Digital Communication
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Module 01
Lecture 22
Baseband and Passband Digital De-modulation: General issues and Concepts

Today I am going to talk about de-modulation in digital modulation schemes.

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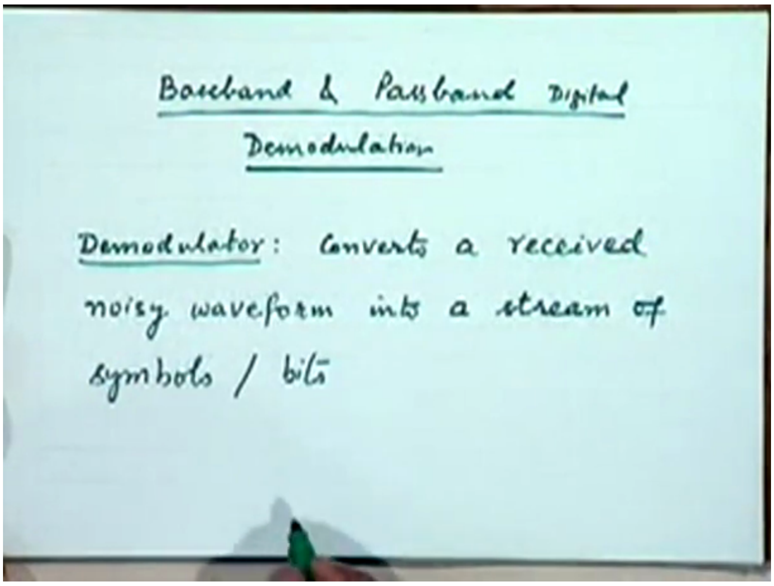

So basically our subject of interest now is the de-modulation of Baseband and Passband Digital Modulations (oh let me) yes Baseband and Passband De-modulation maybe we can add digital here digital de-modulation alright. So will now come to de-modulation so far we have looked at all the possible or various possible ways in which we can put data onto a waveform, for communication through a waveform channel right.

This modulation scheme could be a baseband modulation scheme or it could be a passband modulation scheme depending on what kind of channel you are finally going to use. So far we have just looked at the mapping from data to waveforms. Now we like to go from waveforms which are finally being received at the other side and its conversion back to data right that is the job of the digital de-modulator its function is precisely the opposite kind of mapping that a transmitter or a modulator is doing for you.

It is not how a just an inverse mapping, you should appreciate that whereas the job of a modulator is comparatively simpler because you have to have a pre-design set of waveforms pre-decided set of waveforms and the modulator essentially carries out the implementation of a rule from data bits to choice of one of these waveforms right. Unfortunately for the de-modulator you will not get one of these sent waveforms precisely in the same waveform in which it was sent, it will be received along with noise, distortions another kinds of impurements and therefore the job for de-modulator to carry out the so called inverse function namely interpretive waveform into a sequence of binary bits is relevantly non-trivial one as compared to the job of the modulator which is a rather straight forward function right.

So basically a de-modulator converts a received waveform which should be typically noisy so I will say a received noisy waveform into streams of symbols or bits that is a basic job.

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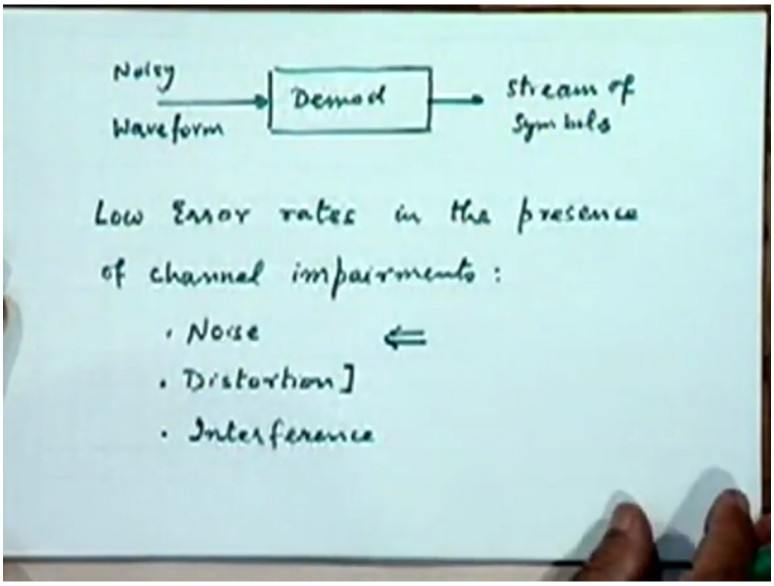

So let me put it in the form of a picture, if you have this de-modulator input is a noisy received waveform and output is stream of symbols or stream of bits ok Now how well a de-modulator is doing its job will be really therefore a (de-modulator) what we are talking about is how to judge whether you got a good de-modulator or not so good de-modulator right.

The performance for a de-modulator in a digital communication environment is really judged by its ability to give you very low error rates as low as it is possible what you call probability of error. So what (would) really like to have is, low error rates when our received waveform is impaired by so in the presence of channel impairments. Various kind of channels impairments. For example these impairments could be noise which we have talked about it could be distortion, you know your channel may introduce a lot of distortion in the transported waveform.

Student: sir how do you distinguish between noise and distortions?

Professor: I think the answer should be known to you by now, distortion is a deterministic transformation of the waveform by the time, depending on the restriction on the channel encasement for example the channel may not support that particular waveform perfectly because of its bandwidth limitation its frequency response may not be flat in the bandwidth of interest or its phase response may not be linear with respect to frequency in the bandwidth of interest. Any of these is this waves of distortion you know, even discussed in channel distortion ever.

You can have amplitude distortion you can have phase distortion and you can have non-linear distortion right. Amplitude distortion is caused by the fact that in the bandwidth of interest the channel does not have a flat frequency response alright. Phase distortion is caused when you have a non-linear phase with respective frequency. If you have a linear phase that will imply a constant delay of the input waveform to the channel outward right.

If you have a non-linear phase response different frequency components are delayed by different amounts and therefore the resulting output waveform is not a same as input waveform ok. Similarly you can have non-linear distortion, if there is a non-linearity in the communication systems somewhere, it maybe in the transmitter, it maybe in the channel, it maybe in the receiver right that will cause creation of inter-modulation terms. So you can have various kinds of distortions which are distinctly different from additive Gaussian Noise or any kind of additive noise ok. So is it clear, what is the difference between noise and distortion?

They are two distinct kinds of phenomena. So in the presence of noise distortion or interference from other sources, distortion would include inter symbol interference, inter symbol interference would come within the preview of distortion right, because afterall when you have let us say you have non-linear phase response right that might cause what is called spread of the signal a long time because different frequency components are coming at different time instants right. I mean they have to come with different delays and therefore the received waveform will be perhaps broader signal as compare with the incoming pulse.

So as far as inter symbol interference is concerned that effect is included in this term distortion. Now when I am talking about interference next is, interference from other communication systems which are operating in the neighbourhood of your frequencies alright extra no interferences which would be therefore somewhat similar to noise in the sense they are also additive in nature this other signals also adding onto your transmitted signal but they differ from noise in their (())(9:48) characteristics ok. Noise is a different kind of (())(9:52) characteristic whereas another signal which somebody else is transmitting and is interfering in your signal will have a different kind of (())(10:00) characteristic ok.

So these are the kinds of impairments that you may typically encounter in a any communication environment be it analogue or be it digital right and in as far as digital de-modulator is concerned what we really like to have is a low error rate de-modulator in the presence all this impairments. Offcourse for the time being we will primarily concern ourselves with this impairment that of additive noise right. Will not really look into these other two impairments and their impact on the design of a de-modulator at this stage.

Maybe will pass some comments here and there but broadly will be concern with this most important channel impairment namely that of noise alright.

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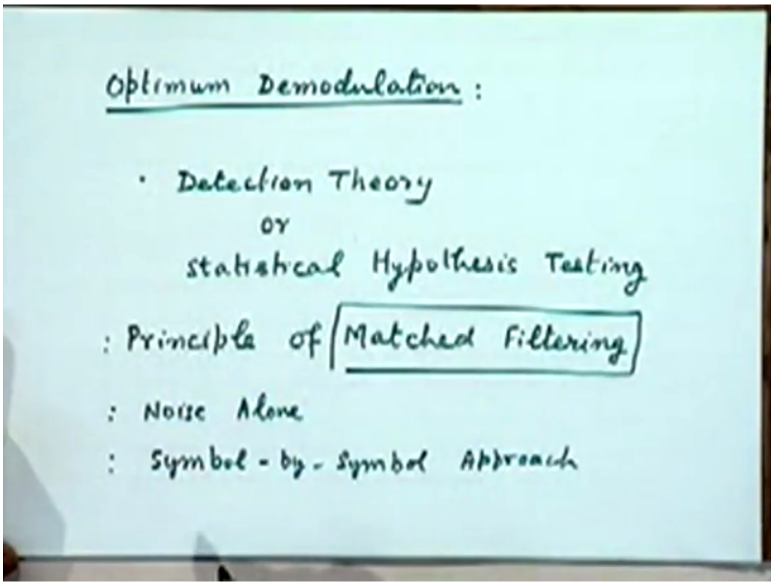

And our concern here will be to study some kind of optimum methods or as good methods as possible for de-modulation ok. Now actually we have very well developed theoretical tools available to study the subject of optimum de-modulation. These theoretical tools are studied typically in advanced courses of the kind which are known by the names of the say things like Detection Theory or what the Mathematicians would like to call Statistical Hypothesis Testing.

And the primary idea there in this theory is to look at the answer the question that your received waveform in the absence of noise would be one or in any every interval of interest would be one of many possible set of waveforms which are defined by the transmitter right, which are defined by the modulator, for example if you have an M-ary modulation scheme then you have a dictionary of M possible waveforms which could have been transmitted and therefore one of these waveforms is being received in the presence of noise a typically hypothesis distinct approach to this problem will ask, which of the M waveforms was most probably transmitted based on the current based on the received waveform that you have got.

So given the particular waveform that you have in a particular interval which of the M hypothesis is true? Namely the first waveform as transmitted or the s second or the third or the Mth alright. So that is a subject which is studied under this tittles Deduction Theory Statistical Hypothesis Testing and things like that. However to take their approach one has to go through a bit of formal background for which we don't have time so will really look into will take an approach which is slightly more simplistic and which embody is some of the very basic concepts in this, basic results in this alright which will try to motivate both mathematically as well as intuitively.

So will take a slightly more simple and therefore slightly Ad Hoc approach but it will be still quite satisfactory I can (gurant) or assure you of that. The most basic principle that finally comes out of these theories is quite simple and concise and essentially it is a principle of match filtering which is what we really study in detail now ok and this provides the answer to de-modulation of a very wide variety of digital modulation waveforms. So if we study the basic principles of matched filtering will be well equipped to study optimum de-modulation for digital modulations right.

Just to complete my statement here, as I already said will primarily be looking at optimum de-modulation in the presence of noise alone right, noise alone will be considered, one could pose the problems of optimum de-modulation in the presence of noise and interference and distortions and all these things right, which becomes a obviously more complex problem to study and it is usually simpler to study them in a systematic step by step manner and the first step always is to study without any other uncompressed right that is with noise alone and one assumption that we will make here is that as far as the channel is concerned it is not doing anything funny except adding noise.

So it has sufficient bandwidth so that whatever the transmitted signal it reproduce faithfully except for the noise right and therefore we can do de-modulation of a waveform one symbol at a time right, because there is no channel impairment there is no interference from one interval to the next interval right no inter-symbol interference and no cross interference from other channels all we have to therefore do is study de-modulation of one symbol at a time provided we know the start starting point of a symbol and the ending point of symbol right.

The waveform corresponding to one symbol interval offcourse another assumption therefore that we have made here that we have perfect synchronization between a timing signal relation between the transmitter and the receiver so that we know average symbol perfectly, symbol duration perfectly right. So a symbol by symbol approach ok.

So the problem is considerably simplified if you make this assumptions because all we are looking at is one symbol interval in it once in given one symbol interval we know we are like it in care to one of several possible waveforms and we like to obtain some method for deciding which and will simplify the problem still further by trying to just look at a much simpler problem of detection of a signal the presence of a signal in a given network ok.

So will say let us only look at the very simple problem in which N observation interval may have a signal or it may have only noise let us try to find a method by which we can detect the presence or absence of a signal then will try to generalize this to detecting the presence of one or more of possible waveforms, the presence of this or that or that particular symbol but to start with let us look at the deduction of simply presence or absence of a signal in a particular observation into a much simpler problem to look at.

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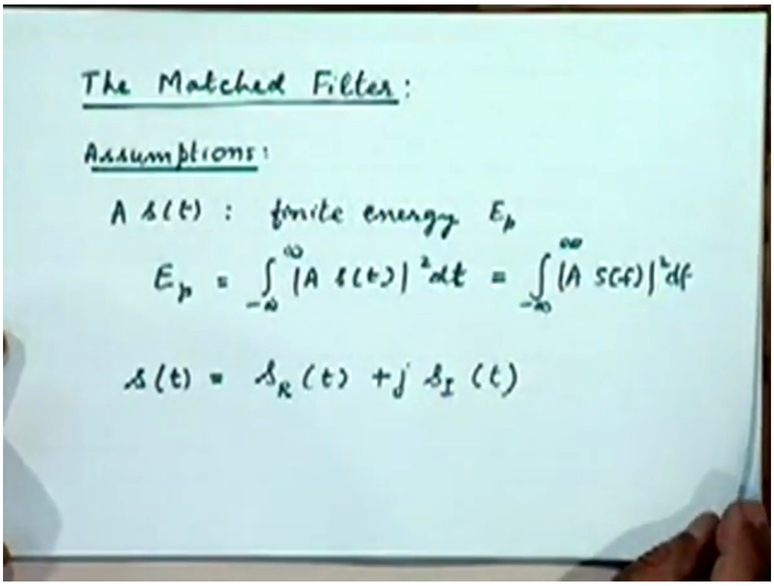

So that brings us really to the concept of matched filter and will study that in some detail now. We have been talking about it often on at various places in the past and I start by writing down a few assumptions regarding the signal and noise so let us say we are transmitting a pulse or trying to receive a pulse of shape S T with some amplitude scaling factor A which is of finite energy so A S T is a finite energy pulse with energy E sub P. Since a pulse of finite energy E sub P where obviously E sub P would be equal to integral of or in the frequency domain right.

In general because we have seen that we can represent our various kinds of modulations schemes very conveniently using complex notation in general will permit this pulse shape to be complex, so it has a real part S sub R and an imaginary part S sub I because we will be using complex notation. Now this pulse when received is contaminated by noise right, so let’s talk about that noise.

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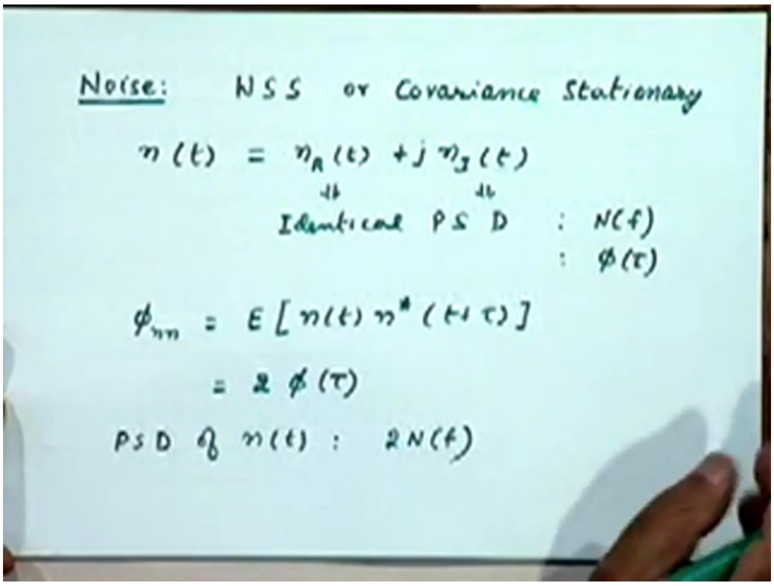

Will assume that the noise is wide sense stationary or covariance stationary right. Wide sense stationary or covariance stationary you remember what that means, it is the same thing, this a basically if you remember the definition of wide sense stationary that precisely the definition alright.

That is its covariance function depends only on Tau not individual time instance at which the two random variables are the samples right. If you look at expected value of N T1 into N T2 this is function only of T1 minus T2 rather than separately of T1 and T2 that kind of a noise process are random process is called either wide sense stationary or covariance stationary. Offcourse the mean is constant or zero. So in general we can take the noise also to be a complex random process complex valued random process having a real part and a imaginary part and will assume that is a reasonable assumption that both the real and imaginary parts if they exists have identical power spectra identical power spectral density function right.

Let us call them each N f, right N R t, as well as N sub I T both have power spectral density function which is the same that is the real and imaginary components have the same kind of spectral characteristics power spectral characteristics, it is a very reasonable assumption for most kinds of noise encountered in real life. So example the real and imaginary components might come in when you produce the wait, can you see the immerse keep in mind the physical picture behind each of these complex cycles.

These are the kind of things that you might see on the quadrature channels after you have done the let us say down conversion of the incoming passband signal into a baseband representation. It should be a quadrature signal and it may contain in phase noise and the quadrature phase noise. The typically they will have identical characteristics or spectral characteristics. We could also talk about the auto correlation function of these process I will say since they have identical power spectral characteristics obviously they will have identical auto correlation functions.

Let us denote that by Phi Tau so auto correlation this is auto correlation function this is a power spectral density function for each of these two components. So if phi tau is auto correlation function of this or this what will be the auto correlation function of this noise process, can you tell me? Exactly same or with some difference let us take quickly write it the mathematical definition is N T into N conjugate T plus Tau right, so what you will get?

Student: (())(23:45)

Professor: It is N R plus J, ok it is very simple to check that this will simply turn out to be 2 Phi into Phi Tau under the assumption that N R and N I are uncorrelated or independent offcourse uncorrelatedness would imply independence only under the assumption that N T is Gaussian right. So all those things, that is just to elaborate that I mention the fact that we will have to assume them to be either uncorrelated or independent, uncorrelatedness of two random variables would imply independence on the random variables if and only if the two random variables are Gaussian and in this case it means if the noise process is Gaussian.

So that is a relationship between the complex noise of the correlation function to individual components right. So if your noise was strictly real valued its auto correlation function would be Phi Tau if it is complex valued it will be 2 Phi Tau that is only, scaling factor with come with picture and similarly the P SD of N T would be simply given by 2 N A right and what is the relationship between N F and phi tau?

Student (())(25:14)

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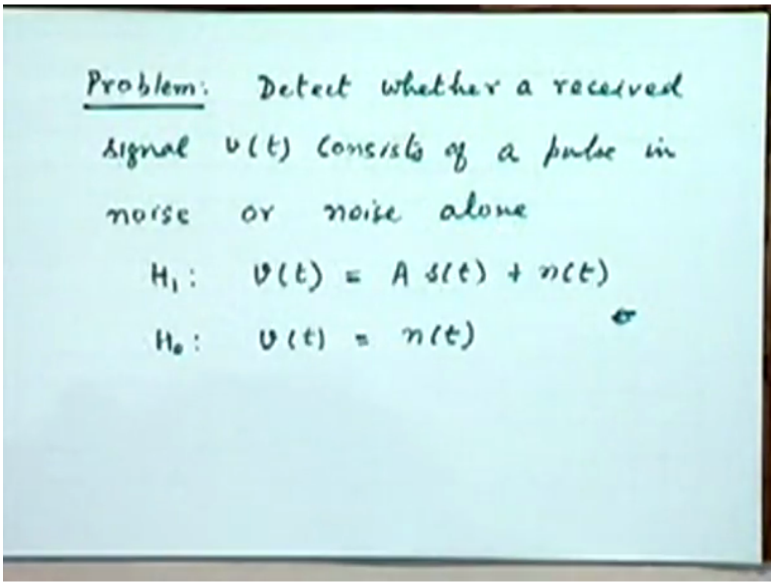

Professor: That is your (()))(25:17) ok. So as I said our immediate problem of interest is relatively simper one that is detect I will try to evolve a strategy to detect whether a received signal V t which is noisy version of the transmitted pulse A S P contains that pulse or is particularly noise right consists of a pulse in noise or noise alone, that is a problem limited problem in which we will deal to start that is we are really doing hypothesis testing with two possible hypothesis right.

Under this hypothesis we are saying that our received signal which we are denoting by V t consist of the pulse A S t plus noise N t right and under this hypothesis V t consist only of the noise N T the question is, which of these is true? All we have with us is observed waveform V t, we want to go from this observed waveform over some interval but at the moment specify interval but maybe infinite interval to start with.

Let us say we are talking of the tension of the single pulse occurring somewhere right and all we have is this V t, we need to process it and finally produce our decision right, whether this or this is true. Now if you want to take the proper mathematical approach one would start just with this problem and with the assumption that you already make a nothing more than that, that is called a non-structural approach that is just start from the problem and the solution will itself tell you what is the structure of it processing that we have to do.

Structure of the processor that one has to finally adopt, but because we are not really equipped here with twist of such typical hypothesis testing theory here or detection theory or will have to take this structural approach, and the structural approach starts by saying or saying something about what kind of a processor you will use, what kind of processing will carry out right and the reasonable processing that comes to our mind, it offcourse can be shown to be optimum is in the non-structural choice is, that you will pass this signal.

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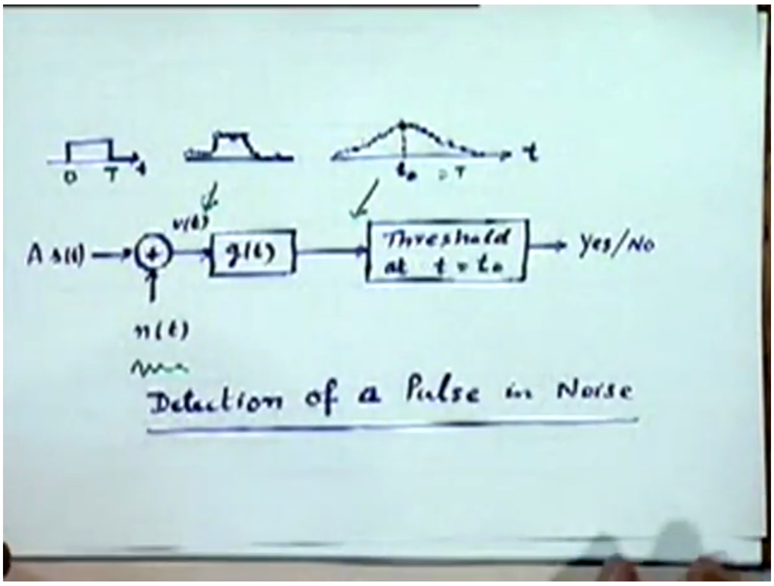

See I have just illustrated here by diagnose here, this is your pulse which is contaminated by some noise and this is how it might become after contamination so this is what you are observing this V t, the sum of this signal plus noise, random noise right and what the structure we are proposing here is that will pass it through a filter which ideally should do what? Should pass the signal completely without attenuation and should reject the noise completely, there is something that is going to be impossible to do right.

So at best what we will try to do is we will pass it through a filter and then sample the output of this because all we have to eventually is take a decision right. We don't really look at a complete waveform. So we will pass it through a filter such that as much of noise as possible is rejected whereas at a certain point in time if we sample it we should see a larger contribution due to signal and therefore we will sample it at that time and compare the value with a threshold at this time instant and if that sample value X C through threshold will declare that the pulse was indeed present.

If it doesn't exceed the threshold value we will say it is absent alright

Student: One graph is showing sampled output the third one

Professor: What is your question? No this is the yeah this is the output of the filter, this is input to the filter this is output to the filter which you are sampling at T equal to T0 right.

Student: (())(30:17)

Professor: Yes, I will tell you what that point typically will be, suppose this pulse if of duration T right, typically this pointer T0, it datively you should know, it should be at a point by which you could have seen most of the signal right, infact it should have observed most of the signal infcat all of the signal. So ideally T0 should be greater than or equal to the duration of the signal. You should have try to sample it before it , before the whole of the signal has try to pass through this filter.

So the sampling instance should be located beyond T or atleast the smallest value where you can choose is, the duration itself equal to T, so it could be equal to T if this is starting at zero and ending at T right.

Student: (())(31:21)

Professor: Basically something intuitively something similar to what we discussed long time ago that basically the idea is to reject noise one way of rejecting noise would be to integrate it out right, no we nor really doing integration here we are doing something more general the integration, integration is a very special kind of operation we are not putting any constraint on what that operation here should be. Allowing the filter to be as general as possible right. Integration will turn out to be a very special operation but roughly yes you can see it from that point of view that you want to integrate noise out over the symbol duration, over the pulse duration.

So if you see the observe the pulse after you have done that for the whole first duration and if you are integrate that out over the pulse duration at the end of the integration interval you will probably except to see a peek building up at the output of the integrator after the symbol duration right and therefore that is a right point to sample it and try to take a decision make a decision. So that is the basic philosophy of what is called a linear structure that we are going to use here comprising of a filter which will try to optimize and determine.

Strictly speaking I should have shown a sampler here which samples the output of this at P equal to T0 which I just written here and then compare this sample output with a threshold which is somehow pre-calculated and that produces a decision yes or no whether the signal is present in this interval or is absent. Yes

Student: (())(33:05)

Professor: In general it will not be, if a pulse shape is of no concern to us infact that I have shown that here, this is your input pulse the output pulse it obviously a convolution of this and this as going to be different from the input pulse, that is not also a highly concern to us, all we want to is convert tis into a value which we are going to sample as you are going to obtain by sampling such that it helps us make our decision as correctly as possible.

Student: (())(33:42)

Professor: Right, other word we just want to produce for a single (())(33:50) when generalisation to distinguishing between two signals or more signals it is not going to be very difficult again assure you that once you understood this basic concept. O any questions regarding this? So this was the broad frame work in which we are going to work and our concern is to look at the velvet G T.

Student: (())(34:18)

Professor: Will see that that is what precisely we are eventually leading to. So let us see how you can go about it. What will try to do is, will try to understand with this kind of input, which contains a signal plus noise what kind of output you are going top get particularly at some pre-chosen sampling instead because ultimately the behaviour of this receiver is going to depend on not really this output waveform but on the nature of this value. How much of this value has been contributed by the signal the pulse and how much of it by noise.

If example there was no noise this value will be completely contributed by the signal alone right, but the present of noise would disturb the value that she would have seen if signal alone had contributed. It might decrease it, it might increase it, the presence of noise. It will perturb the ideal value right and depending upon the magnitude of noise that perturbation maybe quite large right. So basically what we like to do is design a filter such that the signal output signal contribution becomes as large as possible, but the noise perturbation remains as small as possible right.

There is how we will this G t right, the filter G t should be selected so there at this point most of the value most of the contribution is coming from the signal within possibly a small perturbation coming from noise and if you can maximize the signal contribution to noise perturbation ratio right then will have done a good job and we can hope to make reasonably reliable decision, is it fine? So therefore let us look at the nature of the contribution here due to the signal and due to the noise alright and that will help us in designing the G t.

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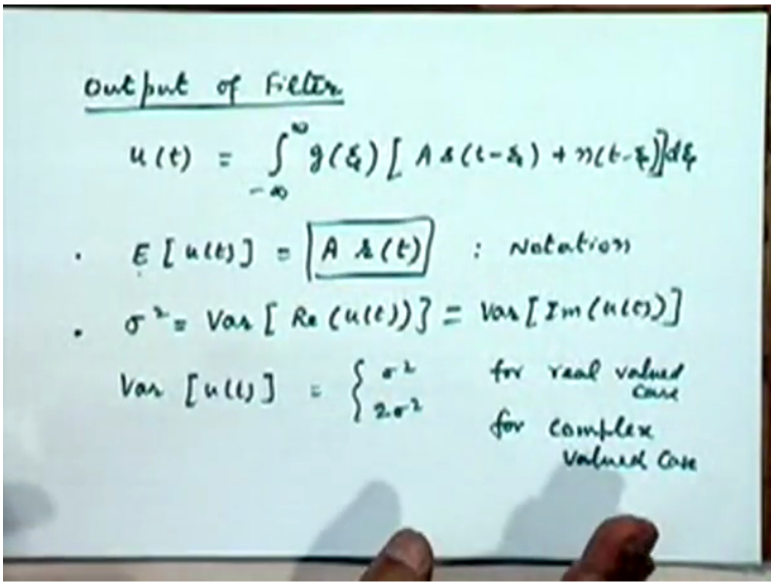

Let us look at the output of the filter, which whose impulse response is G t and input is A S T plus N t right. Let us denote the output by U t, it will be convolution of the filter impulse response with the input which is A S T plus N t, so it will become A into S T minus zeta plus N t minus zeta D zeta alright.

Now is it fine, simply the convolution of the input there should be a bracket here, convolution of this input with the impulse response ok. Since we have random process in a in-push namely this noise process this U t therefore is actually a random signal like I illustrated here right, which is the realization of it random waveform. However what is really important to us is the mean value a signal here because different noise waveforms will cause different output waveform and when we are trying to do optimization we like to optimize on the basis of average properties right.

So will write it we didn't like to look at the nature of this random process in terms of its mean and variance at the sampling instead right just look at what the mean value is and what the variance is, the variance will represent the perturbation over the mean value and hopefully the mean value would be contribution of a signal alone right, infact we can see that very easily now, so first let us look at let me just introduce some notation the mean value of U t I am going to denote by some A times R t, just a notation right and variance of U t, variance of real part of U t is going to be dictated by sigma square either a real part or imaginary part.

Because if you remember N t is in general complex with identical variance for real and imaginary parts similarly U in general is a complex random process will have identical variances for real and imaginary parts.

Student: (())(39:33)

Professor: That is what is the variance is our concern it doesn’t matter. Ok it is more or less obvious I don’t know whether it needs any explanation because as will soon see the mean is really govern by the signal because we will going to we are going to assume here this is a very reasonable (())(40:10) assumption and I can always enforce this kind of an assumption that N t noise is a zero mean process right, this is a zero mean process its mean value of this is going to really dictated by the first term right and the variance is this is not going to contribute to the variance, variance is always around the mean.

The waves to be contributed by the noise, I am just (denote) calling it some R t, will see soon see what a R t is right, yes basically that is what it is you are right ok. So we are denoting the variance of the real and imaginary parts both with sigma square obviously the variance of U t itself will be either sigma square or 2 sigma square depending on whether it is real or it is complex ok if U t is real that is if N t is real then U t is real that if you are assuming everything is real then U t is a real process real valued process in which case the variance will be simply sigma square.

If our everything is complex valued then the variance will become 2 sigma square as we have seen earlier. So variance of U t, will be either sigma square or 2 sigma square depending on whether it is real valued or complex valued ok. Now let us consider this quantity what is the (meaning) this are two important things, the mean value and the variance ok,.

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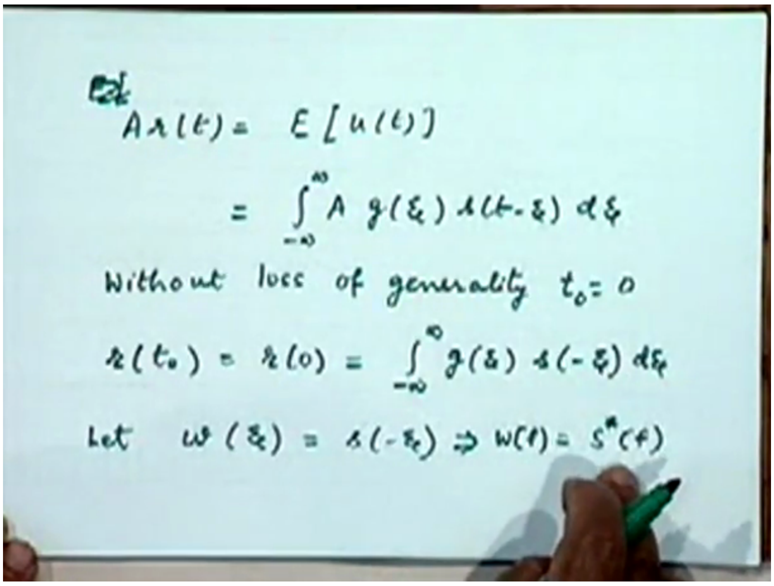

Let us look at the mean value therefore I mean just look at A R t, which is by definition expected value of U t, alright from this integral what we can say about it if you take the expected value of an integral we can expectation is a linear operation I can exchange it with a integration operation take inside the integral the second term becomes zero because we assume this to be zero mean so essentially it will be this way constant, there is no nothing random in this right.

So it will be simply integral of A g zeta S t minus zeta and it will mention essentially convolution of the input signal with the filter impulses response right. Now out of this waveform the value which is going to really matter to our performance is the value at the sampling instant T equal to T0 right. So we don’t really look at need to look at this mean value for all values of time. We only need to look at the mean value of the sampling instant value right.

Now sampling instant is in under our control right and if you are assuming that our pulse is sufficiently in the past let’s say somewhere less than T equal to 0, without any loss of (()) (43:39) I will assume actually this assumption is not really required but slightly simplifies my analysis it is not really required we can assume T0 equal to 0 right that is a sampling instant T0 (())(44:02) chosen I am calling that some instant zero right, that has become my reference instant obviously the pulse would have occurred somewhere before that right.

Student: sir it is lot expression part yes sir how do you get this expression?

Professor: It is not clear?

Student: (())(44:17)

Professor: Yeah left with expected value of G zeta into S t minus zeta both are deterministic terms there is nothing random so the expected value of a constant is equal to a constant itself I mean I didn't think it was required in a elaboration therefore I didn't reveal of it, is that ok? There is no random process here therefore it is a constant process, constant with respect to the random domain right, there is nothing random in it, so not constant with respect to time, it is a time varying function.

Ok can we come back to this point that it hardly matters, what is the value of T0 I select as long as I select it to be beyond the signal interval right, so without any loss of generality let me choose T0 equal to 0 that gives me your R t 0 equal to R 0, how can you obtain that? By putting T equal to 0 here right, so that will become minus infinity G zeta S minus eta T zeta ok. Let me define this as some W zeta now you remember the fully transformed properties if I define W zeta to be this what will be the fully transform of W F?

Conjugate of S F, right, fully transform of this, is simply as conjugate F for a general complex signal alright, if you reverse the time access that is equivalent to conjugation in the frequency term. This equal to the transform properties therefore let us look at the implication of this property on this integral R0.

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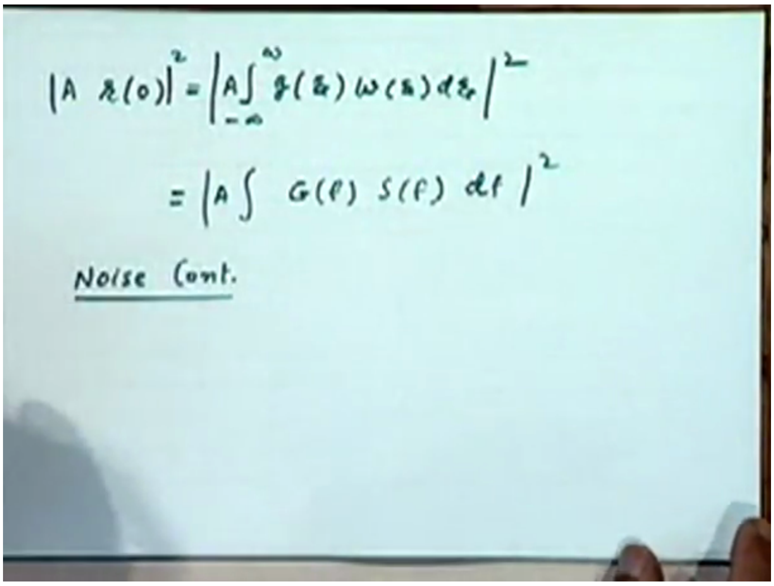

Therefore your R0 actually this something that I could have just told you intuitively but I hardly need to just go through the whole thing which is G zeta W zeta D zeta by Parseval theorem if you apply Parseval theorem what will you get?

Student: yes sir random is in the convocation G and S

Professor: I am coming to frequency domain now, so far I am not something frequency domain it is all time domain I am coming to frequency domain now is that ok, so to come to frequency domain I am just use the Parseval's theorem. This will become let me write A here A also here (everywhere) and let me write mod square of this the reason is because ultimately I am going to compare it with noise variance right, so I am squaring it up and calling the signal output signal power just for the sake of comparison ok.

So this will be the value and by Parseval theorem this will be simply what will this be? Fourier transform of this into some star will come

Student: (())(48:44)

Professor: I think there should be no ok

Student: (())(49:03)

Professor: It is correct but we have made a mistake, W F is equal to A star F so ok, right that is how it is. So this is the final answer which I could have easily obtain directly intuitively, how? Input is S t whose Fourier transform is S F, right, let me tell you a one line another one line method of obtaining this result without having to go through all those steps. Input is S F output is obviously this G f into S f right, and the in time domain the output is inverse Fourier transform of this which you are evaluating at t equal to 0 right, so that E to the power term becomes small. So essentially go get this ok.

So even without using Parseval theorem you can obtain this result just precisely in one line if you do things properly. Let us now look at the noise contribution, I think will stop here and look at the noise contribution and then compute the signal to noise ratio next time. Once we have got the signal to noise ratio expression what we really do is choose a G t which maximizes that is signal to noise ratio such maximizing S L R filter is what we call a matched filter ok and will see what its form is.