Digital Communication.
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Lecture-3.
Digital Representation of Analog Signals: Pulse Code Modulation.

Professor: We have talked about one method of converting analog information into digital form and that is Delta modulation. It is a very simple technique by which one can represent analog messages in the form of a sequence of positive and negative pulses which is what is eventually transmitted onto a channel. And let us say you can carry out digital communication in this way. Another method of doing this which is also in fact, which is perhaps somewhat more popular is the pulse code modulation by which you are partly familiar in the context of Analog to Digital conversions in other courses.

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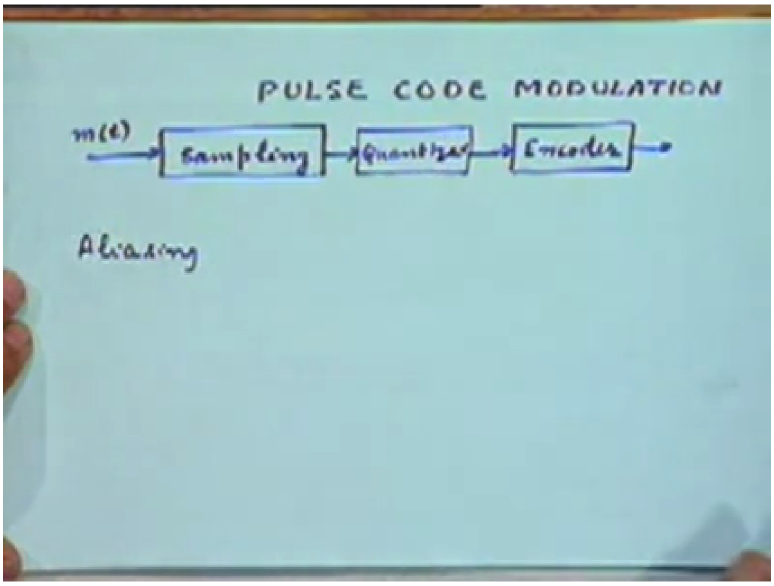

And that is what we will discuss today in the context of digital communication. Like Delta modulation, this is also a digital pulse modulation scheme. But the approach is quite different as I will show in this block diagram. You start with sampling the signal at a suitable rate, so that is a sampler, the message signal mt is passed through the sampler. Of course when you do sampling of any signal, it is assumed that to do it at a sufficient rate so that you do not lose any information. And you know what that rate is, the nyquist rate. That is if the signal mt has a bandwidth W, then you must use a sampling rate at least equal to 2W samples per second.

In fact also it is desirable that if you are are sampling and to W samples per seconds, you make sure that mt does not have any component about W, other way round also holds. If you do not do that, you are going to have some kind of distortion with which you are familiar, aliasing distortion, right. Take for example the case of your voice signal, voice signal does contain components even beyond 4 kilos hertz. But for most purposes the quality of voice signal hardly suffers, if you were to band limit the within 3.3 kilos it or so, right, therefore we can say that, we can sample at twice the maximum component which is fitted into 4 kilos hertz and therefore you sample at 8 kilos hertz.

But then you must make sure the voice signal before being sample the lowpass filtered by filter whose bandwidth is about 4 gigahertz or less, right, about 3.3, between 3.3 and 4 kilos hertz, because it may contain components of higher frequencies which may not be of interest to us but which allows, if allowed to represent will cause aliasing distortion which we do not want.

Student: (())(4:28).

Professor: Aliasing distortion. Maybe I should just remind you, these are the spellings, aliasing. After sampling the signal the most important step that you carry out in a pulse code modulation system is that of quantisation. There is a quantiser, will soon see what a quantiser does and immediately following that there is an encoder. These 3 basic building blocks are the key components of a pulse code modulation system. Excuse me. The main function of the quantiser is to take each sample and represent it in one of prefixed number of levels.

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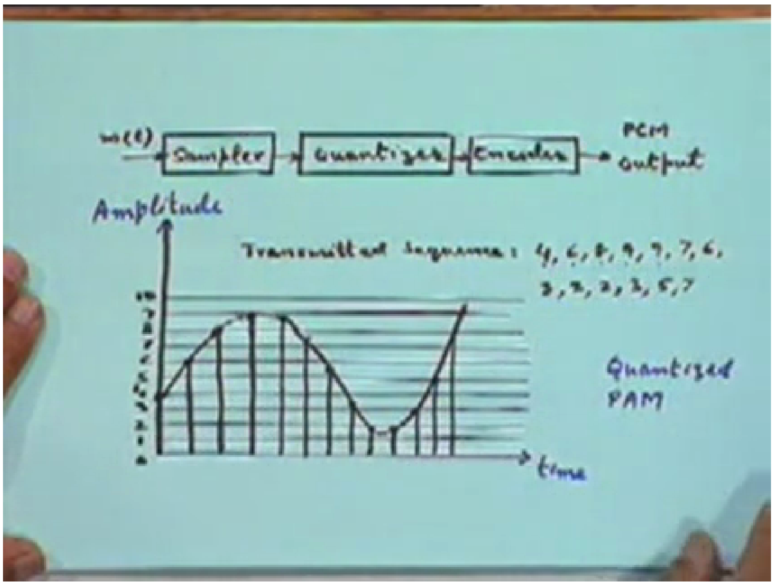

That is unlike the sample value which can take any value over an infinite range, the quantiser converts this into or transforms this continuous valued variable into a discrete valued variable. I think it is best illustrated by means of a picture which I have here. It is the same picture that you have when we talked about the sampler, the quantiser and the encoder. Let us say this is your continuous time continuous valued signal, right. So it is continuous both in time as well as in amplitude. The 1st thing that you do is sample it, so we are talking about sampling at nyquist rate and let say these are the samples that you have obtained after sampling, right, these are values that you have.

Now these values lie on a continuous axis, amplitude axis. What you now do is assign each of these sample values to one of these discrete levels which I have indicated by horizontal lines. And the manner of assignment is you assign each sample value to the level, to the value which lies closest to it, okay. So for example this sample will be given the level number 4, assigned to level number 4, this one to 6, this one to 8, this one to 9, next one also to 9 because it is close to 9, next one to 7, next one to 6 and so on. And therefore now we will be transmitting not the actual sample values but the levels to which these sample values are close and these levels are discrete in nature.

Further these levels which are levelled out, which are coming out of the quantiser are not transmitted is such. One could do that, if you do that, then instead of PCM is what you get is quantised PAM. If you were to transmit the quantiser output directly, you could do that, it would be, see the sample output is normal analog pulse amplitude modulation scheme. After the quantiser it becomes some kind of a quantised pulse amplitude modulation scheme. And even that could be regarded as some kind of a digital pulse modulation scheme because of the process of quantisation.

But usually it is preferred that instead of transmitting in the quantised PAM form, we carry out some encoding of each of these levels and transmit finally in a binary form. So the job of the encoder is simply to give or introduce a binary representation to each of these levels and typically the binary representation will use a certain number of bits in a word. And those bits of the word are transmitted in the form of a sequence of binary pulses. So the PCM output that we have got here is really a binary files representation of each of these numbers. I have shown this here.

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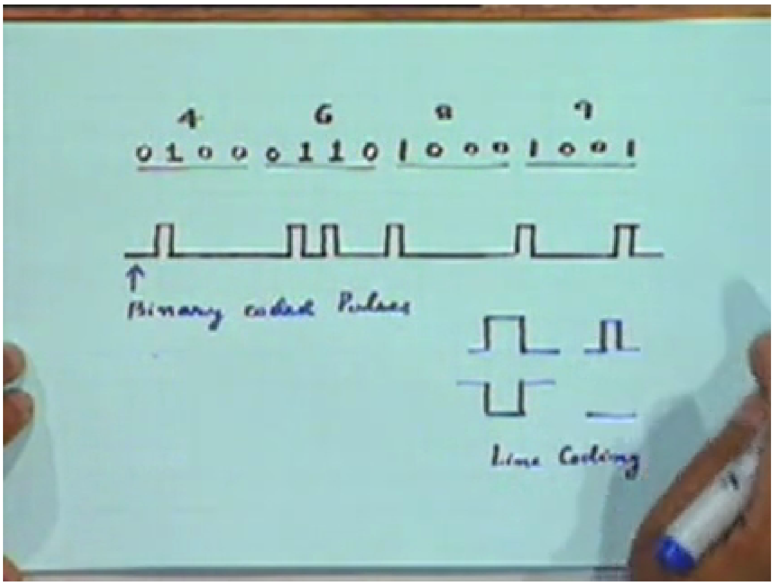

I have taken a few of these samples, the 1st few sample 4, 6, 8, 9 and represented each of them buy a 4 bit word, assuming that my total number of levels is only 16. The number of bits that you will require for representation will depend on how many levels of quantisation I have, right. Assume that we have only 16 levels of quantisation, then each sample would require 4 bits for representation and we may have sequence of binary digits like this which you eventually have to transmit. This is the representation for 4, for 6, for 8, for 9 and finally what you make transmit is a pulse sequence like that. So these are the binary coded pulses that form the output of the PCM system and which are finally transmitted onto the channel.

Student: Excuse me sir.

Professor: Yes please?

Student: (())(11:05).

Professor: Well, it depends on how you represent each one, how you represent 1 or 0, there are many ways of doing it. I could represent a 1 like, like this and a 0 like this, right. In which case what you are saying is right, I could also represent a 1 by a pulse like that, of half a duration and a 0 like that. Right. Anyway we will discuss various possible representations of digits onto pulses and that is what we will discuss when we talk about line coding, the various representations possible. And depending on which representation you use will decide the actual nature of your pulse train that you finally have to transmit. Any other question? Okay.

Now, so this is a very, conceptually a very simple process, the process of converting analog signal into a binary representations by the PCM process. It is a very very simple, conceptually simple process, so implementation why that is much more complex than let us say Delta modulator because you have to really build an analog to digital converter. The implementation of quantiser is a nontrivial operation and I am sure you are all familiar with that fact. However at the moment we are interested in looking at not the implementation of A to D converter with which we are already quite familiar but about that communication aspect of PCM.

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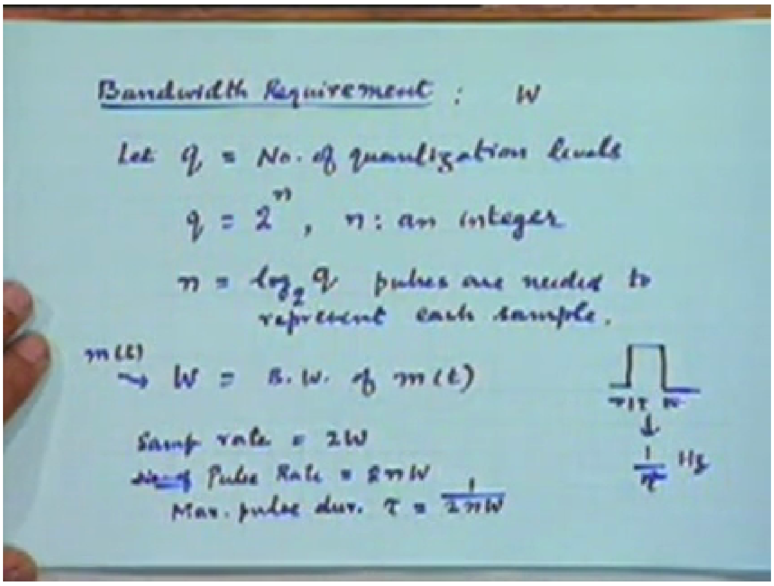

And the 1st thing that we like to look at is the bandwidth requirement of a PCM system. You have a, to start with you have an analog signal, let me talk about bandwidth requirement. As I said to start with I have an analog signal, let us say of bandwidth W. The question is do I continue to use a bandwidth which is of the order of W or do I eventually end up using a bandwidth which is much larger? So that is the question we would like to address and workout quantitatively how much it is. To do that let us introduce a few parameters of the quantiser.

Let us assume that the quantiser of the, associated with the PCM system is interests has q number of quantisation levels. And for convenience let us assume that q is chosen to be a power of 2. It will be convenient to do so because we have to eventually give a binary representation to each level. So efficiently to do that in choosing q to be power of 2, 2 to the power n , where n is an appropriately chosen integer, which implies that n will be the number of bits in a word that will be kind to represent each level, right. The encoder will have to use a word length of n bits, that is implication. And n is obviously equal to log of q to the base 2.

And these many pulses are needed so represent each samples. Let us say that your signal has bandwidth W, your message mt has a bandwidth W, so this is the bandwidth of mt. And what will be the minimum sampling rate that you will have to use?

Student: 2W.

Professor: So sampling rate is 2W. Now what determines the bandwidth of a pulse train? Or roughly we will say, well, ideally we all appreciate that we are talking about an ideal pulse train, the bandwidth is going to be infinite, right. So we are talking about some more practical measure of bandwidth here than the theoretical, a Fourier transform extends the span of the Fourier transformed signal, that will be of course infinite. We have perhaps talking about the bandwidth in which most of the signal energy lies, right. And suppose we have a pulse of a certain kind, let us say an ideal pulse of width Tao, roughly what can you say about the bandwidth of this? Any idea? I am sure you can say something.

Student: Sir the Sinc function.

Professor: Fourier transform of this is a Sinc function. One rule of the thumb is most of the energy life in the main lobe of the sinc function whose width happens to be the reciprocal of the pulse width, right. Makes reasonable sense? So we say roughly we can say that if we are using a pulse train with each pulse having a bit of approximately Tao seconds, than the bandwidth associated with that person will be of the order of 1 by Tao hertz. That is most of the energy of the pulse train will lie in this bandwidth. It is essentially a convener method of bandwidth rather than precise method of bandwidth.

Alright if your sampling rate is 2W and I am going to represent each sample by how many pulses, n pulses. So number of pulses or pulse rate at which you have to transmit pulses is going to be 2nw. If I am using a pulse train of this rate, what is the maximum pulse width I can have? Obviously of the same duration, maximum pulse width I can have is 2nw, 1 by 2nw. So maximum pulse duration Tao is of the order of 2nw or 1 by 2nw, sorry. Is that okay?

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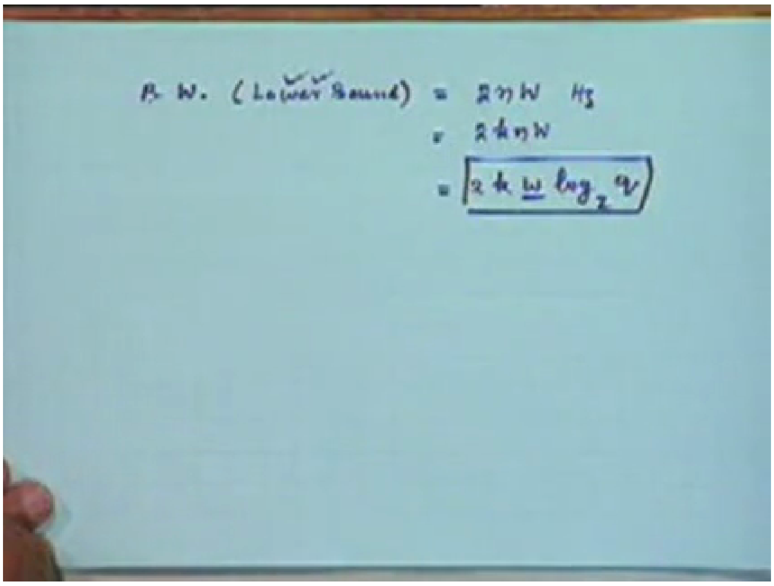

And therefore what is the bandwidth of the this will be the minimum bandwidth because you are talking of the maximum pulse width that we can have. So this is some kind of a lower bound on bandwidth that we are talking about, it is of the order of…?

Student: 2nw.

Professor: 2nw. In general if you want to be more precise, it will be some constant times this 2 nw, where K is the appropriate constant of bandwidth depending on what definition of bandwidth they precisely use. Whether we decide on the main lobe bandwidth or some other measure of bandwidth. So it may vary between a factor of 0.35 to 1.2 or something like that. Depending on the precise definition that we use to say, to decide, you know what is the definition of bandwidth you are going to use. Yes please, question?

Student: (())(19:19) the rate of pulses that are to be sent but the bandwidth required for one individual pulse might itself be larger than… So the bandwidth requirements can be more.

Professor: This is, this is if I would have said this is the lower bound, right, if your pulse width happens to be less than 1 by 2nw which you can use if you wish, you will end up using more bandwidth. But you cannot use a bandwidth less than this, right. So this is some kind of a lower bound on bandwidth that the PCM system must be associated with. You have precisely got it right, it is a lower bound or in other words I can say this is 2KW log q to the base 2. So therefore you are increasing the bandwidth from W to this factor which is in the order of magnitude increase and depends on how many quantisation levels you have, right.

The larger the quantisations, number of quantisation levels you have the final the quantisation you carryout, of course that will lead to a more accurate representation of your signal, right. But it will also introduce a larger bandwidth sacrifice, it will also require larger bandwidth for transmitting the signal.

Student: What is K?

Professor: K is a suitable constant which will descend, which will depend on the precise nature of definition that you use to, use for understanding what bandwidth is. Like for example the argument that I gave that a pulse may be considered to have bandwidth equal to the width of the main lobe of the Sinc function associated within the frequency domain is one possible destination, you could use some other definition, right. For example you could decide to use 99 percent energy bandwidth, right. That is bandwidth, in which that portion of bandwidth and 99 percent of the total signal energy lies, power lies. So in which case your value of K will turn out to be different.

But nevertheless the order of magnitude will not change, right. This is one aspect of PCM when we talk about using it for communication, digital communication. Now the other aspect is that the process of quantisation introduces errors, introduces noise, right at the transmitter itself. This is unlike other communication processes, you do not introduce noise at the transmitter and deliberately, right. Here it seems you are not sending the precise value of the sample but only an approximation of it in terms of the quantisation, you are actually introducing or interpreting certain error of representation directly at the transmitter itself in the process of encoding , which may sound to be an undesirable thing to do. Right.

But the advantage is, in spite of this is being introduced, this is a controlled amount of error which you can control by looking at our requirement and deciding on how many conservation levels I must use. The advantage that I get is that of digital communication that is I am able to eliminate noise that is introduced later in the channel, right, very very significant thing. But nevertheless we must be aware of this fact that the process of quantisation is a noisy one and we must also be able to figure out how much noise is being introduced, how much energy is being introduced.

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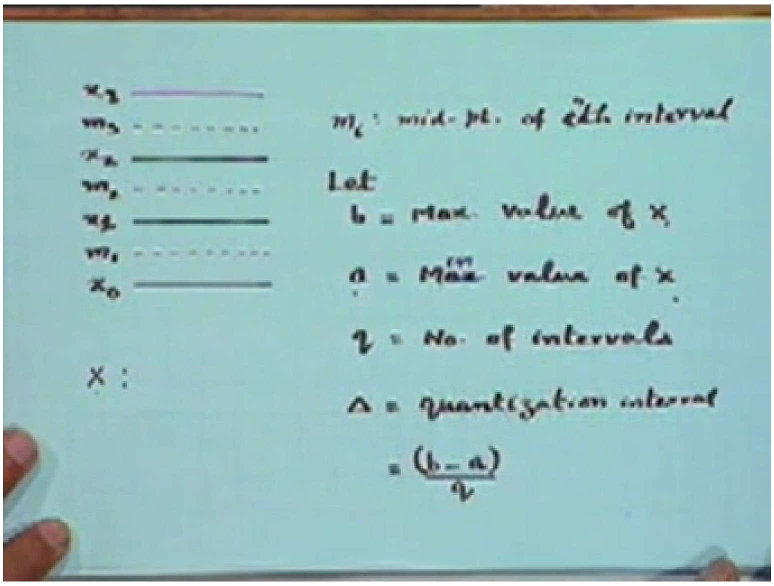

So the next thing that is of interest to us is to be able to calculate how much quantisation error or quantisation noise as we call it is being introduced. Let us try to calculate that. In doing this I will be using this simple set of definitions. The solid lines are denoting here the boundaries of various quantisation levels. So x0 is some lower boundary and x3 is some upper boundary. Well I have got only 4 levels your let us say 4 regions here but you will have more, right. And M sublet I is, M1, M2, M3, they represent the middle point is shown by the dotted lines of the radius, the Ith interval. So M sub I is the midpoint of the Ith interval.

Let us assume that your signal sample values which I am going to denote by x has an excursion between the lower limit of A and upper limit of B, all right. Each sample coming out of the sample which I am going to represent by x can take the maximum value of B and a minimum value of A, all right. So this is the maximum value of x and therefore that will be represented by x3 or whatever, whatever is the largest, highest level you have and A is the, this should be minimum value of x. And let us say there are q number of quantisation intervals and therefore each interval, each quantisation interval is obviously equal to B - A upon q.

So we are going to use this framework for our calculation of quantisation error. If the signal value happens to be here, the level that will be assigned to it will be the middle point, right. And therefore this much error will be introduced in the process of representation. If it happens to be here, this much either will be introduced, right. So we can see that the error that is going to be introduced is going to be random in nature depending on what the value of x turns out to be, as going to be both positive, equally likely to be positive or negative, right.

So we will model this error to be some kind of a random variable and then calculate what is its mean square value, okay. And that will give us some idea of the quantisation noise that is being introduced, that is the procedure we are going to follow. Any questions? So now just start this computation.

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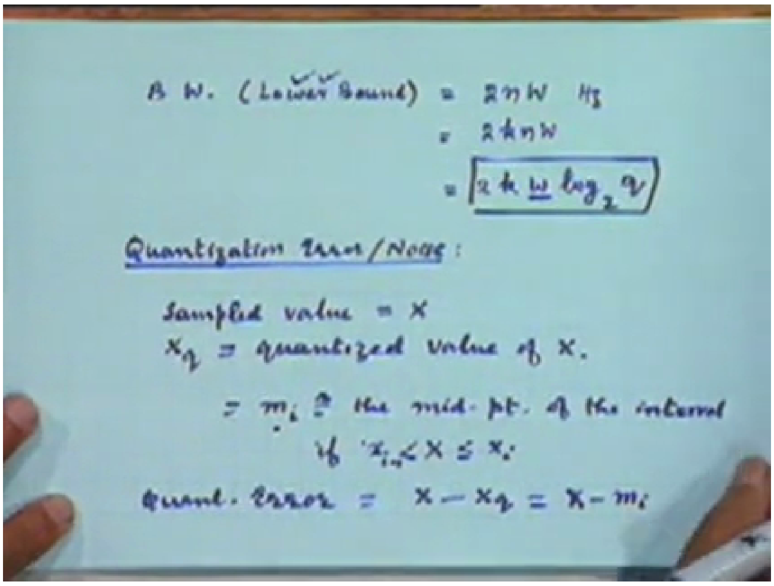

So let us say that the sample value is x, okay and then this is quantised and the quantised value is represented by x of q, quantised value of x. It is obviously going to be equal to M sub I which is the midpoint of the interval in which x lies. Or I could say that it is equal to M sub I is x lies in the interval x sub I -1 and x sub I, right. Because the middle point of, the interval whose boundaries are x sub I -1 on the lower side and x sub I on the higher side is precisely equal to M sub I and that will be the quantised value of x which I am going to represent by xq.

What is the quantisation error? It is going to be x - xq or x - M sub I. Is that okay? Any questions? Nothing. Very complicated, we solve very straightforward. Right, what is the quantisation noise power or mean square value of this error?

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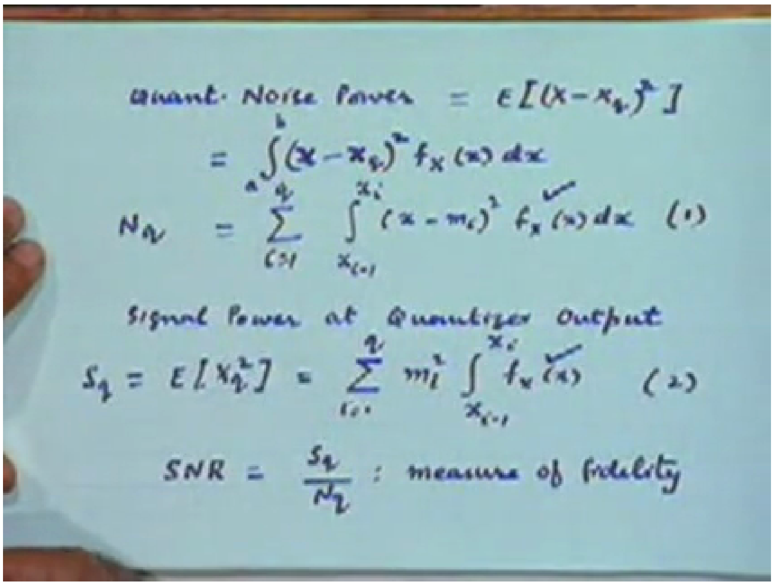

So quantisation noise power I am going to define it is expected value of x - x sub q square, right. You are familiar with these kinds of operations which will be, take x - xq, X is being used to denote the random variables, the actual values that they will take will be denoted by small letters, so small x - xq whole square fx x dx integral of this. Where what is fx x, take the probability density function of the random variable x and x is a sample value at the sampling instance. Now we have to evaluate this integral.

Student: Sir the limits from a to b?

Professor: I have deliberately left the limits to be open at the moment. I am now going to rewrite this integral is the sum of 2 integrals. Because the total integration range of course is going to be from a to b which I can divide into various sub integrals, the ith of which going from, goes from x I, x sub I -1 to x sub I. X - M sub I also is fx x dx. Fine. Actually we will be interested in computing the signal to noise ratio that comes out of the PCM quantiser.

So let us also do a similar exercise for the signal itself, that is let us look at the signal power available to you at the quantiser output. That will be obviously equal to expected value of x q square because the signal power, signal sample that is coming out is x q, which again I can write in this form, xq equal to M sub I, so M sub I square and that is a constant. I just skipped one step of this kind here and directly in the 2nd step here, Right. We call this n sub q, call this S sub q and therefore the signal-to-noise ratio is, and we need to compute this so that we get some kind of a measure of fidelity of this representation.

This is a useful measure of fidelity, how good the representation coming out of the quantiser, yes, the signal. The larger the value of the signal-to-noise ratio, the better your fidelity, the more accurate your representation or less amount of noise that is being added due to the quantisation process. Any questions?

Student: (())(32:46) it does not have to be the…

Professor: Yes, that is, that is why we must look at this, is not it. We must look at the signal power…

Student: (())(33:02) is it due to the transmitted signal power or this will be actual baseband power…

Professor: It is only slightly different from it, in the sense that it represents the power in the quantised signal.

Student: Which might be far from the original signal?

Professor: Not far, it will be close to it but in many case, this is the one which is of interest to us because it is this power switch is going to the receiver. Right and not the unquantised power.

Student: Sir it is of interest to us only when we know that the original signal to this noise that we are introducing ratio is high. Why do we have S - M divided by M?

Professor: In any case since usually S and S sub q are going to be so close to each other that it does not matter very much because the number of quantisation levels typically is going to be reasonably large, this question is only of academic interest. But nevertheless what I am holding still is that it is this power which the receiver is going to look at and therefore the signal-to-noise ratio that is of interest to us is this, should involve this power, the quantised signal power and not the unquantised signal power, right. And in any case there is not going to be much difference between the quantised between the quantised signal power because we are assuming that the approximation is going to be reasonably good, right, of x with xq.

So we are assuming that noise power is still going to be typically much smaller than the signal power, all right. Now to proceed further we have to make some assumptions regarding the nature of the signal itself, right. The values of these powers will turn out to be different for different kinds of signals, we must appreciate that, right. In fact strictly speaking even the nature of quantiser that I use should be dependent on what kind of signal I have. The kind of quantiser that they are looking at right now is called a uniform quantiser. Why is it called a uniform quantiser?

Because the intervals, quantisation intervals is same as across the whole interval, right. That is Delta is constant across I, it does not depend on I, the Ith interval, each of the interval has values, step size of Delta, independent of I. Now this kind of uniform quantiser is actually designed on the assumption that your density function of the signal is actually uniform over the range of its values. So therefore let us consider a case, simple case, when we assume that fx x corresponds to a uniform distribution function. Okay.

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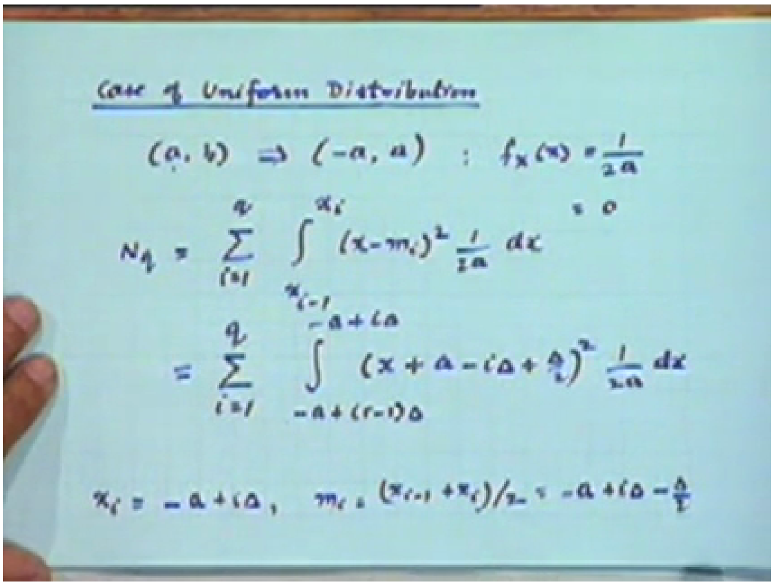

So we will take the case of a uniform distribution and complete the computation. Let us assume that your lower value, let us assume that your interval A, B is actually, I am going to simplify it and say it goes between some - a to + a. So your lower limit is - a and upper limit is + a, assuming it to be asymmetrical. And therefore your as from the uniform distribution assumption, your fx x is going to be 1 by 2a when x lies in this interval, x equal to 0 if it lies outside this interval, right. Based on this let us assume, let us compute n sub q.

Now I am going to replace fx x by 1 by 2A, this becomes , all right what will be x I -1? You have to replace now for x I -1, x I and x sub I and as you can see x I -1 in general that we write down the expression for x I, obviously it will be the lowermost value + I Delta, right. So this will go from - a + -1 Delta to - a + I Delta. Also what would be, what about x, M sub

I? M sub I will be x sub I -1 + x I divided by 2, right because it is the midpoint of the interval. If you do this computation and substitute here, this is what you get.

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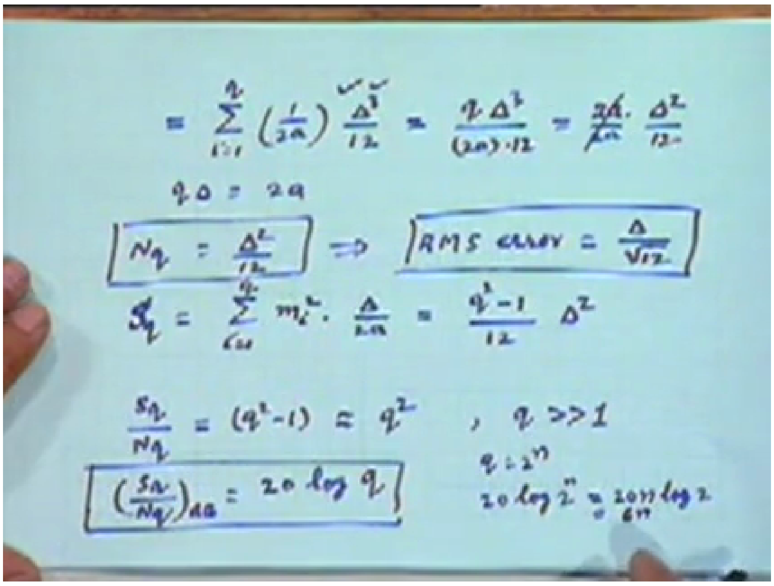

X + a - I Delta + Delta by 2 whole square 1 by 2A dex, implying that your M sub I is equal to - a + I Delta - Delta by 2, this is something which you can easily check by direct computation or direct substitution. Now this is a bit of algebra which I will skip and I am sure you can go through the necessary explanation and simplify finally to get… If you compute, it turns out, each of these integrals turns out to be equal to Delta q by 12, right. And the q times this summation, this is a constant inside the submission, it simply becomes q times Delta q upon 2 into 12.

Using the fact that q times Delta, what is q times Delta? It is the total length of the interval, which in this case is 2a, we get the quantisation noise n sub q. So q into Delta is 2a, substituting that here, we get n sub q equal to Delta square by 12. Very simple and neat expression for the quantisation error, okay. This is the only step that you need to just confirm, the previous entry about, that I talked about here, this one, with change of limit, with change of limited turns out to be very simple integral, by substitution for x when you get a very nice close form expression like this.

Similarly coming down to S sub q, that is equal to M sub I square and inside was the integral of 1 by 2A from x I -1 to xi, right. That is simply Delta by 2A, right, that is obvious, because value of density function is 1 by 2A, we are talking about this, we are talking about this one, this expression here. This is 1 by 2A, value of this integral is simply Delta by 2A where Delta is the width of this integration interval. Again substituting for M sub I, the way we did just now and simplifying, I am skipping again those simplification of steps, that will involve a series summation, please check on your own.

We will get a very simple equation also for S sub q, that is equal to q square -1 upon 12 into Delta square. And we now get a very nice simple expression for quantisation signal-to-noise ratio which is q square -1, it should divide this by this, Delta square by 12 will cancel out and you are left with q square -1. And I am assuming that q is a sufficiently large number, which it is going to be in typical applications, it is really of the order of q square, is q is taken to be much larger than unity. In dbs, normally we talk about single to noise ratio in decibels, this is power, so we take 10 log on both the sides, we get 20 log to the base 10 q, right.

So these are the 2 main results, one is the result for signal to quantisation noise ratio and the other is this which equivalently sometimes is just given in the form of the fact that RMS error that you may expect is Delta upon square root of 12. These are the 2 results or important results to remember or quantiser characteristics, uniform quantiser characteristics, yes. Please speak out if you have any questions.

Student: DB is 20 log (())(43:47).

Professor: It is 20 log if you are talking of voltage ratios or current ratios, it is 10 log if you are talking of power, power ratios, right. Any questions? This result can also be made slightly more interesting into a rule of thumb kind of result. If you remember the q is power of 2, right, so if you take q to be equal to 2 to the power n, then this will become 20 log 2 to the power n which you can write as 20 n log 2, or 6n, you right, 20 log 2 is says 6, so that is n, right. So that is a very interesting result and that gives you a very simple rule of thumb for calculating how much signal-to-noise ratio again in dbs depending on the number of levels you use, right.

If you use for example number of levels corresponding to 7 bits, right, you are going to get a 42 degree signal-to-noise ratio which is very good, right. Usually for what is communication we use the signal-to-noise ratio between 40 and 50 degrees. So 7 to 8 bits are adequate for representing voice signals in PCM. But the number of levels that were talking about 10 is 128 to 256, right. So very useful relation to keep in mind.

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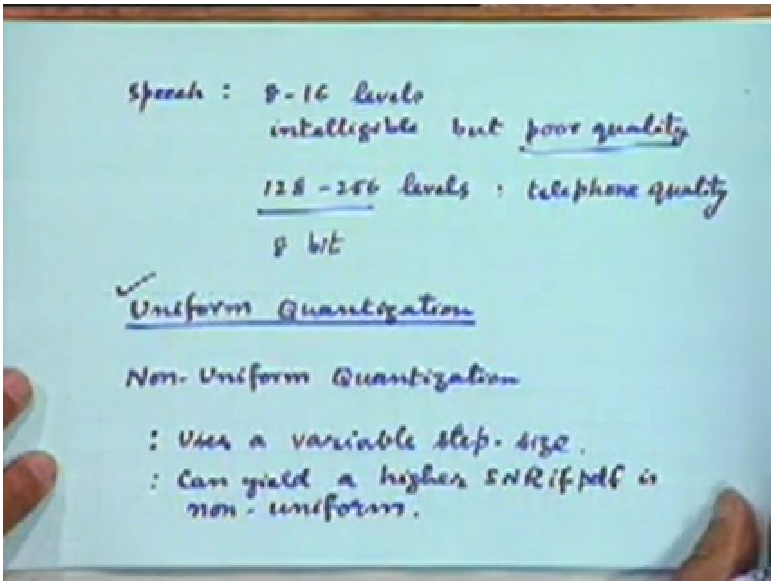

So for speech if you use 8 to 16 levels, which will correspond to 3 to 4 bits, right, the quality is very poor but the intelligibility is very good, right. You get quite intelligible speech but poor quality. That is you will hear also a lot of distortion along with the information. 128 to 256 levels is quite good quality, what we call telephone quality speech.

Student: (())(46:42).

Professor: Very rarely use it, very rarely 8 to 16 levels. It is just to give you some information as to how many levels are necessary, right. If you use these many levels, you lose a lot of quality because both are important, not only intelligibility but also quality. So usually in most PCM, commercial PCM systems that you use, you have actually an 8-bit representation, right. Any questions? I can stop for a minute just to answer all kinds of questions you may have so far.

Student: You have not performed the analysis in the delta modulation.

Professor: I will talk about it, I have planned to do it after doing this for the PCM, right, we will be doing that for Delta modulation also. And we are not yet finished with Delta modulation, will be coming back to it. Any other questions? Because what we like to do is compare on the whole how delta modulation and compare with each other and compare both of them, from the point of view of bandwidth requirement, from the point of view of signal-to-noise ratio that you can get and then get a feel for which is better if at all.

Now this discussion that we have done was for the case of uniform quantisation as I mentioned just now a few minutes ago. And it makes sense to use uniform quantisation, provided that you have reasonable basis to say that your signal is of the kind where its values are uniformly distributed over certain interval, right. If for example you know that most of your signal values are going to lie in a smaller range and the larger values are going to be much less frequent to occur, the probability of occurring, having larger values of the signal is small, let us say just for the sake of assumption.

Then intuitively you may feel that uniform quantisation noise, uniform quantisation process may not be a good idea to carry out, right. Because you are going to use or waste a large number of levels in the region where the signal has a very low probability of occurring anyway, right, and you are wasting your number of quantisation levels. Given a number of quantisation levels, it makes sense to make them more finely distributed in that region where the signal is likely to take values with larger probability, than in those regions where the values are going to occur with smaller probabilities. Right.

So therefore this is not a good idea to work with if your signal is known not to have uniform distribution, right. And then you go for what is called nonuniform quantisation. Yes please?

Student: The derivation only applies to the uniform derivative, uniform quantisation?

Professor: The derivation that we have done applies only to uniform quantisation and also assuming that distribution function associated with the signal is uniform one, right. But still even if it is not so, this is still a good rule of thumb to work with, okay. Now let us come back to the concept of nonuniform quantisation. So basically what we are going to now do is, will not use a uniform step size across the whole interval of quantisation but use a variable step size, right. And it can yield a significantly higher SNR if you do this higher signal-to-noise ratio when your PDF or the probability density function is nonuniform, which is the usual case in practice, it is very rarely uniform interest this.

Another way of looking at why this may give better results is, one which I have already given you is by putting less number of levels and let us say larger amplitudes, you are going to incur larger errors when the corresponding signal values are also large, right. So that also masks away the error quite a lot. You have large error associated with large sample values and small errors associated with small sample values, there is another way of looking at it. But I still think the better way of looking at it is that large values occur less frequently in practice and we can therefore occasionally tolerate larger errors and they will contribute much less to the mean squared value than the small values which occur more frequently. And if they are associated with large errors, then your overall mean square is going to go up.

Student: Can we do some adaptive quantisation?

Professor: Yes we can, that is if your signal, it is a very good suggestion and in fact it is practically use in many applications. That is you can make your quantiser characteristics dependent on the input signal that is coming along and it can be made adaptive to the extent that you may not know what you are a Priory density function is, it can adapt to those characteristics. But usually this kind of thing is complex, more complex and outside the scope of our discussion here, but that can be done. Let us take the case of the signal having a Gaussian distribution.

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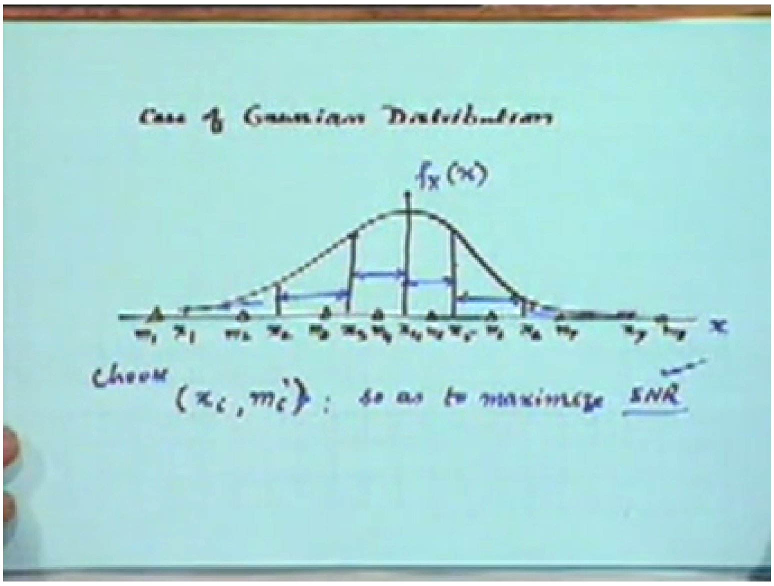

I have a couple of minutes more, so we will just talk about it today, little bit, we will take it up in detail next time. Let us say we have signal with Gaussian distribution, so as you can see this is a PDF, this is the value of x, this is the density function, I have plotted in the density function. As you can see that small values of x have much larger probability associated with them than larger values of x which lie in the tail of the distribution, right. So it does not make sense to crowd the quantisation levels equally across the whole interval. And usually what you will do is you will have closely spaced levels in these intervals.

Of course I have drawn a very good pictures help with a very small number of levels, in practice number of levels will be much larger. But you can see that these intervals are smaller than these intervals and this is a good thing to do, right. In practice we will like to space them out as you go farther and farther away from the mean value of the signal, if you have a Gaussian distribution, right. So the boundary is now x1, x2, x3, x4, x5 and so on are not uniformly spaced. Not only that, it may not be the best thing to do to choose the quantisation level to be the middle point of the interval, right.

It may be proper, it may be alright perhaps to use a slightly different value than the midpoint because you do not have a uniform distribution even within, even within the interval, right. So when you have nonuniform distribution, the proper way of designing the quantiser would be to ask the question how to choose the values x sub I's and the values of M sub I's so as to, so choose these set of values so as to maximise the SNR, right. That is what a proper quantiser should worry about, right. And that is how nonuniform quantisers are actually designed.

By looking at actual distribution and then asking this question as to for this distribution how to choose this xi's and Mi's for a given number of total quantisation number of intervals so as to maximise signal-to-noise ratio, okay. We will take up this discussion next time in detail.