Digital Communication
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Module 01
Lecture 23
Digital De-modulation: Matched Filters

If you may recollect the subject that we have been talking about before this has been some gap now was digital de-modulation right and specifically we are going to look at a very important component of we in the process of looking up looking at a very important component of digital modulators which we call them the matched filters.

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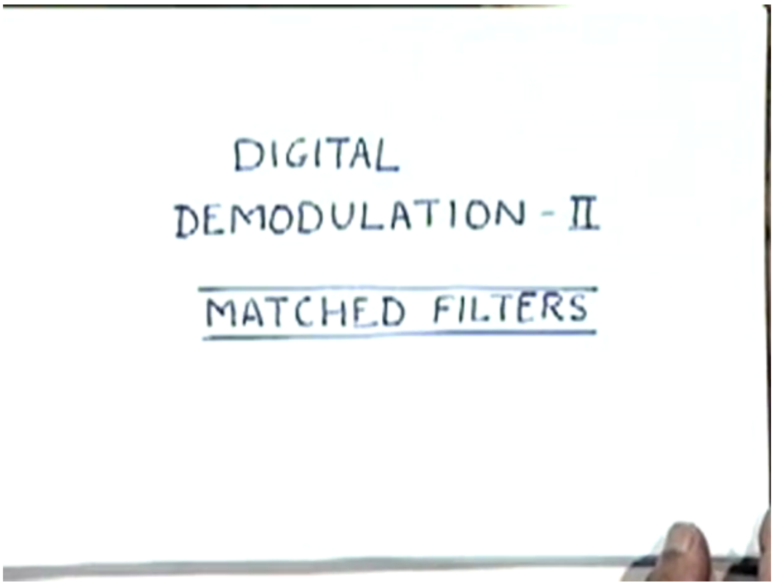

Just to quickly recapitulate for you, what we have been talking about so far this is just a very quick review of what we have talked earlier because there has been some gap.

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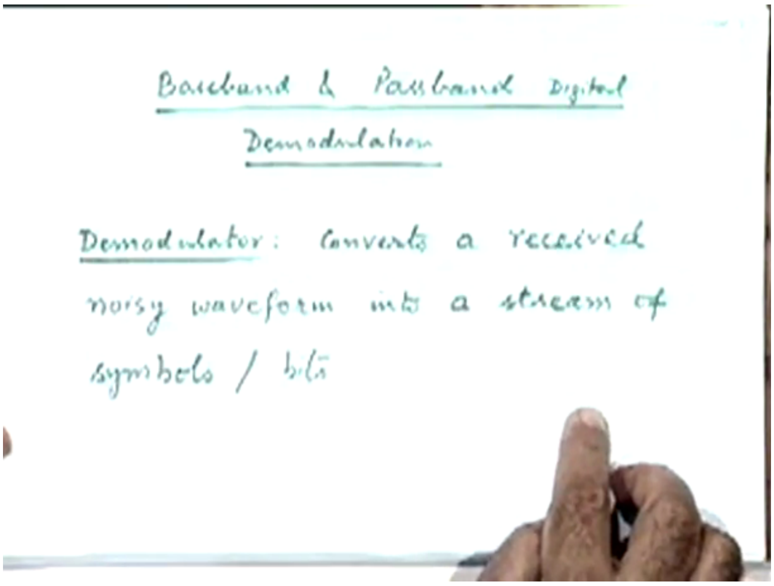

The job of a de-modulator is to convert the received (signal) noisy waveform into stream of symbols or bits which were initially transmitted right and to cut the story short of our last time discussion the way we said this will be done is to pass this noisy signal that you have through an appropriate filter.

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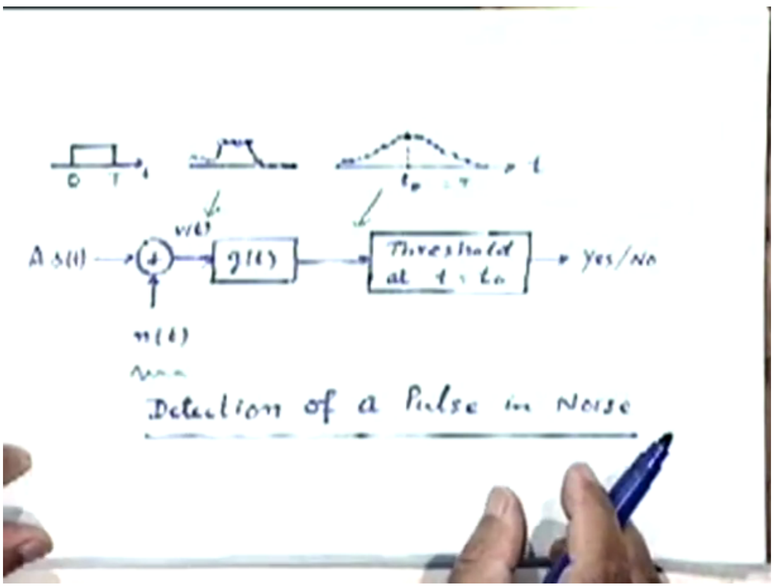

Such that it will pass as much of the signal as possible and reject as much of the noise as possible and since ultimately we are only interested in each interval symbol interval the detection of that symbol whether it is this value or that value we need to sample this output at a specific time instant and it is a that time instant that we like the signal to noise ratio to be the largest right.

Other sampling instant and after this is sampled we pass it through a threshold at that specific time instant rather we compare with the threshold and then decide whether this value corresponds to a particular symbol value or some other symbol value right. This is the approach that we have to with that we have decided to follow and the crucial component in all this is this filter G t, right, because it is the job of this filter to produce an output which frame sample at a specific time instant will contain most of the contribution from the signal and as smaller contribution is possible from noise right.

So basically that is the important thing we need to look at how to design such a filter and we therefore went through some little bit of exercise as to what this filter does to the signal and noise that is coming along right.

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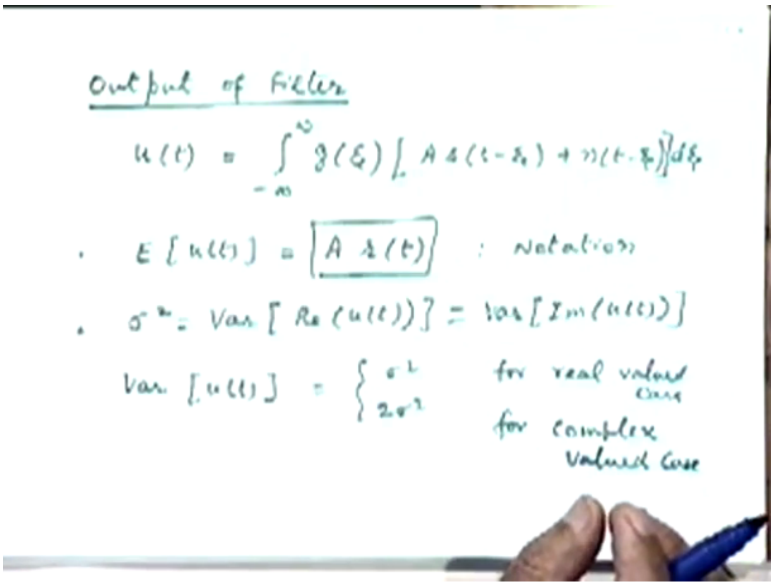

The specific problem that we have presently decided to work on is the detection of a pulse in noise, we are not even looking at the complete digital communication problem here right. A simplified our problem of interest look to that of finding out whether or not a pulse exists in a given interval of T seconds right, because once we know the answer to this question how to solve this problem we can extend it through the de-modulation of sequences where you may have multiple valued sequences (avail) possible of in a given interval.

Example a pulse could have multiple amplitude levels it could have different shapes you know we could take care of all those things but to start with to simplify the discussion the detection problem you are looking at is a very simple one you have a pulse coming along with noise but you are not sure lets say whether this pulse exists or not in this interval of T seconds and you want to detect whether what you are receiving is contains only noise or contains that pulse and noise in the presence of noise.

So this is a process this is the problem we are looking at. Therefore what we do is we start by looking at the output of this filter in terms of the input V t, which in the presence of signal will be consisting of these two components right the signal will be A S t plus N t when pass through the filter will get an output which is a convolution of these two. We have gone through this derivations before I am just repeating for you to capitulate.

Therefore when the signal is coming along with noise we have really at the output of the filter contributions both from the signal as well as from the noise right, the contribution of the signal comes in the form of the average value of the output because the average value of noise we have assumed to be zero right. So the average value of output will obviously corresponds to the signal which is deterministic right, because it will be essentially equal to this component whereas the average value of the noise will be zero right.

So therefore to look at the signal contribution we look at the average value but to see the effect of noise we have to look at the various of this output around the average value right because that is uncertainty associated with the output value and that will be somehow dependent on the second term the noisy term right. so we will therefore like to look at both the average value of U t which contains information about the signal and the various around the average value at the sampling instant which contains information about the how much noise has managed to pass through the filter right essentially that.

Student: (())(06:48)

Professor: R t is that yes A R T represents the first convolution right, that is, because average value of U t will be simply the first term right which I am denoting by R T, so R T is the average value of or expected value of U t fine.

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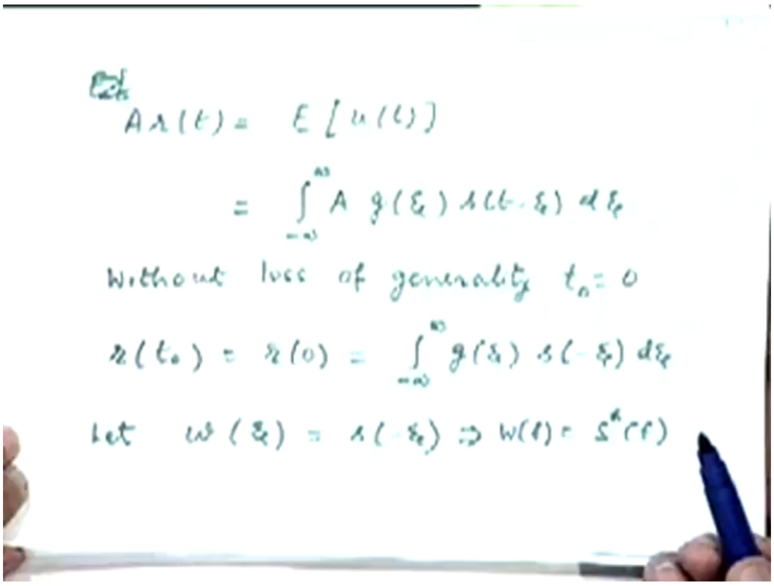

Ok and I think if you go through the steps that we went through we finally arrive at an expression for R T which was this which is offcourse straight forward which is a basic point and then through a little bit of manipulation and the assumption that will take T0 the sampling instant to be 0 right just for a, it can be really arbitrarily chosen, only thing is depending on how you choose it, you will either get a casual filter or a non-causal filter.

At the moment will not provide too much of our that aspect. So depending on where you select your T0, you will either land with the casual filter or a non-causal filter but just to simplify things lets say T0 is equal to 0

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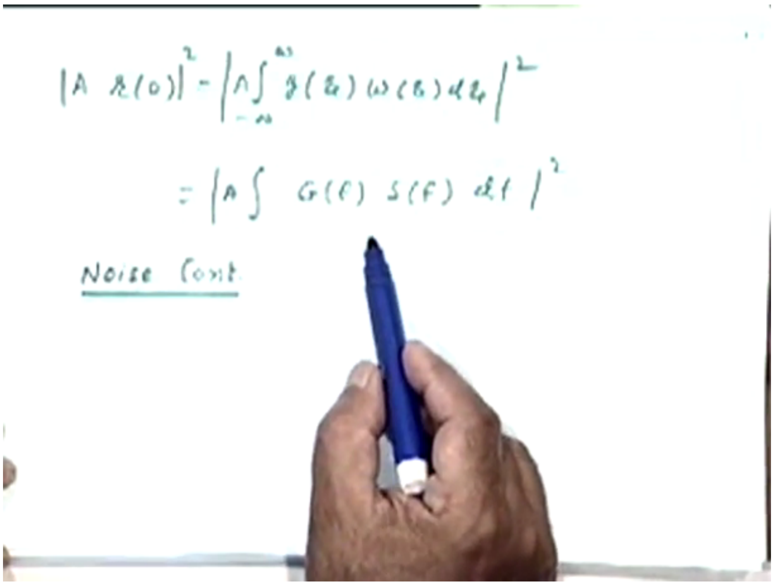

And then essentially your sampling sample value at T 0 equal to 0 is either this or in the frequency domain by Parseval relation is this. This is a point at which we had stopped last time. I think go through all this because there was some gap and I thought we should all be where we were quite nicely so that we can proceed from then onwards.

Ok so this is an expression for this gives us an idea about the signal contribution to the output average value at T equal to 0 or T0 equal to 0 sampling instant chosen to be zero right. we like to now look at what is the corresponding noise contribution in terms of noise variance because that will help us to setup to write down an expression for the signal to noise ratio will define the square value as a signal power right the square value of a mean as a signal power and the variance as the noise power and the signal to noise ratio will be define as a ratio of these two quantities but larger this ratio the better it will be for our power detection right.

So this is a point at which we have stated. As far as noise power calculation is concerned that should be extremely simple to appreciate let's look at this picture again.

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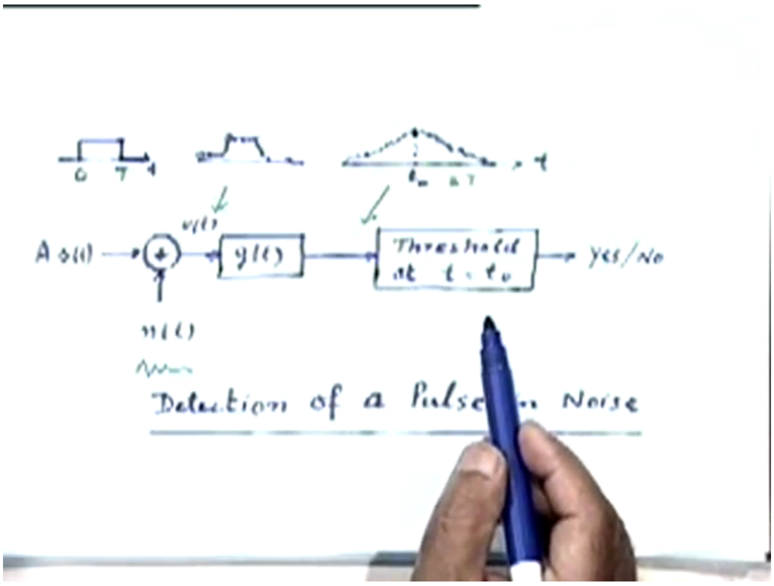

This V t contains signal and noise hence being passed with G t, best way to calculate the noise variance that the output of any filter is to simply work in the frequency domain right.

Suppose this has this noise has a power spectrum N f alright, then what will be the power spectrum of the noise coming out the filter? The input power spectrum multiplied by G f mod square right and the variance of the output noise will be simply the area under its power spectral density function right. Whatever the power spectral density function of this noise the variance of noise which is the total power in noise total average power in noise is nothing but the area under its power spectral density function right.

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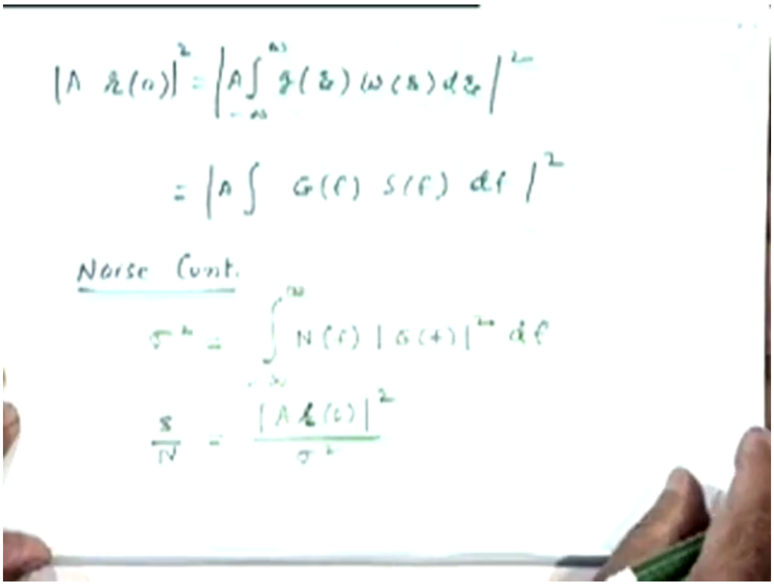

So the most convenient way of writing down this expression is to go to the frequency domain and write sigma square directly as equal to first we write down the expression for the output power spectral density function which will be N f which is a input power spectral density function multiplied by G f mod square and the area under this density function which is a integral of this quantity is the output noise variance. So that is very straight forward.

So we now have an expression in the frequency domain for signal power output signal power at the sampling instant an expression in the frequency domain for the output noise variance or noise power, we all agree with this? Is there any doubt about any of this computations? We can therefore set up an expression for the signal to noise ratio by defining this to be the representative of the signal as A times R0 mod square upon sigma square which then obviously is a ratio of this two phonetics ok alright and our job is to choose a filter transfer function G F.

What is the function which is under our control? Or which we want to fix, is the filter right. So we want to optimize this transfer function G F so as to maximize this. Ideally what we would like to have is, the filter which will completely eliminate the noise and pass only the signal but we know that will be impossible, why? Because the signal and noise spectrum partially overlap with each other isn't it? The noise spectrum maybe wider but there is some amount of noise present in the same band as the signal is present right.

So it will be impossible to eliminate both of them or eliminate noise completely and pass the signal only right. So at best, the best compromise we can hope to get is maximization of this quantity which will be a useful thing to do alright. So lets therefore write down this expression in detail and see what we can do about it.

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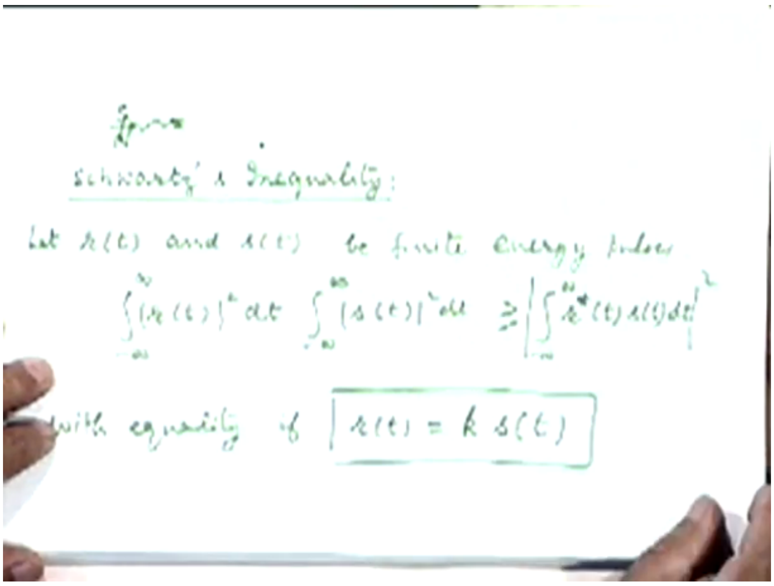

Or before I do that I think that much is clear has to what this expression will turn out to be. To do this maximization it is clear what we have to maximize, we have to maximize the ratio of these two quantities with respect to the function G F right.

Now this is a slightly different kind of maximization or minimization that you might have done in your calculus, have you done the calculus of variations? No, you don’t know, you are not familiar with calculus of variations which tells you how to maximize functions of functional, the function of functions right. Because here is a function signal to noise ratio of a function G F, when you choose a function G F so as to maximize this parameter right. Well that is a slightly different kind of area in mathematics and since we are not that familiar with it in your class we will not use that. Will use a different result which is very simple result and I will not prove it, I will just state that result from math’s.

It is known by the name of Schwarz inequality, are you familiar with Schwarz's Inequality, ok, so that is no problem there. Where have you used it? In what context did you used it?

Student: (())(14:41)

Professor: Just as a result by itself, ok so all I need to do is recalculate what that result is for you and then we will see how that is very useful in obtaining the derivation of our matched filter right or in doing that maximization that we want to do alright. This results as follows, suppose we have R t, so let R t and S t be our finite energy pulses right, they maybe complex signals both R t and S t maybe in general complex. It maybe real valued or complex valued. Then this result holds take amplitude of R t square integral of this function with respect to T multiplied by S t mod square D t integral of this, this product is always greater than or equal to the integral of R conjugate T S t dt, mod square thank you ok.

With equality do remember when are the two sides equal? If and only if well R t is or we say constant time S t, scaling factor doesn't matter right. So the two sides will become equal if that is way. In other words suppose I take the ratio of this upon this, I take this to the right hand side ok I will come to that a little later. So that was the statement of Schwarz's Inequality and will like to now use this to do the maximization that you want to do. Remember this is our starting point, why we are interested setting up the ratio S by N from these two expressions.

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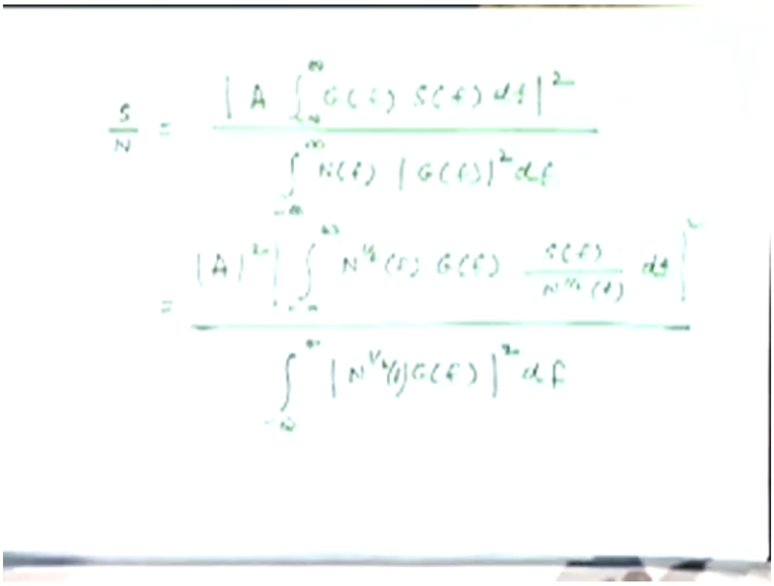

Now let write down this expression S by N will be mod A integral GF SF DF square upon NF times now do you see how to apply it and that is obvious.

Student: data or numerator.

Professor: Some manipulation is required before you can apply it straight away because you have to identify functions.

Student: (())(18:09)

Professor: Yes that is true, so we can write it as mod A square but will do is will rewrite the function here let me see the approach is the following will identify this as what? As a product function Rt St ok and we have to find out we have to then sorry, this is one of the functions where this is a product function, this is a product (func) we have to take this as a product function and this as one of the functions and therefore we like to rewrite this in terms of that product right. With this as the square of another functions, this is square of another functions right.

So we will write the square root of this into GF, into SF upon right, DF mod of this whole square and obviously you can write this as root NF GF. Now you can see the connection between Schwarz's Inequality and what we want to do ok and it is nice if I could have figured both of them in front but I think only one of them I can assure a time.

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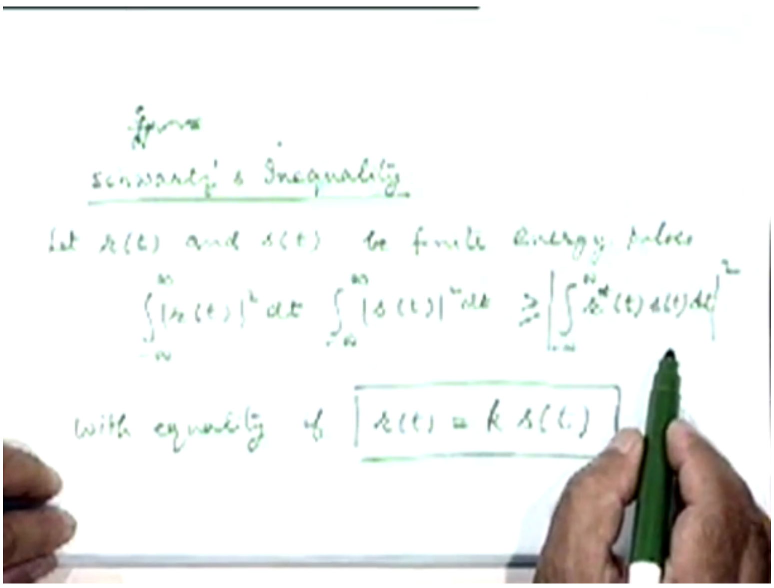

So I have constructed one of this functions this is in the denominator and this is in the numerator right.

So what we are saying is we have the ratio of this upon lets say this, this upon this, this ratio is going to be less than or equal to a specific quantity which is this right always, at best it can be equal to this right. So what is the maximum value of this ratio?

Student: it is equal to the two functions from S and equals

Professor: There are two things, one is at what point is the maximum achieved the other point is what will be the maximum value, right. The point at which maximum value achieved is, when this condition is satisfied right and the maximum value is that maximum value of the ratio is well whatever it is alright. So these are the two things. First lets look at the maximizing point, right, in the maximizing function.

The condition under which this ratio will become the largest which is what we want to do, we want to maximize this ratio right the condition for that is that let me write down. This quantity is constant times this quantity right actually not. Some other, see the original result is R conjugate T St, so the conjugate of this quantity should be equal to a constant times this quantity right.

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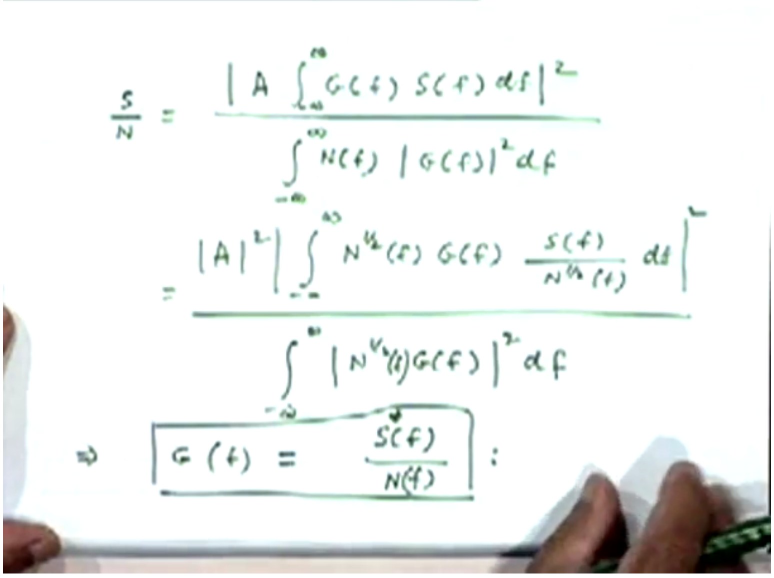

In other words GF maximization would be done for Gf equal to Sf S conjugate F upon N f is a real function, why? Why N F is a real function?

Student: we have assumed Gaussian Noise.

Professor: No it is the power spectral density function, a power spectral density function has to be real right. N f depends on noise, power spectral density function any power spectral density function has to be real positive function right. So that is why infact we can talk about a square root otherwise we can't even talk about its square root right. So it will be, what is this? This is the function G f which will maximize this ratio. How does it come? It comes from the condition that the conjugate of this or this function should be the some constant times a conjugate of this.

Student: the denominator is taken only of the product terms. Sir basically times the product of two integrals that R t square multiplied by

Professor: That is what I have tried to explain to you this basic result, I will take of one of this here right which is then there is a perfect matching between this and this all we saying that this ratio is less than or equal to certain quantity which is a constant which depends on S t, we are talking about the what the value is.

Student: maximum value will be the other functions.

Professor: Right, we are not looking at the maximum value here we are looking at the condition under which this maximum will be achieved right, that condition will be this condition R t equal to K S t. We identified what is R t and what is S t right. We have identified in this what is which corresponds to R t, which corresponds to S t right and then we are using that fact in arriving at this result, is it clear?

Student: you put as a quantitative only.

Professor: Yes why? Because this result here says R conjugate T St, right, so one of them has to be identified as a conjugate function or as a unconjugated function. I have combined that. Actually root N f at G f is equal to S conjugate F upon root N f and then I have taken root N f to another side and finally we get this result ok. So this is expression for filter transfer function which will maximize this signal to noise ratio at a sampling instant T equal to zero, T0 equal to 0 alright,

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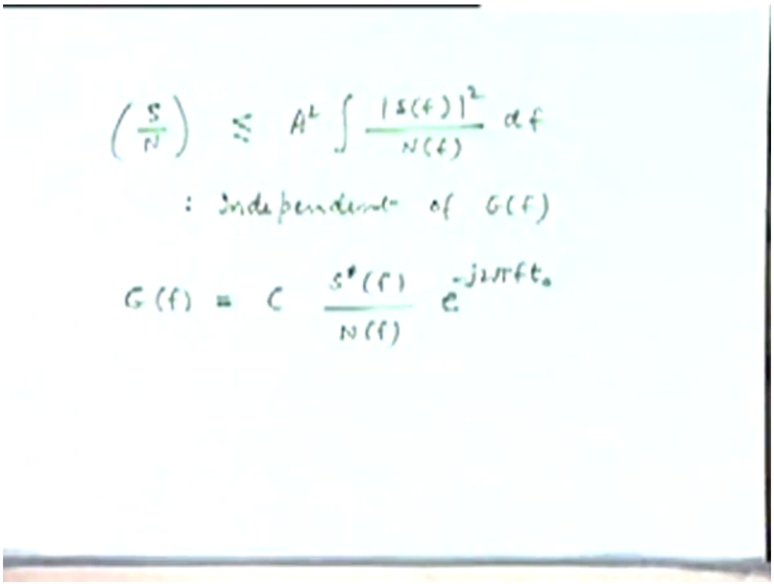

And the maximum value is maximum value of the signal to noise ratio is obtain from the fact that this is going to be less than or equal to well, now we can do that maximum value we can obtain by substituting this in this expression.

This transfer function will maximize the signal to noise ratio at the sampling instant T sub 0 equal to 0, the maximum value of the signal to noise ratio can be obtain by substituting this expression for G f over here in this expression and that we can easily check, will be simply equal to S f mod square upon N f ok and the interesting result that you will see here is that the expression for the signal to noise ratio therefore obviously does not depend on G f right.

So maximum value of signal to noise ratio is it is a maximum possible value and it is independent of G f offcourse it is been derived from the fact that G f has taken that specific value, yes please.

Student: (())(26:13)

Professor: Alright, this is it.

Student: in this thus what is special in (())(26:19)

Professor: This is one of the functions and this is the other function correct the other function is still on the left hand side right, we are looking at this ratio right. Now we want to find out under what condition will this ratio be maximum, what the Schwarz's Inequality tells us is, that this numerator quantity is less than or equal to this into the other quantity right. Therefore this upon this is less than or equal to the other quantity alright and therefore the, therefore this is what we need to maximize.

We don't have to worry about what the other quantity is right and the point that which is maximum will occur is this point alright, it is on the right hand side not on the left hand side I said it is on the left hand side ok fine. Now let us look at this equation again, suppose I want to change the condition to I want to eliminate the condition that T0 equal to 0, what modification in the result you will notice? So our optimum filter is some constant times S conjugate F upon N f, E to the power minus J 2 Pie f t right.

This will be the additional factor we have to consider, if we want to maximize the signal to noise ratio at some arbitrary time instant T sub 0 rather than equal to 0 ok.

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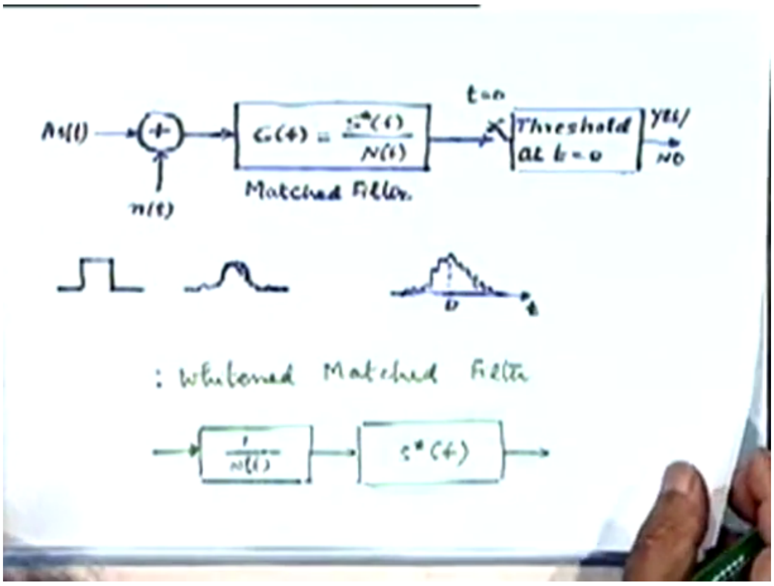

Now let us look at before I do any discussion let me show you the picture that we have in mind, here is your signal coming along with noise, so noisy signal is coming along we pass it through this filter which is whether we put the scaling factor there are not it hardly matters ok, because it will not affect the signal to noise ratio right. The scaling factor will affect the signal and the noise in the same way, so we not bother about it.

We sample the output of this at whatever sampling instant we need T equal to 0 or T equal to T sub 0 right and threshold it, that is compare it with the threshold this value and then decide whether that pulse was indeed present or whether the input signal contains only noise ok. If the value output value is below a certain threshold value that you have decided earlier then you can say that the there was no pulse present. If it is above that value that we can say some level of confidence that it is due to signal right. This output is due to the signal.

Therefore the complete receiver design really speaking not only involves things in this kind of a match filter but also deciding on a suitable value of threshold because that is going to affect your performance quite significantly. If you choose for example two high a value of threshold, what is likely to happen? Sometimes when the signal is indeed present, because of noise the level may be less and you may wrongly decide that it is absent right.

On the other hand if you choose two lower value a noise pipe might come along then there will be nothing else but noise and sometime because the noise is sufficiently large because occasionally it can be large, it can produce at that sampling instant a value which is sufficiently large to exceed the threshold and thereby causing a false decision that the signal is present.

When indeed, none is present. So we have the false alarm probabilities on the probability of missing a pulse and these are kind of dependent on how you choose a threshold.

Student: here we are maximizing signal to noise ratio but we are not maximize the probability of error.

Professor: Yeah the, you are absolutely right in pointing out that fact, the ultimate performance criterion in a digital communication system is not just signal to noise ratio, it is really probability of error because that is what we finally, but fortunately for most problems of interest one can express the probability of error directly in terms of this quantity, this parameter signal to noise ratio right. So if you maximize this effectively that is equivalent to minimizing that probability of error. Will see that when we take up digital communication problems in particular right.

Right now it is a general problem and basically I wanted to introduce to you the concept of a matched filter which is an essential component in the de-modulator. Now let me just to a very brief discussion and make you appreciate how this really functions, how this match filter so called actually this is what is called a whitened matched filter. Strictly speaking this function is called a Whitened Matched filter right. But I will elaborate on why it is so called, a few minute later. First of all how it works? Lets discuss how it works, or how it tells how it maximizes signal to noise ratio or white maximizes signal to noise ratio intuitively we can try to appreciate that.

Basically what you notice is that the transfer function will have large values at those frequencies where the signal to noise ratio is large, it will have small values at those frequencies where the signal to noise ratio is poor, input signal to noise ratio is poor right. so basically what a matched filter is really doing for you or a whiten matched filter really doing for you is, weighting the input signal in the frequency domain according to signal to noise ratio's present at different frequencies. Components which have most contribution from signal only are highlighted are given more weightage and components which are more noisy those frequency components which are basically getting information from noise or which is basically contain energy from noise are the once which are suppressed ok.

So basically choose a weighting function or a transfer function which depends on the input signal to noise ratio at different frequencies that is the basic idea and the reason why it is called a whiten matched filter is simply this, there I can think of this operation as a cascade of two filters one is this and the other is this. As if I have two transfer functions right in cascade like that, the if you just think about it if you look at the output here or lets say input contain only signal only noise, what will be the power spectrum of noise here ? It will be constant equal to one.

It will become white noise here, right. So whatever noise is present is whitened here and then offcourse if the signal is also present there also gets modified to some extent and then we pass it through a filter which is really matched to the signal itself right. So we call it whitened matched filter and this is just a term it doesn't really comes from this kind of a interpretation alright.

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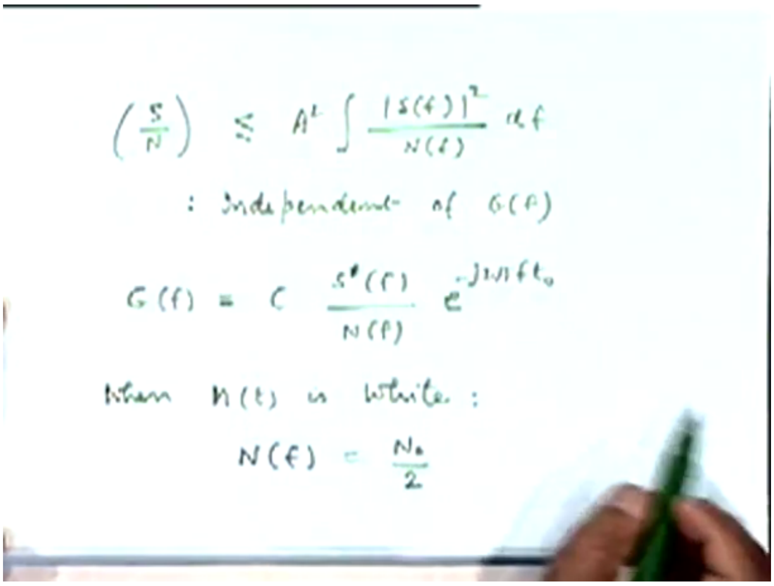

Let us take a case specific case when your noise to start with is white Gaussian noise, you already assumed this Gaussian but we are now in making a further additional assumption that your input noise itself is white right.

What does it mean? That N f to start with this equal to some constant say equal to N0 by 2 alright. Then what will be your optimum filter? You substitute that here it is C times S conjugate F upon N0 by 2 hence is N0 by 2 is a constant you can combine it with this constant C, so essentially it is equal to S conjugate F E to the power minus J2 Pie F T0 alright.

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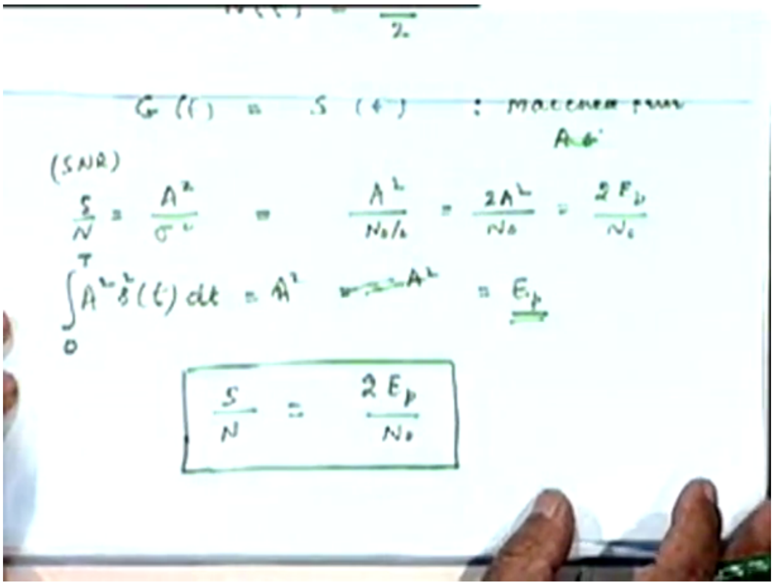

So in this case the filter that you get is G F I will forget about the constant it is simply equal S conjugate F and if you are take T equal to 0 again that is in the state right. This is really what is called a matched filter right. The other one is whitened matched filter it is more general the whitened matched filter is obviously more general than the matched filter right. Because this is for a specific case when noise is input noise is white right ok. The next thing I like to do is write an expression for the output signal to noise ratio. Which is S by N which will be essentially equal to A square by sigma square because we are assuming that your signal is A S T, let me write down if your signal is A S t such that S t is of unit energy then this is equal to A square right and noise variance output is sigma square.

Now we have an expression for this already here with us, for the general case this was the expression you have put N F equal to N0 by 2, then what will I get? What will be simply is A square sorry, this will be A square by N0 by two right or 2 A square by N 0, I think a better expression to write is let me denote this we can think of as signal energy in its time interval suppose signal is of duration T pulse is of duration T, this A square is nothing but E sub P alright. If provided integral S square is 1, we are assuming that the integral S square t is 1 or whatever S t may be finally lets say this energy is E sub t then this is simply 2 E Q by N 0.

Anyway this is expression I wanted to really write and there are some important lessons to be learned from this expression and from the previous expression for signal to noise ratio. Let us see what this lessons are, is this ok?

Student: (())(39:18)

Professor: This is rarely awkward I have computed the signal

Student: ok in general this not therefore particular, you are not doing a particular case of white noise

Professor: This is the expression for signal to noise ratio in a general case right at the output, offcourse I should remove this and say equal to when you are using a match filter right. When I put when I take the case of white noise N f becomes equal to N 0 by 2 and this is really A square integral S f mod square which is E p right. So it becomes 2 E p by N 0. So I have two expressions one the general one and the special one for the case of white noise. Lets look at the special one first, what you will find here is, that this output signal to noise ratio once you have used a matched filter does not even depend on the pulse shape.

We appreciate that fact it only depends on how much energy is contain in a pulse right the specific pulse shape is of absolutely no consequence in the determination of the output signal to noise ratio whereas that is not the case if your noise is not right you can see that . In this case the output signal to noise ratio depends on the pulse shape also depends as much on the power spectrum of noise as on the pulse shape itself. So the pulse shape itself is of little importance in situations where mostly dealing with white noise and that is the general situation for us in most communication applications right.

Because the noise is sufficiently broadband so as not to worry about the specific pulse shape. However we already seen the pulse shape is important for us for other reasons like for example inter symbol interference right and things like that. In any case it is nice to know that we don’t have to worry about pulse shape as far as performance against noise is concern as long as we make sure that there is no inter symbol interference and as long as we give sufficient energy in the pulse to have a good signal to noise ratio. Only parameter that is really importance is energy contained by the pulse ok.

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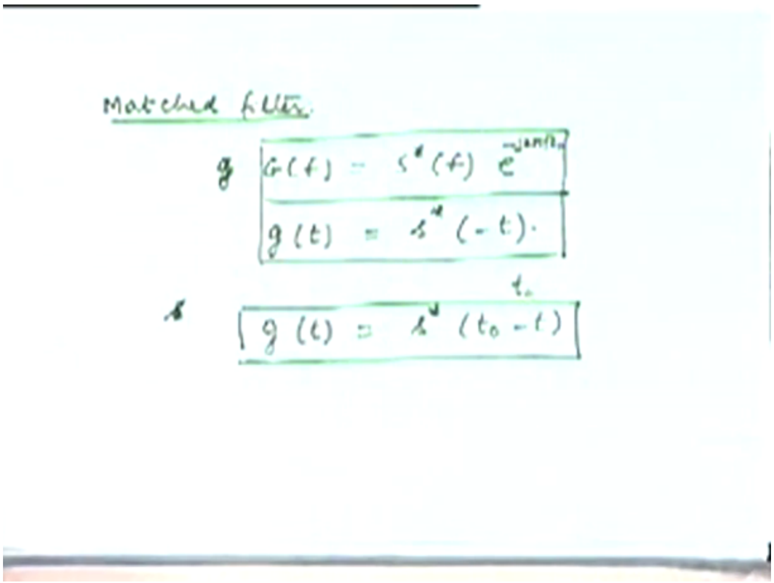

Let me write down an expression for the matched filter in time domain. We start with G f equal to S conjugate F what should be expression for G t? Can you straightaway tell me.

Student: (())(42:24)

Professor: Or will it be S conjugate minus T? In general if the signal is complex S, this is what I wanted to check. Now lets ok since there is some confusion lets avoid the talk, some here to here, from Fourier transform properties, then whatever the special case you are referring yeah but, here considering general (())(43:05) ok but you want me to do it or do you think it is ok, fine. So and if I want to generalize it further and make sampling instant arbitrary rather than things equal to 0, how will it change?

Suppose I wanted to introduce a non-zero value of (())(43:28) right, here it will be multiplied with E to the power minus J 2 Pie F t knot the corresponding thing here will be this become S conjugate T 0 minus T or is it ok?

Student: minus T 0 minus T 0.

Professor: No-no this corresponds to yeah so minus T plus T 0, which you can write T0 minus T. So that is the most general expression for the matched filter in time domain and that is in the frequency domain. Let us take a few examples, before taking examples few are the remarks.

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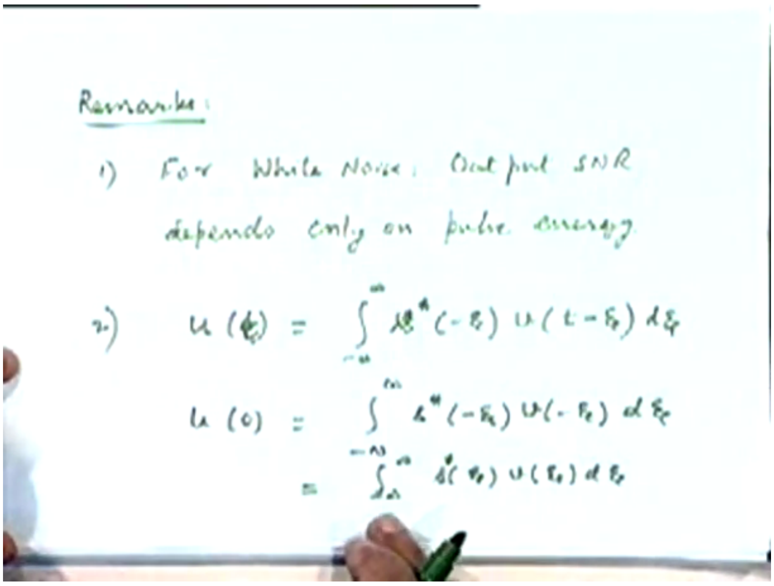

So first remark if you remember that the for the case of white noise output SNR depends only on pulse energy right. Second important point is, the nature of the matched filter output, by considering the output let us say at time instant zero again what this will be?

Your impulse response is let me write down the general expression first, U t is equal to output is input or impulse response is S conjugate minus zeta into input V t also V t minus zeta right D zeta convolution between the input and the impulse response, impulse response we have seen is S conjugate minus zeta the matched filter impulse response right thus just using this expression G t is equal to S conjugate minus t alright. At T equal to zero what will happen? This is the sampling instant suppose we chose this as the sampling instant because that is this corresponds to sampling instant T0 equal to 0.

This will become S conjugate minus zeta T is equal to 0, so V minus eta D zeta which by change of sign of the change of variable in the integral here can be simply S conjugate zeta V zeta D zeta or V t S t S conjugate. What is this quantity? What kind of operation you are performing? Correlation between the received signal and replica of the transmitted signal right. (())(47:25) so what does it tell us that the output of the matched filter at this sampling instant this is a very straight forward expression but the significance is that the value that you are sampling and based on which you are going to take your decisions can be regarded as haven in obtained by correlating your received signal with the replica of the transmitted signal right.

So you could as well replace your matched filter for the purpose of the de-modulation by a co-suitable corolator is it clear? Whose job will be to multiply the received signal with a local replica of a transmitted signal right and integrate between the with all the duration of this lets take zero to T or whatever and then sample it at ok, so you can replace the matched filter with a corolator will talk about this point again.

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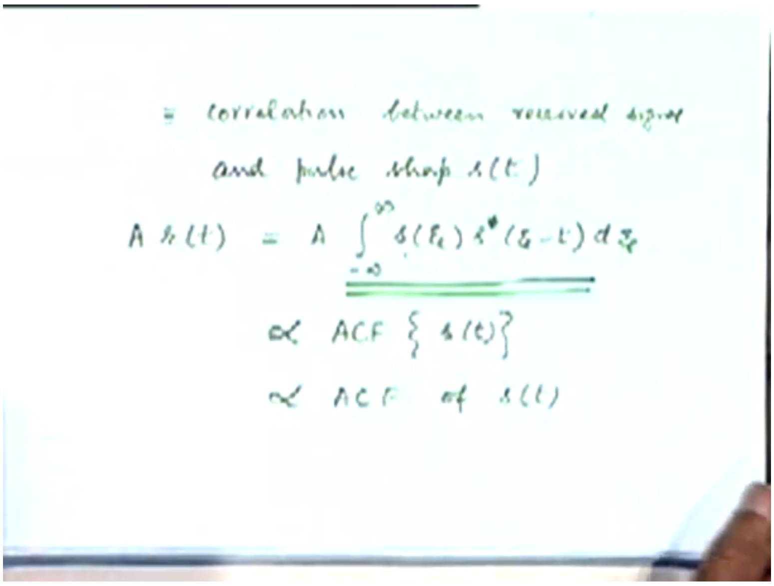

So this is nothing but the correlation between the received signal and the pulse shape S t. So basically matched filter helps you to calculate this correlation. Infact if you were to if you have asked to describe the nature of the that filter output in terms of its waveform, we can now say something about that. Let's talk about A R T the mean value of the output, right only the mean value we are not looking at the noise contribution we are only looking at the signal contribution. Suppose I had only a signal of shape S t feeding the matched filter output will be obviously R t, the mean value because there is no noise right. Then that is going to be equal to we have just seen that alright I made a change of variable ok.

I am rewriting that right, which is nothing but, is this expression familiar to you? S t is auto-correlation function of S t ok. So is proportional to the ACF of S t, the matched filter output when the input is only signal the pulse shape to which it is matched right will produce an output which is nothing but the auto-correlation function of the input waveform and where are you sampling this auto-correlation waveform at its peak value which corresponds to zero line right. Basically that is what you are doing the matched filter for the case of situation where the input we have only signal produces an output which is nothing but the auto-correlation function of the input signal right because the impulse response is matched to the same (signal) alright.

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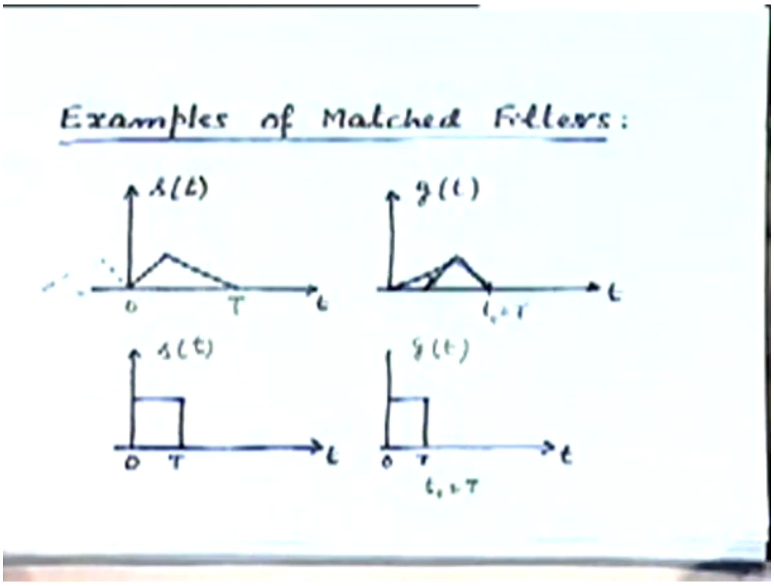

So finally I will end up this discussion by just discussing a few cases of matched filter, I will, these all cases are for real signals here the situation where this is your pulse shape, right, lets say from 0 to T, then matched filter will be alright it depends on where you select your T 0, I have selected my T0 equal to T. How do you obtain the matched filter in pulse response, you first you know in the same operation that you do in convolution first reverse the time access so this will go here right and then shift it by T0 whatever it is.

If I have taken T 0 equal to T, this is what is looks like ok. For a rectangular pulse of this kind and T0 again equal to T will again get the same rectangular pulse as the impulse response in a matched filter right. So this is your S t this is your corresponding matched filter offcourse I am considering here only real examples. Suppose I had taken T0 equal to 0, what would have happen, what would be the impulse response on the matched filter? Essentially the same in shape which is a same only thing is it is non-casual that is why it is best to choose T0 equal to T.

Infcat it is, this is the optimum value in a digital communication environment because every T seconds you are sending a new pulse right so better therefore choose a sampling instant equal to or the end of every pulse signaling interval right, this is what is typically done. That is why a matched filter base de-modulation is really a symbol by symbol de-modulation. You look at one symbol at a time sample the output at the right instant and infact there is a theory that we used even when we discussed Nyquist pulse shaping because we assumed that we are going to look at the output of some filter in this case is nothing but the matched filter every T second that is why we wanted the output pulse shape to be such that the regularly space zero cross it every T seconds.

Student: (())(54:04)

Professor: Same thing, no-no-no that will depend on the specific kind of memory signal you have, will discuss.

Student: ok suppose we have a M different pulse shapes

Professor: If you have M different pulse shapes, yes then ye but not all memory system use M different pulse shapes ok so you have to be will see all those things. We are only looking at the basic natural thing showing here but the next thing will be application of this to digital communication. We are going to look at lets take a third example where I have deliberately chosen a pulse shape which goes from 0 to infinity right.

Now how do I choose a matched filter? There is problem here you can see that, isn't it?

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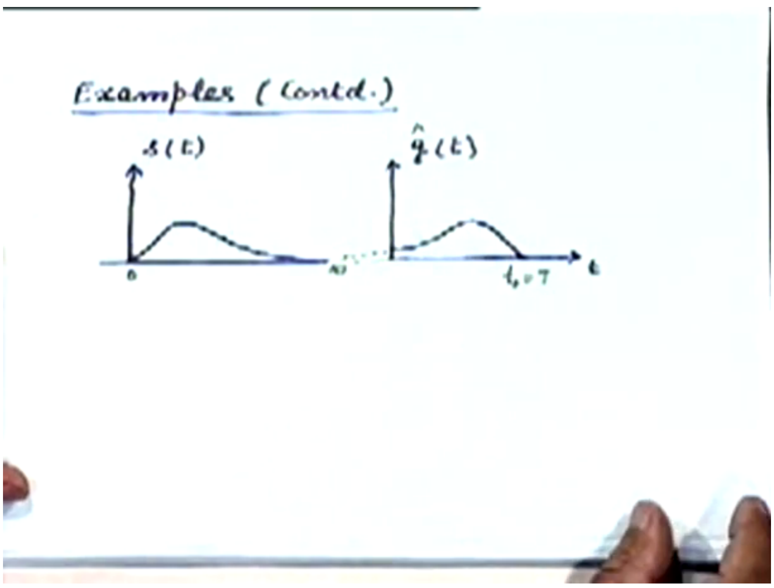

Particularly in choosing a value of T sub 0 such that the filter will remain it doesn't matter I can always chose any T sub 0 and the proper matched filter will offcourse, suppose I chose T 0 equal to some value T, right, the proper matched filter really is this it is non-casual right. It extends into the negative time excess and goes on and become zero at T 0 equal to T but obviously if I want it to make it casual now, there is no way out but to choose a sufficiently large value of T sub 0 such that what I am ignoring when I do the causality of approximation is the tail of the impulse response right this tale of the impulse response.

So that it doesn’t have significant energy right. So in this case what I have plotted here is you can say some kind of an approximating matched filter rather than an exact matched filter alright. So think this is where we stop today and we will next time see how do we invoke or use the matched filtering theory in designing de-modulators for different kinds of modulations schemes that we have considered earlier ok thank you.