Digital Communication
Professor Surendra Prasad
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Module 01
Lecture 25
Coherent Demodulation of Binary Waveforms Binary Waveforms

We have discussed what matched filters can do for us and we were now getting ready to discuss how they can help us in designing de-modulators for various kinds of digital modulations schemes that we have considered in the past and specifically we will we discussing today which we had started doing yesterday coherent de-modulation for binary waveforms. So toady I will basically take up binary waveforms of two kinds namely the antipodal kind like binary phase shift keying and the orthogonal kind like frequency shift keying ok.

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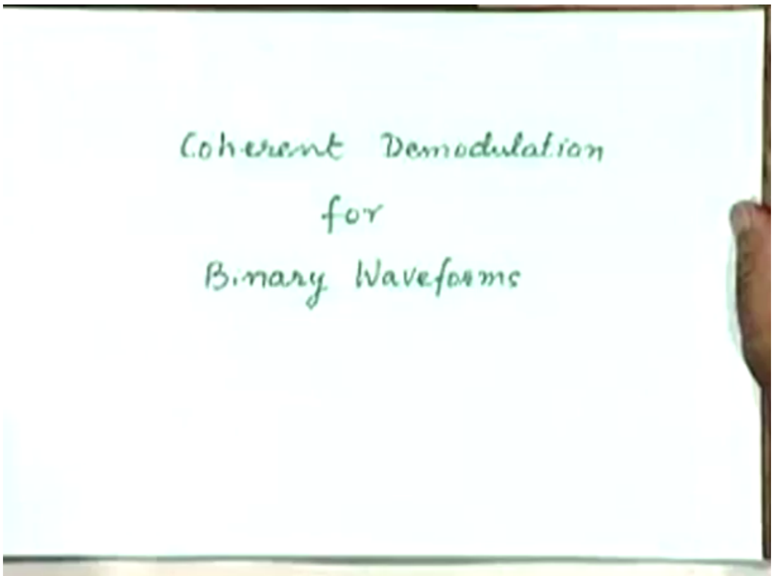

So will take up these two but before I get into this I just want to point out to you that matched filters have applications in many fields. Not only in optimum de-modulation in digital communication there are applications in any kind of problems where you need to detect signals right, what are the? Can you name it few application where you may need to do that other than communications other than digital communications, where you need to detect signals weak signals and noise in the presence of noise. I am sure you know a few applications radars for example you needed you transmitted narrow pulse and it gets reflected back comes back very weak in the presence of a lot of noise in clutter and the radar receiver has to decide in a particular at a particular range whether a pulse is present or not right.

So as to decide whether the target is present or not at a particular range. Again another property of matched filters can be used in such applications namely the output of the matched filter is the auto-correlation function of the waveform right. Now obviously in such applications like radar and sonar you would like the output pulse to have a, to be broad or narrow? The output pulse to be narrow, why? So as to be able to resolve nearby targets right because the resolution ultimately depends on how broad is the output pulse right. So you can design a pulse shape there such that its auto-correlation function is very narrow and thereby you can increase the resolution.

You could offcourse get the same resolution by transmitting a narrow pulse but the disadvantage of that is narrow pulse will it be power limitations will have finite amount of energy and therefore it will affect the detectability of the pulse right because ultimately the outputs SNR depends on the signal energy once the noise power is fixed. So it will be useful to transmit a white pulse but so design that's auto-correlation function is narrow right.

Student: why we can't have a narrow pulse in the first phase?

Professor: Because if you have a narrow pulse and if you have a peak power limitation which most of its applications will have that it will put limit over how much energy you can put in each pulse, which will in turn affect the output signal to noise ratio of the matched filter right and therefore affect your performance in terms of detectability right. Therefore it will be useful to design pulses which are let's say broad to start (with) while transmitted or broad but after matched filtering they become narrow right.

So such waveforms which have such properties are called pulse compression waveforms they are very useful in these are the kind of signals which are used for obtaining good resolution in applications like radars and sonars right. The other applications of matched filtering also even in digital communication not only for de-modulation but for other purposes. Will perhaps have time to discuss our things later but maybe we don't have time so will just briefly mention some other applications of match filtering.

If you may recollect timing synchronization is going to be an important aspect of our digital communication systems whereby we need to know precisely when the message will when a particular sequence of messages starts right. Whenever you start transmitting you have to you don't want to lose anything and therefore typically before you actually start transmission of a given message you will be prefixing it with some kind of a preamble sequence right. The purpose of transmission of this preamble sequence will be to designate very clearly the time instant from where onwards the message actually starts right.

So will be transmitting some well-defined (())(6:22) known bits to the receiver and what the receiver will try to do is based on this transmitted (())(6:31) preamble it will match filter that so as to have a very narrow sharp auto-correlation function output to pin-point where the next message actually starts. So it is useful in carrying what is called frame synchronization. So matched filtering is used also in frame synchronization. Similarly match filtering has a lot of other applications in spread spectrum communication systems however we don't have time to discuss how long those things but the main point that I wanted to mention before getting a bit deeper into the application of interest towards was that match filters have a very larger class of applications right this is just one of these classes of applications alright.

Have any questions on that?

Student: (())(07:21)

Professor: It will be quite digressive so I think maybe in some other platform will do that right it will be quite digressive at this point so lets first get to this business. So lets talk about coherent de-modulation for binary waveforms, a few things we talked about yesterday (on the) one thing that we talked about was how to take care of the phase, phase offset that maybe present in the received signal right in the carrier of received signal. All we have to do is to compensate for it by multiplying the complex envelope of the received signal by simple E to the power minus J theta. So after that if our phase estimation is good we can more or less assume and this is what we shall assume for the present discussion that will have we have no phase offset effectively alright.

Because we have perfectly compensative right offcourse in real life will never be able to do this estimation perfectly right even your carrier recovery circuit will be affected by noise, so it will not be able to do a perfect estimation of the actual phase. So there will be some phase data which is still left over and will affect the performance of your system but at the moment for the discussion of looking at the (opt) the de-modulators for these waveforms will assume that we have perfect phase compensation possible will therefore ignore it.

Student: sir how can you (())(08:57)

Professor: Yeah theta is quite to be known to us provided we do something at the receiver and that is something is carrier recovery circuit right that is a circuit which is specially design to track the carrier frequency and phase. So we are assuming that such a circuit is in operation at the receiver and from its output we have an estimated of theta available which we shall use to compensate for the value of theta that is taking place phase shift that has taken place.

So this precisely why I am saying that since this estimation is not going to be perfect in practice in practice will not be able to compensate for you perfectly and therefore this assumption of zero phase offset will not be strictly valid in practice however we shall assume that it is valid for the present treatment ok.

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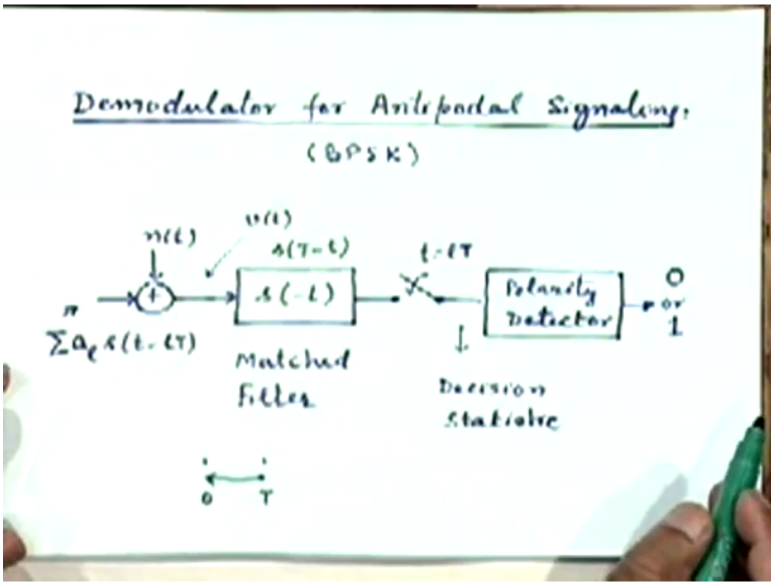

So to start with let me discuss this picture for the de-modulation for antipodal signals. So I am taking the case of an antipodal signals here of which BPSK is an example binary phase shift key is the main example of antipodal signaling in more spectacle applications.

Although there could be in general antipodal signals can be defined in terms of any arbitrary waveform S t right, so recollect that we can represent a general binary signal which uses the pulse shape S t as a sequence of such pulses right L going from minus infinity to plus infinity A sub L's are your bits right taking the values plus minus 1 in the case of antipodal signaling right and this is your basic pulse shape which could be of finite duration or it could be of infinite duration depending on whether it is lets say rectangular pulse or a Nyquist pulse right it could be anything.

This sequence of positive and negative pulses gets corrupted by noise and that is what you receive at the receiver in the form of the signal V t alright and the receiver or de-modulator for this kind of digital modulation scheme is obviously very trivial now after our discussion of matched filtering, what we do is pass it through this sequence of pulse phase coming one after another is actually de-modulated one bit at a time. Suppose this is a bit interval right we transmit we de-modulate it one bit at a time. So every incoming pulse excites this match filter produces an output which is expected to peak where that T is equal to (zero) no-no this is where you have to be careful offcourse I have shown here at T0 T equal to 0 but is actually going to peak at I should really put here S t minus t.

Because we wanted to peak after the pulse duration right after every pulse duration we wanted to peak right but symbolically I will typically write as S of minus t right. So typically at the end of every bit interval which is suppose the output of the matched filter suppose to peak and that is where I will sample the output of the match filter and if it was a positive pulse I will expect its peak output to be positive. If it is a negative pulse, pulse shape is same whether it is positive and negative. So it will keep on producing auto-correlation function the peak value of the auto-correlation function at the output except for the polarity of it right.

Polarity will depend on whether A sub L is positive or it is negative. So if I get a positive large peak that implies lets say a 0 or a 1 must transmit either way it is a matter of how you map the signal and if it is other way around it is the other bit right.

Student: What did you say about S of capital T and a small t?

Professor: That was to do with where the peak will occur right, suppose I am considering the very first bit interval zero to T right the if I want peak to occur at T equal to capital T the impulse response in the matched filter should be S capital T minus t, you T sub 0 is capital T right and so on for average bit interval. So really speaking I would like it to be the peaking every T seconds and I will like to keep on sampling the output of the match filter every T seconds right. Is it ok? However you must note that the fact that it is peaking at T equal to L t for every T seconds interval does not imply that the match filter output is zero elsewhere for a particular excitation interval right.

It may not be zero elsewhere infact even if your pulse is of finite duration the minimum duration of the matched filter output will be, suppose this is strictly of duration T (in) each incoming pulse is strictly of duration T, what is the minimum width of the matched filter output?

Student: 2 T

Professor: 2 T, because the impulse response is of duration T input pulse is of duration T the convolution will be of duration thus which is a sum of these two durations which will be 2 T right. So although it is peaking at T equal to capital T and therefore then capital 2 T and so on. We do have the distinct possibility of inter symbol interference here right because the output of a match filter response the match filter response does not become zero in the next bit interval it remains finite and so on for subsequent bit intervals right.

So what we have to make sure is that this pulse shape indeed not at the input but the output satisfies the Nyquist criterion ok, that this pulse shape that we choose be such that at the sampling instance the Nyquist criterion is satisfied that is when we the output of the match filter is peaking due to the current pulse the response of the same match filter to all previous pulses is zero at that time instant right. We don’t care what the response is at other time instance but at multiples of T seconds the response becomes zero.

This is something we are familiar with we have discussed that separately at length when discussing the transmission of pulses in bandlimited channels alright, so we are familiar with that fact. Therefore the net result is if the Nyquist criterion is satisfied then their match result is the output is output can be detected now offcourse there will be variation around this peak due to the presence of noise. Will not get the true peak value which would have expected if signal alone was the input to the match filter right.

If signal alone was input to the match filter there would be no problem right but since the signal is associated with noise this peak values will be associated with some variability due to noise with some variance and therefore there is a need to put a threshold detector there and all we can say is if the peak is positive take one decision if the peak is negative take the other decision right this makes sense.

Student: sir what is that decision?

Professor: This sampled value is called the decision statistic on the basis of which you take a decision ok, this is the terminology I picked up from detection theory. In detection decision statistic it is written there ok, this is the (samp) the sample values that are coming over here every T seconds are called the decision statistics because it is over basis of this sample value that you decide whether a 0 or 1 was transmitted alright. So this term has come from there. So broadly the scheme for antipodal de-modulation is clear? Ok.

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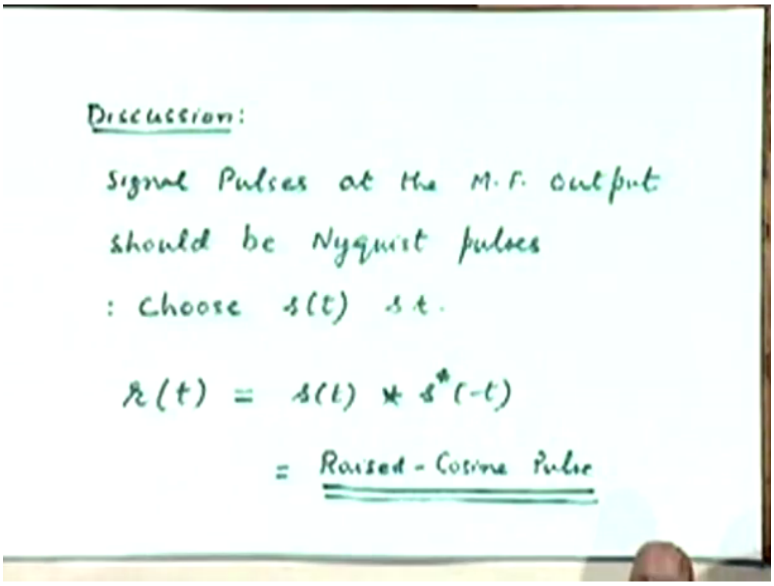

So just briefly let me mention this fact that I have just discussed with you that the signal pulses at the output of the match filter must not exhibit inter symbol interference right. The signal should be so design in this queue of things that the pulses at the output of the match filter although exhibiting a peak every T seconds should also contribute no inter symbol interference right. So signal pulses at the output at the match filter output, in other words they should be Nyquist pulses right ok. Which essentially means that we must choose for our antipodal signaling a waveform or a pulse shape S t such that the match filter output due to the signal which you remember I had denoted by R t right which would essentially the convolution between S t and S conjugate minus t right, this should be equal to one of the Nyquist pulse shapes that we discussed.

For example it could be a raised cosine shape a time waveform corresponding to that is cosine pulse.

Student: (())(19:34)

Professor: Assuming the channel to be ideal, infact that is how we have modeled the channel for this discussion right. All I am saying is that the channel does not distort the signal right if the channel distorts the signal also then corresponding wave match (filter) I mean your receiver structure has to change then and that is a situation that I am not discussing at the moment, I am, I have idealized the discussion a bit I am assuming that channel is ideal and therefore the waveform does not change and channel itself does not introduce inter symbol interference right. So the inter symbol interference that we have taken care of is the one which could have been infuse by the transmitter filter and the receiver filter alright and therefore if there is such a distortion we have to take care of separately our receiver becomes more complex right.

If we have time I will discuss the issue separately ok. So now you have some idea about what one does with that raise cosine pulse that we designed earlier right at that stage you had just left it there that in when we discuss transmission of pulses in bandlimited channels we just said that our overall pulse shape should be such that it exhibits inter no inter symbol interference and from Nyquist considerations we said that such pulses could possibly be of these kinds of shapes right.

But now we know something more than what we did at that time. We now know how to split that pulse shape into the transmitted pulse shape and the receiver into the transmitter pulse shaping filter impulse response and the receiver impulse response right, that splitting should be such that convolution of these two should be equal to that pulse shape right. At that stage we did not discuss this issue but now you know something more that somehow these two things have to be put together.

Student: (())(21:41)

Professor: No at that stage we did not infact choose S t yeah we only discuss what was the final shape pulse shape we wanted now we can say what is a actual transmitted pulse shape that we should use right at the transmitter. It is linked with its match filtering later at the receiver right. The convolution of the actual pulse shape that we use and the match filter impulse response should be equal to the Nyquist pulse it did not be raised cosine any Nyquist pulse shape will do ok, raise cosine is just an example. Any point to discuss or any doubt here? Ok

Now I have discussed one basic realization for antipodal signaling which is essentially that each incoming pulse is match filtered and its polarity detected at the sampling instant that is what the optimum the de-modulator (())(22:51) Infact it can be shown to be optimum from the probability of error point of view in Gaussian noise I am not going to be do that it is very simple to do that. Infact when we look at the performance of receiver it will become obvious that it is optimum. Now there are other possible implementations of this receiver.

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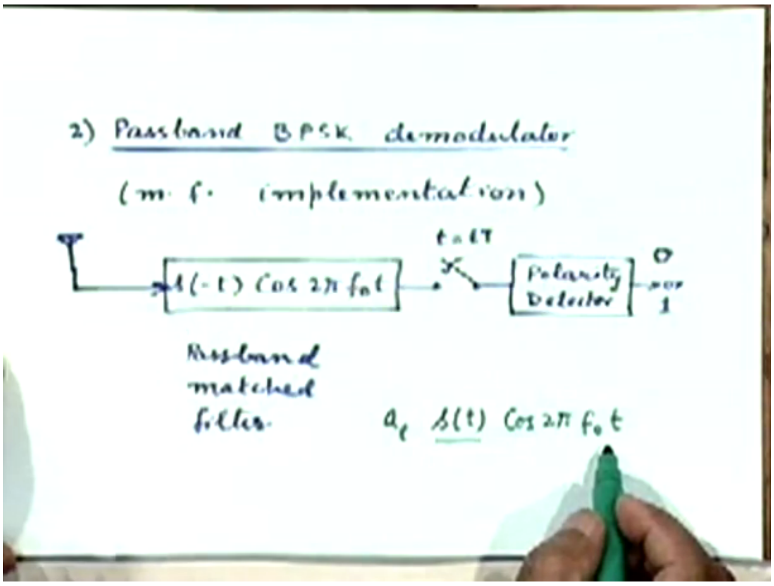

Particularly when the pulse shapes that we used lets say a passband pulses right that is they are emburied on to a carrier. Right now I choose a general pulse shape I didn't discuss whether it was a baseband pulse or a passband pulse it could have been anything right, the result would have been the same. So the need to discuss that issue separately is important because when we are discussing passband pulses lot of other issues come up and I like to atleast pin point some of those issues. The basic structure does not change, the basic processing that we need to do remains the same whether this is a baseband pulse or a passband pulse right.

Basic operation has to be the same in match filtering followed by polarity detection after sampling right. But some of the other issues pop-up which are important and therefore I would like to spend a little bit time on alternative realizations particularly in the case of passband. For example if you really talk about binary phase shift key the presence of a carrier is implied right therefore we talking about a passband signal ok. One could have the same match filtering implementation like I have got over here, the received signal is coming in the match filter because (your) I am again assuming carrier offset to be (zero) phase offset to be zero.

So that if you transmit a signal S t cosine 2 Pie F knot t but this is your baseband pulse shape which is emburied to the carrier right and this will come along with positive or negative amplitude that is what binary phase shift keying is either cosine 2 Pie F t or minus cosine 2 Pie F knot t right. So depending on whether A sub l is plus 1 or minus 1, this is what your receiving the corresponding match filter will be a filter with impulse response S minus t cosine 2 pie F sub O t alright.

Student: sir previous case was general it was not BPSK

Professor: It was general antipodal signal right, BPSK is a special situation where the pulse in emburied onto a carrier right that is when we talk we can talk about phase shift only when you have a carrier right isn’t it? Without a carrier it is meaningless to talk about phase shift we just say it is antipodal right BPSK is an example of antipodal signaling and that is that obviously implies the presence of a carrier.

Student: when the output of this match filter (())(25:58) another passband in output.

Professor: Yes it is a passband pulse

Student: sir it will be (())(26:02) and if we sample it at some

Professor: But we know that even that passband pulse is suppose to peak at T equal to capital T right. We have seen that yesterday you remember that?

Student: (())(26:26)

Professor: Yes let me say if I can get (())(26:21) picture which I present to you yesterday. Just let me give me a second please.

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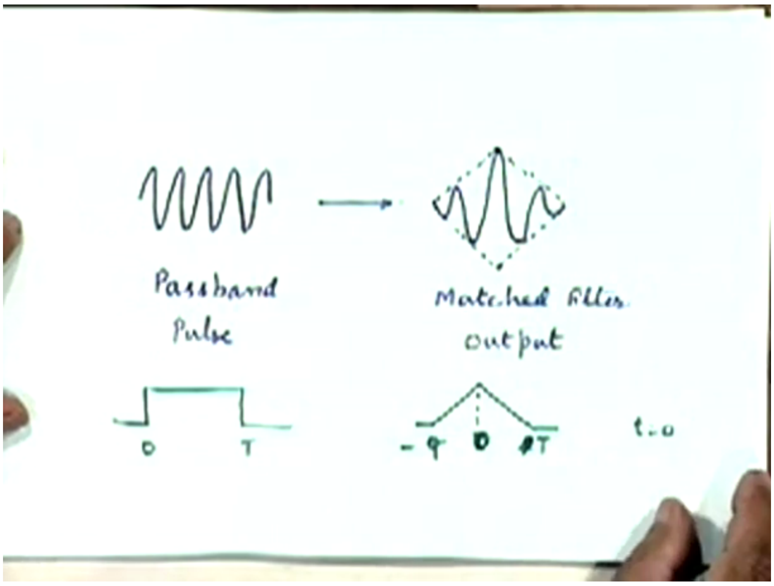

We discussed that yesterday, right, this is a passband pulse after match filtering this is how it will become and you are going to sample it here right and if it is now if the polarity is changed the phase is changed by 180 degrees, what will happen? A whole thing will come upside down and this peak will be seen here.

Student: sir because it is very fast varying output (())(26:59)

Professor: That is a separate problem and that is a problem I really want to discuss, that is good, you could think of that. Yes because this is because of this fast variations in the pulse carrier at the output it is going to be sensitive to timing error right you got it very good. That is the point I was coming to. So the base (())(27:29) is obvious just replace that match filter with this with corresponding match filter here and the problem as cyclic pointed on by Varun is that this kind of arrangement is going to be a bit sensitive infact quite sensitive to timing error, lets see that in some detail.

The pictorially it is quite obvious right from the picture that I have drawn it, but lets do some analysis and try to appreciate to what extent there will be problem.

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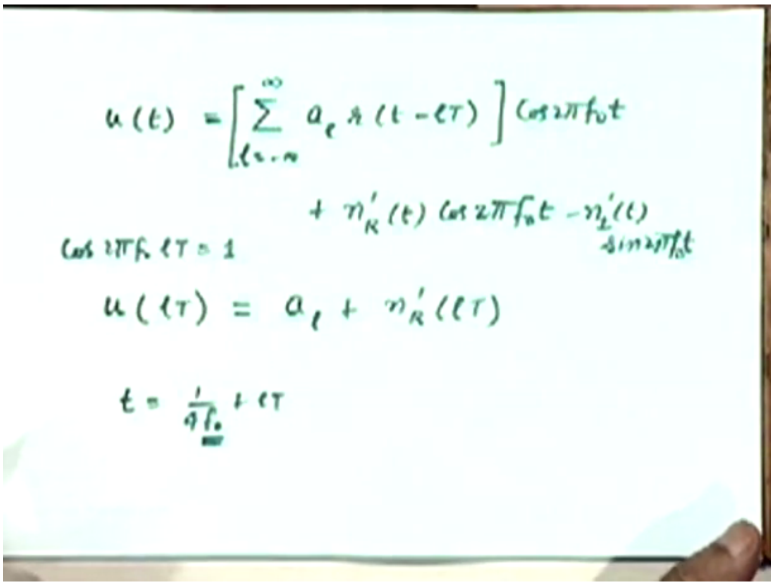

The output of the match filter corresponding to this situation can be written like this, A sub L this is quite obvious so I am not going to explain this any point unless you would like me to ok. This is the signal (())(28:40) filter output which essentially comes this of the baseband output pulse modulating the carrier right and that is the noise part where N R prime and N I prime are the convolved outputs corresponding to noise.

What really is of interest was is the sample value of this output waveform at T equal to l T at the sampling instance right. What will be this output at T equal to l T? If cosine 2 Pie F if we design cosine 2 Pie F knot l T to be 1 right, and if we make sure that this output pulses satisfies the Nyquist criterion then there are only contribution that will come at T equal to l T will be from A sub L right, because all other bit values will not contribute to the value as sample at T equal to l T alright. So it will be A sub L we also assuming the T equal to L t cosine 2 Pie F knot L t is equal to 1 right.

Similarly this will become N R prime some value L T and this will be obviously 0 then because if cosine is 1 sine will be 0 at that point right and that tells you the sensitivity, for example if you sample it at somewhere else for example if you sample at t is equal to 1 by 4 F knot plus l T what is going to happen? What is going to happen to this cosine? It is going to become sine right Pie by 2 phase shift. So your signal will disappear altogether right and F if F knot happens to be high if your carrier frequency happens to be high, that is a very small timing offset ok.

So this just mathematically demonstrate the same issue that we saw pictorially.So what is the solution there are two possible solutions, one is to use a carrier frequency which is small enough offcourse that is not going to be under our control that will really depend on the communication system in which we are with each you are working. If it is a microwave system it has to have microwave carrier frequency right if it is a voice band channel then the offcourse F knot will be relatively lower above the corresponding data it is also smaller right.

So in any case the timing error problem is can be solved by relatively changing the value of S sub knot and one way to do that is known to you, what is that? Use an intermediate frequency receiver right that is don't do the de-modulation passband de-modulation straight away first bring it down to a lower carrier frequency so that the sensitivity to timing error reduces to some extent right. Of course you are not exactly solving the problem you are just passing on the problems somewhere else, can you appreciate that?

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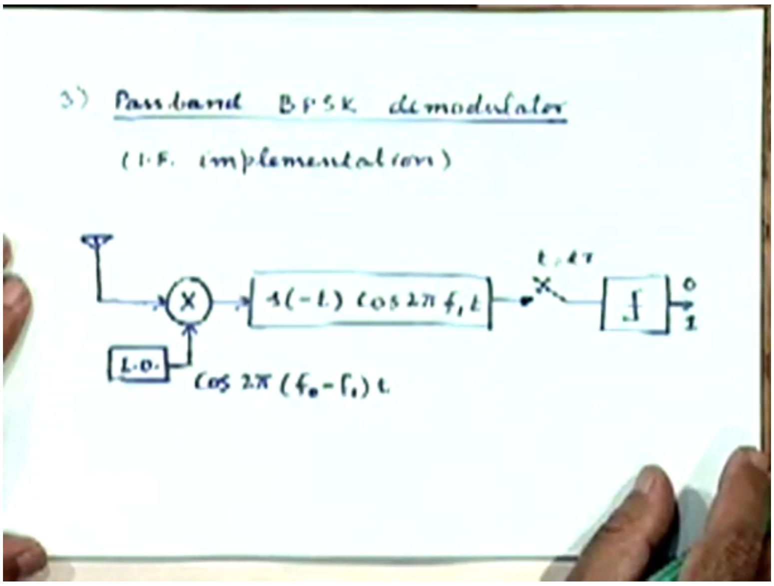

So this is an alternative passband BPSK de-modulator which is the same as before except that instead of a match filter being matched to S minus t cosine 2 Pie F1 t, F0 t it is matched to some other frequency F1 t which is an intermediate frequency of operation.

So obviously it is imply that your first translating the incoming signal down to this frequency by passing it through by beating it with a local oscillator frequency of F knot minus F1 ok and everything else is same.

Student: sir why not we bring it right to the baseband.

Professor: An alternative is to bring it right down to baseband, the pros and cons sometimes it is this is easier sometimes that is easier. Now you just to make sure you appreciate this fact (I like to) again need to write that we have not solved the problem of timing sensitivity. All you have done is you have shifted it to now this place here right. Suppose there is a carrier offset here with a local oscillator that will again manifest itself in the form of a timing offset there right. Therefore all you have done is ok the timing cycle you see you have in any digital communication system they are now you appreciate two kinds of synchronization you have to do, one is the timing synchronization.

You should know where a new pulse is starting and ending right that is the timing sensitivity aspect, the other aspect is a phase synchronization right, we had discussed now both of these briefly I mean what they are about how they are done we have not discussed, but what they are about we have discussed. So we still have a problem of phase estimation and basically we are shifting the timing sensitivity problem to phase sensitivity problem. Fortunately the phase techniques circuits are easier to design as compare with timing synchronization circuits right. So it is worthwhile shifting this problem from timing to phase tracking from time tracking to phase tracking and intermediate (frequency) use of intermediate frequency helps us to do that precisely.

Any questions? Vivek, fine? Now we have discussed therefore three different implementations for de-modulation of antipodal signals one general and two for the case of BPSK.

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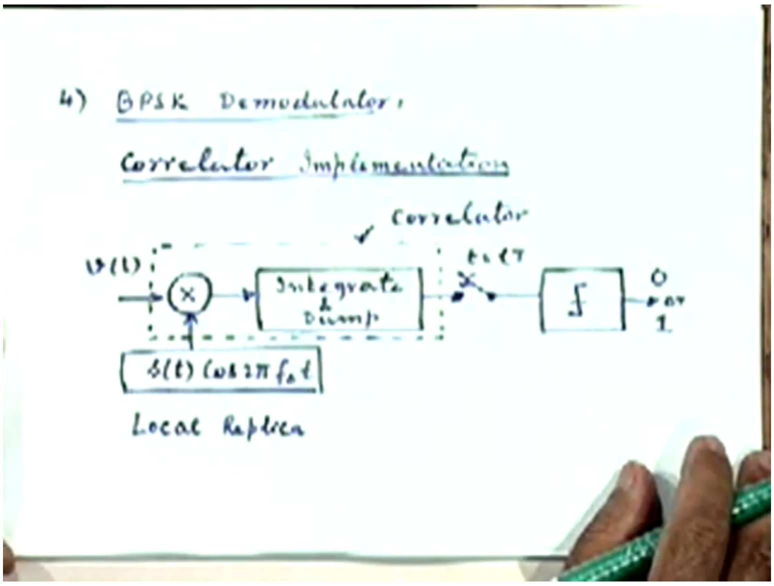

I will also like to discuss one more implementation which is sometimes used and that is called relative implementation ok. Before you rather picture let me just briefly discuss the idea behind it, I think you already know the idea behind it. You recollect and discuss this in the context of match filtering.

Student: Match filtering is same as modulator.

Professor: It is not the same match filtering operation is not the same as corelator operation however if you confine your attention to the output at t equal to capital T or t equal to 0 or whatever your output sampling instant is designed to be right. Then we can regard the two to be equivalent right. The output waveforms are not the same but the waveform value at the sampling instant of interest can be the can maybe the same.

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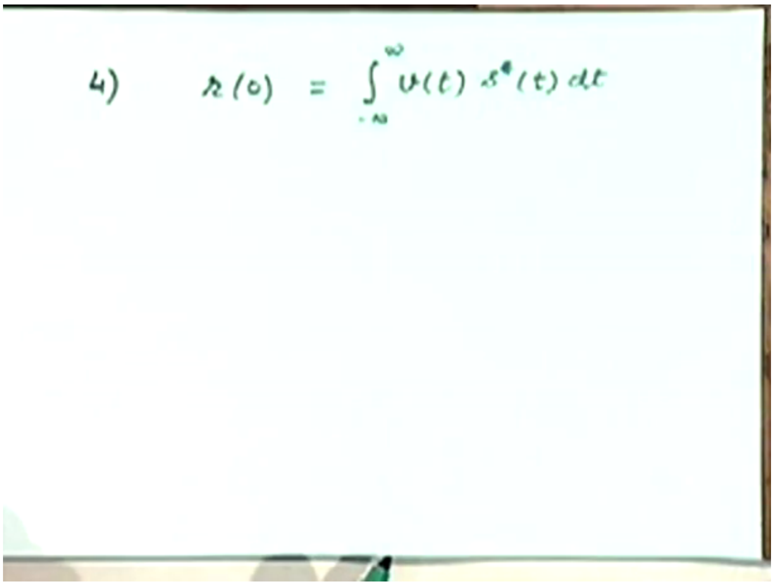

Because suppose (we are) our designated sampling instant is t equal to 0 then we have appreciated before we try to discuss that several times that this value R sub 0 R of 0 is nothing but the correlation of the received signal with the transmitted signal right.

So we can as well replace match filtering operation by a correlation operation, the advantage of doing that is suppose your pulse duration the pulse is strictly of a time limited nature right. This correlative implementation becomes very-very easy. See the match filters that I have talked about in the BPSK case for example the so called passband matched filters, they are essentially tune filters isn't it? When I say a filter with this impulse response it is essentially a tune filter right with a certain kind of transfer function which is dictated by this shape because a Fourier transform of this is going to be the actual transfer function right.

Whereas in a corelator I don't implement any filter at all or I do is take the incoming signal multiply it with this a local replica of the expected signal and integrate and integration is a very simple operation to implement. So implementation wise it is very convenient.

Student: we have to reset the integrator

Professor: That is right you have to restart the integration every bit interval if we want to share the same integrator from one time interval to the next offcourse we could have a number of integrators for different time intervals that will not be very convenient right.

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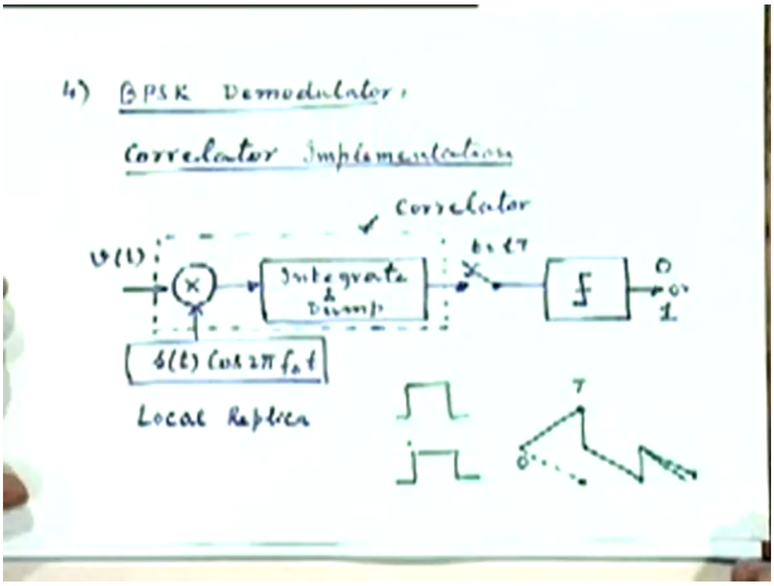

So what will do is have an arrangement like this, this is your incoming signal multiplied by the replica of local replica of the expected signal pulse shape what you transmitted if it wasn't passband pulse it will be S t cosine 2 Pie F knot t and then integrate the result and the result of integration at t equal to L t every T seconds is what you are interested in and if you want to share the same integration over and over again you must reinitialize this integrator every T seconds that is how we call this integrate and dump operation.

That is integrate for T seconds suppose I have a rectangular pulse what will happen? Lets just try to appreciate the fact from a rectangular pulse. Suppose you are transmitted pulse was a rectangular pulse I will even forget about the carrier for the moment. So your local replica is also rectangular right you will multiply the two and integrate it. Product will be a rectangular pulse again and (())(39:02) you will build up the integration like that right. Some zero to T you want to sample this pulse, if one was positive and the other was negative for example if you transmitted pulse was really negative and this replica is offcourse the same then it will build up like that right.

So in any case I need to sample this value and then I must ready to for the next integration and therefore I must reinitialize the integrator back to zero after sampling it right, that is called a dumping operation. So that you are ready for the next maybe this was a positive pulse this was a negative pulse so you dump it again and next one also could be negative pulse and so on. So you keep on sampling these then dumping the integrator output to that is reinitializing the integrator at that angle operation and the rest is all the same. So this is yet another possible implementation for BPSK de-modulation.

However note that this kind of thing will be almost impossible to do if your pulse shape is not strictly time limited, you appreciate that? Right for example the Nyquist pulses that we have been talking about if they are using those Nyquist pulses then integrating dump is ruled out, that is we can't share the same integrator over successive time intervals right. So you must appreciate that fact, any questions? Can we go onto the next? I think that more or less covers binary phase shift keying de-modulation or antipodal digital de-modulation right.

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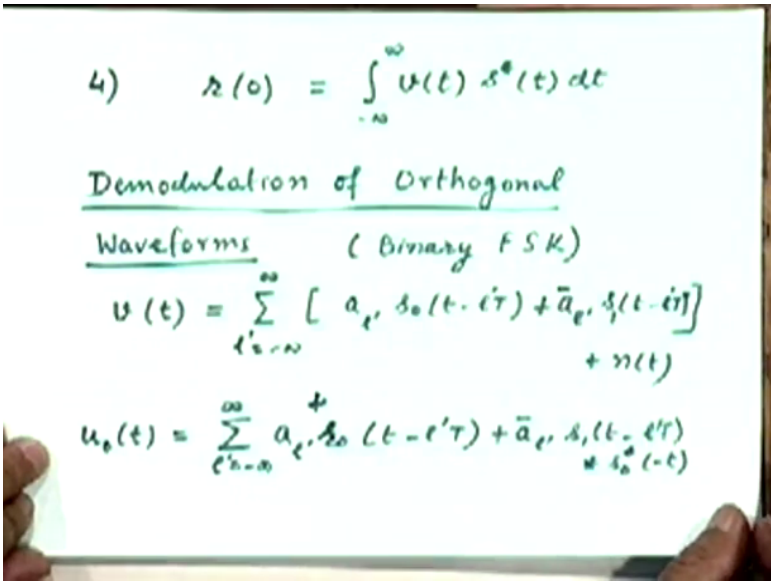

Let's now come to de-modulation of any issue that you would like to discuss? The other kind of binary waveforms you can have namely orthogonal waveforms ok of which the most important example is binary frequency phase shift keying right but first I will take a general I will do a general discussion irrespective of whether it is frequency shift keying or something else I mean as long as the waveforms are orthogonal and if you remember binary FSK is a kind of general nomenclature for this class of waveforms even though your orthogonal signalling scheme may not involve frequency shift right.

It may be some other kinds of orthogonal waveforms right, so therefore first I will do a general discussion and then will mostly concentrate on FSK alright. I hope you remember that discussion of orthogonal waveforms well you would have a minor recently same as we are remembering something. Now before I go on to telling you what the de-modulator should be like in this case can you make a guess? Can you has any guess? (())(42:43).

Well in this case we talking about binary we have two, we have the only two right well lets first talk in terms of matched filters right. We need two match filters, one matched to each other to waveforms we discussed that yesterday also in the context in its slightly different context, but now we are doing it in the context of de-modulation and this is what your de-modulator will look like for orthogonal signals.

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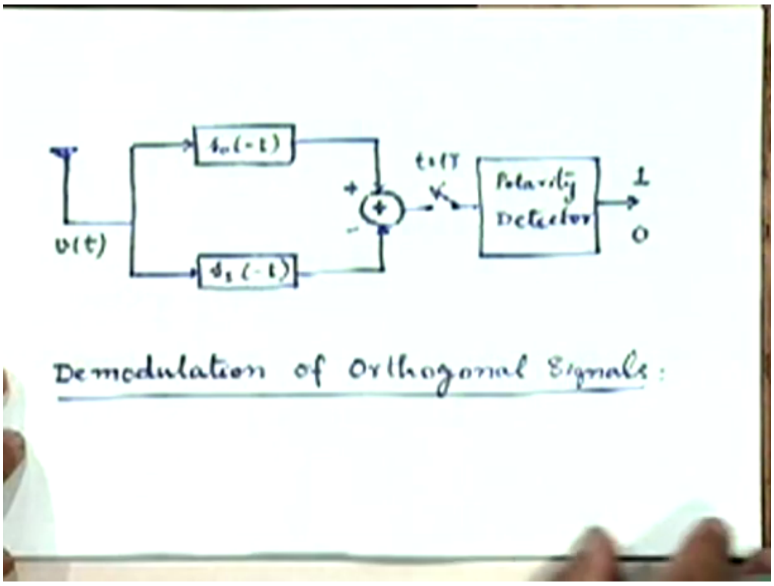

Here received signal is processed by two matched filters now simultaneously right, obviously one of them is matched to it right depending on which one was transmitted if I see a transmitted waveform is one of these two you don't know which one you pairing the whole idea of de-modulator in this case is to find out which of these two waveforms was transmitted because that tells me whether a 1 or 0 was transmitted right for the binary case and therefore what I need to do is I expect the match filter output the right match filter output to be larger than the other match filter isn't it?

Because in one case you remember we show that yesterday in our match filter properties if you have this orthogonal filters you produce an amplitude A in one and zero in the other right. Of course there will be noise in both but since the mean of one is suppose to be larger than the mean of other by considerable amount in general we will expect the correct match filter to produce the larger output and therefore we just subtract the two depending on the polarity of these we again decide whether this or that was the correct transmission and therefore whether 1 or 0 was transmitted that is the intuitive (picture), is it ok.

So max will take the case where the received waveform is V t which is signal plus noise right so let us see what kind of conclusions we can draw mathematically this is the physical picture but let us just write down a bit of maths so that we can do the analysis later when it needs to be done. The received signal can be written as the sequence of orthogonal pulses like this, alright the either transmitting S knot or S1 right, now A l prime is taking only the logical values 0 and 1. In this case we are not allowing this A L's to take positive and negative values but only the logical values of 0 and . So either we transmitting S knot for example if A L prime is 1 we are transmitting S knot if A L prime is 0 we are transmitting S1, that is how one can represent this plus the noise that is coming along right plus N t.

So your output of the match filter output of the lets say the first filter, suppose this is passed through a bit we are passing this waveform to both of these right. So lets look at the output of it first filter that is the top filter here, that will be given by U knot t A L prime S sub O I can replace this with r right t minus L prime T and what this will be, will leave it like this for a moment A L prime S1 t minus L prime T convolve with we are convolving with S O right plus N t convolved with the same thing let me call that N prime, there is no place to write it here so I will write it on the next page, is it fine?

This R knot represents a convolution of S knot with S knot right this is the convolution of S1 with S knot and plus will have N t convolved with S knot I will call that N prime, N sub O prime because this convolve with S sub O.

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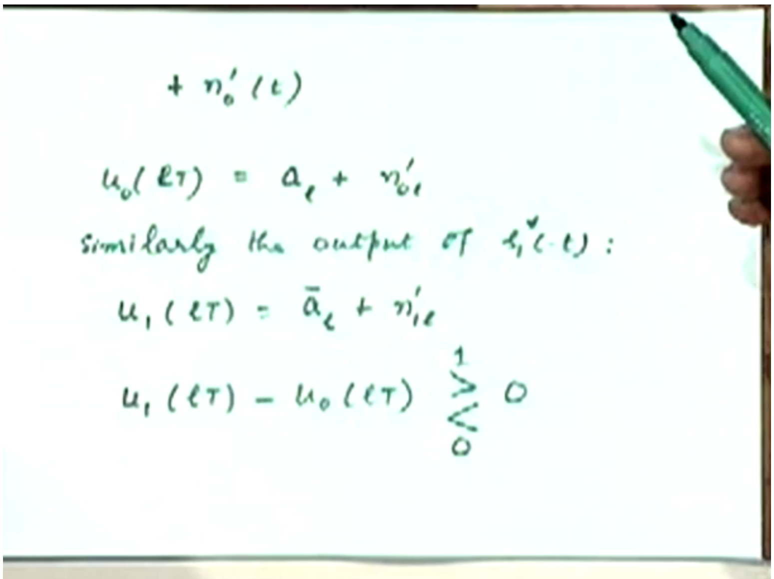

So plus N sub O prime t ok, in particular if you sample this output at t equal to L T, what will be the output?

Student: (())(48:27)

Professor: This lets try to just appreciate this, what will be this value? This convolution, if S1 S0 are chosen to be orthogonal, this will be zero right, so really it will be getting contribution only from this right. So U knot L T is simply A L plus because all other values of L prime will not contribute if again we have chosen the pulse shape carefully so as not to exhibit inter symbol interference right. So out of this summation only the Lth value will be of interest or will be what is called is shown out at the output. So this will be this plus n I will just call it N O L prime which is the sampled value of this.

Similarly the output of the lower filter the other filter the same thing is being crossed by both the filters right, the output of this other match filter which is S1 minus t you can appreciate will be let us call it U sub 1 is going to be it will be now A L bar plus lets call this N1 L prime alright same argument and now your decision is going to be based on sampling both of these or and deciding which one is larger right, this is one way of taking the decision or you could subtract one from the other which is what we are sure in that picture right.

So as use a threshold detector. So decision statistic could be either choosing the larger of the two values and then deciding which is larger or alternately lets say subtract U1 l T and U knot l T depending on whether this is greater or this is greater we decide I mean whether the result is positive or negative you may decide that we transmitted a 1 or we transmitted a 0.

Student: (())(50:59)

Professor: A pulse is going to be there what the match filter will do is it will try to minimize the amount of this noise.

Student: (())(51:15)

Professor: No the two noises will be un-correlated remember that, so that is not cancel the waves will affect become double right when you add or subtract two un-correlated or independent noise components the variance will be double right. So a subtraction is not going to help at all in that process the only way the noise is being taken care of is to the fact that you are doing match filtering and therefore these values would be as small as possible right and this is a best one can do right, nothing better one can do about it.

So that is a good point but S1 has to live with this noise and obviously your remark is correct in the sense that it is well known that a binary orthogonal signalling scheme in general will have a poor performances as compare with a corresponding antipodal scheme right and this is linked with what you are saying ok.

Because of the fact the noise will be more making more sensitive. So this is the maths behind the same structure that we discussed ok. Now this discussion is for a general yeah I will stop in a minute this discussion is for a general orthogonal signalling scheme, will try to look at some special aspects of it or in the context of FSK particularly because when you are talking about FSK we talking of a carrier again right and what kind of issues come up and how we can simplify the implementations etc. One general question I have is lets say Varun remark with one point that I could have a baseband implementation of the BPSK receiver right, can you tell me how could you have a baseband implementation?

Student: (())(53:18)

Professor: Just did it wave F1 and then you have baseband de-modulation. Now when you do a general one can also do it for a general situation and I will discuss that next time in the context of orthogonal signalling where it is not necessarily BPSK but for any arbitrary passband signal that you may have you can do the de-modulation at baseband and the way to do it would be to first represent the passband signal by its complex envelope right. So an important question is how we derive the complex envelope from the passband signal, we have discussed that before by just doing two of these modulations, namely one with a cosine carrier and one with the sine carrier. So that gives the complex envelope complex representation after that we can work in a complex baseband domain everything is baseband after that right.

So and will use that for doing de-modulation of FSK at baseband as well as at passband thank you very much.