Digital Communication
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Module 01
Lecture 28
Error Rates for Binary Signaling: Coherent Receivers

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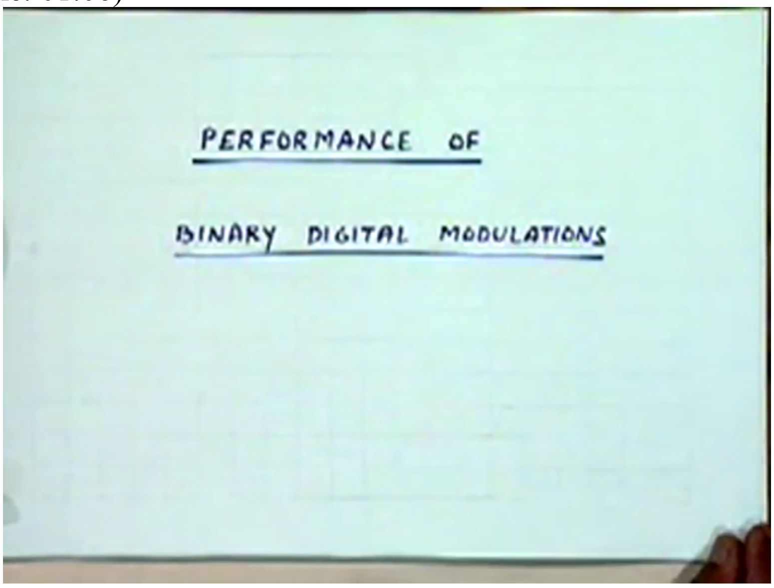

Binary Digital Modulations as a preview to doing this performance analysis we had discussed yesterday the properties of the sample output on the basis of which you are going to make decisions in typical in binary digital de-modulators receivers right and specifically what we appreciative was that as far as coherent signals are concerned we continue to work with either zero mean or non-zero mean Gaussian random variables but for non-coherent receivers we may have to deal with other kinds of distributions the Rayleigh distribution and the Rician distribution right.

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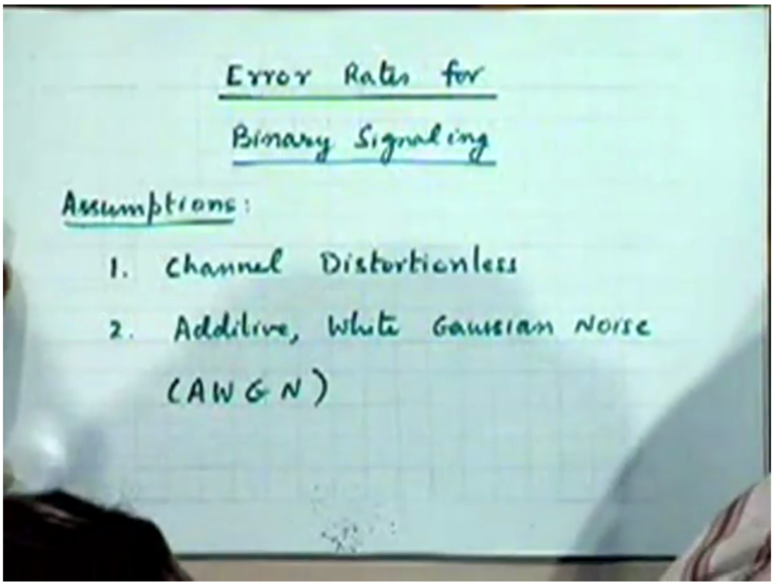

And we were in the process of doing a performance analysis based on this assumptions that your channel is distortion-less and the noise is additive and white Gaussian in nature.

So it is an Additive White Gaussian Noise channel, I am just quickly recapitulating what we did yesterday.

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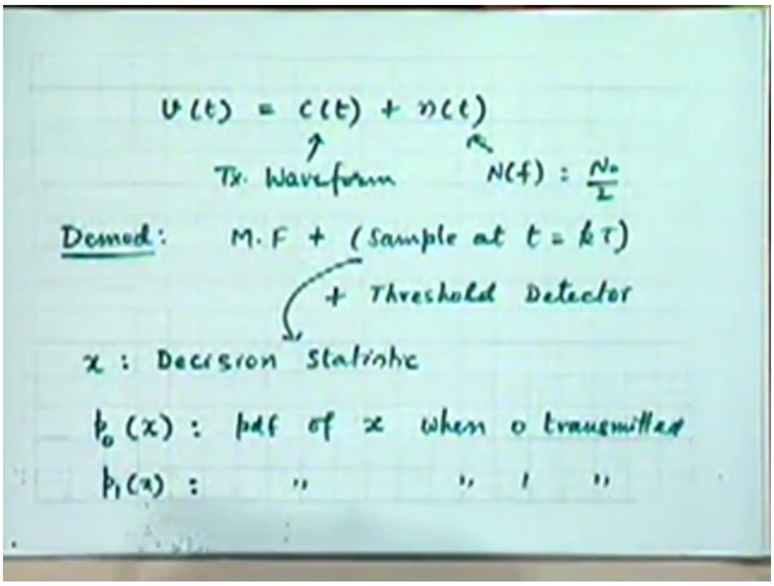

So will assume the received waveform to be a sum of a transmitted waveform as such without any distortion plus noise with power spectral density as given here N 0 by 2 and the de-modulator consists of the matched filter we are looking at coherent receivers first so it consist of matched filter, no envelope detection just thus matched filter output is sampled at every bit interval at the end of every bit interval and this is followed by a threshold detection operation right.

Let's denote this sample value by x which becomes our decision statistic ok and what you have to appreciate is that this x will have in general different distribution when a 0 is transmitted and when a 1 is transmitted right. They maybe different only in means if they are all Gaussian then they different only in means for example but they will be different right. So will denote by piece of 0 x as a PDF of x when 0 is transmitted and P sub 1 x PDF of x when 1 is transmitted. I think this is a point at which we stopped yesterday and will start from here today, any questions on this? Fine, ok.

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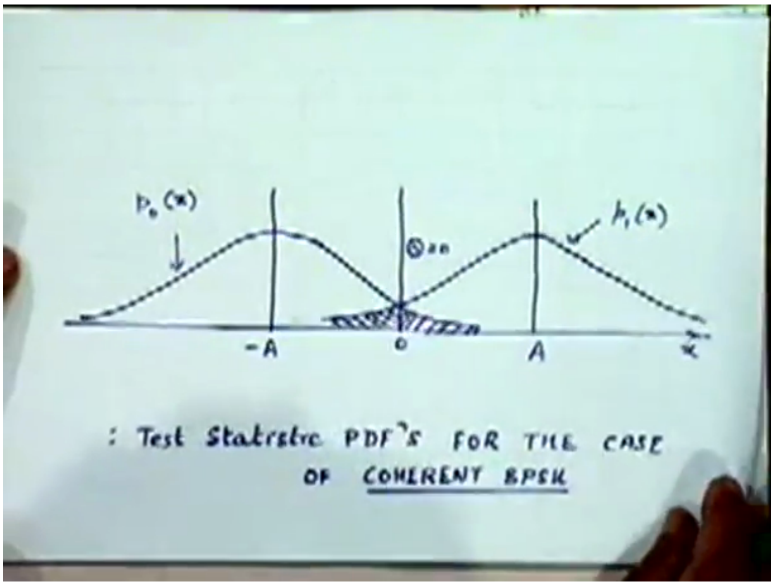

So this is the picture for that ok, in think I will start with a slightly different picture which is here, we have another one yeah I think that is ok.

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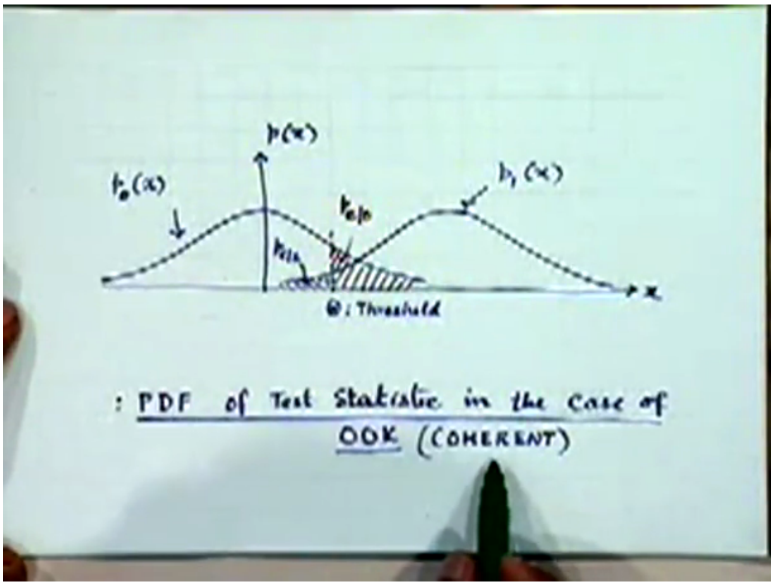

So I have displayed where as an example the case of coherent On Off Keying right, the matched filter output in this case will be a zero mean random variable the zero mean Gaussian random variable when a zero is transmitted because in that case the matched filter is getting only of the input when you have On Off Keying for a zero transmission nothing is being transmitted right. So the matched filter output when sampled at t equal to K T will only be a zero mean Gaussian random variable which is the distribution P 0 x shown over here alright.

When a one is transmitted what is going to happen to the matched filter output? It will be shifted the in mean, the distribution will be shifted in mean by an amount lets say equal to A where A is the amplitude at the output of the matched filter alright. So will have a amplitude due to the signal the nose will continue to be govern by this distribution as shown here except that the mean is shifted to K right.

So the random variable x which we are looking at either has this distribution or has this distribution right. So if it is has this distribution typically this is a range of values over which x will go right, well typically with a average spread equal to the variance and things like that. When signal is coming along when the one is transmitted the range of x of values of x that you will look typically see is govern by this distribution and our decision on whether a zero or one is transmitted is going to be on the basis of comparing this value of x that we have observed on the basis with a threshold value theta right, which we have I have arbitrarily shown to be here somewhere, somewhere between these two distributions right.

Obviously it has to have a location of the kind that I have shown here so that we don't make too many errors right and the error probability in terms of these curve is shown by the two shaded areas. For example this blue shaded area right from here towards minus infinity this side this gives a probability that error occurs when a one was transmitted but you will wrongly but you are wrongly thinking that the zero has been transmitted right because although we are considering this distribution that is the distribution corresponding to transmission of a one this small little shaded area in blue is the probability of the event that the value of x will still turn out to be less than the threshold and therefore will make a wrong decision that the zero was transmitted, is it ok?

Similarly the black shaded area is the probability of the error where a zero is transmitted right that is your X is really governed by this distribution but the noise at the matched filter output the noise sample at the matched filter output turns out to have a value larger than threshold that you have selected and therefore the probability of that event which is the shaded area shown here is the probability P E given zero right. So this are the two kinds of errors that we can have a zero going to 1 and a 1 going to a 0 with a corresponding probability as shown by this two shaded areas.

Student: (())(7:55)

Professor: No I have not deliberately shown, this is the point I want to discuss, so I will just come to that point. The question then naturally comes, where should this threshold be right, this is one of the important things you will have to worry about, where you should looked it the threshold. One of the there are several ways by which you can look at this problem, one is one typical approach is well obviously our overall interest is to minimize the total error probability right and what will be the total error probability? Well you might like to say it is this plus this, but you have to be careful, these are conditional probabilities right.

Total error probability will be the half of this plus half of this right, let me write down a few things and I will come back to this discussion of how to select the threshold. Let's go through a bit of maths and I will come back to this picture again.

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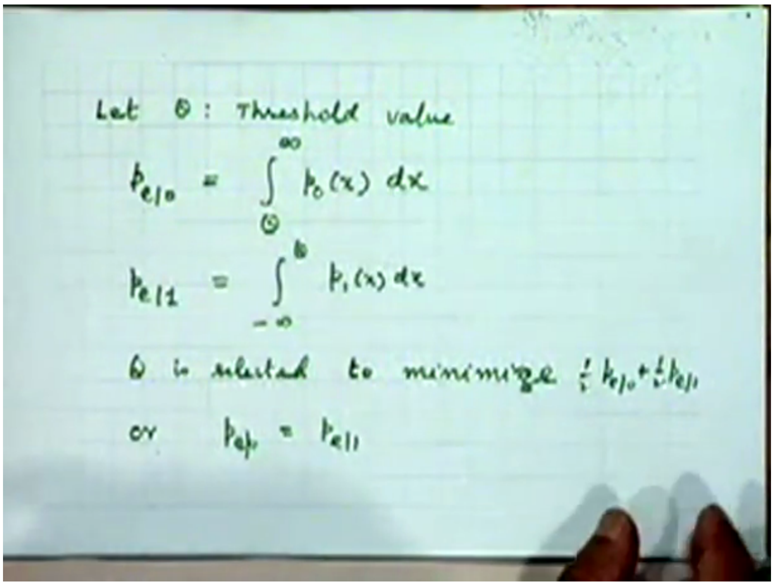

So let’s denote by theta a threshold value as I have shown there then mathematically the error (prob) two error probabilities are error when a zero is interpreted as a 1 that is you actually transmitted a zero but error occurs right, that will be governed by the distribution that will be governed by the tale of the distribution of P 0 x right.

P 0 x integrated between theta and infinity right dx, similarly P e given 1 will be the area under P1 x between minus infinity and theta. Now from point of view of minimizing the total probability of error assuming that both 1’s and 0’s are equally likely, what can you say about the location of theta? Can make an intelligent guess? It should be at the point of intersection, it is obvious yeah that is one way of doing it, the other way is this picture automatically shows it, isn’t it? Isn’t it obvious from this picture that the minimum probability of error will occur when it choose a threshold to be at the crossing point.

Very good you have got it right, see if you put it here what is the area of that you are considering? The area under this still here and area under this still here if you put it anywhere else you will anyway have all this area plus you will be adding some more area to this, either this way or this way right. So that is the logic which makes it obvious that from this point of view from the point of view of minimizing the overall error probability particularly when 0’s and 1's have equal probability of transmission right. The optimum threshold should lie at the point of intersection of these two distributions.

You can even as Varun was rightly say differentiate this two areas and mathematically check that indeed that will be the result. However occasionally people use a different criterion for selecting the threshold occasionally depending on their application environment. Occasionally rather than making sure that the total error probability is minimum, sometimes you just like to make sure that the number of 1's and 0's that you the probability of declaring a 1 to a 0 and the probability of declaring a 0 into a 1 have equal values, so that at the receiver output 1's are not there are not too many 1's as of compared to 0's and there is no bias for 1's or 0's right.

So occasionally you may have a different criterion but most, in most of the situations one uses the criterion which I just mentioned and (if you take) for symmetrical distribution need to anyway doesn't make any difference which criterion you select right, because if these two distributions are symmetrical and have identical variances it is obvious that if I put the threshold here not only I will have minimum probability of error but also the two kinds of error probabilities will also be equal right.

Except of the (())(12:43) which is the case for most of the coherent situations right, but non-coherent situations that situation may not apply, so is it fine alright. Lets proceed with the so usually the threshold theta is selected either to minimize lets say of P e 0 P by 0 plus right or simply make P e 0 given 0 equal to right on of this two criteria's is typically used and for symmetrical distributions the two criteria will turn out to be the same. Now while evaluating while proceeding further you notice that will have to evaluate this two areas to obtain the expressions for these two error probabilities will have to evaluate the areas under this tails right of the Gaussian distribution.

So the area under the tail of a Gaussian distribution is a very important function with each will have to continue to work for developing expression for error probability.

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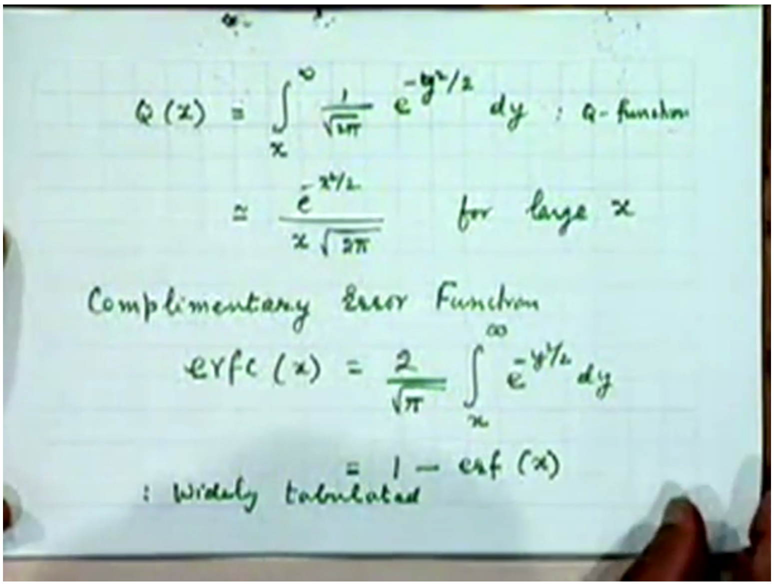

I am sure you are familiar with that function but just to complete the discussion let me define for you an integral Q of X is called the Q function which is the integral of this kind 1 by root 2 Pie basically you take the standard Gaussian distribution let me make it Y here because I have put X here right.

If the standard Gaussian distribution, the value of the integral from X to infinity which is the area under tail of the distribution from X to infinity right, that is called the Q function. Now this has not exactly this some related functions have been widely tabulated this is an integral which you cannot evaluate in a closed form right, this has to be evaluated numerically but asymptotically for large values of the argument X here that is when you are looking at the real tale of the distribution going very far away from the mean this can be approximated by e to the power minus X square by 2 upon X into square root of 2 Pie for large values of X ok.

Now Q x is related to a very standard function maths, which is known as a complimentary error function, I am sure you are familiar with the complimentary and basic error functions, it is denoted by E R F C argument is X and define as 2 by square root of pie X to infinity E to the power minus Y square by 2 dy and this is complimentary error function because this is actually define in terms of another function which is called the error function one minus error function is (function) ok.

This function the error function has been widely tabulated in maths so one can obtain the values of this function error E R F C x from those tables and therefore the value of Q x also from those tables. We consider there is only a scaling factor difference between the two right.

So this is widely tabulated, between Q x and this you know here the scaling factor is 1 by square root of 2 Pie here it is 2 root pie otherwise everything is same right.

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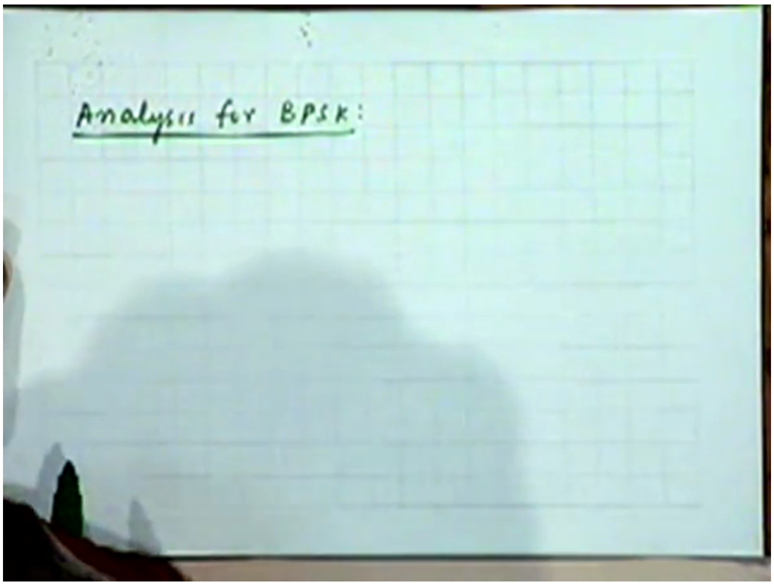

So now with this background lets take up specific modulation schemes and try to do some analysis and the first one I will take up is binary phase shift keying.

You know even that example of On Off Keying that I have discussed is only for the point of view of illustrating the approach, now we are taking up the calculation for BPSK alright. So BPSK is the situation is very simple it has to be a coherent de-modulator right that means we are basically looking at matched filter followed by sampler followed by so what are the two distributions that will be coming to the picture here?

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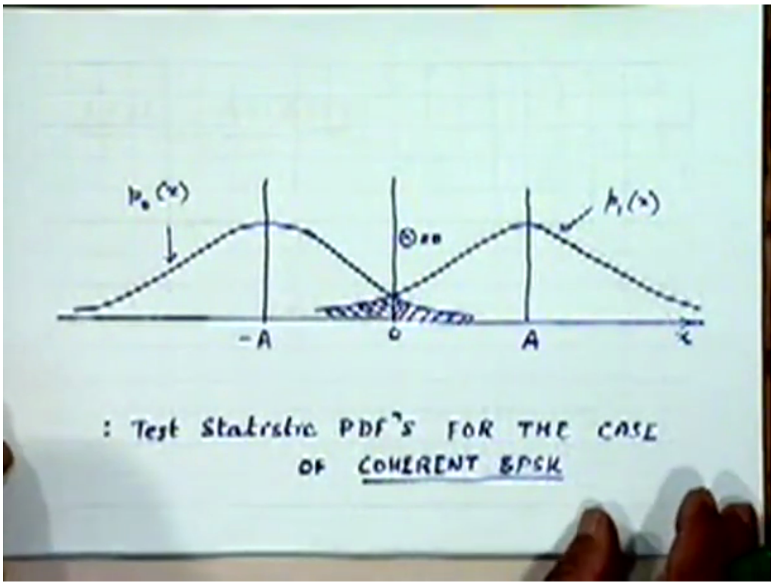

Both will be Gaussian but it is an antipodal modulation scheme so under the situation where a zero is transmitted, we expect the matched filter output to be minus A right.

Because the signal that is transmitted is either A cosine omega knot T or minus A cosine omega knot T matched filter will be matched to a pulse the R F pulse cosine omega knot T and you are going to sample the peak value of this output that will be either plus A or minus A right. So basically you are going to have P 0 x is a Gaussian distribution with mean of minus A and P1 x as a Gaussian distribution with a mean of plus A where A represents the amplitude of the output pulse at the output of the matched filter.

So I think the calculation is straight forward obviously the threshold should be selected at, at X equal to zero right that is you are looking at the matched filter output at t equal to K T and comparing it with a threshold zero, if it is positive it is greater than zero will declare a one is negative will declare the zero and the shaded areas are the two kinds of error probabilities we have discussed ok.

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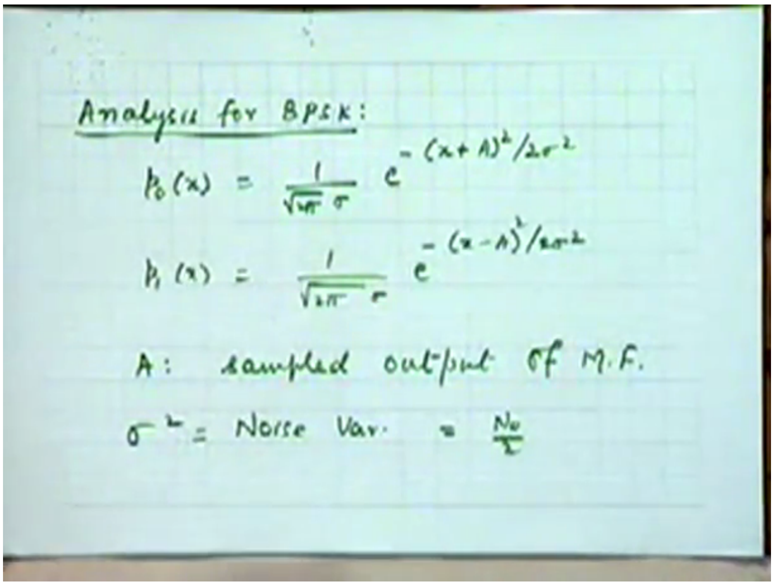

So what is P0 x here mathematically we have 1 by root 2 Pie sigma E to the power minus X plus A whole square by 2 sigma square and P1 x is same thing except that the mean is A, where A is sampled output of the matched filter and sigma square is a noise variance which is equal to N 0 by 2. Any questions? Fine.

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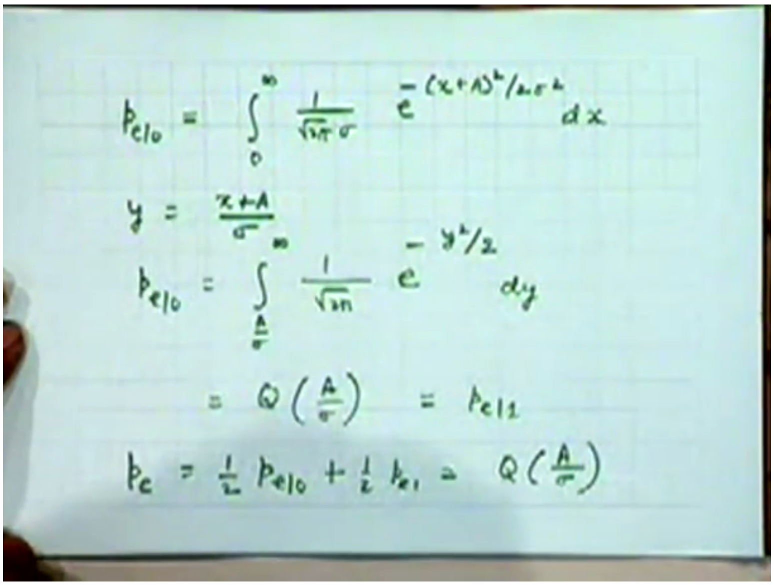

So what will be your P e 0, P e given 0 is the integral for zero to infinity of 1 by root 2 Pie sigma, which distribution? E to the power minus X plus A whole square by 2 sigma square dx, is it ok? Divided by, where, oh that is when I consider the total error probability then I will write half of this plus half of that right (but) this is the probability of making another 1 when 0 was transmitted this is conditional probability right.

From conditional probability when you come to a complete probability will bring in that factor of R using base theorem and all that right, fine. Now whenever you have integrals of this kind the typical way to do handle things will be two put it back in the standard form and then you will be able to get in the form of in terms of the Q functions of the error function right and typically the way you will do it is the Y a new variable Y lets say equal to X plus A upon sigma. If you do that what will be the value of this integral?

Let's go to the thing, what will be the lower limit with respect to Y? A by sigma right, upper limit will be infinity and it is obvious that this will be now because d Y will be equal to d X by sigma right, so d X by sigma can be replace with d y so 1 by root 2 Pie sigma will disappear minus Y square upon 2. So now that is precisely in the standard form and this X is to be identified as A by sigma. So this is nothing but the value of the Q function evaluated at A by sigma right, it is obvious so on the symmetry of the problem that P e even 1 also will be the same, isn't it?

It is obvious that this two I areas are equal, whether I consider this area or this area, so both have to do the commutation again which is also equal to P e given 1, fine. Therefore your total probability of error which I will just denote by P sub e for base theorem is the (()) (23:40) probability of zero transmission multiply with a corresponding error probability similarly for this, which essentially means half of this half of this which is again Q A upon sigma. Now what is you're A upon sigma?

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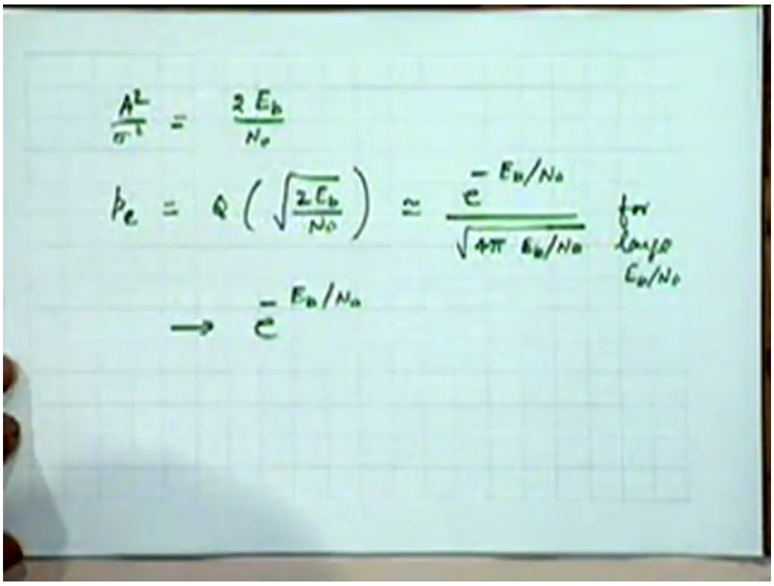

Remember you're A square by sigma square the SNR that we discussed at one time is 2 E sub b by N 0 right because we discussed that before let me not discuss it again. Therefore your P sub e is square root of this quantity Q function evaluated at ok, that is the mathematical expression to describe the error rate or the error probability for binary PSK, the precise mathematical expression which for large values of the signal to noise ratio when E b by N0 by 2 large right can be approximated by the asymptotic formula we discussed as this quantity for large SNR and you might observed that the asymptotic behavior is really governed by this factor E to the power minus E sub b by N0 that is asymptotically as a signal to noise ratio is made larger and larger the error probability can be made smaller and smaller approaching zero right.

So asymptotically is governed by this factor, now if you go to plot this curve that is error probability versus the signal to noise ratio and you will find in typical curves of this kind typical error rate curves the basic parameter against which curves are plotted is this quantity E b by N0 right, because this is fundamental thing which h is which you can extract from your transmitted signal and compare various modulation schemes on the basis of same E b by N0 that is for a given signal energy for a given noise variance that we have to deal with that is why given matched filter output, what is the various kind, what is a error rate?

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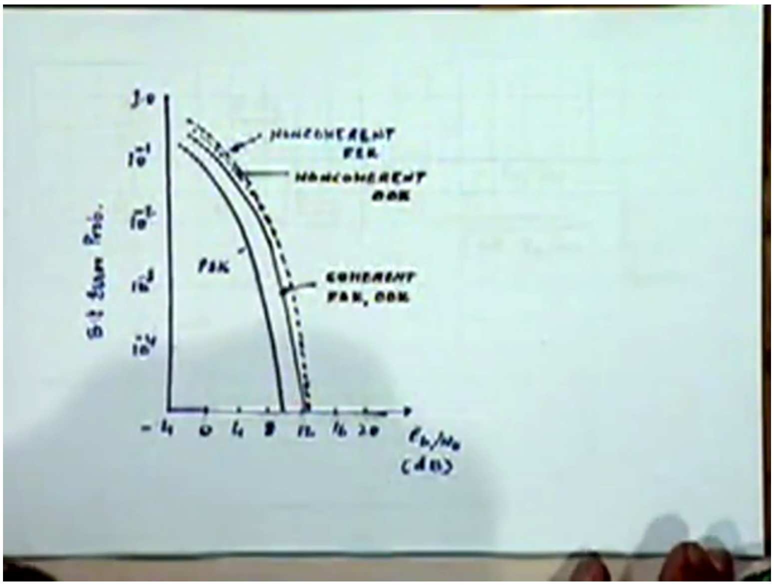

So you will find typical error plot curves as shown over here, you only have to concentrate on this curve at the moment because I am not discuss here a modulation schemes yet. So E b by N0 is on this, this axis, this has been taken in dB's right, this are some right some typical values and some typical values of P what are called bit error probability right and again there are on a log scale as you can see. Both this as well as this are on the log scale (right) because otherwise you can't put them nicely in a small picture like this.

So typically this is (10 to the) error probability of 10 to the power minus 4 that is you expect if you choose this error rate for design, what you are saying is 1 bit in 10000 bits is allowed to go in error not more than one bit in 10000 bits and so on. One in thousand, one in hundred, one in ten right, so typically what you will see is that if an error rate of 10 to the power minus 4 you require SNR of something like 8.4 dB, 8.4 decibels right. For an arbitrary rate of 10 to the power minus 4 this point is around 8.4 dB right.

So this is a standard curve which becomes a benchmark for comparison where actual performance which you obtain for your system that you have designed right. Offcourse in reality even if you were to use coherent PSK your error rates will be higher that this slightly higher than this because of various imperfections you may have in the system, but this becomes a benchmark for you to compare how badly you are with respect to what you could have done ideally right. Now let's go to, is there any question about this? Before I will go to the next binary modulation scheme.

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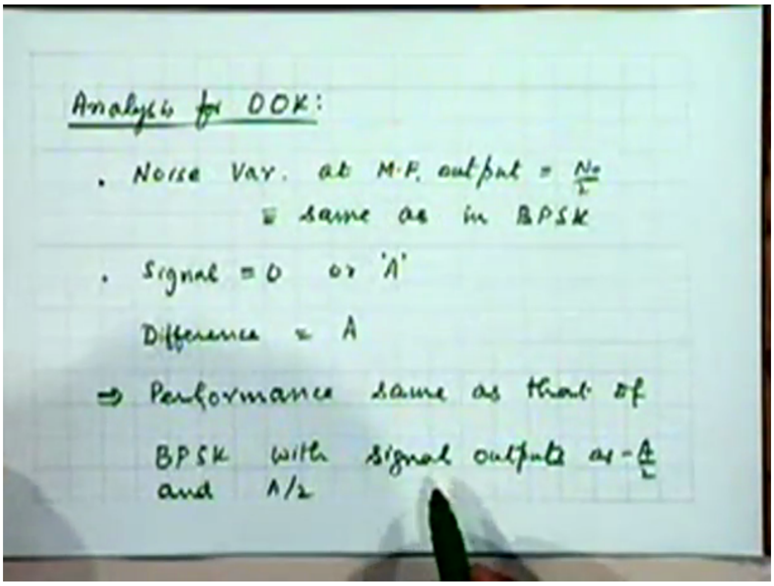

So its in (comparison) performance analysis quite simple for the case of BPSK of lets do the same thing for On Off Keying now.

The one can go and repeat the whole exercise as we have done for BPSK alright but one does not have to do it, one can derive the result for binary On Off Keying from the result that we have for binary PSK very easily. Yeah the only difference is in one case the P0 and P1 have mean minus A and plus A in other they have zero and A right. So one can go out and do it and obtain the expression which you can do and verify the result that I am going to tell you but another way of looking and it is the following.

No matter whether you using BPSK or On Off Keying, the noise variance of the output of the matched filter in each case is same right. Because the two distributions and everything else we are assuming all conditions is a same so at the matched filter output the noise variance remains that N0 by 2 or whatever right. That is same as in BPSK, it is a signal output which differ in the two distributions right. In one case, in the (OOK) On Off Keying case signal output is zero or A depending on whether you transmitted a 0 or transmitted a 1.

And the difference between the two means is A right, that means the performance of this scheme would be the with this difference would be same as that of a binary PSK system with amplitudes of minus A by 2 and plus A by 2 right, is that clear? So therefore performance would be same as that of BPSK with signal outputs at matched filter as minus A by 2 and plus A by 2 right. So therefore as far as that is if I have this situation On Off Keying with amplitude A and BPSK with amplitude minus A by 2 and plus A by 2 are the matched filter outputs I will get identical bit error probabilities right.

Now I want to convert that into an expression for the error rate. What we can do is, we can look at what is the signal to noise ratio here I wanted a signal to noise ratio here in this situation right and that gives us some simple way of obtaining the expression right. What will be what is the signal average energy that you transmit here?

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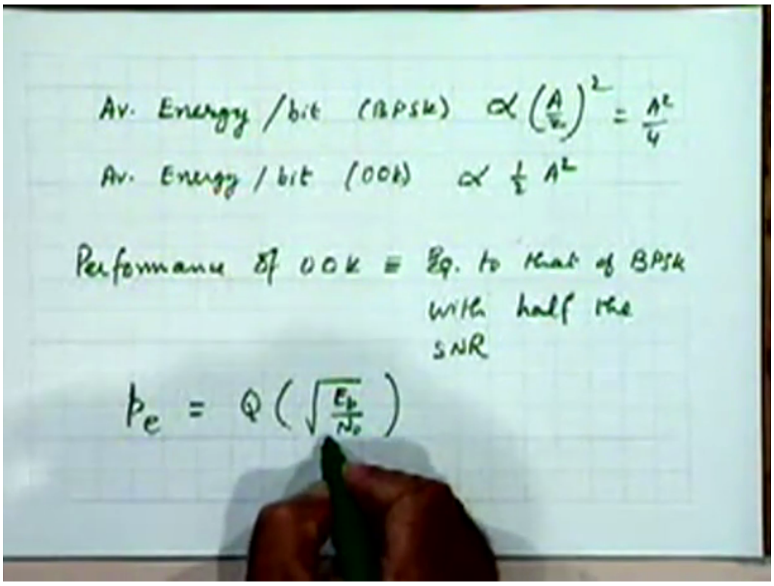

Average energy per bit for a case of BPSK would be how much? It is proportional to A by 2 square right, lets call it A square by 4. For a case of isn't it? Equivalent BPSK which gives you the same error probability right.

Similarly average energy per bit for the case of On Off Keying is proportional to half A square right. So what we can say is that performance of OOK is equivalent to that of BPSK with how much SNR? With 3 degree lower SNR or half the SNR right. So now tell me how can I write P sub e, it will be Q function evaluated at, that is right, E b by N knot it was 2 E b by N0 for PSK right for the same error probability it becomes or for the same SNR it becomes corresponding to half of that SNR right, so I is argument clear?

Offcourse one could obtain this (())(34:31) in the normal way that we did for BPSK there is no difficulty about that. But this gives you a feel for a comparison between On Off Keying and phase shift keying in the coherent case, that to obtain the same probability of error in the two cases On Off Keying would require 3 dB more power than phase shift keying is something that we try to appreciate much earlier intuitively but now we have a very clear mathematical understanding of what that means.

When we say that On Off Keying is an inefficient scheme it does not use the power efficiently now a very clear idea, how it does not use the power efficiently right. That is we are unnecessarily wasting 3 dB of power as compared to BPSK we unnecessarily transmitting twice as much power for getting the same performance, same error rate.

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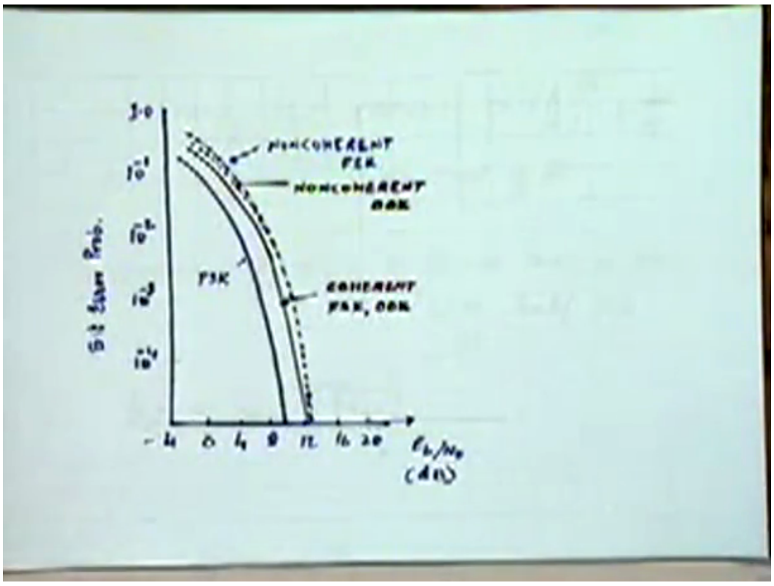

So if you look at the curves this is a curve, this next black curve is a one for both coherent FSK as well as coherent OOK but I will discuss FSK separately right.

At the moment you look at OOK but you will find is that if you were to draw a horizontal line anywhere right, the corresponding SNR readings could be separated by 3 dB right. If you have to draw a horizontal line anywhere for any error rate. So the curve is higher than the PSK curve. For a given error rate the On Off Keying would require three degree more SNR as compare to the corresponding PSK, that is a conclusion of our discussion right. So if I draw a horizontal line anywhere on this curve the two points on the two curves will be separated by three degree along the X axis right any questions? No questions? Ok good.

So we have the expression for BPSK, we have the expression for coherent On Off Keying. We now come to coherent or non-coherent On Off Keying, that is slightly more involve but not really all that much. Let's (I am) ok before going for sorry I have to do one more coherent scheme before I go to non-coherent schemes and that is coherent FSK.

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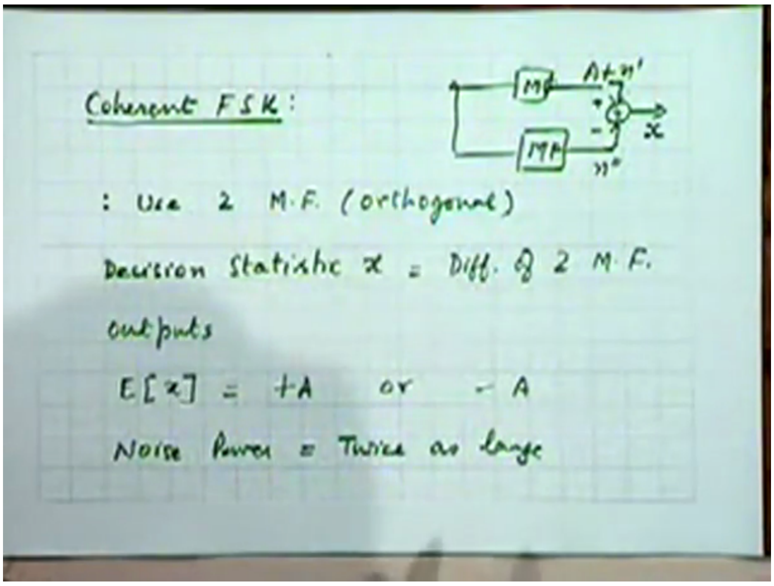

The Coherent FSK has a same kind of performance as coherent On Off Keying but for different reasons, can you guess what is the reason?

Student: sir, distance between the two signals, equivalent that of

Professor: Is equivalent to that off?

Student: for the same (())(37:49)

Professor: Let's now talk in terms of matched filter outputs because the way we are doing the processing you have to see right.

Student: sir the output is zero or A? (())(38:00)

Professor: I think the best way to look at it is if you remember we have two coherent matched filters one corresponding to each of the two waveforms and then we are taking the difference of these two right and then comparing with the same kind of threshold problem like in BPSK right. Now what is going to lets say one of them contains an (ampl) a signal one of the, the signal is one of those signals on of the two matched filter signals then the correct matched filter will produce a output will be of A right and the other one will be just noise.

So what are you going to do? You are subtracting the two, the result of subtraction will produce a n output which has a mean of either plus A or minus A depending on whether the top filter or the bottom filter produce the amplitude A, but the noise variance is twice as much because the two noises in the two orthogonal matched filters are uncorrelated and independent. I don't have the picture here but I think let me write it down. So in this case we use two matched filters orthogonal matched filters right and this two matched filters outputs are being subtracted from each other before being apply to the threshold detector right.

The process of subtraction basically X the noise variance increases the noise variance by a factor of 2 right. So the decision statistic that we are working with X which is a difference of the two matched filter outputs only one of them has the signal and the other has only noise, one has signal plus noise other has only noise right. You have one matched filter here another matched filter here. One is producing signal plus noise the other is producing only noise and the noises are independent you are subtracting the two right, that is what you are doing.

This subtracted value if you look at this value as X the subtracted value and the mean value of X is obviously either plus A or minus A right, precisely the same situation as in BPSK, the mean value in BPSK in also either plus A or minus A clear. But the noise variance noise power is twice as large, that is essential point we should because both this noises have equal variance and (())(41:27) when you add them basically you are adding you are using a random variable with twice as much variance ok, that is very elementary statistics that you are familiar with.

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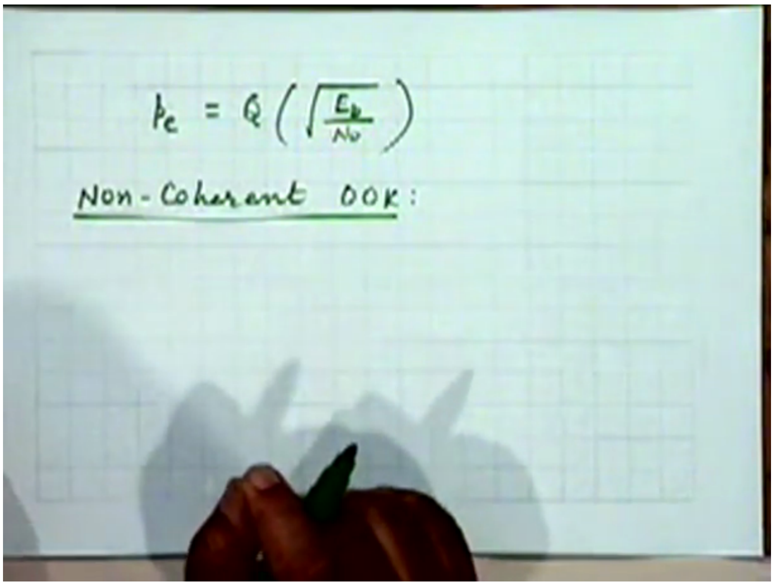

And therefore your expression for P sub e your noise variance becomes from N 0 by 2 it becomes N 0 effectively so E b by N 0.

So which is the same expression that we had for coherent On Off Keying, offcourse from another point of view we expect them to behave similarly because we can some say that both are orthogonal modulation schemes right so they have identical performance ok. Now lets come to non-coherent situations and the simplest to consider is offcourse we can't have non-coherent BPSK right, so we don't consider that. We consider On Off Keying case whether for coherent case or for non-coherent case is being considered only out of academic interest and also because it helps us to understand things in a proper perspective.

Must understand we don't use On Off Keying in almost any application (())(42:52). So it is just for the purpose of developing our analysis nicely and again non-coherent On Off Keying just for the sake of completeness of discussion we are going to it where rarely it is used.

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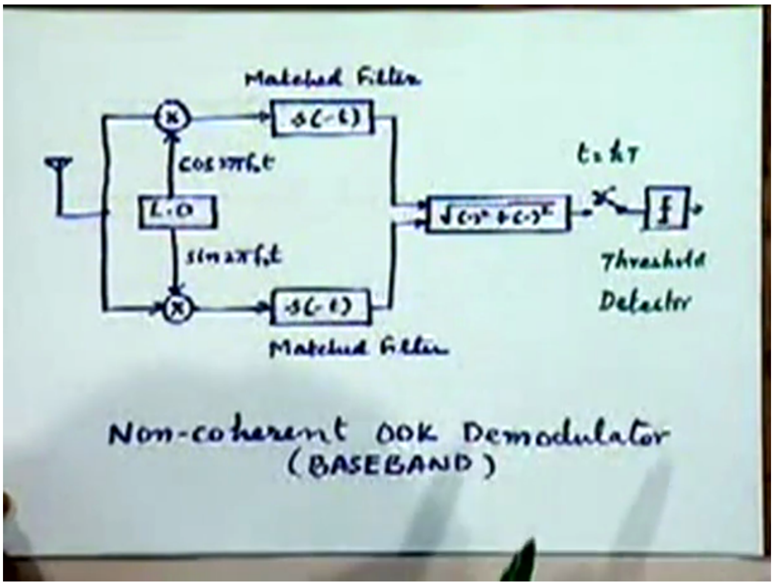

If you remember your non-coherent On Off Keying receiver was something like this at the baseband right, that is you have the baseband output coming out of the I Q de-modulators right and we are assuming that your transmitted signal is A times S t right or it is either there or not there, either with amplitude A or with amplitude 0.

So this complex input signal is being passed through both the real in part as well as the imaginary part have been passed through S minus t, the S t, t minus t knot minus T the matched filter corresponding S A and these two outputs are being combined by envelope detector I square plus Q square, square root of that sampled and (output). But this was I know you remember this but just to make you precisely understand what is happening, basically therefore if you look at these input here, in this output of the two matched filters you have two random variables right and we can describe them by a two dimensional Gaussian Distribution Function because both are individually Gaussian.

I remember what is this output like A K cosine theta plus J times A K sine theta plus noise right. So this is A k plus noise this is also A K plus noise, actually A k cosine theta this is A k sine theta with noise independently there.

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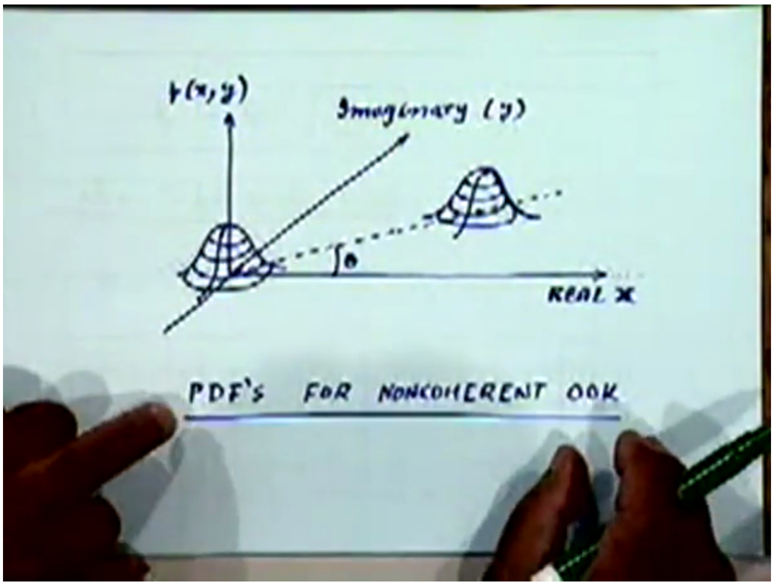

Therefore you can think of the two variables X and Y if I call this X and call this as Y, the real part is X and the imaginary part is Y right, we have a joint distribution of X and Y both of it means you are considering a two dimensional distribution both are individually Gaussian so it is a joint Gaussian distribution of this kind and either if we are transmitted a zero both of them will have zero mean, X as well as Y will have zero mean, because the input is containing only noise both the inputs right because this is a complex representation your actual input.

If your actual input contains only noise, only noise is going into both of these right and therefore both X and Y will be zero mean clear. On the other hand when a signal is coming along in that pulse duration they will have non-zero mean, the means will be A k cosine theta and A k sine theta. In this case will be A k cosine theta this will be A k sine theta. So you can think of in the X Y plane 2-D distribution which for zero transmission has a 2-D density function 2-D Gaussian density function like this and for a one transmission have a has a Gaussian density function whose mean is A k cosine theta along X and A k sine theta along Y, so somewhere here.

So the same distribution function has been shifted here right. This is the PDF's which come into the picture for non-coherent On Off Keying. Let's see what we would have done in the case of coherent On Off Keying in this picture. In the coherent case we would have somehow known the value of theta right this theta would have been known to us and therefore I could have aligned my that's say the X axis by rotating it by along this direction and the value of the imaginary component would have been (())(47:32) kind of no use to us right, this is what we do in coherent On Off Keying any coherent system because when we do that carrier compensation E to the power J theta multiply by E to the power minus J theta essentially what we are doing is we are aligning ourselves with this axis over here which separates the two density functions at this line and then only you know how the variable behaves in one dimension is enough we don't have to look at the other dimension which is what we did just now right.

But right now when we do non-coherent situation the parameter which is important is the radial distance or the distance of the observed value from the origin right because we are going to say if that, that is what analogue detector does, it will really not look at just along this line it will look at what is the observed value along X, what is the observed value along Y at can compute a radial distance from the origin, whether it is from this distribution or from this distribution right and that radial value radial distance from the origin we have try to appreciate last time, either has Rayleigh Distribution in this case or has a Riciaan Distribution in this case right, this is what we try to appreciate.

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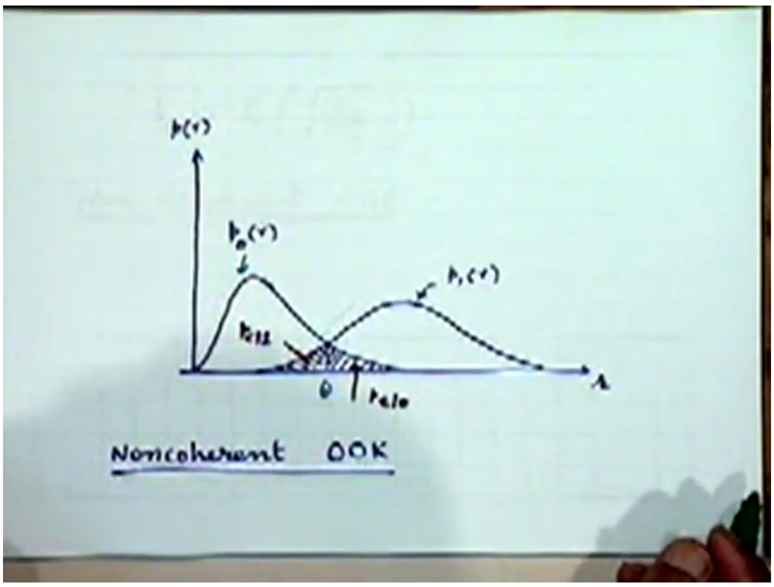

In other words we are now governed by a distribution function of R which is square root of X square plus Y square right which has for a zero transmission a distribution like this which is Rayleigh and for a transmission of a one as a distribution like this which is Rician right which for large SNR's we know resembles like example like Gaussian distribution function. So that is a picture re really have to consider because we are working with R you are not working with two dimensions directly. You are converting the two dimensional X and Y into a variable R right (())(49:43) which is square root of X square plus Y square which is governed by this two distribution.

Again we can choose a threshold appropriately and go ahead and do the error probability calculation. So conceptually is there any problem? For the case of non-coherent On Off Keying, I will just go through a few mathematical steps and I will not really do the complete analysis in this case for two reasons, one is it is of only academic interest and the second is because when the two density functions are different in one case you have Rayleigh in other case you have Rician, determination of the optimum threshold as well as the corresponding error probability become fairly involved just algebraically involved exercises which I don't want to waste time on.

You can read about it yourself and (())(50:48) from the books and I will just like to basically give you the performance (())(50:54) corresponding to this. What do you expect? The performance to become better or worse than coherent On Off Keying, worse, because you are not making use of the phase (information) and that is precisely what you have here.

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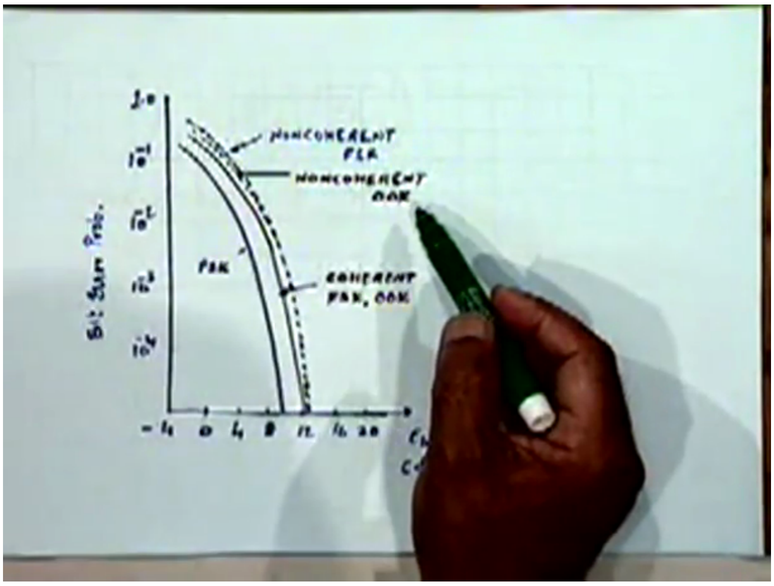

This black dotted curve is what you get for non-coherent On Off Keying ok. Now non-coherent On Off Keying is we already seen it is not very good because it waste 3 dB of power for you, you will got it into more power but you could say that FSK also uses 3 dB more power and still in some systems we do use FSK even binary FSK.

How come we have such a strong disliking for On Off Keying that we don't use it at all, whereas we continue to bear this non-coherent FSK, coherent FSK or non-coherent FSK, we have to discuss non-coherent FSK still, we have only discussed coherent FSK right, I will discuss non-coherent FSK next time because I don't have time now but between these two how come one of them we totally reject and the other we still continue to work with, can you give any argument? DC null, no from the performance point of view you have any reason to give, from this (())(52:26) point of view can you say?

Actually On Off Keying is a frequency shift keying whether coherent or non-coherent right. Lets take the case of coherent On Off Keying, something that should strike you immediately from this plot for I got. Most importantly the fact that a threshold depends on amplitude that fact is a single most factor, single factor which makes a lot trouble for us because we have to know a priory what the signal amplitude is going to be, right which is very-very difficult for a receiver designer to work with, he doesn't know what amplitude to design the receiver for, how to set the threshold, or he has to experimentally to do it as a signal is being received right.

Because he don't know what your received signal amplitude going to be exactly right. Particularly in a faring channel lets say where amplitude maybe even time varying, received amplitude maybe even time varying, if there is a the fare occurs occasionally the amplitude may become very low or in occasionally it may reinforce itself right. So the fact that your optimum threshold depends on because it has to be somewhere in the middle of this and this, whereas in binary PSK or infact for that matter for FSK after you have done the subtraction, the threshold is always fixed at zero.

It is independent of the signal amplitude which h is received. So that is one the single most important factors other than the ones which we have discussed earlier which also discourages the use of On Off Keying in practice. I think this is a good stopping point for us will continue this discussion.