few slides that we used there, we saw that we can view differential PSK differentially M'ary PSK as a kind of orthogonal modulation scheme over two bit intervals which can be non-coherently de-modulated in the absence of phase in the absence of knowledge of the phase right.

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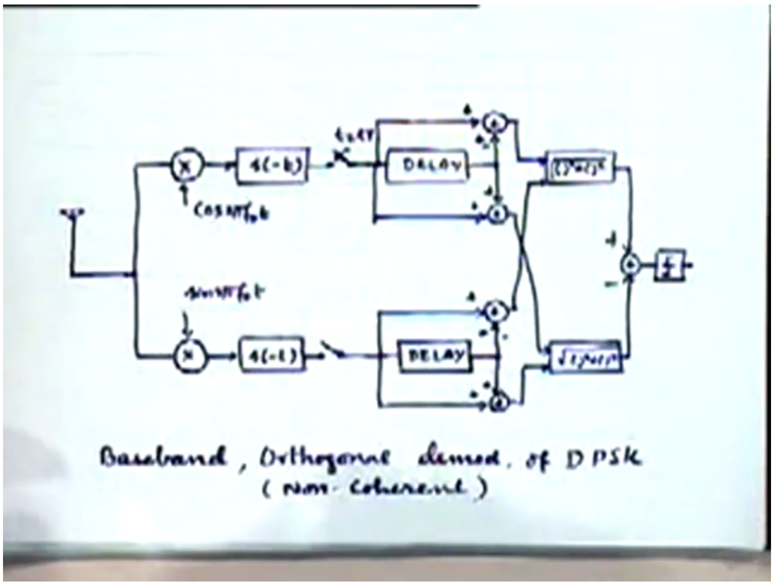

And keeping that view in mind we arrive at a receiver structure which is essentially that of that base on the concept of de-modulation of orthogonal waveforms right, this was the receiver structure that we looked at.

Where we obtained the output corresponding to waveform S t plus S t minus L t here, S t minus T here and S t minus S t minus T here.

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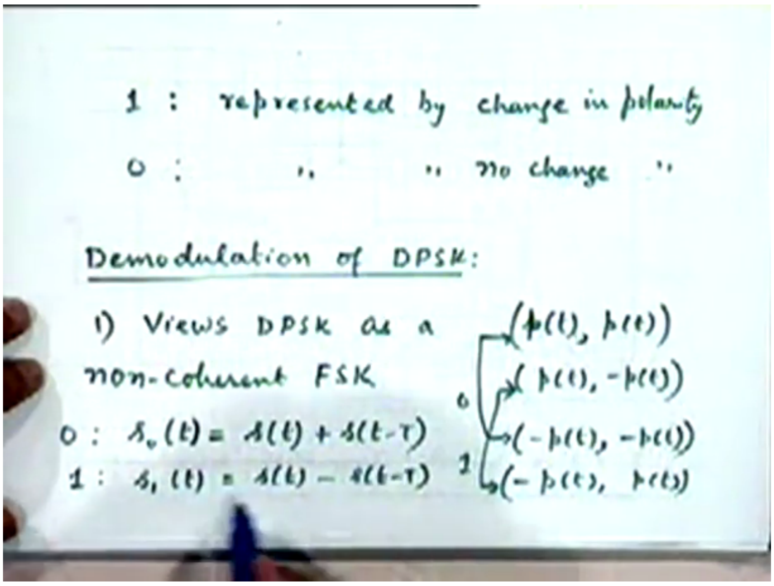

Because what we discussed was that we could think of now is zero being represented by the transmission of S t followed by S t minus T the same waveform delayed with the same sign right, whereas a one could be represented as a waveform in which the polarity gets changed as you go from one symbol to the next symbol. So over two bits intervals the received waveform can be represented as either this waveform or this waveform. So based on this appreciation we arrive at this receiver structure right.

We recollect all this at about three four days ago so help you can remember what we did and this is a non-coherent receiver as you can see I have combined the real and imaginary parts like this here and like this here, this corresponds to the output due to S knot t, this is the output you will get here will correspond to what you should get at the output of a matched filter followed by an envelope detector when the filter is matched to S knot minus t, similarly here the output will correspond to the situation for the same input but the filter followed by envelope detector is such that the filter is matched to S1 minus t, where S knot and S1 are as given over here right.

Now we also saw that this also implies that we have an SNR advantage over non-coherent FSK by how much? 3 dB So just like coherent FSK and coherent PSK different performance by 3 dB similarly non-coherent FSK and non-coherent PSK differentially (())(4:35) PSK different performance by 3 dB that is differential PSK requires about 3 dB a lower power than the corresponding non-coherent FSK for same error rate ok. So this is a summary of what we discussed that day and I like to simplify this receiver structure a little bit by a simple trick. Let's try to imagine or lets try to appreciate the outputs that we are looking at here and actually finally we are subtracting this two right.

We are looking at the sign of this difference and deciding whether a zero or one was is being decided. So starting from here lets see whether you can simplify this receiver structure a little and that simplification is intuitively quite satisfying. So let me go over to continuation of the discussion of DPSK with a view to simplifying the receiver structure further.

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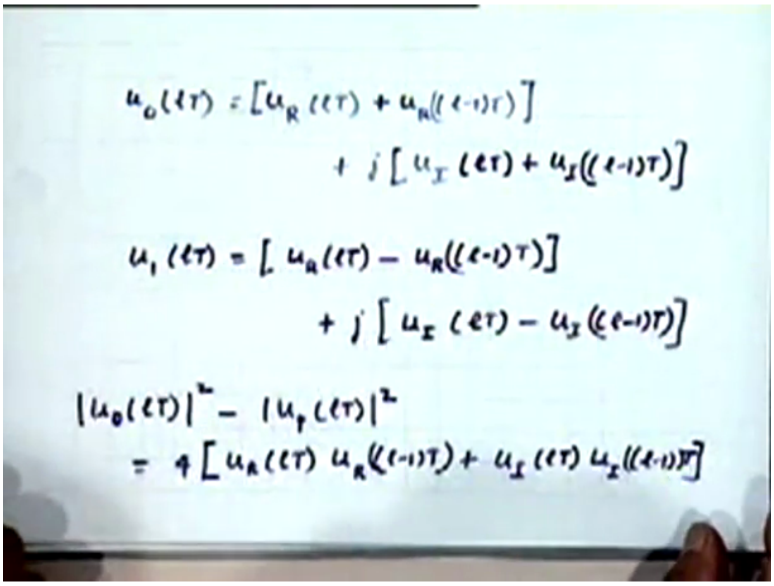

In order to that let me represent the output of this matched filter S t as U t as I have always being doing and lets look at the output of the matched filter corresponding to the signal S sub O at a time instant LT ok.

As you can see, you see this is a complex input right this is the complex matched filter assuming a real pulse because S minus t we are assuming in a real pulse but the input has been represented as a complex waveform by I Q de-modulation alright. This output here then comprises the complex output of this matched filter alright to a complex input because a filter itself is really real. So the both I and Q components are being filtered by the same filter. I am representing the output of this filter by U t right. Obviously it is comprise of a real part and an imaginary part U sub R and U sub I. Now I am intrusive writing down the output at this point which I call the or lets say the output of this point, this complex output representing this and this.

This is the real part of the final output, this is the imaginary part of the final output corresponding to the filter S sub O here the filter is S, here the filter is S sub O right and so the output due to S sub O will be U sub O L T and this is a complex output comprised of this real part and this imaginary part right. So I can write it as alright, this corresponds to the difference component that is where S sub O I think this corresponds to S sub 1 and this corresponds to S sub O because S sub 1 is S t minus S t minus T and S sub O is S t as I have depicted here, it really doesn’t matter either way but since I have this is what I have written I must change this and this I should call the output due to S sub 1 and this I should call the output due to S sub O alright.

So what is this value? This value is this minus this, right. What is this? This is the output of this matched filter at time T equal to L T, the real part of that. Similarly I have a corresponding component for the imaginary part right. So I can write this as I think it is quite obvious now I can just more or less straight away write it and you will agree with me. The real part is obtained by summing the I am not writing S sub 0 by summing the real parts and also the imaginary parts. So this gives you the overall real part this gives you the overall imaginary part, I will use a different pen, ok, you agree with this? What is this output? This output strictly speaking represents because I am not talking about S sub O, S sub O corresponds to this and this.

This is obtained by taking a real part here U r Lt to that your adding U r L minus 1 T right, is it clear? That is giving you a real part here. Similarly the imaginary part is being obtained from the corresponding imaginary parts E sub I t plus U sub I Lt plus U sub I L minus 1 T alright, this is what I have written here. Similarly the output of the matched filter matched to waveform S sub 1 T can be written at time T equal to L T the only difference will be, this will be ok. Now what are we doing with these two, lets consider this quantity, ultimately if I ignore the square rooting operation just assume that I am taking the real part and imaginary part summing the squares of both of this and similarly for this.

So basically I got the magnitude of U1 here and the magnitude of U knot here which I am subtracting, let me look at that ok. Suppose I ignore the so I take the magnitude of this minus the magnitude of this ok. Can you tell me what is, what this result is going to be? This is very trivial take the square of this magnitude square of this which is this square plus this square and subtract from this, this square plus this square right, it is very easy to see very trivial with the answer we four times U r L T times U r L minus 1 T plus U I L T into U sub I L minus 1 T alright. There is a minus sign here it should I think be plus, alright in that case let me make it this minus this alright ok. Now what does that tell me?

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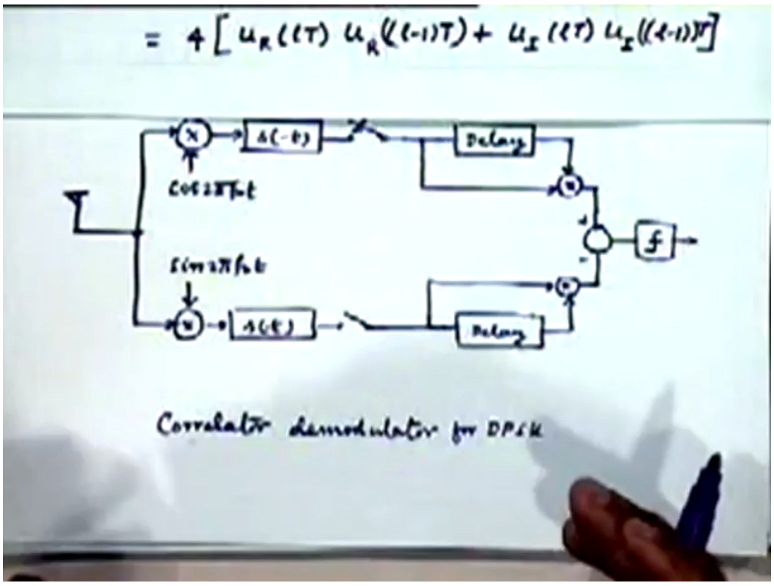

That brings me to this kind of a receiver structure, this is a receiver structure that is implied, it is really so simple. You are taking Ur LT you see what I have doing, you are taking Ur LT and multiplying it with a delayed version of the same thing right, so this is your LT and multiplying now you are not adding or subtracting you are multiplying the two right. Similarly you are taking U I LT and multiplying with a delayed version of the same thing and subtracting the difference right.

So in effect I can carry out a decision regarding a 1 or a 0 transmission based on this quantity rather than the quantity coming out of the previous receiver. The receiver structure looks quite different isn't it? Because it is much simpler for one thing and secondly intuitively it is quite appealing if you really think about it, because what you are saying is that you can detect this sign off that difference by just taking complex output of this receiver and multiplying with a delayed version of that complex output right and then checking the sign of the real part. Basically that is a real part or the product of this two this complex signal delayed with itself by (())(14:39) just think about it, isn't it?

This quantity is a real part of the complex output of this matched filter. Suppose U t is your complex output of this matched filter right it is in generally U t plus U r sub U sub r T plus J times U sub I T you are multiplying that with its delayed version and taking a real part of that will produce this right, forget the scaling factor agreed. So all you saying is if you, you want to determine see afterall in a DPSK what are you doing? Intuitively is much more satisfying as a receiver structure than the previous one which we had concocted so much by forcing a similarity of that scheme with an orthogonal modulation scheme.

But here things are very much straight forward to look at, you have a complex output at time T, t equal to LT you are also looking at the complex output at time L minus 1 t and in DPSK what we want to know is whether there is a phase shift phase change or not, so I am multiplying the two waveforms (())(16:01) real part of that right. If there is positive then implies is a phase change if there is negative it implies no phase change right ok. Do you appreciate this point intuitively?

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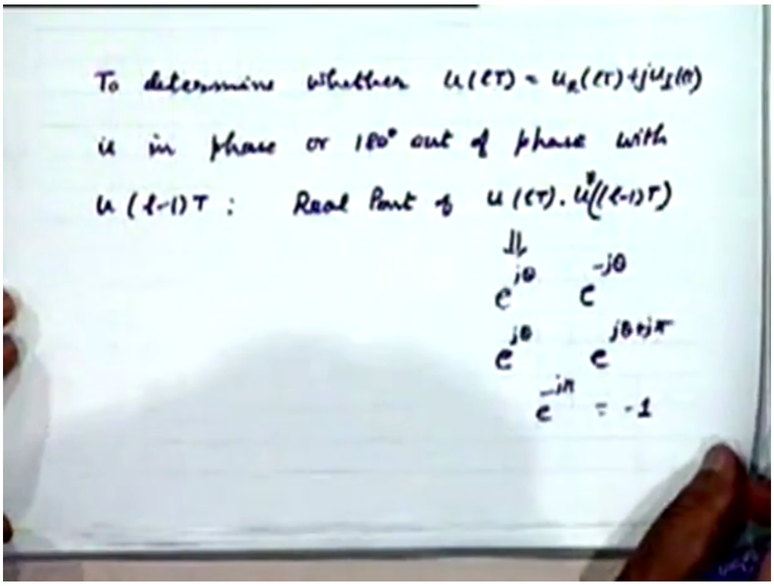

That is to determine whether U L t equal to U sub r LT plus (U) J times U sub I L T is in phase or 180 degrees out of phase with U L minus 1 T or you need to do is U multiply the two take the product and see the real part of the product.

Oh yeah it should be conjugate you are right, yes, that is what right and that makes sense because suppose U LT at some for E to the power J theta with some amplitude right and suppose there was no phase change right then this will become E to the power minus J theta and the product will be 1, if on the other hand there is a phase change this will be E to the power J theta plus J pie right now you multiply the conjugates the answer will be E to the power minus J pie which will be minus 1. Basically that is what you are doing taking the looking at the real part.

So that was an alternative implementation of the DPSK receiver which is what is really commonly implemented rather than the scheme based on orthogonal implementation right, orthogonal waveform implementation. So this is (accor) this is also called a correlator implementation for DPSK, as if your matched filtering the received signal and your correlating the present bit interval output with the previous bit interval output right. So some kind of a correlation (inter) operation is implied and that is what is called a correlated de-modulator for differentially (())(19:01) PSK ok.

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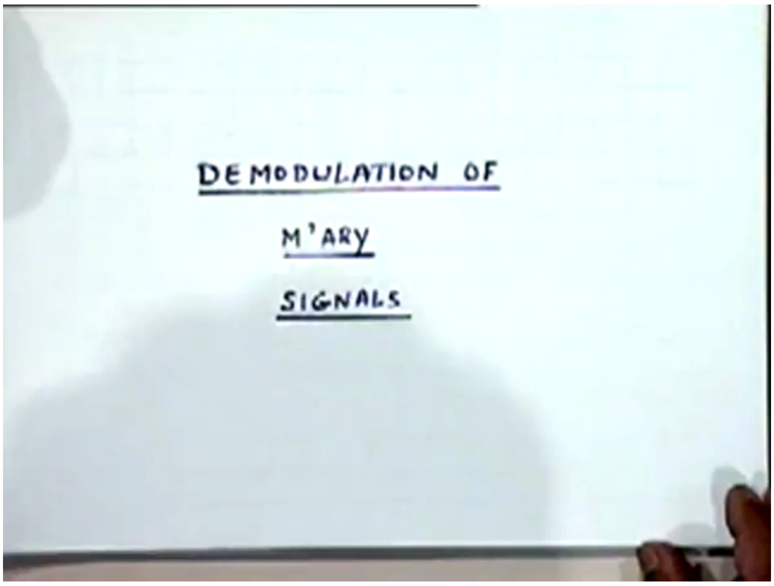

Now we come to de-modulation of M'ary signal right, I think we have more or less taking care of all kinds of binary signals that we will ever encounter. Now if you may recollect there are two way in which we go from binary signaling to M'ary signaling essentially right, one by means of orthogonal schemes and orthogonal signals derived schemes (())(19:34) by orthogonal and things like that and the other is going for two dimensional M'ary schemes in which you have a signal constellation in a two dimensional plane where the two dimensions are typically the quadrature carrier components right, the cosine and the sine of the carrier that you are using.

So will take up each of these two kinds of modulations separately because they require slightly different kinds of receiver structures. So first just look at the de-modulation of M'ary signals based on orthogonal waveforms.

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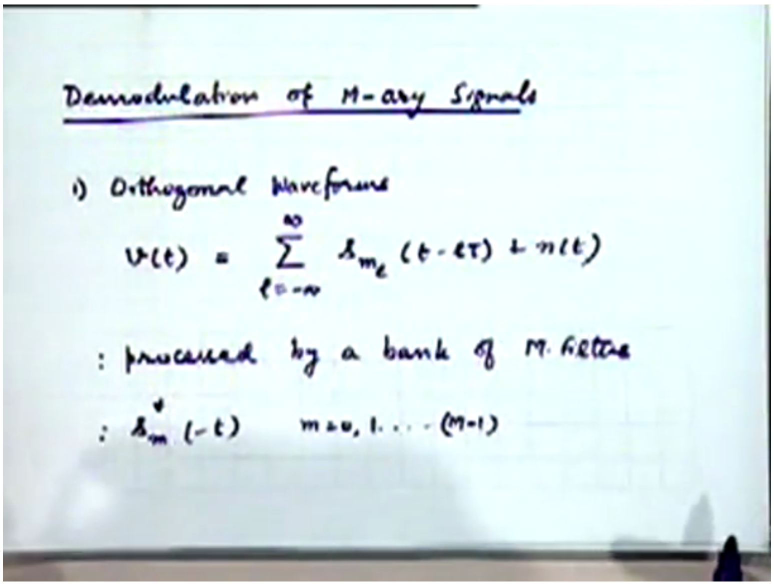

So de-modulation of and first we look at orthogonal waveforms. Now for this case the receiver structure is a trivial extension of what we have been discussing for the binary case as should be obvious right, because what you are doing is, in each symbol interval you are transmitting now will use the term symbol interval rather than bit interval because if you are using a non-binary scheme it is implied that you are transmitting more than one bit of information in one signaling interval right and therefore typically we represent the entity of information that is being transmitted not as a bit but as a symbol because it may carry information regarding several bits right.

So this is a usual terminology difference is used between binary and M'ary schemes. So will now refer to the basic transmission interval signaling interval as a symbol interval rather than as a bit interval. So in each symbol interval we are transmitting one of M possible waveforms which are predefined and which are mutually orthogonal right. So what should be the receiver structure? Obviously a bank of matched filters or corelators. So lets say your received signal is V t equal to ok. Now this is processed by a bank of matched filters, one for each of the M possible waveforms that you might be transmitting in each signaling interval right. So these matched filters will take the form of S m conjugate minus t in general at baseband for M going from zero to lets say M minus 1 ok.

And the implementation could be either a passband in nature or baseband in nature, you could have passband matched filters followed by analogue detectors or we could have a baseband implementation based on first converting the input signal into a complex representation followed by further process. So there will be one matched filter of this kind for every possible S sub L M going from zero to M minus 1. As we know the outputs of this matched filters will have to be looked at t equal to L T at which time instant we expect the outputs to be real right, from the properties of matched filters we have discussed before. So it is a real parts of these matched filters outputs that we looking at which are given as follows.

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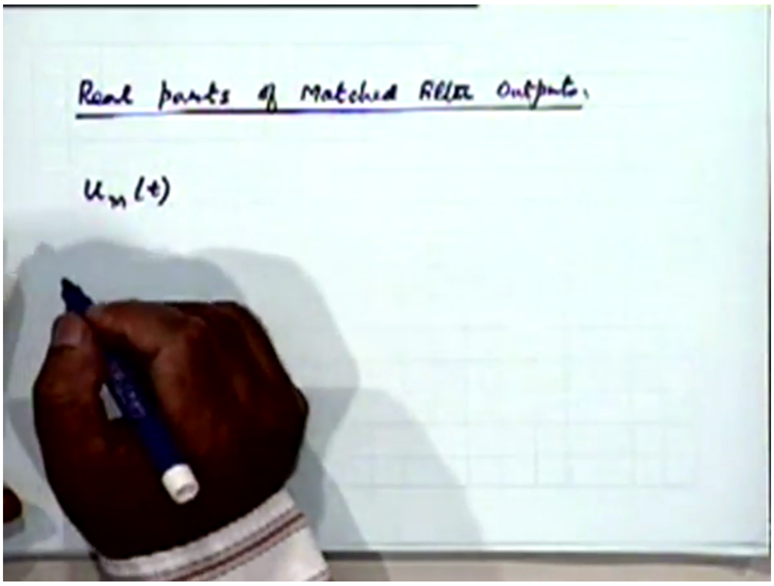

Let's take the output of the Mth matched filter denoted by U sub m t.

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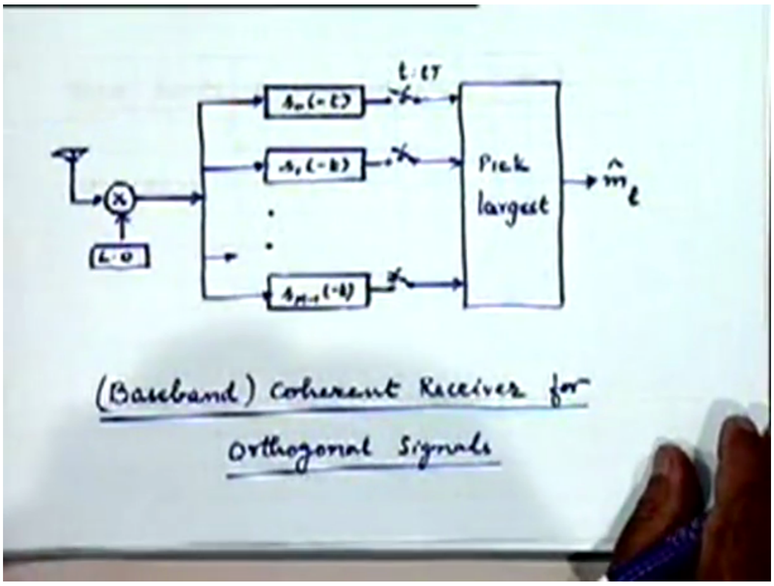

I think before I do that let the picture be very clear this is what you are talking about right, this is the receiver structure and I will come back to this after doing some maths, don't start copying it right now, I will just comeback to it in a few minutes. We are looking at the output of the Mth such filter right corresponding to an input waveform which is like this, which were each successive signaling interval carries some specific waveform S sub M sub L right.

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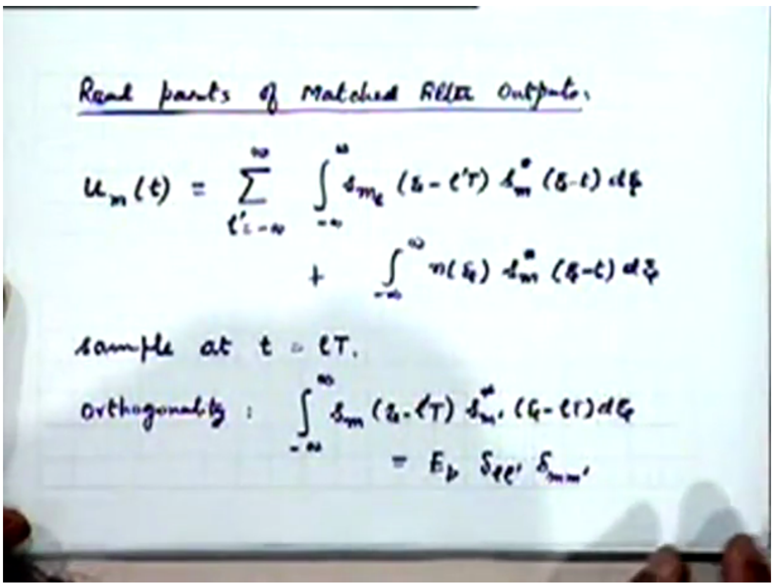

So U m tis going to be some L prime going from minus infinity to infinity, the output of the matched filter is represented as a convolution input is this right and impulse response is right plus the corresponding noise output.

We appreciate what is happening, just a simple equation which tells us what is the output of a specific matched filter which one? S sub m, right, corresponding to this input ok. Obviously it will be this convolved with that matched filter impulse response plus this convolved with same matched filtering impulse response which is what I have got here right and offcourse I have taken the convolution integral inside the summation. Where the summation refers to successive (symboling) signaling intervals at going from minus infinity to plus infinity alright, and this is what you sample at I am going to this maps although the answer we already know as to what kind of receiver you should use, I am going to maths because later on this is going to be important for us when you do performance analysis.

Because for performance analysis we want to precisely be fine the quantity that we have sampling mathematically and the operations that we are carrying out for making a decision as to which symbol was transmitted. So we must be able to precisely define how we are carrying out the comparison, what precise competitions are being done. So we are going to compute this matched filter outputs for each value of M sample them at some specific time instant lets say T equal to L T and now using the orthogonality property right, the orthogonality property tell us that these integrals, what is the value of these integrals going to be?

Let us consider S sub m zeta minus L prime T into S m prime is just general result I am writing. What will be the value of this? This will be zero if either L prime is not equal to L or M is not equal to M prime right, either way we are talking about general orthogonality where waveform is not only orthogonal with respect to other waveforms in the dictionary but is also orthogonal to each of the shifted versions of the waveform by a signaling interval. It is also orthogonal to itself after it has been shifted right by T seconds, the T is a basic (())(28:23) interval.

So it is that kind of orthogonality that we have in mind so will say this is equal to E sub P where E sub p is a energy of each of the waveforms into (())(28:35) delta function delta L L prime also delta sub m m prime, so this is the orthogonality property that we are keeping here, assuming here based on which we can now simplify this expression considerably alright. So what will you observe when you put T equal to L T here? What you will get?

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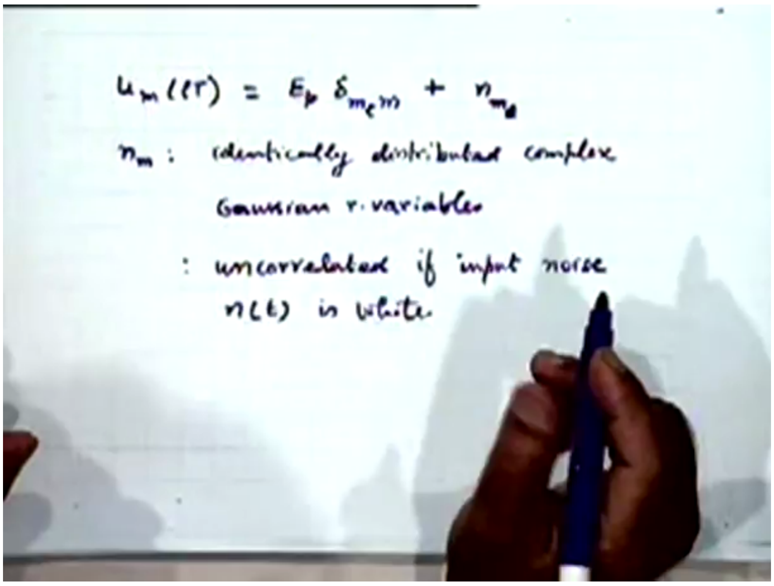

Suppose I am interested in the value U m LT U sub m LT, based using this orthogonality what will be left here? Apply this over here right, this will be E sub P will be contribution only from L prime equal to L right because of this (())(29:39) delta function that is the term L prime corresponding to the sampling instant is the only one that is corresponding signaling interval pulse, is the only one which will contribute to the output and the value of the contributed output will be E sub P provided also this the transmitted symbol is the same as the corresponding matched filter impulse response with which we are correlating it, with which we are through which we are passing it right.

So I have to also put a Kronecker delta function m sub Lm is it clear? First of all the summation goes because of delta L L prime only one term from the summation counts that is L prime equal to L right secondly after the summation has been removed we are left with this integral with a index m sub L here and some index m here, this represents the actual symbol value in the Lth interval, this represents the matched filter in the bank of matched filters through which you are passing it. You are looking at the Mth output Mth matched filter output. So the actual symbol is m sub L and the corresponding matched filter through which you are passing it is corresponds to the Mth waveform.

So obviously the since they are orthogonal we have to write this is in this form alright. Vikram are you lost? It looks like, how many people are lost, let me see, all of you, no only a few of you fortunately ok. Let's explain it again, lets try to understand it very clearly, this orthogonality definition is ok, with everybody? Are you lost here also? This is fine, we just applying this orthogonality definition to this integral over here right. Lets look at the implication of each of this two Kronecker delta function that we have over here right.

Well the implication of this Kronecker delta function is that unless its I am putting t equal to LT so this t and this t become equal to LT right, now apply tis definition the only terms out of all this infinite number of terms which will contribute to a which have the possibility of contributing a non-zero output is, the term L prime equal to L right. So this summation goes except for the value L prime equal to L agreed. So I have only one term to consider. Within that term also this integral be non-zero only if m sub L is same as m, this is the actual transmitted waveform in the Lth signalling interval.

You are processing it by a bank of filters right, we are looking at the output of each of these of this filters in the bank, is it clear? What we are saying is only one of the filters in this bank will contribute an output right, which will correspond to m equal to m sub L, the other filters in the bank will not produce an output due to the (()))(33:07) that is all we are saying, that is a implication of (())(33:11) good thing the noise integral separately, I am only looking at the first another mode I am only looking at this another mode right, this is to be taken separately, is it alright now? Everybody who said was lost, is with me now? Ok, fine plus the noise term, obviously the noise term is, well we can represent it either way lets call it m sub m L or m sub it doesn't really matter m sub m yes right.

I am just denoting it but we know what kind of noise is this, if this is White Gaussian Noise with a particular variance right we know that, this will also be Gaussian Random variable with the same variance right, we already seen that and these variance noise random variables are going to be at the outputs of different matched filters because they are orthogonal are going to be mutually uncorrelated. They all going to be Gaussian Random variables and therefore going to be uncorrelated and independent. So n sub m are identically distributed complex Gaussian Random variables and there also be uncorrelated provided that what is the condition under which they will be uncorrelated? If the input noise is White, any questions regarding this equation.

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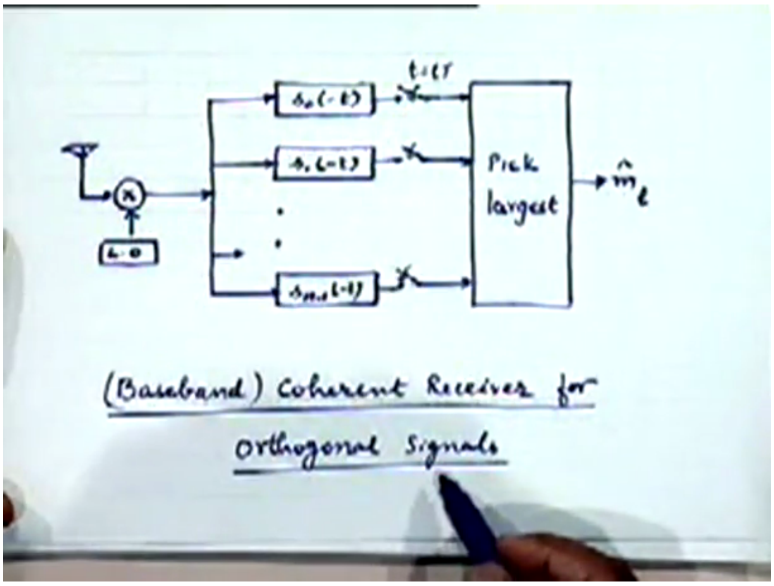

Basically what we are saying is that each transmitted pulse will produce a significant output only in one of these filters right, because the pulse is orthogonal to every other filter, every other filter impulse response. So obviously the receiver structure will correspond to during this matched filtering, sampling the outputs of these matched filters and then finding out which of this matched filters has produced the largest output right. So you have a decision logic here which looks up these m numbers besides which is largest and produces an index m sub L corresponding to the largest output which is taken as an estimate of the symbol that was actually transmitted right.

Offcourse this will be finally converted into a binary form because typically the number of waveforms in an M'ary signalling scheme is chosen to be a power of two so we can represent basically what we have done is looked at each symbol as an L bit binary information sequence where 2 to the power L is equal to m right, something (())(37:02). So that is the baseband coherent receiver for orthogonal signals. I have taken in this particular implementation this filters to be real but you could as well have taken complex receivers. You notice something that I have not although I am calling it a baseband implementation I have not shown the Q outputs at all, why?

Seriously speaking or properly speaking if I am talking about baseband implementation I should first represent the incoming waveform into an I Q form, I have only shown a single look-loss letter here, what is the reason? I have not looked at the Q output at all, the imaginary part at all. I should have really taken well I would do that provided these are in general complex waveforms right. If the, this orthogonal waveforms were complex the baseband representation of this orthogonal waveforms were complex then I should have done that. But if this are real, I don't have to do it, why?

Because the imaginary part will only produce an imaginary output corresponding to because the same filters will get repeated right, they only contribute an imaginary output at the signal at the sampling instant I am only interested in the real output real part of that, so that is the reason why I have not shown it. However if this were complex then you have to modify the receiver structure accordingly right. You will have to put conjugates here and then retain the real part finally and then pick largest of the real part. Now I think you all understood this, is that ok? I am waiting for you to absorb it sufficiently so that I don't rush to the next thing, is there any question, if so please speak out?

What I mean by this? What I mean by this is I have to look at each of this outputs, find out which matched filter is producing a largest output and decide therefore which index m sub L has been, is the correct index right, which is a waveform which was actually transmitted. Because the waveform that was actually transmitted the corresponding matched filter will produce the largest output right. So that gives me an estimate of the symbol that was transmitted in that particular symbol interval, is there anything else that you would like me to explain here? All it has to do is take decision every signalling interval.

Lets that is the real least complicated part of it really, this is much more complicated implementing so many matched filters

Student: Sir what was the question?

Professor: His question was that it should be able to pick the largest value from the given values before the next set of outputs is produced surely right. Sure but we have sufficiently fast processors available and this kind of task is relatively simple right, all it has to do is look at some numbers whereas these filters have much more complex task, they have to perform filtering right, implementation of this via processors is a much more challenging operation than just picking up the largest value alright. So if that is fine we have only looked at the coherent receiver for orthogonal waveforms we know that we can also look at non-coherent receivers right and I think its there is no need to spend too much time on it, it is quite obvious what we should do, right, very good.

So what we now have is, we now have to go through a complex I Q representation right we can't of worried now because we don't the phase right, the phase is unknown so we can't compensate for it.

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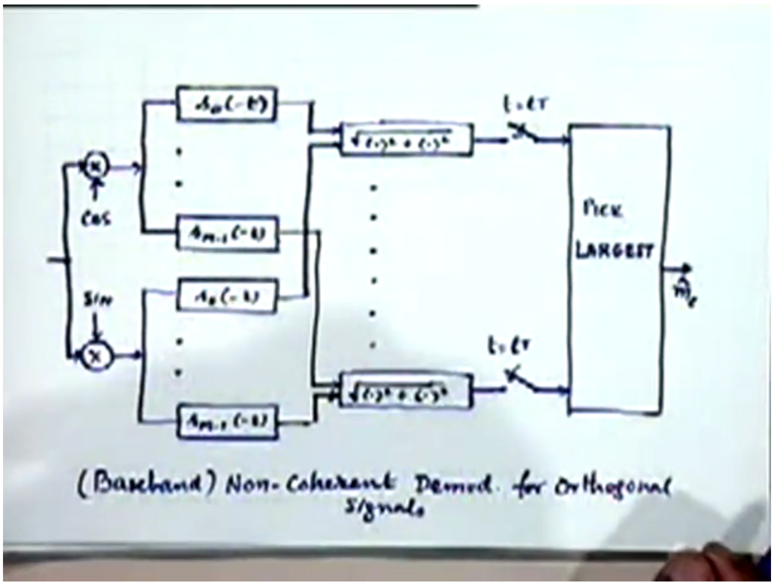

So we have I Q output the bank of filters in now I have again shown this picture for the case where the orthogonal signalling waveforms are real, if they were complex this picture would be slightly more complicated or atleast you will have to put conjugates everywhere right. If it real then what is the matched filter corresponding to this any particular complex input, well this it will be filtered by the same filter in the real and the imaginary parts alright and then they will be combine through this envelope detector operation.

So take the magnitude square and take the square root. You will do this for each of the M possible waveforms that you could have transmitted sample this and pick the largest as before. So that is the baseband non-coherent de-modulator for orthogonal signals. Is there any question here? Can you tell me how will the thing get modified, if I were the use complex pulses rather than real pulses at baseband? How will I modify this for complex pulses? No the imaginary part is there, here also, we are producing the imaginary part here also, right.

Basically what will happen is this, yes, it will become more complicated. This complex number will be operated upon by a complex filter right, so schematically let me represent one filter. Suppose I represent the complex input as a doubled line arrow, showing that it has a real part and imaginary part, then we have to pass it through a complex filter with impulse response some H minus t right, this will produce a complex output in general right.

Therefore really speaking, how many filtering operations you have to do? Four, suppose I was to ask how many equivalent real filtering operations I have to do? Yeah obviously four, because this is complex, this is complex when you multiply or convolve a complex number, a complex signal with another complex this thing there will be four real such operations really, so that is our essential increase in complication.

However remember that I am only interested in the real output at the sampling instance. So I only need to produce a real part and not the complex output. So that is how things will be different but in principle they will be broadly the same, is that ok?

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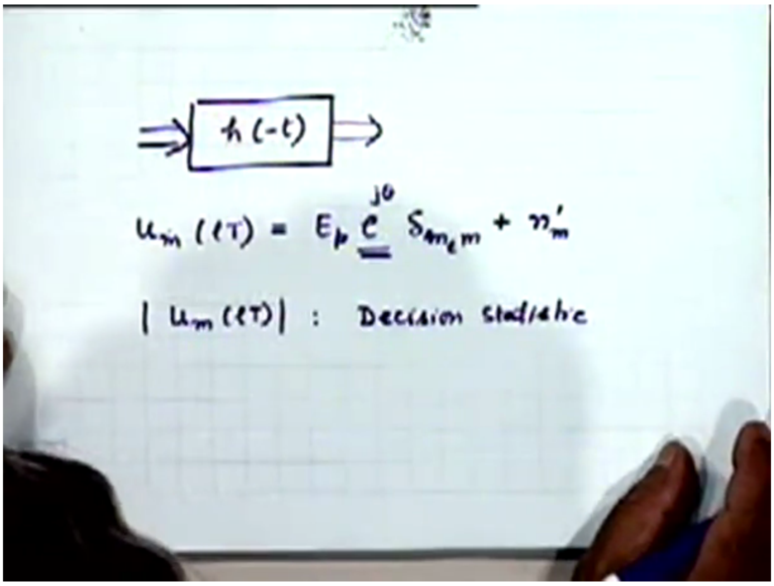

Mathematically the waveform that your, the sample output that you look at, at each of the filters can be represented as U sub m at the time instant t equal to L T will be E sub p this everything else is same this was there earlier also will be an additional factor that comes to the picture, what is that? E to the power J theta plus n m prime right and therefore, what you really look at is the magnitude of this is your (())(46:07) statistic because we don't want to consider E to the power J theta right.

That is if you just look at this numbers here, they are represented like this, finally you only looking at a magnitude ok. Now we come to the situation where we don't have orthogonal waveforms I am leaving out as an exercise for you to do yourself what we shall if you had to work we lets say M'ary simplex waveforms or M'ary bi-orthogonal waveforms ok think about it yourself, how will you de-modulate signalling schemes based on M'ary simplex waveforms or M'ry bi-orthogonal waveforms right, they will be very simple and they can be obviously derived from this bank of matched filters right because both the simplex set as well as the bi-orthogonal set is define in terms of some orthogonal set.

So if I have the outputs of the orthogonal filters I can construct a required outputs which will be used for de-modulation of the simplex set or the bi-orthogonal set as applicable, is it ok?

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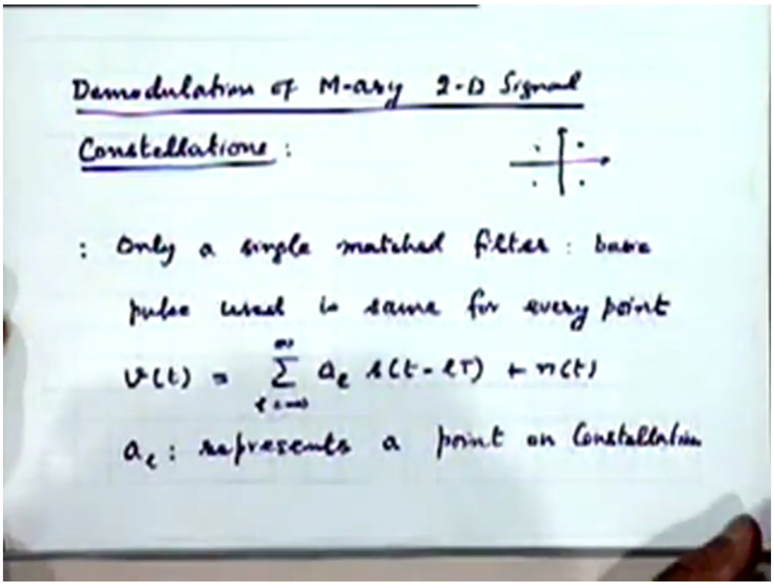

Therefore I should really take up only one more system and that is de-modulators for M’ary signal constellations in two dimensions which includes the very important class of modulation schemes because modulation schemes like QPSK, offset QPSK, M’ary PSK, amplitude phase shift keying along with multiple points large number of points in the constellation, they all fall within this class right.

We have seen the variety of constellation we can have, we can have on them on a circle, we can have them on a grid right, in a mesh or we could have other combinations which we have seen. Now this de-modulation the de-modulation of this waveforms is quite different from the corresponding orthogonal waveforms that we have just seen, the reason being the basically each waveform that we are transmitting is essentially the same in this case isn’t it? The only thing that differs is the associated complex number complex amplitude with it, which defines the point on the constellation that you are transmitting.

But the basic (transmitted) transmission entity is the same pulse right. So whether you are suppose QPSK right, just associating with a cosine and sine of the pulse a different complex number complex amplitude, therefore that is a different amplitude in a different phase, but essentially the basic pulse shape is the same. So really speaking how many matched filters you require? Only one, you don't require whole right your offcourse a complex matched filter right. You require a simple complex matched filter and after from that you can produce a complex number based on which you can take a decision as to which particular symbol was transmitted, depending on where that complex number lies right.

Intuitively it is quite obvious that this is what we have to really do. So one needs only a single matched filter because the same basic pulse shape for every point ok. So if you're receive waveform is as before A sub L S t minus L T plus n t so I don't have an index (coress) with the S here right, because there is only one basic pulse only this symbol is denoted by A sub L which is some complex number on this two dimensional plane right, is that clear. For example if it is QPSK, this A sub L will take one this four possible complex values but in each case the basic pulse shape is same alright and offcourse will assume Nyquist signalling A sub L represents a point on the constellation ok, in the Lth time interval signalling interval.

So if you assume that the matched filter output is going to be pass through a matched filter corresponding to the pulse shape S t if we assume that is denoted by r t as before, if you assume that output satisfies the Nyquist Criterion then at the sampling instant t equal to L T the only pulse that will contribute to the output is corresponding to that signalling interval.

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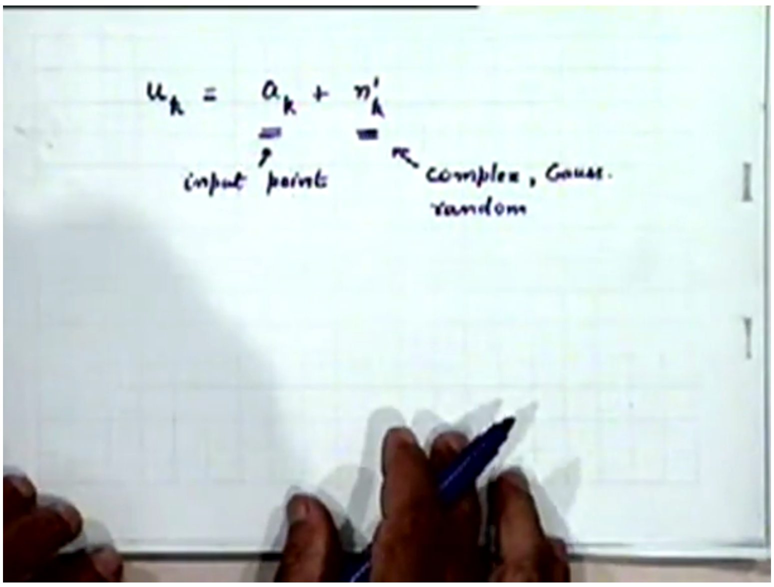

And therefore we can simply write U sub K in the at T equal to k t, what will be that equal to? A sub k plus some n prime k right, is it obvious because this is being matched filtered the output will be A sub L r t minus L t, that is being sampled at T equal to k t and if we use a

Nyquist pulse that is r t's Nyquist then everything else will be zero or others r t's sample at T equal to K t will be zero except for the Kth bit interval.

The corresponding noise waveform is also filtered let denote that by n prime right and again you are sampling the noise waveform at T equal to K t to produce a random variable n k prime which will be obviously Gaussian random variable. So in general this is a complex number corresponding to what you transmitted this is a complex random variable right. This represents the input symbol, input point on the constellation which is a complex number, this is a complex Gaussian random variable. You understand what that means? Complex Gaussian random variable? That is it has a real part and the imaginary part both of which are Gaussian and since they also uncorrelated the joint density function is also Gaussian. Even if they were not uncorrelated due to Gaussian but very simple form of Gaussian number over here.

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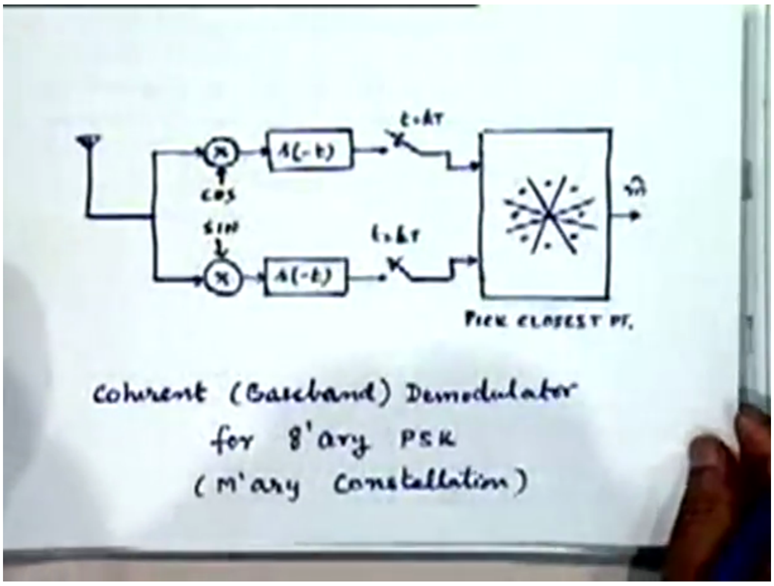

So what does one have to do now? This is shown here you take the input signal to the I Q processing to get the baseband representation you do the matched filtering right, this I am assuming again that our basic pulse shape is real here to produce the real part of the output and the imaginary part of the output that defies the complex number which we are calling V sub k right which is going to be equal to A sub k plus n sub k ok. I have illustrated the situation for an eight phase PSK system, 8'ary PSK system in which case the A sub k should come from this 8 point constellation on a circle right.

Now our job is to decide what was transmitted, intuitively what you think we should do? We should find out the point to which V sub k is the nearest in this constellation right and that will be the best thing to do, infact one can prove that this is optimum thing to do and that defies in this two dimensional plane the a whole lot of regions corresponding to 8 points it defines 8 regions such that in each of this regions this point is closest to the given constellation point.

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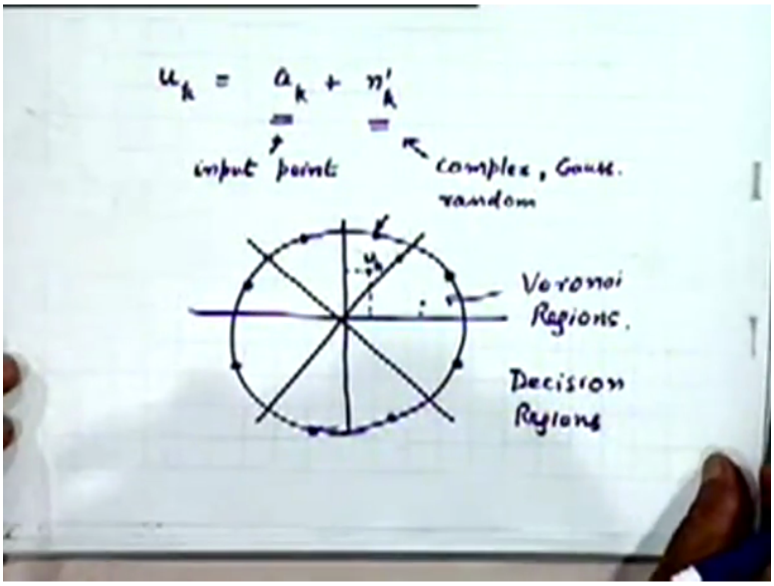

Professor: No not the modulus, (())(55:48) independently, please try to understand let me fully describe what it is being done and then I think you let me do up this picture a little bit offcourse this is a bad circle but imagine this is a circle ok, I am sorry lets assume that this are your constellation points this different for the picture that I have drawn this slightly different, the points are shifted with respect to the previous set of points, but what we are saying is if this is a valid constellation point and you receive a V k which if plotted on this two dimensional thing lies here right, that is it has a real part like this and has a imaginary part like this.

So obviously this is the plot of the point V sub k. If it lies in this region it implies that this is closest to this point. So our decision will be that this particular symbol was transmitted. So both the real part and the imaginary part play a role in deciding where it lies right of receive V k. Similarly if it turns out to be here will decide that this was transmitted. These regions into which the constellation diagram is divided such that in each region every point will be closest to the specified constellation point, this are also known by the name of Voronoi

Regions right ok. They define the decision regions corresponding to a given constellation diagram, decision regions or Voronoi Regions right.

The property of Voronoi Regions is that corresponding a given point that we have fixed in this region any point in this region will be closest to only this particular point and not to (any the) any other specified points like this. That offcourse is a matter of prior (you know) bias you may put it either way it doesn't really matter ok, but you can predefine that if it lies between here to here will correspond to this because the probability that the point will lie exactly on the boundary is actually zero. So one doesn't really have to worry, yes because it refers to the probability of a specific value right, that is, yes noise having a specific phase right, no problem still zero because the continuous random variable.

If it has a discrete random variable it will have a finite probability because (())(59:11) random variable it has zero, so think we can stop here.