

**Digital Communication**  
**Professor Surendra Prasad**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**  
**Module 01**  
**Lecture 24**  
**Matched Filtering & Coherent Demodulation**

We were talking about matched filters for the case of white noise that is signal in white noise and signal in non-white noise in the later case we call it the whitened matched filter right because we can think of matched filtering operation as composed of first whitening the noise and then filtering it for maximization of the signal to noise ratio right. In any case the main purpose of the matched filter we saw was to maximize the signal to noise ratio at the sampling instant and the manner in which it achieves this objective is by weighting the different frequency components in accordance with the signal to noise ratio present at those frequencies.

That is those frequencies components which have which are dominantly consisting of signal as compare to noise or given more weightage by the filter as compare with those frequencies components when the noise is more dominant right and this is a manner it achieves maximization of the signal to noise ratio. Also we learned that the output signal to noise ratio is a function only of this signal energy in the case of white noise that this is when signal is of stage 1 limit white noise.

The actual signal pulse shape doesn't contribute to the or does not matter does not affect the output signal to noise ratio. The only parameter of the signal that matters is the energy component present in the signal to ratio whereas if the noise is non-white if it is a non-white Gaussian noise then the pulse shape will also contribute to the output signal violation and not only its energy right. So these are a few important things that we learned last time about natural test. We have any questions any about any of these?

Student: sir one problem, signal to noise ratio will be define for a base of a white noise,  $(\sigma^2)$  (3:15) that is the point sir, I mean that is what I am confused and signal power take  $A^2$  I mean in the constant, though a filter is it will be  $S$  of conjugate of  $d$ , so the noise part is the actually  $A_0$  by 2 times the I mean the bandwidth, when we take  $N_0$  by 2 only so that we get something like.

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$$SNR = \frac{2E_p}{N_0} = \frac{P \times T}{N_0 \times T} \quad \left[ N_0 \times T \leq \frac{1}{T} \right]$$
$$A^2 \int_0^T s^2(t) dt = E_p^2$$

Professor: It is actually  $\frac{1}{T}$ , if you remember the expression for the signal to noise ratio or put signal to noise ratio that we (achieve) we designed was actually  $2 E_p / N_0$  this is the expression which is more educational upon  $N_0$ , well if the other expression is slightly misleading because it doesn't bring out the energy it doesn't bring out the units properly. This is the one which brings out the units properly because this is energy which means actually  $A^2$  square into the time duration right, which is which should be which probably implicit somewhere you know implicitly use it if they say something about the set signal rather be unity or something or not.

Student: we use a five filter pulse as a  $V$  equal to 1 or  $E_p$

Professor: Right, so we said that  $\int_0^T s^2(t) dt = 1$ , right, so which is essentially saying that because if  $S$  or let us say this equal to  $A^2$  square right

Student: the point is I mean when you substitute,  $S$  upon  $N$  for  $\sigma^2$  I mean  $A^2$  square comes from

Professor: No, what is the problem that you are facing in your mind?

Student: sir  $\sigma^2$  shouldn't be replaced by 0 by 2, but it should be 0 by 2 times of bandwidth.

Professor: The bandwidth is implicitly built in here, let us what I, from this expression it is not obvious but from this expression it is obvious as you can see because this is signal power times, what is energy? Power times the pulse duration right and that is an  $N_0$  by 2. So if you

bring it down that is  $N_0$  by 2 into  $1$  by  $T$ , that is equivalent to noise power spectral density multiplied by the bandwidth. So it is really speaking this expression is nothing but the signal to noise ratio in the bandwidth of the matched filter because effectively the band with a matched filter will turn out to be  $1$  by  $T$ .

So it is built in into this expression. We saw it from very obvious term this expression that we wrote otherwise but if you really interpret the signal to noise ratio properly which I tried to tell you last time it is really speaking the more fundamental expression is this expression right, the output signal to noise ratio of a matched filter in the presence of white Gaussian noise is  $2 E_p$  upon  $N_0$ , where  $E_p$ ,  $E_{sub P}$  is a pulse energy ok. So

Student: in the denominator we have energy term and in the numerator we have power term.

Professor: No-no will have power only, this is power spectral density multiplied by bandwidth which will give you the power in that angle right and which is a power which will really be which will appear at the output of the matched filter.

Student: That  $E_p$  is the denominator then how does that  $T$  correspond to the bandwidth, bandwidth didn't correspond to bandwidth.

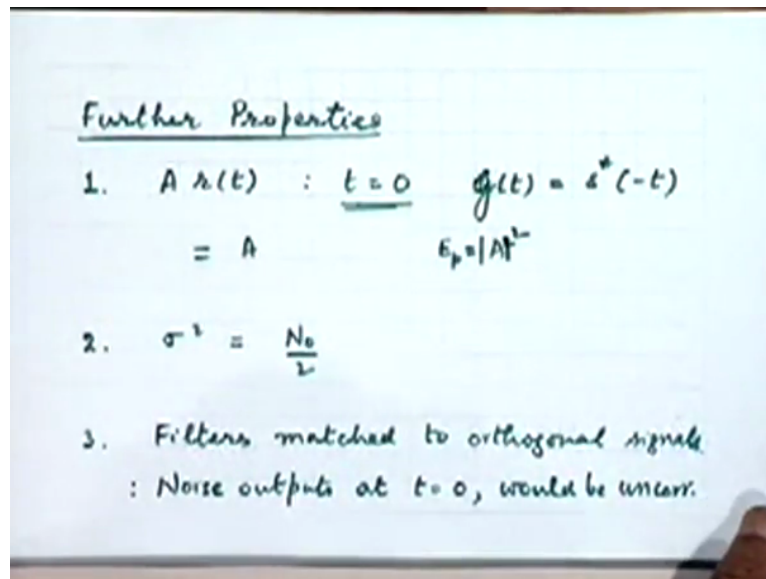
Professor: It has a units of bandwidth, I mean it has a units of frequency let us put it that way. It's a power, I think one other one of the problems that we implicitly was facing was the units I didn't say so right.

Student: I mean the problem still remain that, (7:07)

Professor: Ok this is say this is a mathematical expression here derived and an approximate way of interpreting it like this, that is it is effectively signal power that is being (7:25) divided by the noise power in the signal bandwidth because roughly for a signal of duration  $T$  we already discussed many times we can say most of energy will lie in a bandwidth of  $1$  by  $T$  right which from that point of view from an interpretation point of view roughly speaking, any of the questions? So is it clarify? The problem.

Now I will start my discussing some further properties of matched filters very deeply, some of which are more or less obvious to you.

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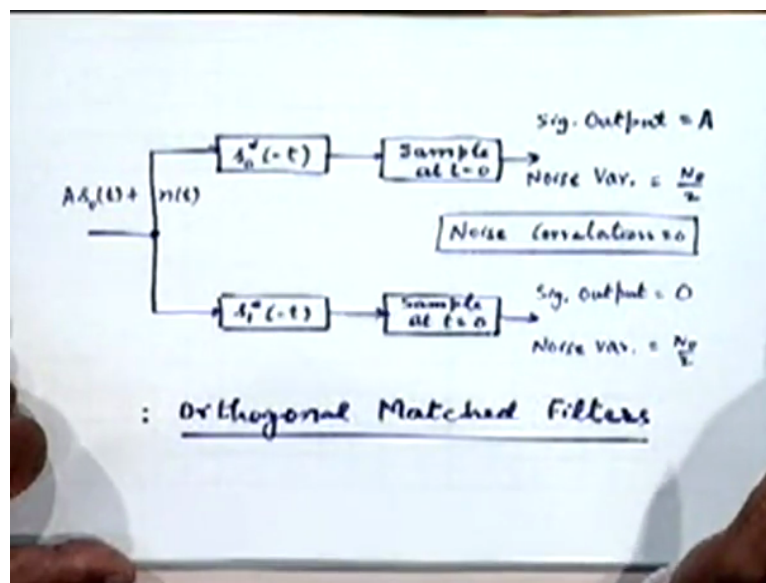
So let me discuss a few important other properties and then we will go on to use of matched filters for optimum de-modulation of a de-modulation of digital modulations. So some further properties the first thing to notice and that I have been infact just talking about it. The signal output  $A r(t)$  this is the signal output and we are sampling this output at  $T$  equal to let us say zero, it could be anything it could be  $T$  equal to some arbitrary time instant  $T$  sub 0, but if we chose that equal to 0, without any loss of generality corresponding to the matched filter  $H(t)$  equal to or is it  $G(t)$ ,  $S$  conjugate minus  $T$ .

If it was  $T$  sub 0, it would have been  $T$  knot minus  $T$  the sample value at  $T$  equal to 0 is given by corresponding to the noise component (equal to) is given by  $A$ , the sample value, its power is  $A$  square energy is  $A$  square  $T$  right, whatever, actually the  $T$  somehow normalize in my discussion all though you didn't explicitly turn out so you could say that  $E_p$  is equal to  $A$  square and I think  $T$  is probably implicitly taken to be 1 or something I don't know how it has come about. Let  $T$  equal to 1 but somehow the way we have been talking about things we have made  $E_p$  is equal to mod  $A$  square.

Any case the signal amplitude at the output at a sampling instant is  $A$  right, and the second thing is that the noise variance  $\sigma^2$  at the output can you say anything about the noise variance? It is equal to  $N$  sub 0 by 2 ok, actually speaking ok I will come back to that and thirdly these are the three properties that I want to discuss together let me just mention all the three of them first. Suppose you have the same, suppose you have two matched filters both of which are matched to two different pulses which are mutually orthogonal to each other. So filters matched to orthogonal signals.

Will individually, suppose the input signal contains one of those two signals to which these two filters are matched and they are simultaneously this input is simultaneously fed to both these matched filters right. You have let us say signal  $(S_0(t), S_1(t))$  and let us say the input signal is  $(S_0(t) + S_1(t) + n(t))$  and you are simultaneously feeding it to both the filters remind you  $S_0(t)$  and  $S_1(t)$  are mutually orthogonal right. Then the outputs the noise components of the two outputs of the two filters will be mutually un-correlated right. So the noise outputs at  $T$  equal to 0 or  $T$  equal to  $T_0$  would be un-correlated ok and I have shown this in the form of a picture whatever we have discussed just now over here.

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Let us look what I am saying is suppose I input is the signal  $(A s_0(t) + n(t))$  right and you are passing it to simultaneously through two different filters matched filters, one matched to  $S_0$  and the other matched to  $S_1$  alright these are the two matched filters impulse responses are two matched filters and we are assuming that  $S_0$  and  $S_1$  are mutually orthogonal right and now sample the output at  $T$  equal to 0 here sample the output at  $T$  equal to 0 here the three properties that I have talked about are summarized here, for this filter the signal output will be  $A$  the noise variance is  $S_0$  by 2 ok.

For this filter here what will be the signal output? Is it obvious it will be zero? Is it obvious? From which discussion does it become obvious? Is it the discussion of previous phase class, remember we also could interpret the output of the matched filter as some kind of auto-correlation function of the signal. In this case it will be a cross-correlation function between the input signal and the other signal and that we are assuming to be 0, because we are taking the two signals to be orthogonal to each other.

So you can prove it very easily that the signal output will be zero here but the noise of course will come out in the same way as over here with a variance  $N_0/2$  right and therefore these are the first two properties, the third property is these two noises although they have the same variances, the same amount of power in it, they will be mutually un-correlated that is the noise values coming out here at the sampling instant and the value coming out here will have no correlation with each other, they will turn out to be totally different from each other right or will have no correlation with each other, that is only we have to put it.

If in addition the noise is Gaussian that will also imply there will be independent noise values that you will sample although they have the same variance that they mutually correlated and the case of Gaussian noise will also be independent. So this is the summary of the three properties that we discussed as far as units are concerned if you compute the energy over  $T$  seconds this will be  $A^2 T$  a corresponding variance value change by  $N_0/2 T$  right.

If somehow this is implied here that we have considering  $T$  equal to 1 but if  $T$  changes from 1 then this will become output will become  $A$  times Square root of  $T$  and this will become  $N_0/2 T$  and so on. So that the ratios will remain the same.

Student: sir how do you get the noise are un-correlated?

Yeah so let me in discuss at now we will as far as a first two properties are concern let us first talk about that, first property is obvious that the signal output here will be  $A$  and here it will be 0, I don't think I need to discuss that in the light of what we have discussed earlier, this is obvious to all of you. Let us first discuss the noise variance,  $R_0$  equal to  $N_0/2$ , can you tell me how it is equal to  $N_0/2$ ? So I am taking the second property now, sigma square

Student: (16:02)

Professor: Yes the sampling instant is a zero,  $A$  by root  $T$  nai,  $A$  into root  $T$ .

Student: (16:20)

Professor: Because this was of same point that we discussed in couple minutes ago,

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The image shows a whiteboard with several mathematical equations written in black marker. The equations are as follows:

$$\boxed{SNR = \frac{2E_b}{N_0}} = \frac{P \times T}{N_0/2}$$
$$\boxed{A^2 \int_0^T s^2(t) dt = E^2}$$
$$\boxed{\frac{N_0}{2} \times \frac{1}{T}}$$
$$\sqrt{E_p} = \sqrt{A^2 \cdot T} = A \sqrt{T}$$
$$\int s^2(t) dt = 1$$

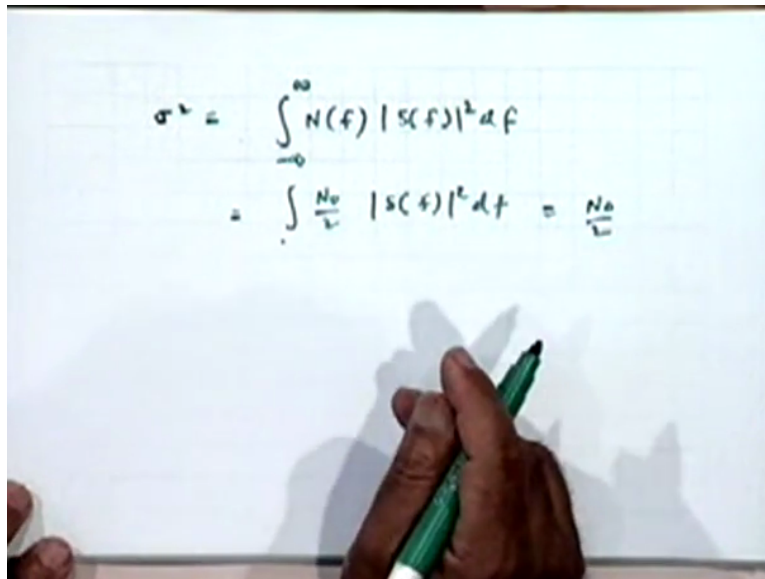
Here the output is really  $E_p$  right and  $E_p$  is  $A^2 T$  right, actually the output is square root of ( $E_p$ ) this is the output power and therefore the output amplitude would be square root of this which is what I was talking about.

Student: ( ) (16:59)

Professor: Yeah that is somehow and this so that its time to correlate for you, I think I am also getting mixed up in this energy normalization business ok I will clarify this point later, I think I will make it very clear because somehow this point has to be link to with the other point that I have been making there  $\int s^2(t) dt = 1$  I will come back to this point.

I think I have mixed up this point a little bit, it is a matter of just taking care of scaling factor which I will do for you separately let me not digress from this discussion I will come back to this point and clarify this at some point.

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$$\sigma^2 = \int_{-\infty}^{\infty} N(f) |S(f)|^2 df$$
$$= \int \frac{N_0}{2} |S(f)|^2 df = \frac{N_0}{2}$$

So let us look at sigma square this is the output noise variance I am interested in which is going to be we are already seen the input power spectral density function multiply to the transfer function matched filter, what is that transfer function in matched filter?

Student: S conjugate of F.

Professor: S conjugate of F, offcourse as far as noise power is concern, we should only multiply it by mod of mod square of that right. So we can as well right S F mod square right and since we have taken again this that particular assumption down to the picture and since we are considering white noise now this will become N 0 by 2 this and since we have assumed that in signal energies unity right, so this simply becomes equal to N 0 by 2 right. Signal power is (())(18:52) ok I think that is yeah I think it's that point is also clarified now, we have been talking a bit confusedly about things, I think we should clarify that once for all.

When I do this, what is this? This gives me power or energy?

Student: (Power) Energy

Professor: It gives me energy, so when I am saying that integral S square let us say it is in between 0 and T is equal to 1 right its effectively we are saying that the signal power signal energy is unity.



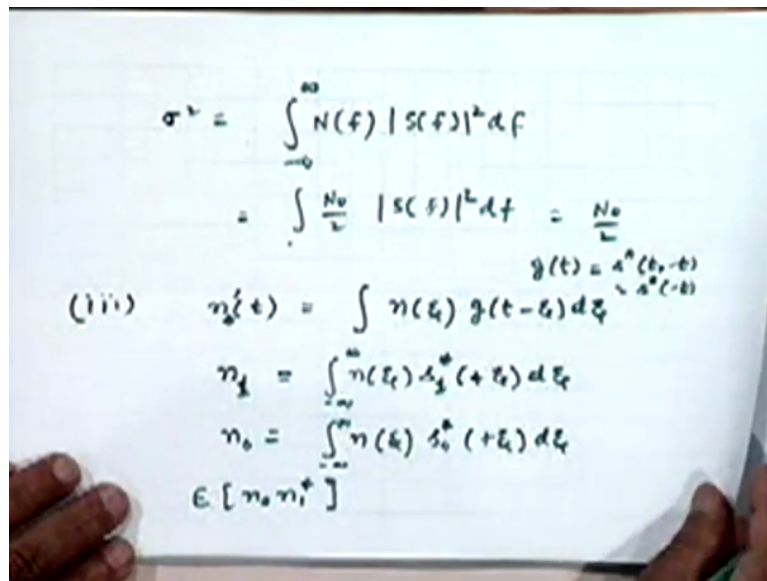
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The image shows a whiteboard with handwritten mathematical derivations. At the top, the equation  $SNR = \frac{2E_b}{N_0}$  is boxed, followed by an equals sign and the expression  $\frac{P \times T}{N_0/2}$ . To the right of this is another boxed equation:  $N_0 \times \frac{1}{T}$ . Below these, a larger boxed equation states  $A^2 \int_0^T s^2(t) dt = E_b$ . Underneath this, the equation  $\sqrt{E_b} = \sqrt{A^2 \cdot T} = A \sqrt{T}$  is written. At the bottom, the equation  $\int_0^T s^2(t) dt = A^2 = \boxed{E_b}$  is written.

So I think that is a point to note, this  $A^2$  upon is not really power it is energy itself. So that is why it's we have taken it to be equal to  $S$  is equal to  $E$  sub. No we still I mean this is the fact that it is energy still imply that we are making a discussion, that its power times the time interval right.

So the all interpretations are still valid, so I think let us keep let us stick to this and then will not bring in this route of (( ))(20:05) there alright will not bring in this route ok Now the third point which Vivek has asked, how do we say that they will be un-correlated in this particular case? That is when you pass the same signal to mutually orthogonal matched filters I call them mutually orthogonal matched filters because they are matched to mutually orthogonal signals and the corresponding output samples at the sampling instance would be un-correlated, let us quickly look at that.

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$$\sigma^2 = \int_{-\infty}^{\infty} N(f) |S(f)|^2 df$$

$$= \int \frac{N_0}{L} |S(f)|^2 df = \frac{N_0}{L}$$

(ii)  $n_z(t) = \int n(\zeta) g(t-\zeta) d\zeta$   $g(t) = s^*(t, -t)$

$$n_1 = \int_{-\infty}^{\infty} n(\zeta) s_1^*(+\zeta) d\zeta$$

$$n_2 = \int_{-\infty}^{\infty} n(\zeta) s_2^*(+\zeta) d\zeta$$

E.  $[n_1 n_2^*]$

Let us take the third property, what is the, let us write down the expression for the noise output, it will be equal to let me call this  $N_{sub O}$ , right or just call it  $N_{prime}$  and it is a simply  $N$  the input noise convolved with the impulse response in the matched filter which will be  $G(t - \zeta)$   $d\zeta$  alright, ofcourse and we are interested in looking at the noise samples at specific time instant corresponding to  $T$  equal to zero alright. Let me call this for one of the signals or one of the filters let us say the first filter I will call it  $N_{sub 1}$  and for the second filter the output at the same sampling instant I will call  $N_{sub 2}$  alright.

So what will be expression for  $N_{sub 1}$ , it will be  $N(\zeta)$  and ofcourse  $G$  is your  $S_1$  or alright let us say  $S_1$  yes conjugate also and I have to put  $T$  equal to 0 right and similarly  $N_{sub O}$  alright it will become plus eta alright, here he is saying  $G(t)$  is equal to

Student:  $G(t)$  is equal  $S$  conjugate of  $P O$  minus  $t$  (())(22:52)

Professor: It is ok, right and this is the definition of matched filter in time domain we discussed last time, please check up your last time notes you will see it there ok and for the second filter output input is same right the noise input is same only the matched filter is different which is  $S_{sub O}$  alright, and look at the expected value of  $N_0$  into  $N_1$  conjugate if you do that it is very easy to check that will be 0.

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$$\begin{aligned}
 E [n_0 n_1^*] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_1(z_1) s_0^*(z_0) E [n(z) n^*(z')] dz dz' \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_1(z_0) s_0^*(z) \frac{N_0}{2} \delta(z - z') dz dz' \\
 &= \frac{N_0}{2} \int_{-\infty}^{\infty} s_1(z) s_0^*(z) dz = 0
 \end{aligned}$$

Let's look at that expected value of  $N_0 N_1$  conjugate that is our cross correlation between the two noise output samples can be written as  $S_1 z_1$  into  $S_0 z_0$  one of them will be conjugate the other will be well I have taken conjugating  $N_1$  star so this will be without conjugation and this will become with conjugation isn't it?

If you conjugate  $N_1$  star this will become  $N$  star and conjugate  $S_1$  alright if you take use that double integral and I have taken the expectation operator inside the integral because it is a linear operator and this will become just a second there should be something that we have to be careful about when we are when you are using two integrals, let me call one of them as  $z_1$  and the other as  $z_0$ , right, or will be  $z$  and  $z'$  it will be simpler, this will be  $z$  it doesn't really matter.

So expected value of  $N z_1$  into  $N z_1'$   $M$  conjugate  $z_1'$ ,  $d z_1 d z_1'$  I have to use two dummy variables right because I am considering two integrals, what is this expected value? This is  $N_0$  by 2 into, into what? This is white noise, so what is the auto-correlation function?  $N_0$  by 2  $\delta(z_1 - z_1')$  alright and that obviously now using the properties of impulse function you can write this as a single integral right, you can eliminate  $z_1'$  we can put  $z_1'$  equal to  $z_1$  itself because that is the only situation where this integral be non-zero with respect to  $z_1'$  and what is this quantity?

Assuming that the two signals are mutually orthogonal this is equal to zero alright. So this is how the third property comes about. Now this was a very useful property to take note of when we are going to work with digital modulations schemes which implies more than one

signal right particularly one more than one set of orthogonal signals right and we do have modulating digital modulations schemes or signalling schemes which are based on orthogonal signals and therefore we are going to have to imply orthogonal matched filters and this property will be useful to remember when we are analysing the outputs of such matched filters alright.

So that was to some properties of matched filters so further properties of matched filters and finally before leaving this properties of matched filters

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Passband Matched Filters:

Passband Pulse  
 $s(t) = s_R(t) \cos 2\pi f_0 t - s_I(t) \sin 2\pi f_0 t$

M.F.  
 $s^*(-t) = s_R(-t) \cos 2\pi f_0 t + s_I(-t) \sin 2\pi f_0 t$

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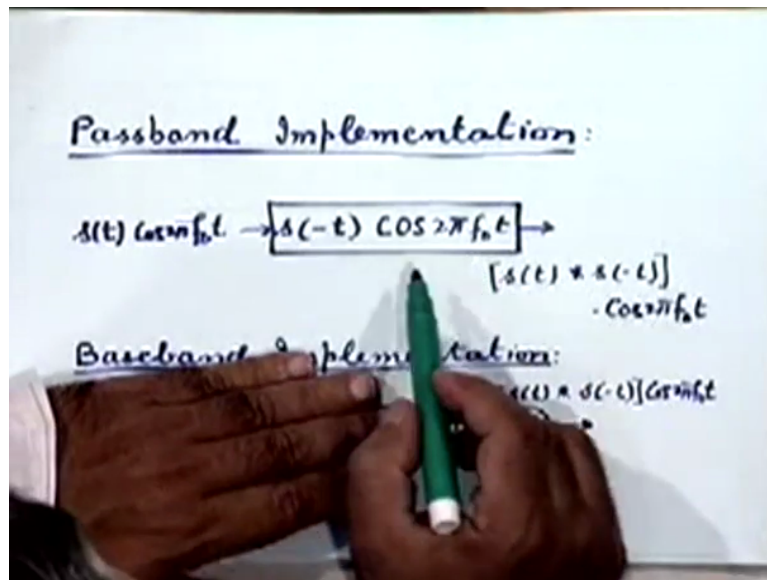
 $s(t) = s_R(t) + j s_I(t)$   
 $s^*(-t) = s_R(t) - j s_I(-t)$

One last point that I like to discuss is regarding passband matched filters because will have both baseband as well as passband modulations to worry about. When I say passband matched filters I am essentially referring to the situation when our input pulse may not be a baseband pulse it maybe a passband signal passband pulse which means it is embedded onto a carrier alright.

So what kind of matched filter will have? Offcourse the definition will be the same but lastly I would like to appreciate both mathematically as well as physically what these, how these qualities can be specified and what will they look like. So typical passband pulse we can write as if you remember as one (( ))(29:02) real part modulating a cosine carrier as well as in imaginary part modulating a sinusoidal carrier at the frequency  $F_{sub O}$ , right, so what can you say about the matched filter? Well if it is a real signal like it is over here will be simply  $S$  minus  $T$  right which therefore can be written as  $S R$  so this will be the matched filter this is the passband pulse this is the corresponding matched filter.

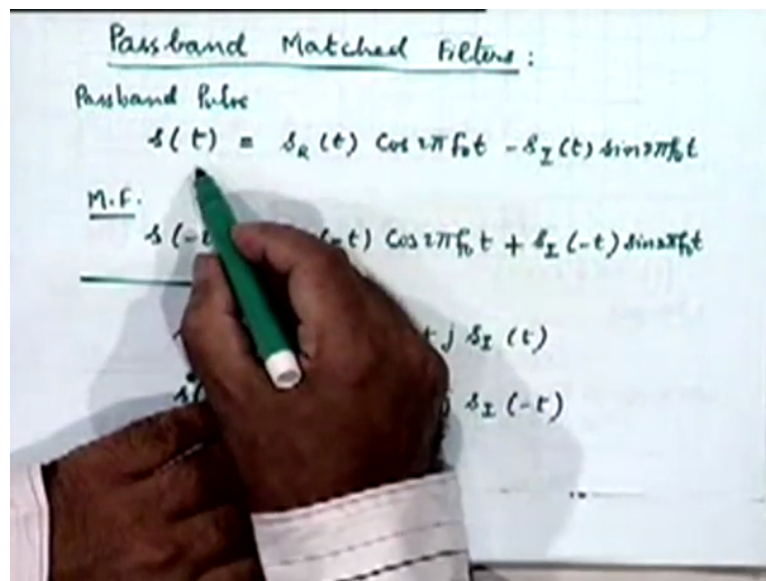
Just put T replace T with minus T everywhere this will remain cosine 2 Pie of knot T and this will become minus S I S sub I minus T sine 2 pie this will become actually plus right. You can as well alternatively think of the signal S t just to recapitulate for you as a complex envelop like that right, isn't it? Corresponding to this we can denote this passband pulse by a complex signal which is baseband essentially in nature and the corresponding matched filter will then be essentially S minus T which will be S R t it will be S conjugate minus t right, it will be S conjugate minus t which will be S r minus t minus J SI minus t ok. So this is the mathematical description of the passband matched filter both at the passband in the passband notation as well as in the complex notation ok.

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Physically this is the I have illustrated or depicted this situation for the case when S I t is zero S sub I t is zero

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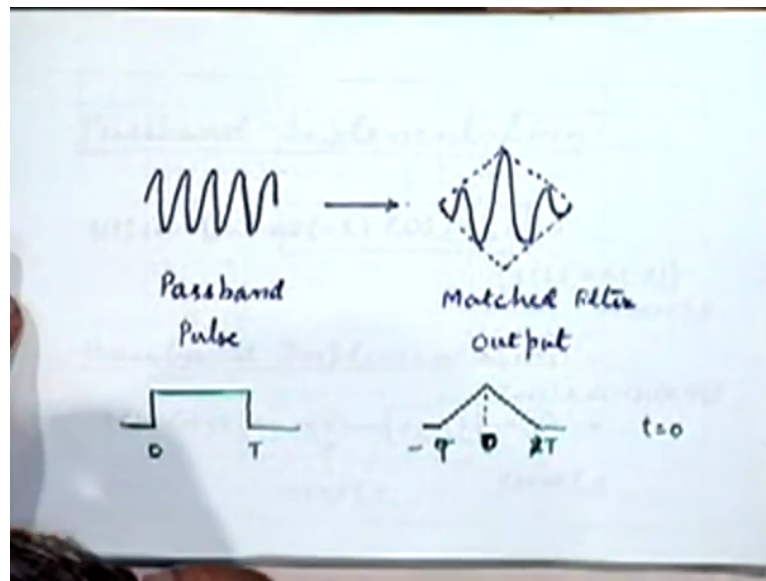


That is we are considering a passband pulse with only cosinusoidal carrier to which the baseband pulse maybe modulating alright. So obviously the corresponding matched filter will be simply this because only the first part of it is there and the output of this I like you to show that the input is this pulse  $S t \cos 2 \text{ Pie of knot } t$ ,  $S t$  is real and infact this whole signal is real and well here also the whole signal is real but I am making a quadrature component zero for this picture here right.

I am essentially taking  $S t$  equal to  $S r t$  equal to  $\sin 2 \text{ Pie of knot } T$  infact I am removing this  $S \text{ sub } r$  also I am just calling the input signal as some baseband pulse into  $\cos 2 \text{ Pie of knot } T$  the matched filter corresponding to that will have an impulse response  $S \text{ minus } T \cos 2 \text{ Pie of knot } t$  and this is what I like you to prove when you pass this through this the output will be again a passband pulse who's envelop will now look like this, it is a convolution between the baseband pulse  $S t$  here and the baseband pulse  $S \text{ minus } t$  this modulating the carrier  $\cos 2 \text{ Pie of knot } t$  right.

Input is a passband signal the filter is obviously a passband filter it is now tune it has its passband around  $F \text{ knot}$  right it is obvious from here isn't it? The output also will be passband pulse whose envelope will be given by  $S t$  conjugate  $S \text{ minus } t$  not really envelope here the output will be this into this right.

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Now let me illustrate this, this maybe your passband pulse right, suppose it was a baseband rectangular pulse, what would have been the matched filter output? It is the auto-correlation function of the input signal right, so what would be the matched filter output corresponding to this? Between  $0$  to  $2T$ , right, the peak coming at  $T$  right.

The same thing when you do in the passband domain that is when you consider passband pulse and you convolve with itself or will not itself but with it is matched filter impulse response, the output will be looking like this ok with the envelope being the same as corresponds to the envelope of this triangular pulse which will be triangle right that modulating the carrier frequency same frequency that's is above.

Student: I wonder a maximum of  $(())$ (34:43) the lower case come s at zero

Professor: Well depends on wave and sampling I am taking a casual filter here but if you are taking the same filter that we are considering earlier yes it will say minus  $T$  to plus  $T$  that is when we are assuming that the peak occurs at  $T$  equal to zero. It depends on where you make the peak occur right, by introducing delay I can always make it occur somewhere else right. So that is really not very important absolute time instance are not important right. For this discussion if you say zero to  $T$  then it is better to call it zero to  $2T$  to keep the causality in picture but if causality is not a point then I can write minus  $T$  to plus  $(T)$

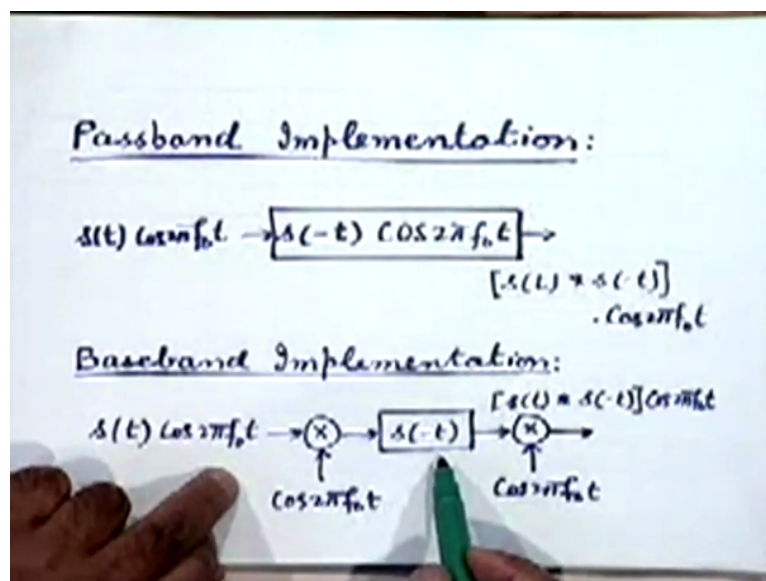
Student:  $(())$ (35:30)

Professor: Yes, this is a very interesting question and I will just come back to that. The answer is that no matter whether you are considering the in phase component other quadrature phase component since the peak has to occur here. See in general the output will be let us look at the complex representation in general the output of the matched filter will not be complex that is will contain both in phase as well as quadrature phase components right.

But at the sampling instant where the peak occurs  $T$  equal to zero for the complex notation this will be purely real for the quadrature notation the quadrature component here will be zero, there will be no sinusoidal component at  $T$  equal to zero they look like this but offcourse the details will change on as you go away from zero, I will talk about this point (in a minute).

So is this ok, this is how the passband situation will be the input pulse of this kind will get modified to a pulse of this kind after matched filtering corresponding to the baseband situation here right after matched filtering will get this kind of thing.

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Now we could as well implement this passband filter by at baseband by first taking this pulse to baseband that is translating the spectrum of this pulse to baseband by multiplying the cosine to pie of knot  $T$  during baseband matched filtering and then re-translating it back to passband.

If we need the thing I mean it depends on what you want, right my purpose is to do matched filtering of this pulse right and to produce the corresponding output one way is to have build



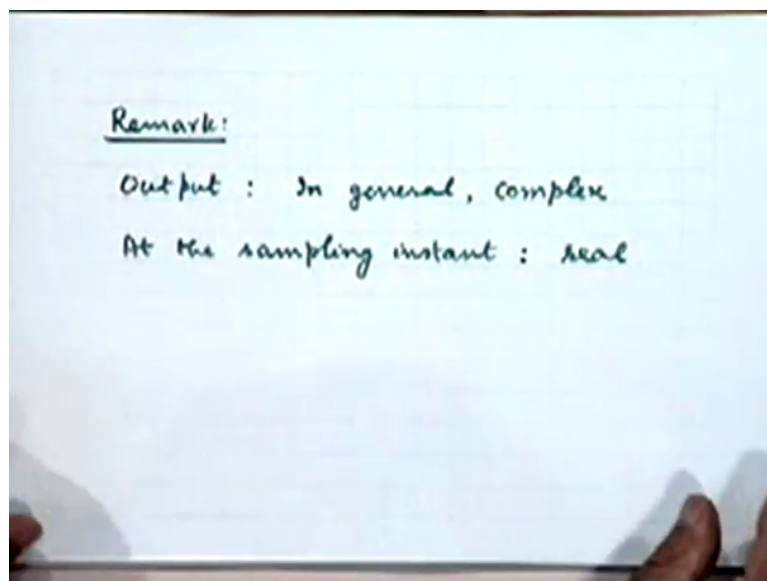
that passband matched filter with this impulse response. Another way is first to bring the pulse to baseband by down translating the frequency, during matched filtering at baseband and then taking it back to the end to the required frequency right. It depends on whether this type required or not, will depend on the application alright to which you are trying this.

Student: ( ) (37:55)

Professor: Ok that is a good question, that will not be required because after all what is a purpose of that low pass filter? To eliminate the  $2F$  component, that will be eliminated by  $S$  minus  $t$  itself because  $S$  minus  $t$  is a impulse response of it baseband filter essentially a low pass filter. If  $S$   $t$  is a low pass pulse, right, so it will automatically eliminate the  $2F$  component, that is a primary purpose of putting that low pass filter to remove a  $2F$  component resulting from this multiplication that will be carried out by this matched filter itself.

So there is no need to show I separate to pass filter, alright.

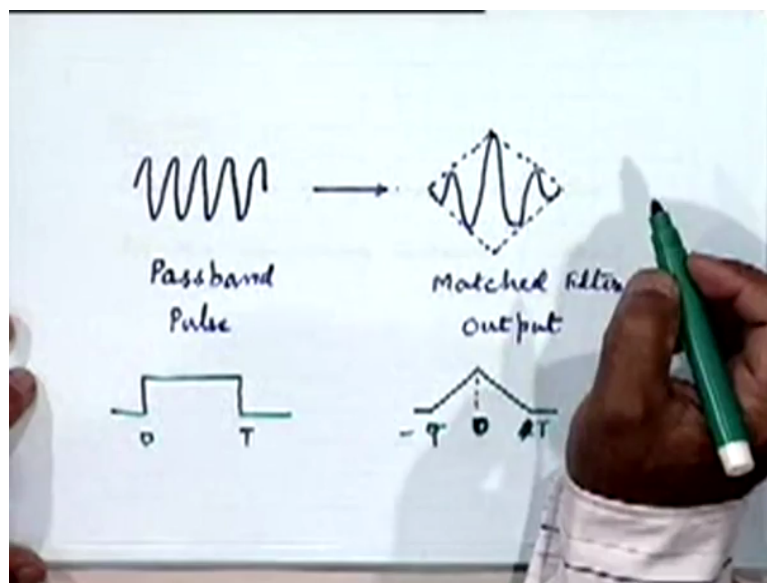
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So finally I will just come back to return to Varun's point that he made by means of a remark that is in general the output let me consider a complex notation, for a passband pulse I can represent it by a complex input distant pulse right the corresponding matched filter will also be a complex valued impulse response alright. The corresponding output in general will therefore be complex because it is going to be convolution of two complex functions complex valued functions.

Input is a complex pulse shape, the impulse response is a complex valued impulse response, the output in general is complex, right, but at the sampling instant, what is going to happen? Is it going to remain complex at the sampling instant, what is the output? We have seen that earlier, it is mod square is  $A^2$  and its value is  $A$ , is always going to be real. At the sampling instant the output is going to be proportional to the energy of the signal right. At the sampling instant the output is not complex it is real. So although the example that I have showed you in these two pictures was for the case where the SI component was missing right and the fact that it is going to be a real at the sampling instant is always going to be true right.

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That is this output is going to peak with a real value at this point as you go away from the peak will have both real as well as imaginary values non-zero imaginary and zero values offcourse then you should draw two waveforms rather than one. The imaginary waveform will go through a zero at  $T$  equal to zero right. It will be a slightly different kind of function. So that when you take the magnitude square at  $T$  equal to zero, the magnitude that is contributed largely infact only by the real part and there is no contribution from a imaginary part.

So that I hope answers your question that is, for a general baseband passband signal which has both real as well as imaginary components or in phase an quadrature components at the sampling instant the contribution will be only the in phase components, will come only from the in phase component and not from the quadrature component. Will come in the form of a what in phase component. Output will be only in phase at that point. Offcourse contribution will be there from both, any questions? Ok

So with these properties of matched filters in the background we now come to our main business and that is de-modulation. Before I come to that if there is any doubt of any kind let us discuss that briefly and get us clarify. Anyone has any question on the various properties of matched filters we have discussed so far? No questions? So let us therefore come to digital modulations and the first class of digital de-modulation we consider or binary (de-mod) binary modulations (and)

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Coherent Demodulation for Binary  
Waveforms:

$$V(t) = V_R(t) \cos(2\pi f_0 t + \theta) - V_I(t) \sin(2\pi f_0 t + \theta)$$

$$= R(t) \cos(2\pi f_0 t + \phi(t))$$

So I will discuss the subject of coherent de-modulation for binary waveforms or binary signals ok and it will be kind of convenient to restrict our attention merely to the passband modulations.

Although baseband modulations also will consider because once we have the results or the discussion for passband modulations the corresponding result for baseband modulations now become trivial particularly you know the fact that even a passband modulation can be expressed in terms of a complex baseband representation. So now the general passband waveform received waveform that you will get at the receiver or in general we had just discussed a few minutes ago will be this ok, whatever difference I have not talking about  $V(t)$ , the received signal which contains both the transmitted signal plus noise right.

But since we are assuming that everything is around I want some frequency of  $F$  knot even the noise can be represented in the same quadrature form right. So  $V_{sub R}$  contains  $S_{sub R}$  plus  $N_{sub R}$  and  $V_{sub I}$  contains  $S_{sub I}$  plus  $N_{sub I}$  ok the, everything else is same as we discussed a few minutes ago except for the presence of  $\theta$ , what does this signify? This

signifies that you might have transmitted  $S \cos(\omega_c t - \theta)$  but what you will receive is that plus a phase shift the carrier will have undergone some phase shift, that is right.

So it is same, I have put it same, yes ofcourse, because afterall what is this? This is just a quadrature representation, you could as well think of this as some envelope  $R(t)$  into  $\cos(\omega_c t + \theta)$  right, this is afterall different way of writing this. Basically the carrier has undergone its phase shift and its quadrature for this is how it will look like. This should be  $\theta + \phi$  I am sorry, the general modulation is given in terms of  $R(t)$  and  $\phi(t)$  right for example you know if it is binary PSK  $\phi(t)$  will be plus minus  $\pi$  right depending on the input pulse sequence this is envelope it maybe rectangular it could be anything and it could be represented the quadrature form like this.

The  $\theta$  is the phase shift that has taken place in the carrier during transmission right yes Deepankar what is your question?

Student: ( ) (46:16)

Professor: Yes it will effect, this envelope actually see depends on what you are talking about. If this is purely signal then this will be purely signal attributes but if contains both signal and noise then those attributes will also appear here.

Student: ( ) (46:35)

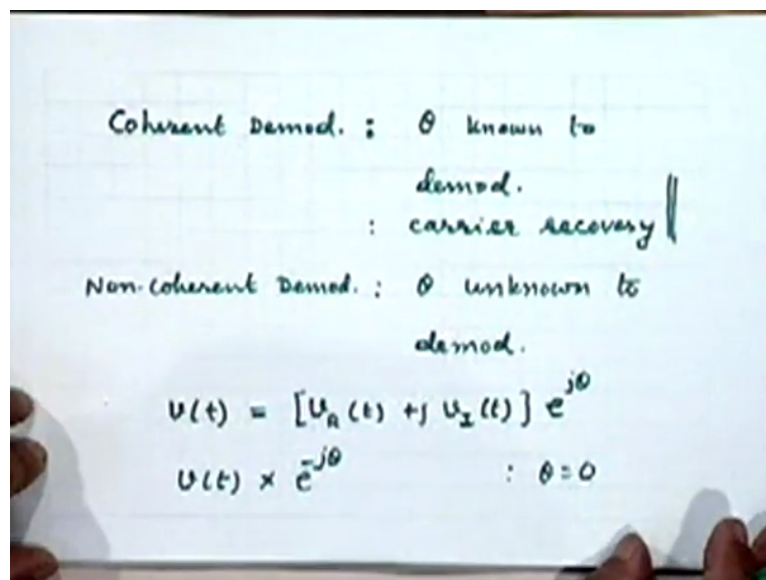
Professor: That is right but therefore if  $V_R$  contains noise if  $V_R$  is  $S + N$  right, then  $R(t)$  will be whatever that is square of that plus square of this that is all they are, we are assuming that this contains both signal plus noise so therefore envelope and phase are both noisy here right then are the two transmitted values they will degraded by noise and this is a noisy envelope and this is a noisy phase but in addition to that noisy phase which is a effect of noise we assuming the presence of a phase shift  $\theta$  which is essentially true with the fact that the transmitter and the receiver are at physically different locations and therefore the carrier in proceeding from one place to another is associate with the time delay and different phase shift right, was it clear?

This source of this phase shift is the physical separation between the transmitter and the receiver right. You might have transmitted the carrier with zero phase but due to propagation delay the received carrier will be with some non-zero phase whose value may or may not be

known to you right, in general it is not known but still even though in general theta is not known still we differentiate between two situations for de-modulation that is theta known and theta unknown. How can theta be known? It can be known provided at the receiver you have a carrier recovery circuit, phase estimation circuit right.

If you have a carrier recovery circuit whose purpose is to find out the phase of the incoming carrier then you can assume it to be known. If it does its job properly alright so for the purpose of the discussion this was the reason I want to explain the term coherent de-modulation with each are familiar in your (48:38) communication process but I want to reemphasize that here for us. When we talked about coherent de-modulation what we are essentially saying that theta is known to the de-modulator. That how can it be known? As I said just now, maybe by carrier recovery circuit.

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So which is a separate task which I will not discuss at the moment but when we talk about coherent de-modulation it is therefore implied that we have a signalization circuit which we call the carrier recovery circuit whose job is to obtain the true value of theta as it is coming in ok. In case for some reason it is not either practical to do it or expensive to do it right, will have a situation where theta is not known. Suppose we have a situation where we cannot make a carrier recovery circuit work properly therefore it is not practical to in that situation to now to find out the value of theta because our carrier recovery circuit doesn't work.

Or it may turn out that we may will want a very cheap implementation of the receiver in which we don't want to do a phase estimation an extra job, then we have the situation of what

we call non-coherent de-modulation. You are familiar with these terms nothing new for you so coherent or non-coherent has to do away a carrier phase being known or carrier phase being unknown. Now suppose  $\theta$  is known because at the moment I am discussing coherent de-modulation right I will, digital communication except for a few very special cases most of the time will be talking about coherent de-modulation. There will be a few situations like F S t where we can talk about non-coherent de-modulation.

But in all other cases like various versions of PSK or combine amplitude phase shift keying because some information is present in the phase which just cannot afford to do non-coherent de-modulation right. Except in FSK where information is not in phase but in frequencies right in that case you can think of doing non-coherent de-modulation, but if the information itself is somehow in buried in the phase either directly or implicitly then there is no question of using non-coherent de-modulation right, so that point you must remember.

Therefore most of the cases will be discussing the coherent de-modulators here and when  $\theta$  is known we can write the received signal in its complex form as  $V \cos(\omega_c t + \theta) + j V \sin(\omega_c t + \theta)$ , so how do I show the phase shift here in the received signal in the complex notation into  $e^{j\theta}$  to the power  $j$  and as I said if it is perfectly known I can remove the effect of  $e^{j\theta}$  to the power  $j$  by simply multiplying the received signal with  $e^{-j\theta}$  right and from then onwards I can assume as if I have received the signal with no phase shift. So we can as well assume  $\theta$  to be zero ok.

So if I having a coherent de-modulator based on the fact that I have a carrier recovery circuit what basically I do is from the carrier recovery circuit I estimate the phase  $\theta$  and then use that estimated phase to multiply  $V t$  by  $e^{-j\theta}$  so as to remove the effect of  $\theta$ , somebody is talking a bit too much please it will disturbs me a little bit quite. So from then onwards from here onwards I will assume that  $\theta$  is zero ok and offcourse we can do so in coherent de-modulator.

Now let us with this introduction to coherent de-modulators let us briefly come to or it is a good stopping point will stop here.